Iss 2 JAN 1968

S.O.R. 51.

HAWKER SIDDELEY AVIATION LIMITED

DE HAVILLAND DIVISION.

THE ANALYSES OF PANELS UNDER SHEAR LOADS.

PARTS 1 AND 2.

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STRESS OFFICE REPORT NO. 51.

THE ANALYSES OF SHEAR PANELS.

PART 1. THE ANALYSES OF FLAT SHEAR PANELS.

CONTENTS

ITEM:-	SECTION NO.	SECTION PAGE:-	FIGURE FIG. NO.	FIGURE PAGE
Introduction	1	1		
Panel Analysis	2	1		
Buckling stress	2.1.	1	1a,1b,1c and 2	16 - 19
Panel stress	2.2.	2	3	20 - 21B
Panel failure	2.2.1.	2	4a,5	22 - 23
Permanent buckling	2.2.2.	2	4a,5,1c	18,22,23
Panel joints	2.3.	3		
Attachment to Edge Members	2.3.1.	3	4b	22
Web joints	2.3.2.	3	4a,4b	22
Shear panel stiffeners	3	3		
Introduction	3.1.	3		
Stiffener stresses and loads due to tension field	3.2.	4		
Stiffener section constants	3.3.	5		
Stiffener failure	3.4.	6		
Local instability and panel inter-rivet buckling	3.4a	6	6,10,11	24 - 27
Panel instability under a closed section stiffener, at a stiffener intersection	3.4a	7	10,11	26,27
Panel instability at an intersection of vertical and horizontal open section stiffeners	3.4a	7		
Flexural instability of stiffeners	3.4.b	8	9,10,11	26 - 27
Forced crippling of stiffener flange	3.4c	8	12	28
Torsional instability	3.4d	8	10,11	26 - 27
Attachment of stiffener to edge member	3.5.	9		
Attachment of panel to stiffener	3.6	9		
Effect of combined stiffener loads	3.7	9	13	29

......Cont'd

Part 1. The analyses of Flat Shear Panels Cont'd.

CONTENTS

ITEM:-	SECTION: SECTION NO.	SECTION: PAGE:-	FIGURE: FIG. NO.	FIGURE: PAGE
Shear panel edge members	4	9		
Introduction	4.1.	9		
Edge member stresses	4.2.	10	14a, 14b	30
Effective panel shear modulus	5	11	15a, 15b	31
Comparison between test results and predictions			16a, 16b	32
Item		Page		
Notation		12 - 14		
References		15		
Appendix 1 post buckling stress distributions				
Appendix II worked example				

STRESS OFFICE REPORT NO.51.

THE ANALYSES OF SHEAR PANELS

PART 1. The Analyses of Flat Panels in Shear.

1. Introduction.

This report has been written with the following objectives:-

a) To simplify the existing methods of analysis.
b) To achieve better agreement between test results and theory.
c) To devise a method whereby true reserve factors for both panel and stiffeners may be obtained with little or no iteration.

This method of analysis is based on the theory as presented by Refs.1 and 2 with certain additional material. The presentation has been rearranged to achieve the above objectives. Simplification has also been obtained by the complete elimination of direct use of the diagonal tension field factor, as this is dependant only on the ratio of panel shear stress to buckling stress.

2. Panel Analysis.

2.1. Buckling Stress.

The two existing methods of determination of buckling stress, Refs. 1 and 2, have disadvantages, the former reference, although simple to use, cannot be used for partial edge fixation, and the latter is very lengthy and at times, very pessimistic.

An investigation was made using the concept that any shear panel could be considered to have simply supported edges at some effective depth and width dependant on the boundary structure and geometry. This investigation showed that this concept could be used with very little loss in accuracy, and that the expressions necessary to determine these effective dimensions were simple. Comparing this approach with the theoretical results, the maximum error is within 4%, which is considered acceptable as this is well within the variations in buckling stress obtained from test panels.

It was also found that the effect of stiffener flexibility could be presented in the same form by rearrangement of the analysis from Ref.6. This is given by a series of curves on Fig. 1c.

The effect of stiffener torsional stiffness was also investigated using the results of Ref.8; this seems to be generally covered by use of the reduced effective width as given on Figs. 1a and 1b for closed section stiffeners, though there may be conditions when the buckling stress could be increased if open section stiffeners have high torsional stiffness. The revised R.Ae.S. data sheet should be used for this condition.

The effect of reducing stiffeners on effective width or depth is partly empirical, being based on the analysis of 20 flat and curved test panels. This is presented on Fig. 1a. The effect of riveting stiffeners is also partly empirical and is based on the analysis of a further 24 panels and is presented on Fig. 1b.

Figs. 1a, 1b, 1c and 2 are used to obtain the elastic buckling stress τ_{bo}.

2.2. Panel Stresses.

The curves for the basic permissible web stress, τ_{all} and the permanent buckling stress τ'_{pb} as presented on the Fig.3 appropriate to material, are based on D.S. 02.03.03, Ref.1. These stresses are obtained directly knowing the elastic buckling stress τ_{be}. A further scale permits the direct determination of the actual buckling stress τ_b, which will be considerably less than τ_{be} when τ_{be} is high.

The current curves, Ref.3 (S.O. 02.03. Sheet 9), are based on a combination of permanent buckling and failure considerations, and have reduction factors incorporated throughout, and have been found to be pessimistic, predicting results about 17% low in comparison with test results. The revised curves incorporate no reduction factors for permanent buckling, however a reduced U.T.S. at 52,000 lb/in^2 (Spec. = 56,000 lb/in^2) has been used for the τ_{all} curves for L 89 Material Fig. 3c, as it was found that using the specification U.T.S. gave optimistic results. For all other materials the specification U.T.S. has been used.

2.2.1. Panel Failure.

The panel failing stress, τ_{FAIL}, is obtained from Fig.4a, and this stress is the actual panel failing stress for the condition of no applied direct stress in the edge members. The effect of combined panel shear stress and edge member direct stress is given on Fig.5.

2.2.2. Permanent Buckling.

The permanent buckling stress, τ_{pb}, is obtained from Fig.4a, having previously used the appropriate Fig.3. to obtain the basic permanent buckling stress τ'_{pb}. This may be over-ridden by the actual buckling stress if stiffeners are flexible, Note 1, Fig. 1c.

2.2.3. **Flanged Webs. Panel Failure**

Failure of a flanged web occurs at or soon after buckling, as the flange is incapable of reacting much tension field loads from the web. Failing stresses for flanged webs are obtained directly from Figs. 3a to 3F, Iss.2. These curves apply to panels where the attachment is made through a flanged web or where the web is flanged to form the edge member. The curves are derived from 45 tests, Ref. 20, 21 and 22.

3. **Shear panel stiffeners**

3.1. Introduction.

Research into information obtained from tests, Refs. 9,15 and 17, has shown that considerable simplification of the existing methods of shear panel stiffener analysis may be achieved with negligible loss of accuracy. Analysis of the measured average stiffener stresses, from these tests, showed that this stress could be taken to be proportional to panel gross shear stress less the buckling stress, $(\tau - \tau_o)$, and that the average stiffener stress could be accurately estimated using a simple expression. This expression is derived from theoretical considerations, summarised in Appendix 1 of this report, and involves the effective stiffener area, panel width and thickness and a factor, determined from the test results, which was found to be dependent on panel depth and thickness only.

The maximum stiffener stress was found to vary with the ratio of stiffener length and pitch and the average stiffener stress. Fig. 8. gives an expression for maximum stiffener stress for double stiffeners and curves for the maximum stiffener stress for single stiffeners, both obtained from the analysis of test results from the above references.

Failing stresses, due to local and flexural instability of the stiffener and forced crippling of the stiffener flange, were estimated for 58 test panels (Refs 9 and 17) using the methods given in this report, and overall agreement between tests and calculations was found to be closer than obtained using Ref. 1.(NACA T.N.2661). Representative material properties were used with these calculations as it is considered that use of minimum specification properties could lead to over-conservative methods of analysis. Comparison between predicted and test stresses are given in Fig.16.

Analysis of some de Havilland test panels (Ref 5), showed that stiffener failure occurred at stresses considerably greater than predicted by either this method or by use of Ref. 1. Extended analysis has shown that the large steel or light alloy boundary members used on these tests reacted almost all the diagonal tension from the panel, causing the stiffener stresses to be considerably lower than usually present in normal aircraft structures, for which this report is written.

A numerical example of the complete analysis of a panel and the associated stiffener and edge member is given in Appendix II of this report.

3.2. Stiffener stresses and loads due to tension field from a buckled shear panel.

The expressions for average stiffener stresses, evolved from theoretical calculations (Appendix I) are:-

For stiffener S_a

$$\sigma_{Sa_{Av}} = \frac{C_s (\tau - \tau_b)}{\frac{A_{Sa\ eff}}{b't} + S/b'} \qquad \text{where } S = 40t, \quad 40\ t/b' < .667 \qquad (3.1.)$$
$$S = \cdot 667 b', \quad 40\ t/b' > .667$$

For stiffener S_b

$$\sigma_{Sb_{Av}} = \frac{C_s (\tau - \tau_b)}{\frac{A_{Sb\ eff}}{a't} + S/a'} \qquad \text{where } S = 40t, \quad 40\ t/a' < .667 \qquad (3.2.)$$
$$S = \cdot 667 a', \quad 40\ t/a' > .667$$

$A_{Sa_{eff}}$ and $A_{Sb_{eff}}$ are the effective stiffener areas of stiffeners S_a and S_b and are taken as:-

A_{Sa} and A_{Sb}, the actual areas, for double single bay stiffeners and for continuous single stiffeners away from the end bays.

A_{Sa}' and A_{Sb}', the effective areas, for single stiffeners extending one bay only and for the end bays of continuous single stiffeners.

a' and b' are the plain panel length and depth, t is the panel thickness and C_S is a constant dependent on panel depth and thickness and is obtained from Fig. 7. .

The maximum stiffener stress, dependent on panel geometry, is obtained from Fig. 8 ..

The average stiffener load is given by:-

For stiffener S_a, $P_{Sa} = \sigma_{Sa_{Av}} A_{Sa\ eff}$ (3.3)

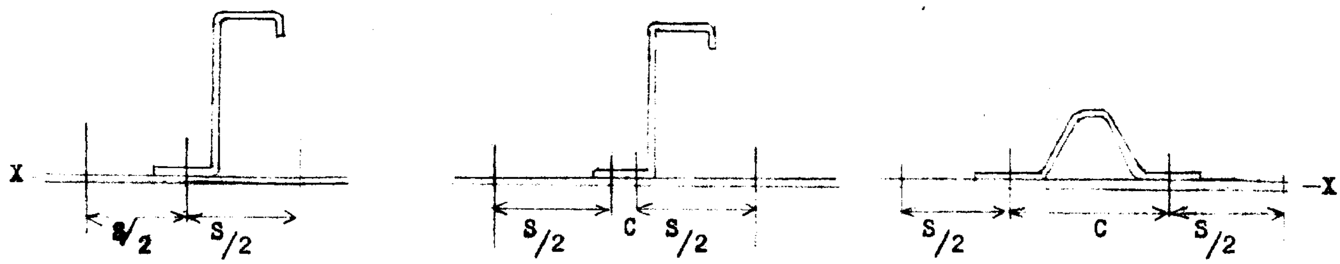
For stiffener S_b, $P_{Sb} = \sigma_{Sb_{Av}} A_{Sb\ eff}$ (3.4)

. . .

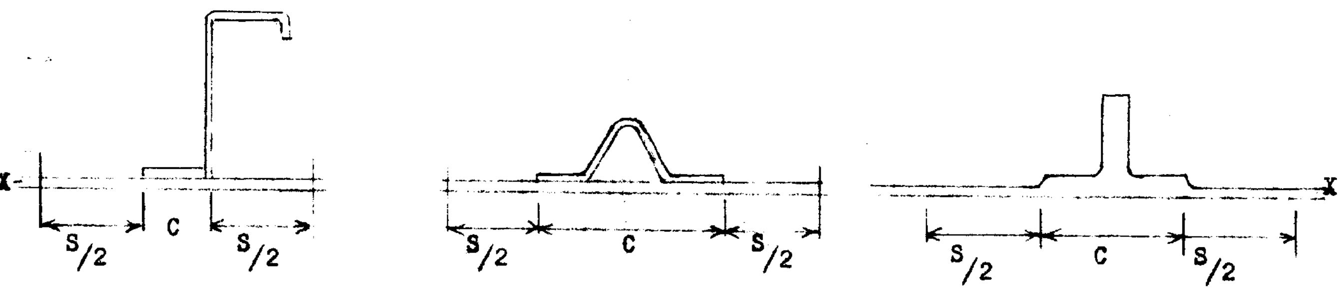
3.3 Stiffener section constants.

As stiffeners are usually single (located on one side of panel only), section constant calculations may be simplified by working about an axis through the median panel plane.

RIVETED STIFFENERS



EXTRUDED OR MACHINED STIFFENERS



NOTATION

(Suffix sa denotes stiffener Sa; when considering stiffener S_b, Suffix becomes sb and b' is replaced by a').

C	=	Panel width directly working with stiffener. in.
A_{sa}	=	Stiffener area (including width C of skin) in^2
S	=	Panel width in addition to C effective with stiffener, given by the lesser value of 40t or .667b'. in.
St	=	Additional area of skin. in^2
$(Ay)_{sa}$	=	1st moment of area of stiffener about axis X-X through skin median plane. in^3
$(Ix)_{sa}$	=	2nd moment of area of stiffener about axis X-X. in^4

The neutral axis of the stiffener plus effective skin is at $\bar{Y}$ from X-X, where:-

$$\bar{Y} = (Ay)_{sa} / (A_{sa} + St) \qquad (3.5)$$

And the moment of inertia about the neutral axis is:-

$$(I)_{sa} = (Ix)_{sa} - (Ay)^2_{sa}/(A_{sa} + St) \qquad (3.6)$$

For panel buckling, sheet 2, S.O.R.No.51, $S = .5b'$ and $(I)_{Sa}$ becomes I'_{sa}

$$\therefore\ I'_{sa} = (Ix)_{sa} - (Ay)^2_{sa}/\left(A_{sa} + \frac{b't}{2}\right) \qquad (3.6.a)$$

And the effective area, A'_{sa} is given by:-

$$A'_{sa} = A_{sa} - (Ay)^2_{sa}/(Ix)_{sa} \qquad (3.6.b)$$

The radius of gyration, i_{sa}, ($\sqrt{\frac{I}{A}}$), is given by:-

$$\frac{1}{i_{sa}} = \frac{(A_{sa} + St)}{\sqrt{(Ix)_{sa}(A_{sa}+St) - (Ay)^2_{sa}}} \qquad (3.6.c)$$

3.4 Stiffener Failure.

Three modes of stiffener failure must be investigated for all types of stiffener. An additional mode, torsional instability, should be investigated for open section stiffeners.

This section is written for stiffener Sa under consideration and when considering stiffener Sb, $A_{sb\ eff}$ replaces $A_{sa\ eff}$, a' replaces b', etc.

The modes of failure are as follows:-

3.4.(a) Local instability of stiffener flanges and inter-rivet buckling and local instability of panel.

Figs. 10 and 11 are used to obtain the local instability stress of the flange adjacent to the panel, for this flange plus the panel and for panel inter-rivet buckling. If the panel inter-rivet buckling stress is greater than the local instability stress, the local instability stress may be used directly as $\sigma_{S\ max.}$. Fig. 8. is then used to convert to $\sigma_{S\ Av}$. The gross panel shear stress at which stiffener failure will occur is then given by:-

$$T_{FAIL} = \frac{\sigma_{sa\ Av}}{C_s}\left(\frac{A_{sa\ eff}}{b't} + \frac{a}{b'}\right) + T_b \qquad (3.7)$$

where T_b = buckling stress lb/in^2.

C_s is obtained from Fig. 7.

If the panel inter-rivet buckling stress is less than the local instability stress, an effective allowable local instability stress is used as $\sigma_{S\ max.}$ This is given by:-

$$\text{Effective } \sigma_{S\ max.} = \frac{\left(\sigma_{L1}\left(\frac{A_{sa\ eff}}{b't}\right) + \sigma_{I.R.}\frac{S}{b'}\right)}{\left(\frac{A_{sa\ eff}}{b't} + \frac{S}{b'}\right)} \qquad (3.8)$$

Where $\sigma_{L.1.}$ = Minimum flange local instability stress lb/in.^{2}
$\sigma_{I.R.}$ = Panel inter-rivet buckling stress lb/in.^{2}

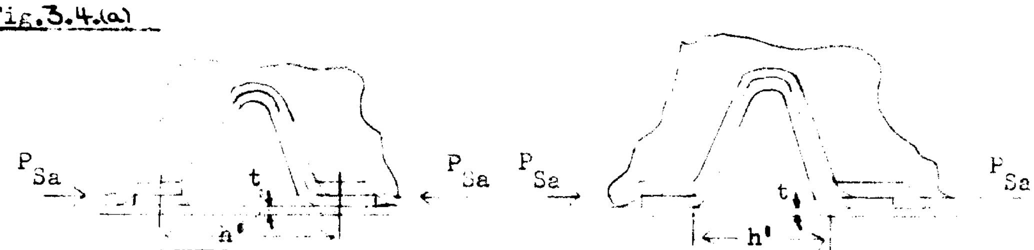
$\sigma_{Sa\ Av}$ and hence T_{FAIL} are obtained in the same manner as for flange local instability. (3.7).

Panel instability under a closed section stiffener.

Analysis of shear panel tests, Ref. 15, has shown that a length of panel approximately equal to four times the plain panel width under the stringer crown may be taken effective reacting the tension field loads. This area of panel may be taken working at a stress given by flexural instability of the element with fixed end conditions. This stress, σ_B may be found directly from Fig. 11. using K_C = 3.62 and $\zeta = \frac{h'}{t}$ (Table 10.(2)). The allowable load is given by:-

$$P_{sa_{all}} = 4h't\,\sigma_B \qquad (3.9)$$

Fig.3.4.(a)



Panel instability at the intersection of perpendicular open section stiffeners

A strap either inside or outside the skins should be provided at the intersection of open section stringers to give load path continuity to the cut-out flange. Test panels with no straps, Ref.15, showed that very early permanent buckling at such an intersection can occur if no strap is provided. A cleat should also be provided to reduce the unsupported length of strap and panel at the cut-out.

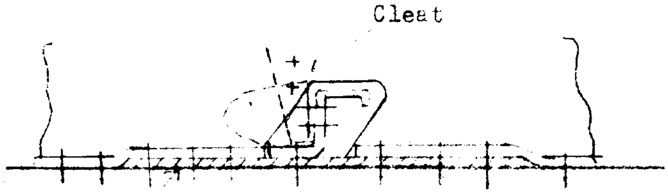


Fig.3.4.(b).

(11)

3.4.b. Flexural Instability.

The effective column length, l_e, is given on Fig.**9**., the radius of gyration by expression (**3.**5.). The allowable average stiffener stress is obtained from table **10**. and Fig.**11**. and this is used as $\sigma_{a_{Av}}$ in expression (**3.**7) to obtain τ_{FAIL}.

3.4.c. Forced Crippling of Stiffener Flange.

The panel shear stress $\tau_{FAIL\ F.C.}$ at which forced crippling of the stiffener flange will occur, is obtained from Fig.**12**.

Effective stiffener areas, A^*_{sa} and A^*_{sb} are used for forced crippling and described in the notes with Fig.**12**.

This method of obtaining the forced crippling stress results from a complete reanalysis of 51 test results, Refs. 9 and 23, from which it was deduced that the ratio of stiffener flange width to panel width in addition to the ratio of flange to panel thickness determines the stress at which forced crippling will occur. Kuhn, Ref. 1, uses the term 'forced crippling failure' to describe failure resulting from severe deformation of the stiffener flange attached to the web, the flange being forced to adapt itself to the shear wrinkles of the web. During the reanalysis, it became apparent that about half of the stiffener failures could be predicted by equating the maximum stiffener stress with the flange local instability stress. The remaining panels were then analysed resulting in the curves given in Fig.1.6.

3.4.d. Torsional Instability.

Open section stiffeners should be checked for torsional instability. The expression for average failing stress is given by:-

$$\sigma_{Av_{all.}} = \frac{GJ}{I_p} + \frac{4\pi^2 E_t \Gamma}{I_p h^2}{}^{*} \quad \text{FOR CLOSED SECTION STIFFENERS AND OPEN SECTION CLEATED STIFFENERS} \qquad (\mathbf{3.}10)$$

FOR OPEN SECTIONS ATTACHED BY ONE FLANGE TO PANEL AND FRAME

$$\sigma_{s_{AV.\,all}} = \frac{GJ}{I_p} + \frac{\pi^2 E_t \Gamma}{I_p h^2}$$

where J = Torsion constant for section.

for or section stiffeners, $J = \frac{A_{sa}\, t'^{2}_{sa}}{3}$

for section stiffeners, J may be obtained from R.Ae.S. Data Sheet 00.07.01.

I_p = Polar moment of inertia of stiffener about centre of rotation. in^4
The centre of rotation should be taken as the rivet line. For reduxed stiffeners take at heel of stiffener.

E_t = Tangent modulus for stiffener material. lb/in^2

Γ = Warping constant. R.Ae.S.00.07.01 - 00.07.06. in^6

* The second term in $\sigma_{Av_{all.}}$ (**3.10.**) may be obtained by use of Fig **11**. using $K = 9.86$ and $\zeta = .5h\sqrt{\frac{I_p}{\Gamma}}$ FOR CLOSED AND OPEN SECTION CLEATED STIFFENERS

AND $K = 9.86$ AND $\zeta = h\sqrt{\frac{I_p}{\Gamma}}$ FOR OPEN SECTIONS ATTACHED BY ONE FLANGE TO PANEL AND FRAME

3.5. **Attachment of Stiffener to Edge Member.**

Taking P_{Sa} and P_{Sb} as the strength of attachment of stiffeners S_a and S_b respectively to the edge members, $\sigma_{Sa\ Av}$ and $\sigma_{Sb\ Av}$ are, from (3.4), given by:-

$$\left.\begin{aligned} \sigma_{Sa\ Av} &= P_{Sa}/A_{Sa\ eff} \\ \sigma_{Sb\ Av} &= P_{Sb}/A_{Sb\ eff}, \end{aligned}\right\} \qquad (3.11)$$

Expression (3.7) is now used to obtain T_{FAIL}.

3.6. **Riveted Attachment of Stiffener to Panel.**

The tensile load per inch run, given by $.075\, t\, \sigma_{ult}$, where t is the panel thickness, σ_{ult} is the U.T.S. for the panel material, lb/in^2, should not exceed the effective allowable rivet loading as given by half the allowable rivet shear strength (D.S.29,10 to 29.14) divided by the rivet pitch.

3.7. **Effect of Combined Stiffener Loads.**

The effect of combined direct stress and panel shear stress on allowable stiffener stress is given on Fig.3.7. The expression, on which this curve is based, is derived from tests on cylinders (Ref.16) and is considered acceptable for use with flat panels, in view of the lack of other test evidence, when the direct stress is uniform along stiffener length.

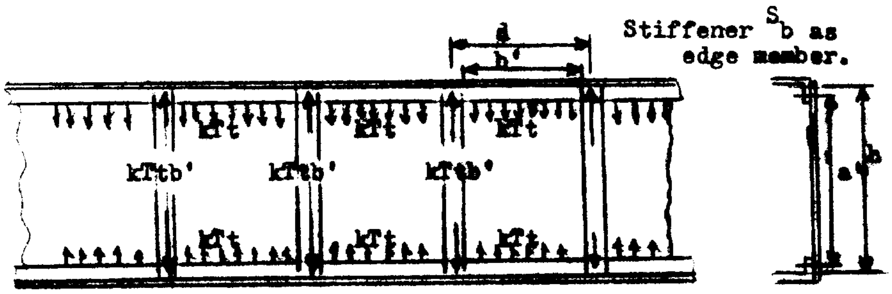
4. **SHEAR PANEL EDGE MEMBERS**

4.1. **Introduction.**

The analysis of shear panel edge members is an extension of the stiffener analysis (Section 3), the only difference being:-

a) The edge member is loaded by tension field from half the vertical panel and half the horizontal panel (if present).

b) The edge member, in redistributing tension field from the panel to the stiffeners, has applied to it local bending moments about an axis normal to the panel. These bending moments are approximately equal in magnitude at mid bay and at stiffeners, but are of opposite sign.



The above diagram shows the applied loading $kT t$ from the panel reacted by the stiffener loads $kTtb'$.

4.2. Edge Member Stresses.

Fig.14a gives the direct stress in the edge member due to tension field (direct stress = $\alpha \times \tau_{app}$). This is derived from the expression (30b), page 20, Ref.1, $\sigma_F = \frac{k\tau}{2A_F/ht + .5(1-k)}$.

Fig. 14b gives the edge member bending moment and is taken from Ref.2, 02.03.08. (Bending moments = $\beta \times \tau_{app} \times t\ d^2$).

The total compressive stress in the edge member is now given by:-

$$\sigma_{Total} = \sigma_{app} + \tau_{app}\left[\alpha + \frac{\beta t d^2}{Z_{Sb\ min}}\right] \qquad (4.1)$$

Where $Z_{Sb\ min}$ = Minimum section modulus for edge member, Sb, in^3.
and at failure $\sigma_{Total} = \sigma_{all}$.

$\sigma_{app} = \sigma_{fail}$.

$\tau_{app} = \tau_{fail}$.

$$\text{Hence} \quad \frac{\sigma_{Fail}}{\tau_{Fail}} = \frac{\sigma_{app}}{\tau_{app}}$$

$$\text{Hence} \quad \frac{\tau_{Fail}}{\sigma_{all}} = \frac{1}{\frac{\sigma_{app}}{\tau_{app}} + \left(\alpha + \frac{\beta t d^2}{Z_{Sb\ min}}\right)} \qquad (4.2)$$

τ_{Fail} will be obtained by method of approximation by first assuming a value of τ_{Fail} and applying this to the L.H.S. of the expression (4.2) using Figs. 14a and 14b.

The 2nd approximation for τ_{Fail} will then be given by R.H.S. expression (4.2), this approximation then being reapplied to L.H.S. from Figs.14. This operation is repeated until the value of τ_{Fail} applied to L.H.S. is equal to value given by R.H.S.

Similarly, if stiffener S_a becomes an edge member

$$\frac{\tau_{Fail}}{\sigma_{all}} = \frac{1}{\frac{\sigma_{app}}{\tau_{app}} + \left(\alpha + \frac{\Delta t h^2}{Z_{Sa\ min}}\right)} \qquad (4.3)$$

And τ_{Fail} is obtained by the same method.

5. EFFECTIVE SHEAR MODULUS.

The curves for elastic shear modulus, including the effect of incomplete diagonal tension, are obtained from Ref.1, Fig.22, and are given on Fig.14a.

The curves of effective shear modulus including effect of plasticity are obtained from secant shear modulus curves on the basis that $E_S/E = G_S/G$ when $\sqrt{3}\,\tau = \sigma$. (Ref.5), these curves are given on Fig.14b.

THE ANALYSIS OF FLAT PANELS IN SHEAR.

NOTATION

A_{Sa}	= Cross section area of stiffener Sa (including effective skin)	in^2	
A_{Sb}	= Cross section area of stiffener Sb (including effective skin)	in^2	
A'_{Sa}	= Effective cross section area of stiffener Sa = $A_{Sa} - \frac{(Ay)^2_{Sa}}{(Ix)_{Sa}}$	in^2	Ref. Sect. 3.5.
A'_{Sb}	= Effective cross section area of stiffener S_b = $A_{Sb} - \frac{(Ay)^2_{Sb}}{(Ix)_{Sb}}$	in^2	
$(Ay)_{Sa}$	= First moment of area of stiffener Sa about median panel plane.	in^3	
$(Ay)_{Sb}$	= First moment of area of stiffener Sb about median panel plane	in^3	
a	= Distance between webs of stiffeners S_b	in	
a'	= Plain panel depth (Between stiffeners Sb)	in	
a_e	= Effective panel depth	in.	
a'_e	= Increased effective panel depth	in	Figs. 1a, 1b,
b	= Distance between webs of stiffeners Sa	in.	and 1c.
b'	= Plain panel width (between stiffeners Sa)	in.	
b_e	= Effective panel width	in.	
b'_e	= Increased effective panel width ($b'_e \neq a'_e$)	in.	
d	= Pitch of stiffeners Sa	in.	
d'	= Free flange width for local instability	in	Table 10
E	= Modulus of Elasticity	lb/in^2	
E_t	= Tangent modulus	lb/in^2	
G	= Shear modulus	lb/in^2	
Ge	= Effective shear modulus	lb/in^2	Fig. 15 a 15 b.
G_{idt}	= Elastic shear modulus	lb/in^2	
h	= Pitch of stiffeners S_b	in.	Figs 1a, 1b.
h'	= Supported flange width for local instability	in	Table 10
$(I_x)_{Sa}$	= 2nd moment of area of stiffener Sa about median panel plane	in^4	Ref. Sect
$(I_x)_{Sb}$	= 2nd moment of area of stiffener Sb about median panel plane	in^4	3.3.

I'_{Sa}	=	2nd moment of area of stiffener Sa plus width b/2 of skin about an axis through the centroid of the combination parallel to plate	in^4	Sect. 3.3.
I'_{Sb}	=	2nd moment of area of stiffener Sb plus width a/2 of skin about an axis through the centroid of the combination parallel to plate	in^4	
I_C	=	2nd moment of area of compression edge member plus width 25t of panel about an axis normal to plate	in^4	Fig 4. a
I_T	=	2nd moment of area of tension edge member plus width 25t of panel about an axis normal to plate	in^4	
i_a	=	Radius of gyration of stiffener Sa	in	Sect. 3.3 and Fig.6.
i_b	=	Radius of gyration of stiffener Sb	in	
K	=	Buckling stress coefficient		Fig.2.
k	=	Diagonal tension field factor (Not used directly)		
l_{ea}	=	Equivalent strut length of stiffener Sa	in	Fig 9 .
l_{eb}	=	Equivalent strut length of stiffener Sb	in	
M	=	Edge member bending moment due to tension field	lb.in.	
P_{Sa}	=	Load in stiffener Sa due to tension field	lb.	Sect. 3. 2.
P_{Sb}	=	Load in stiffener Sb due to tension field	lb.	
.			.	. . .
.			.	.
p	=	Pitch of rivets attaching panel to stiffener	in	Table 10
r	=	Strength per inch run of attachments at joints	lb/in	Sect. 2.3. Fig 4b.
S_a	=	Suffix denoting stiffener parallel to long side 'a' of panel		Fig.1a, 1b.
S_b	=	Suffix denoting stiffener parallel to short side 'b' of panel		
t	=	Panel thickness	in.	
t_{Sa}	=	Combined thickness of panel plus stiffener flange S_a.	in	Fig.1a.
t_{Sb}	=	Combined thickness of panel plus stiffener flange S_b.	in	
t'_{Sa}	=	Thickness of flange of stiffener Sa	in	Fig.1b.
t'_{Sb}	=	Thickness of flange of stiffener Sb	in	Fig. 12

W_{Sa}	=	Flange width of stiffener Sa, if reduced	in	Fig.1a
W_{Sb}	=	Flange width of stiffener Sb, if reduced	in	
$Z_{Sa\ min}$	=	Minimum section modulus for edge member Sa	in^3	Sect.4 Figs 14a, 14b
$Z_{Sb\ min}$	=	Minimum section modulus for edge member Sb	in^3	
α, β	=	Coefficients used in analysis of edge members		
σ_{all}	=	Allowable stress in panel or stiffener when no shear is applied	lb/in^2	Fig. 5 13.
σ_{Saav}	=	Average stress in stiffener Sa	lb/in^2	Sect.3.2.
σ_{Sbav}	=	Average stress in stiffener Sb	lb/in^2	
σ_{app}	=	Applied direct stress in panel or stiffener	lb/in^2	Figs. 5, 13
σ_{FAIL}	=	Direct stress in panel or stiffener at failure under combined effects of direct and tension field stresses.	lb/in^2	Figs. 5, 13
$\tau_{FAIL\ FC}$	=	Panel shear stress at which forced crippling occurs	lb/in^2	Fig. 12.
τ_b	=	Buckling stress	lb/in^2	Fig. 3.
τ_{bo}	=	Elastic buckling stress $(= KE(t/_{be})^2)$	lb/in^2	Fig. 2.
τ_{all}	=	Basic permissible web stress	lb/in^2	Figs. 3, 4, 5, 13.
τ_{app}	=	Applied panel shear stress	lb/in^2	Fig. 5, 13.
τ_{FAIL}	=	Shear stress at which failure of panel will occur	lb/in^2	Fig. 4, 5, 13.
$\tau_{pb'}$	=	Basic permanent buckling stress	lb/in^2	Fig. 3.
τ_{pb}	=	Shear stress in panel at which permanent buckling will occur	lb/in^2	Fig. 4.

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(now Hawker Siddeley Aviation Ltd)
(See attached list of test report numbers and titles)

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N.A.C.A. T.N.2188.

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Ref.15. De Havilland Division test reports

Report No.	Title
T.R. 100002	Wing Skin and Stringer Construction.
100010	Section of Aileron.
100068	Standard English Electric Flanged Holes in Shear Panels.
104004	Fuselage Skin test panels.
104011	Aileron Test Section
106004	Fuselage 20 and 22 SWG, DTD 687 & DTD 546 Curved Shear Panels.
106311	Wing Rib 13 Panels.
1063012	Wing Rib 1 Panels.
1063059	Wing Skin 12G, DTD 687 panel with Reduxed Stiffeners.
1063077	Fuselage 20 x 18 SWG, DTD 546 Curved Shear Panels.
1064025	Fuselage Floor Beams
1064059	Fuselage 19 SWG, DTD 746 Curved Shear Panels.
1064301	Flanged lightening holes in 22 SWG, L.73 Shear Webs.
110001	18 SWG, DTD 546 Panels with Riveted Stringers.
110091	Tailplane 18 & 22 SWG, DTD 687 Reduxed Panel with Riveted Stringers.
110124	Wing Rib 3 Tapered 16G, DTD 546 Panel.
110148	Wing Rib 1, 18G, DTD 546 Panel with Riveted Stringers.
110196	Wing Rib 0 Reduxed 26 & 2[illegible]G Swaged DTD 546 Panel
110512	Wing Rib 0 at 1 20G, DTD 165 Panel with riveted Top Hat Stringers.
112012	Wing Skin 16G & 14G DTD 687 Panels with J.646 DTD 363 Stringers.
121008	Window Shear Test Panel
121054	Rear Fuselage Shear Test Panel
121058	Edge reinforcing for Lightening Holes in .09" DHA 81 Shear Webs
125001	Shear Test on Pressed Wing Rib.
125003	Fuselage Shear Test Panels
T.R.[illegible].400	Window Shear Test Panels.

SHEAR PANELS WITH REDUCED OR INTEGRAL STIFFENERS

EFFECTIVE PANEL SIZE FOR BUCKLING STRESS.

a_e = EFFECTIVE DEPTH in. ($a_e > b_e$)
b_e = EFFECTIVE WIDTH in. FROM FIG 1a (USE MEAN VALUE OF b_e IF STIFFENERS VARY IN TYPE.)
E = YOUNGS MODULUS FOR PANEL lb/in²
t = PANEL THICKNESS in
t_{sa} = COMBINED THICKNESS AT STIFFENER S_a in.
t_{sb} = " " " " S_b in.
W_{sa} = TOTAL FLANGE WIDTH, STIFFENER S_a in.
W_{sb} = " " " " S_b in.

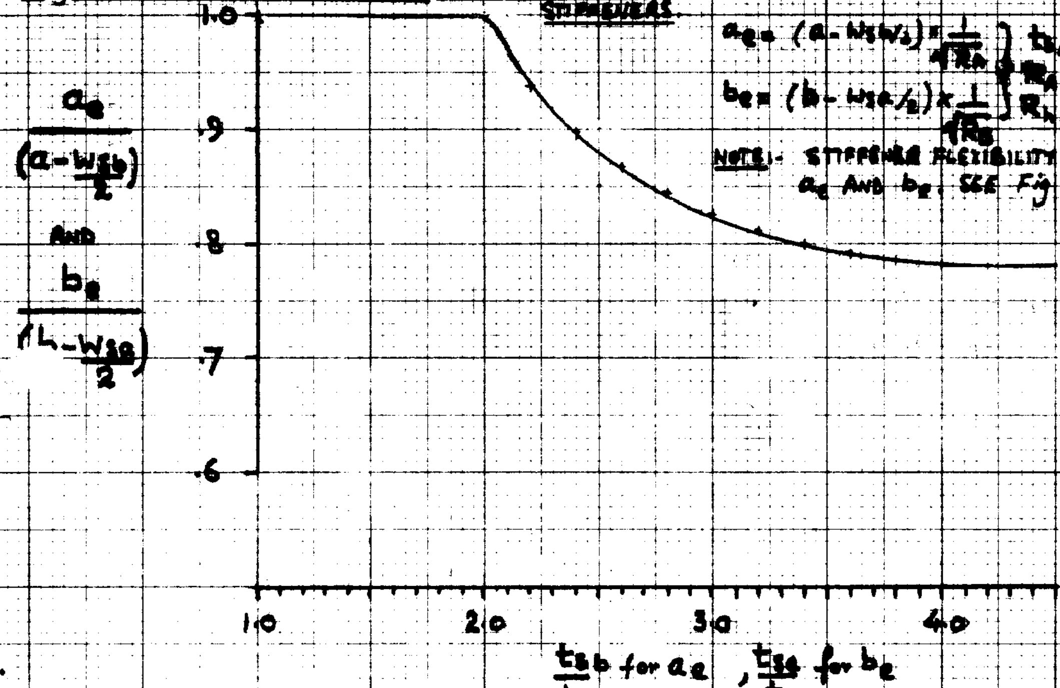
FROM ANALYSIS OF D.H. TEST RESULTS AND N.A.C.A. TN. 2661.

$$\left.\begin{aligned} a_e &= (a - W_{sb}/2) \\ b_e &= (b - W_{sa}/2) \end{aligned}\right\} t_s/t \ngtr 2$$

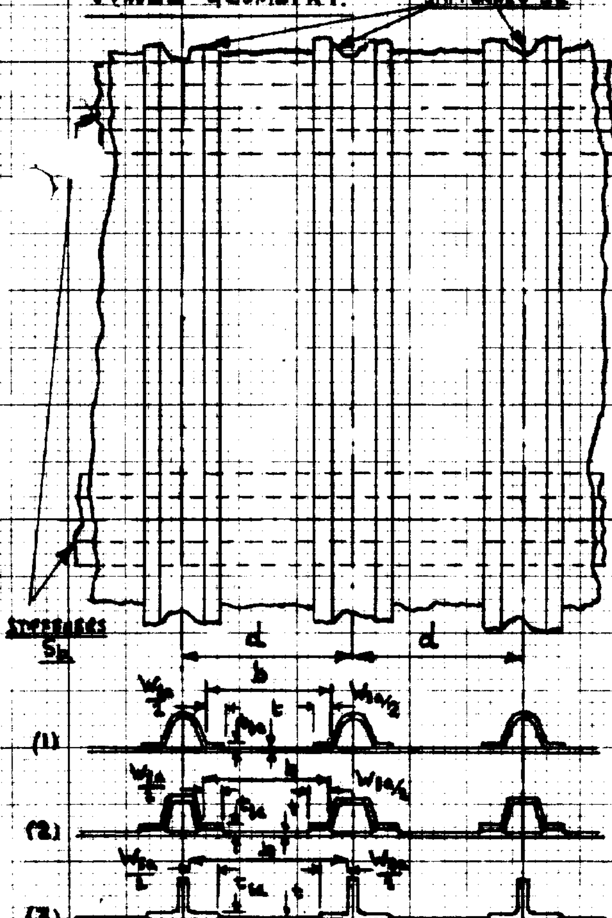
$$\left.\begin{aligned} a_e &= (a - W_{sb}/2) \times \frac{1}{\sqrt{R_a}} \\ b_e &= (b - W_{sa}/2) \times \frac{1}{\sqrt{R_b}} \end{aligned}\right\} \begin{aligned} &t_s/t > 2. \\ &R_a, R_b \text{ ARE } R_c, R_d \text{ NACA TN 2661.} \end{aligned}$$

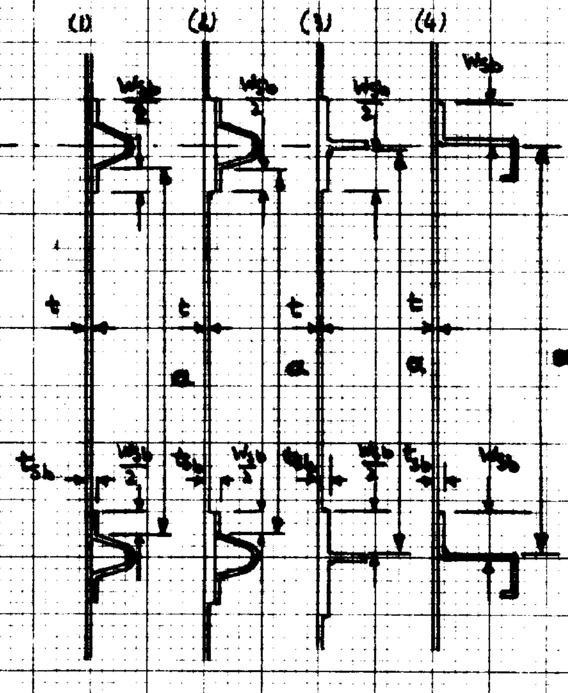
NOTE:- STIFFENER FLEXIBILITY MAY INCREASE a_e AND b_e. SEE Fig 1c.

Fig 1a EFFECTIVE PANEL SIZES REDUCED OR INTEGRAL STIFFENERS.



PANEL GEOMETRY.





TYPICAL EXAMPLES:-

PLAIN PANEL, REDUCED STIFFENERS (1) AND (4) (STIFFENERS S_b SHOWN AS TYPICAL)

MACHINED PANEL, REDUCED STIFFENERS. (2)
(N.B. IF STIFFENERS ARE RIVETED NEGLECT STIFFENER FLANGE THICKNESS IN DETERMINATION OF t_{sa})

MACHINED PANEL, INTEGRAL STIFFENERS. (3)

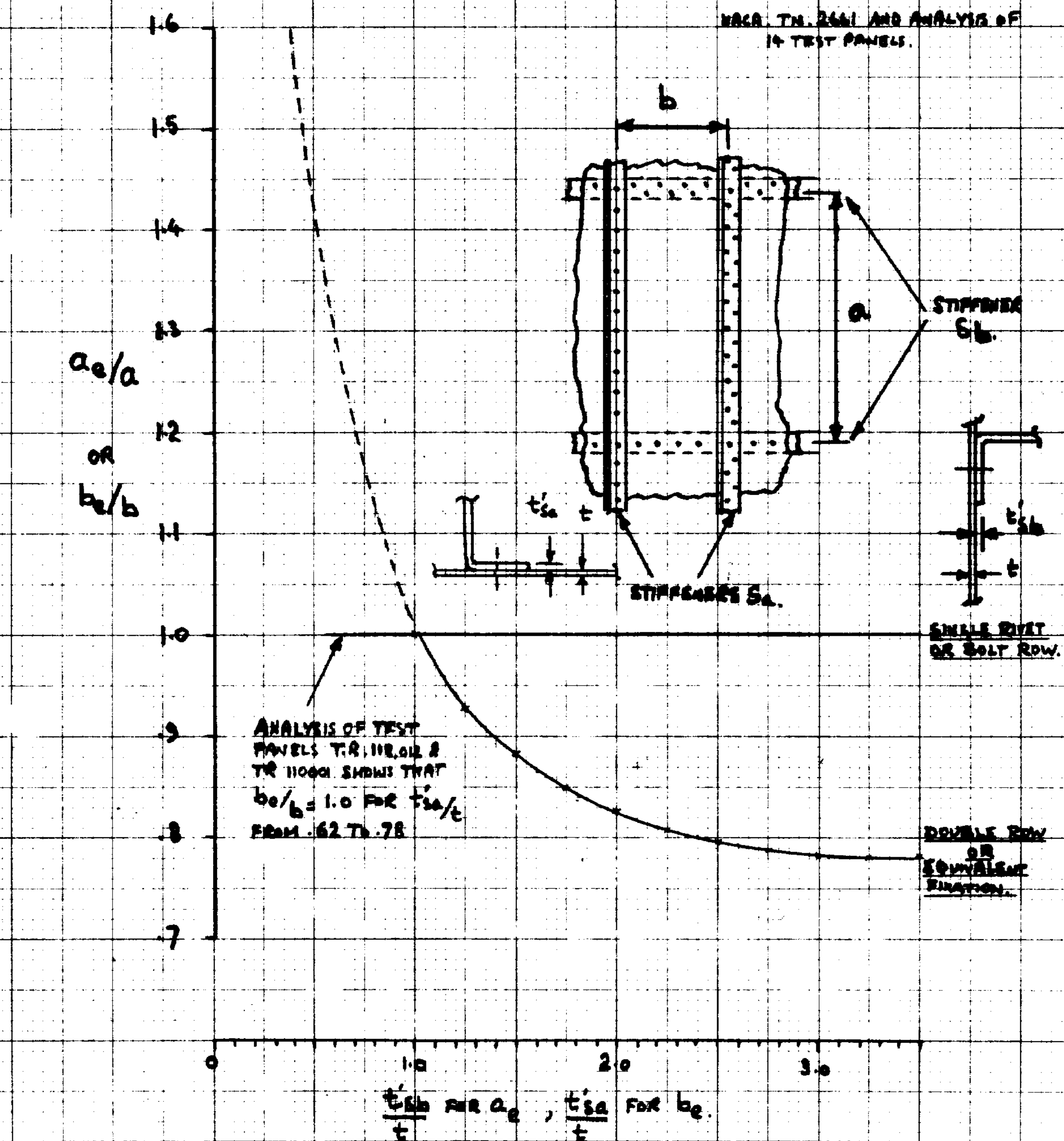
PLAIN PANEL WIDTH $b' = b - W_{sa}$; $a' = a - W_{sb}$.

SHEAR PANELS WITH RIVETED OR BOLTED STIFFENERS.

EFFECTIVE PANEL SIZE FOR BUCKLING STRESS.

a_e = EFFECTIVE DEPTH in. ($a_e > b_e$)
b_e = EFFECTIVE WIDTH in. FROM Fig 1b. (USE MEAN VALUE IF STIFFENERS VARY).
E = YOUNGS MODULUS FOR PANEL lb/in²
t = PANEL THICKNESS in

Fig 1b. EFFECTIVE PANEL SIZES. RIVETED OR BOLTED STIFFENERS.



a AND b ARE DEPTH AND WIDTH OF PANEL, TAKEN AS MINIMUM DIMENSION BETWEEN RIVET LINES.

a_e AND b_e ARE THE EFFECTIVE DIMENSIONS FOR A SIMPLY SUPPORTED PANEL ($b_e < a_e$)
(THESE MAY BE INCREASED DUE STIFFENER FLEXIBILITY, Fig 1c.).

t'_{sb} AND t'_{sa} ARE FLANGE THICKNESSES OF STIFFENERS PARALLEL TO 'b' SIDE AND 'a' SIDE OF PANEL RESPECTIVELY.

Fig 1c. INCREASED EFFECTIVE PANEL DEPTH AND WIDTH DUE TO STIFFENER FLEXIBILITY

a_e = EFFECTIVE PANEL DEPTH in. (Fig. 1a or 1b,
a_e' = INCREASED EFFECTIVE PANEL DEPTH. in.
b_e = EFFECTIVE PANEL WIDTH. in. (Fig. 1a or 1b.)
b_e' = INCREASED EFFECTIVE PANEL WIDTH. in.
d = PITCH OF LONGER STIFFENERS S_a. in.
h = PITCH OF SHORTER STIFFENERS S_b. in.
I'_{Sa} = 2nd MOMENT OF AREA OF STIFFENER S_a PLUS WIDTH $b/2$ OF PANEL ABOUT AN AXIS THROUGH CENTROID // TO PANEL. in^4 } SEE PART 1 SECT 3.
I'_{Sb} = 2nd MOMENT OF AREA OF STIFFENERS S_b PLUS DEPTH $a/2$ OF PANEL ABOUT AN AXIS THROUGH CENTROID // TO PANEL. in^4 } SEE PART 1 SECT 3.

NOTE 1. VALUES OF a_e'/a_e OR b_e'/b_e GREATER THAN 1.0 SHOW THAT THE STIFFNESS OF THE SUPPORTING STIFFENERS IS INADEQUATE. PANEL AND STIFFENER WILL BUCKLE CO-INCIDENTALLY AT BUCKLING STRESS GIVEN BY $T_{b_e} = KE(t/b_e')^2$, WHERE K IS DEPENDENT ON RATIO b_e'/a_e' AND IS GIVEN ON FIG. 2. FOR THE CONDITION WHEN b_e'/b_e IS GREATER THAN 1.0, BUCKLING SHOULD BE CONSIDERED PERMANENT, HENCE $T_{pb} = T_b$ WHERE T_b IS OBTAINED FROM FIG. 3 APPROPRIATE MATERIAL.

NOTE 2. a_e' OR b_e' MAY BE PHYSICALLY LIMITED BY THE PRESENCE OF ADEQUATE STIFFENERS ON EACH SIDE OF AN INADEQUATE STIFFENER. IF A SHORTER DEPTH OR WIDTH IS OBTAINABLE BY NEGLECTING AN INADEQUATE STIFFENER, THIS VALUE SHOULD BE USED IN PLACE OF THE VALUE OBTAINED FROM THESE CURVES.

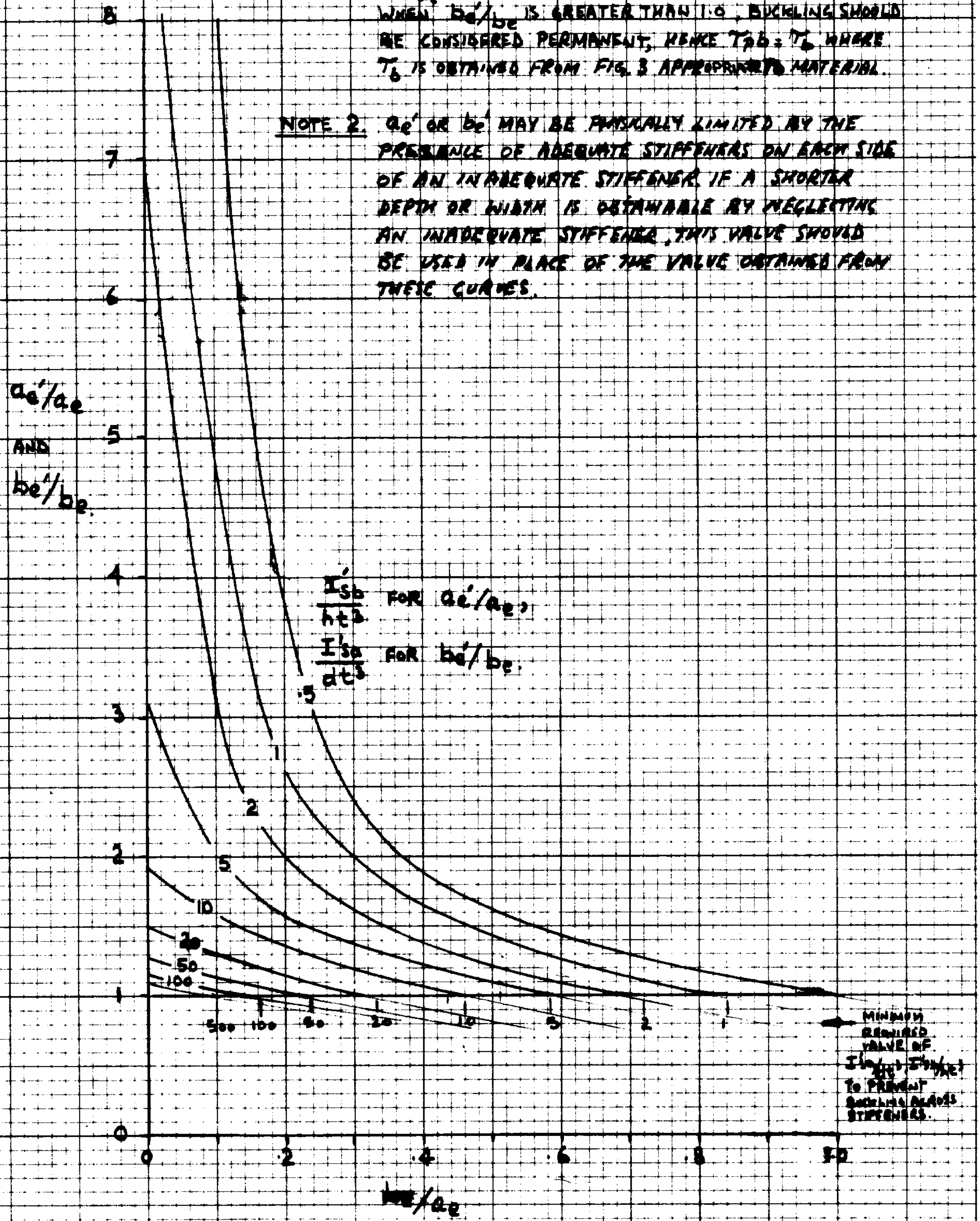
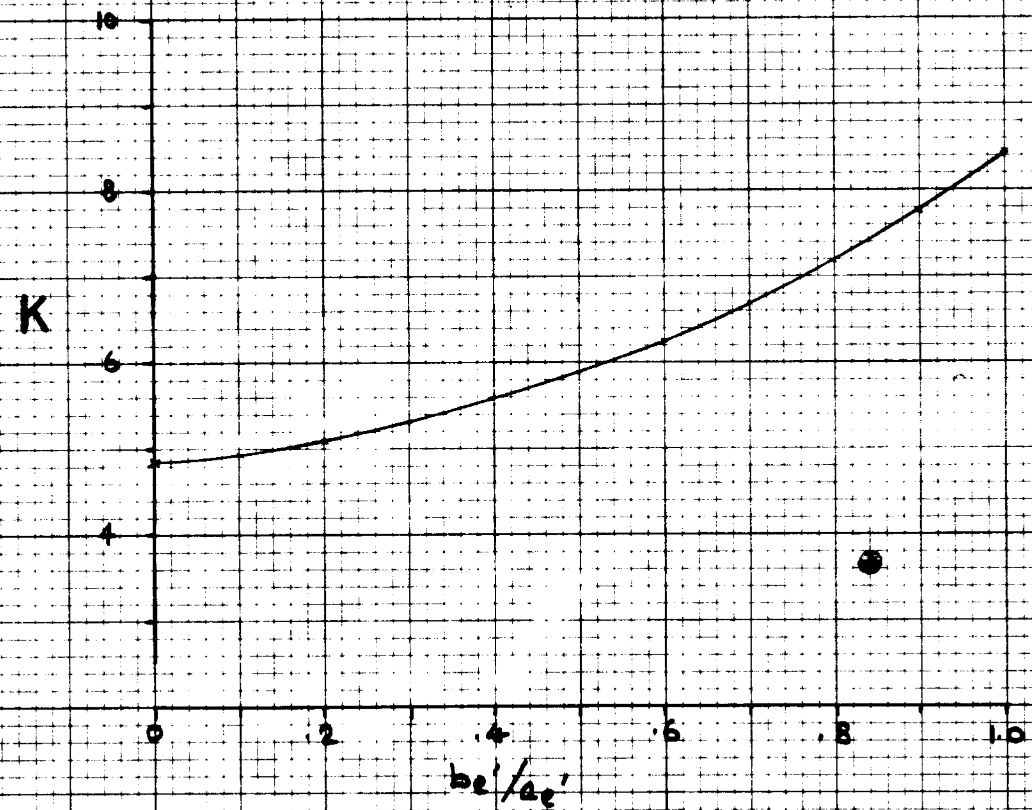


Fig 2. PANEL BUCKLING STRESS COEFFICIENT, K.

a_e' = INCREASED EFFECTIVE PANEL DEPTH. in.
b_e' = INCREASED EFFECTIVE PANEL WIDTH. in. ($a_e \nless b_e$)
t = PANEL THICKNESS. in.

T_{bo} = ELASTIC BUCKLING STRESS $= KE\left(\frac{t}{b_e'}\right)^2$. lb/in².

WHERE E = MODULUS OF ELASTICITY OF PANEL MATERIAL lb/in².



Example of calculation of buckling stress. **Test Panel 25 BS.T.225. T.R. 125003 Add. 2.**

$t = .028"$ (22G L89)

Stiffeners S_a (Stringers, J3009, 22G L90 reduced to panel).

$d = 4.9"$ (Stiffener pitch) $t_{sa}/t = 2.0$

$b = 4.9 - 1.24 = 3.66"$ (dist. between webs). From Fig. 1a. $b_e/(b - w_{sa}/2) = 1.0$ $\therefore b_e = 3.66 - .75 = 2.91$

$b' = 4.9 - 2.74 = 2.16"$ (Plain panel width). $I'_{sa} = .01621\ in^4$, $I'_{sa}/dt^3 = 153$.

$W_{sa} = 1.5"$ (Total flange width).

$t_{sa} = .056"$ (Total flange thickness.)

Stiffener S_b (Frames, J3026, 20G L89, riveted).

$h = a = a' = 22.35"$ (Stiffener pitch, dist. between flanges and plain panel width).

$t_{sb}' = .036"$ (flange thickness)

$t_{sb}/t = .036/.028 = 1.285$ From Fig 1b. $a_e/a = 1.0$ $\therefore a_e = 22.35$.

$I'_{sb} = .1867\ in^4$. $I'_{sb}/ht^3 = 380$.

$b_e/a_e = .131$ From Fig. 1c at $I'_{sb}/ht^3 = 380$, $a_e'/a_e = 1.0$. $a_e' = 22.35"$

at $I'_{sa}/dt^3 = 153$, $b_e'/b_e = 1.0$ $b_e' = 2.91$ {I'_{sa}/dt^3 is only just above min. required value}

$b_e'/a_e' = .131$ From Fig 2. $K = 4.95$.

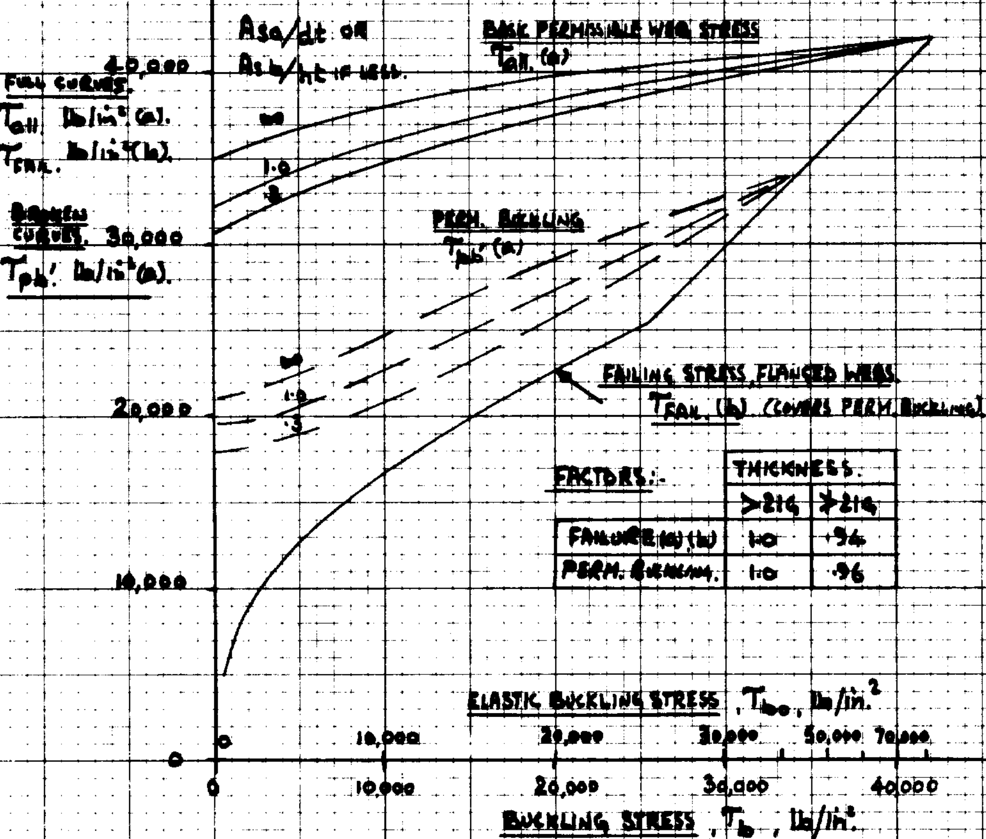
$T_{bo} = 4.95 \times 10 \times 10^6 \left(.028/2.91\right)^2 = 4,580$ lb/in².

NOTE 1, Fig 1c.
THE LOW VALUES OF I'_{sa}/dt^3 AND I'_{sb}/ht^3 DENOTE THAT BUCKLING COULD BE PERMANENT; THIS OCCURRED ON TEST PANEL WITH IDENTICAL STRUCTURE BUT CURVED AT 38" RAD. T.R. 125003 Add 1.

a). BASIC PERMISSIBLE WEB STRESS, τ_{all}, AND PERMANENT BUCKLING STRESS τ_{pb}'. FOR CONTINUOUS PANELS OR PANELS ATTACHED TO SEPARATE EDGE MEMBERS.

b). FAILING STRESS FOR FLANGED PANELS WHERE ATTACHMENT TO EDGE MEMBER IS MADE THRO THE FLANGE OR FLANGE FORMS EDGE MEMBER, τ_{FAIL}.

Fig. 3a. L 88



FACTORS:-	THICKNESS.	
	>.216	≯.216
FAILURE (a) (b)	1.0	.94
PERM. BUCKLING	1.0	.96

FOR RATIO OF $\frac{\tau_b}{\tau_{b_o}}$ SEE CURVED PANELS FIG. 2.2.K

NOTES.

(a) CONTINUOUS PANELS OR PANELS ATTACHED TO SEPARATE EDGE MEMBERS.

A_{sa} - TOTAL AREA OF STIFFR. S_a

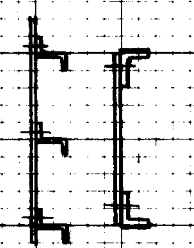
A_{sb} - TOTAL AREA OF STIFFR. S_b.

PERMANENT BUCKLING MUST NOT OCCUR BELOW PROOF LOAD EXCEPT FOR EMERGENCY ALIGHTING CASE. EFFECTIVE τ_{all} IS GIVEN BY:-

$1.50\,\tau_{pb}'$ CIVIL A/C

$1.33\,\tau_{pb}'$ MILITARY A/C

TO COVER FOR THIS CONDITION.



(b). FLANGED PANELS.

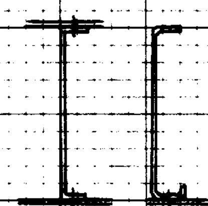
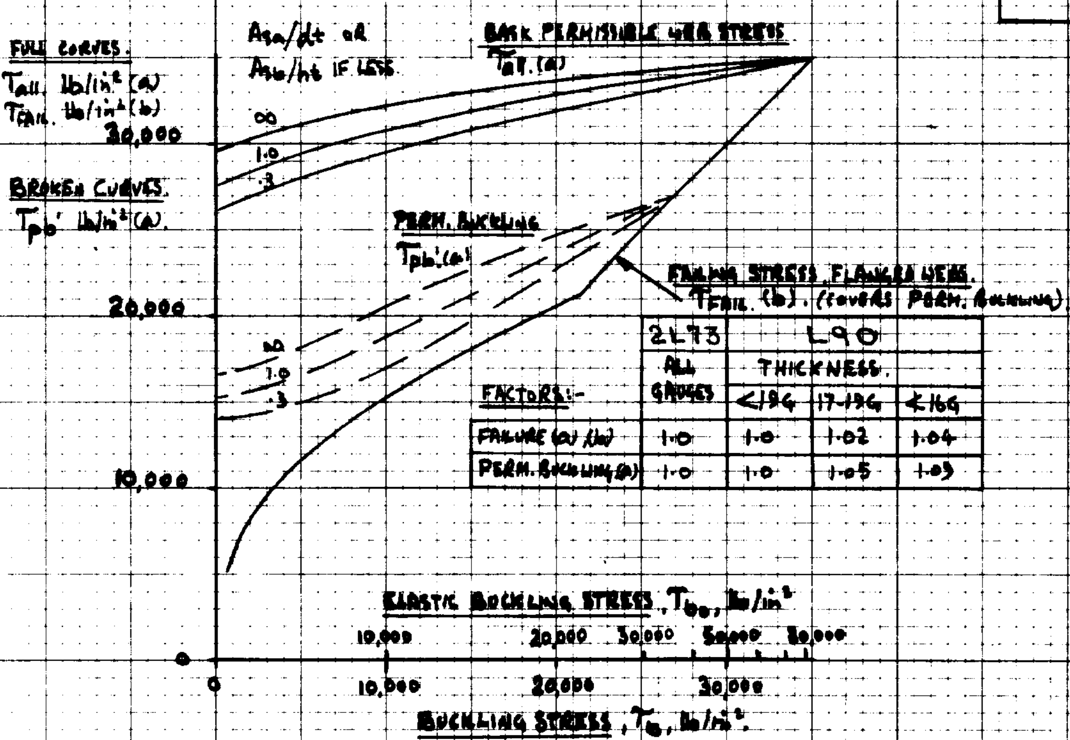


Fig. 3b. 2L73 L90

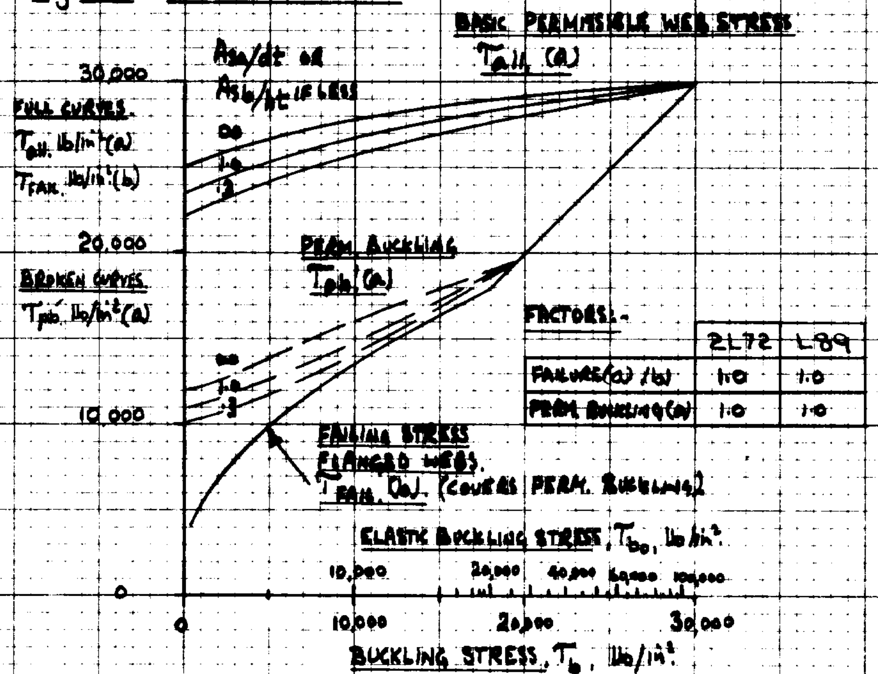


	2L73	L90		
FACTORS:-	ALL GAUGES	THICKNESS.		
		<.196	.17-.196	≮.166
FAILURE (a) (b)	1.0	1.0	1.02	1.04
PERM. BUCKLING (a)	1.0	1.0	1.05	1.03

a). BASIC PERMISSIBLE WEB STRESS, τ_{all} AND PERMANENT BUCKLING STRESS, τ_{pb}', FOR CONTINUOUS PANELS OR PANELS ATTACHED TO SEPARATE EDGE MEMBERS.

b) FAILING STRESS FOR FLANGED PANELS WHERE ATTACHMENT TO EDGE MEMBER IS MADE THROUGH THE FLANGE OR FLANGE FORMS THE EDGE MEMBER. τ_{FAIL}.

Fig. 3c. 2L72, L89

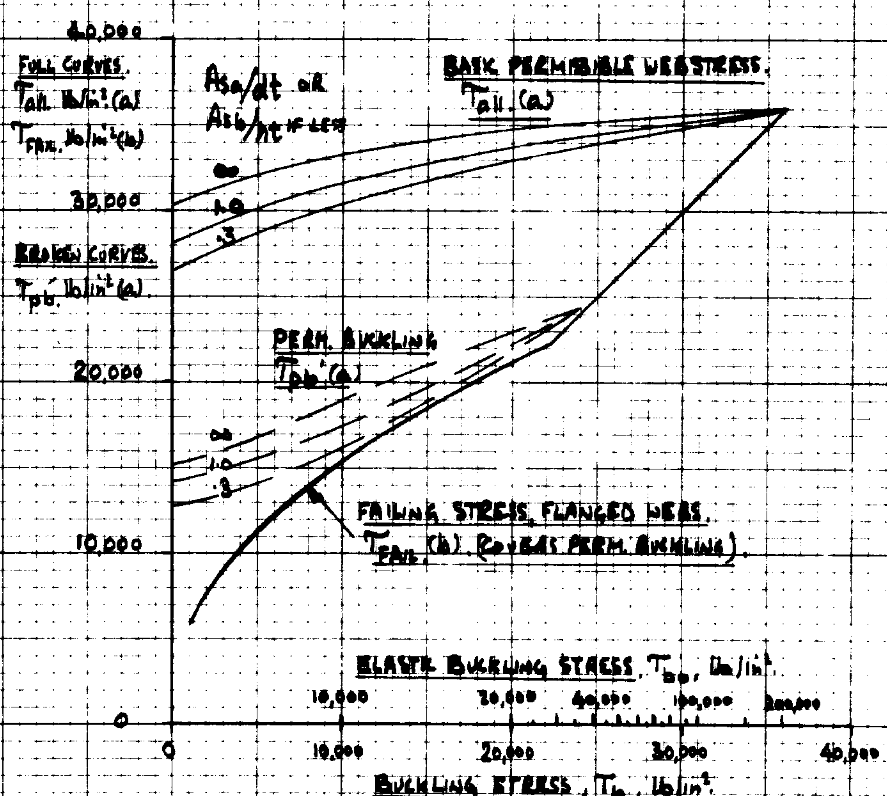


FACTORS:-

	2L72	L89
FAILURE (a) / (b)	1.0	1.0
PERM. BUCKLING (a)	1.0	1.0

FOR RATIO OF $\frac{\tau_b}{\tau_{bo}}$ SEE CURVED PANELS FIG 2.2 h.

Fig. 3d DTD 5100, DTD 5090



NOTES.

a) CONTINUOUS PANELS OR PANELS ATTACHED TO SEPARATE EDGE MEMBERS.

A_{Sa} = TOTAL AREA OF STIFFR. S_a.

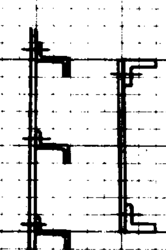
A_{Sb} = TOTAL AREA OF STIFFR. S_b.

PERMANENT BUCKLING MUST NOT OCCUR BELOW PROOF LOAD EXCEPT FOR EMERGENCY ALIGHTING CASE. EFFECTIVE τ_{all} IS GIVEN BY:-

1.50 τ_{pb}' CIVIL AIR

1.33 τ_{pb}' MILITARY A/C.

TO COVER PROOF CONDITIONS.



b) FLANGED WEBS.

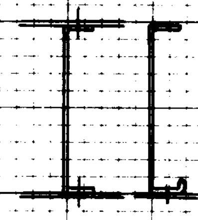


Fig 3e FAILING STRESS FOR FLANGED PANELS IN VARIOUS MATERIALS.

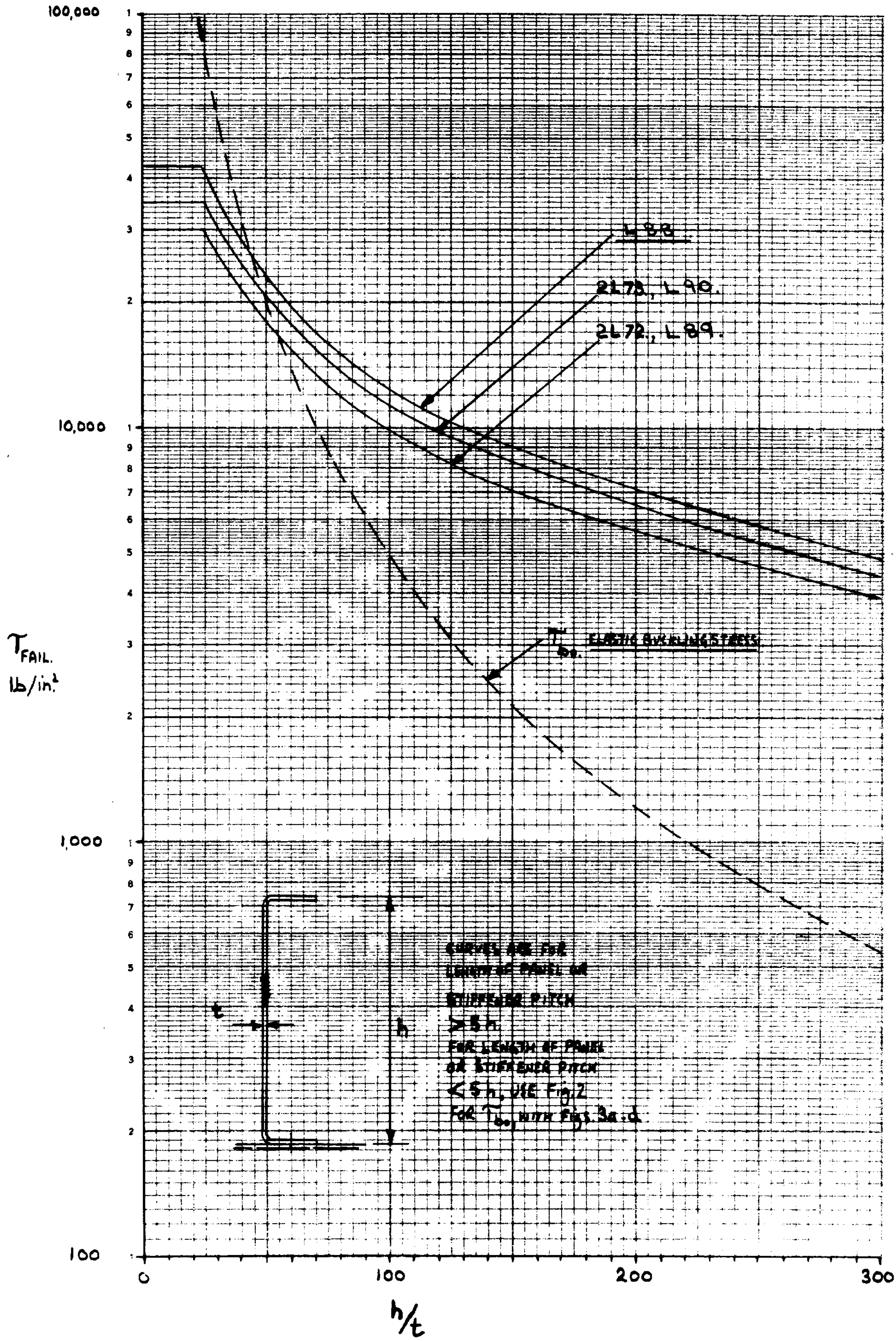


Fig 3f (a) BASIC PERMISSIBLE WEB STRESS, τ_{all}, AND PERMANENT BUCKLING STRESS, τ_{plo}' FOR CONTINUOUS PANELS OR PANELS ATTACHED TO SEPARATE EDGE MEMBERS.

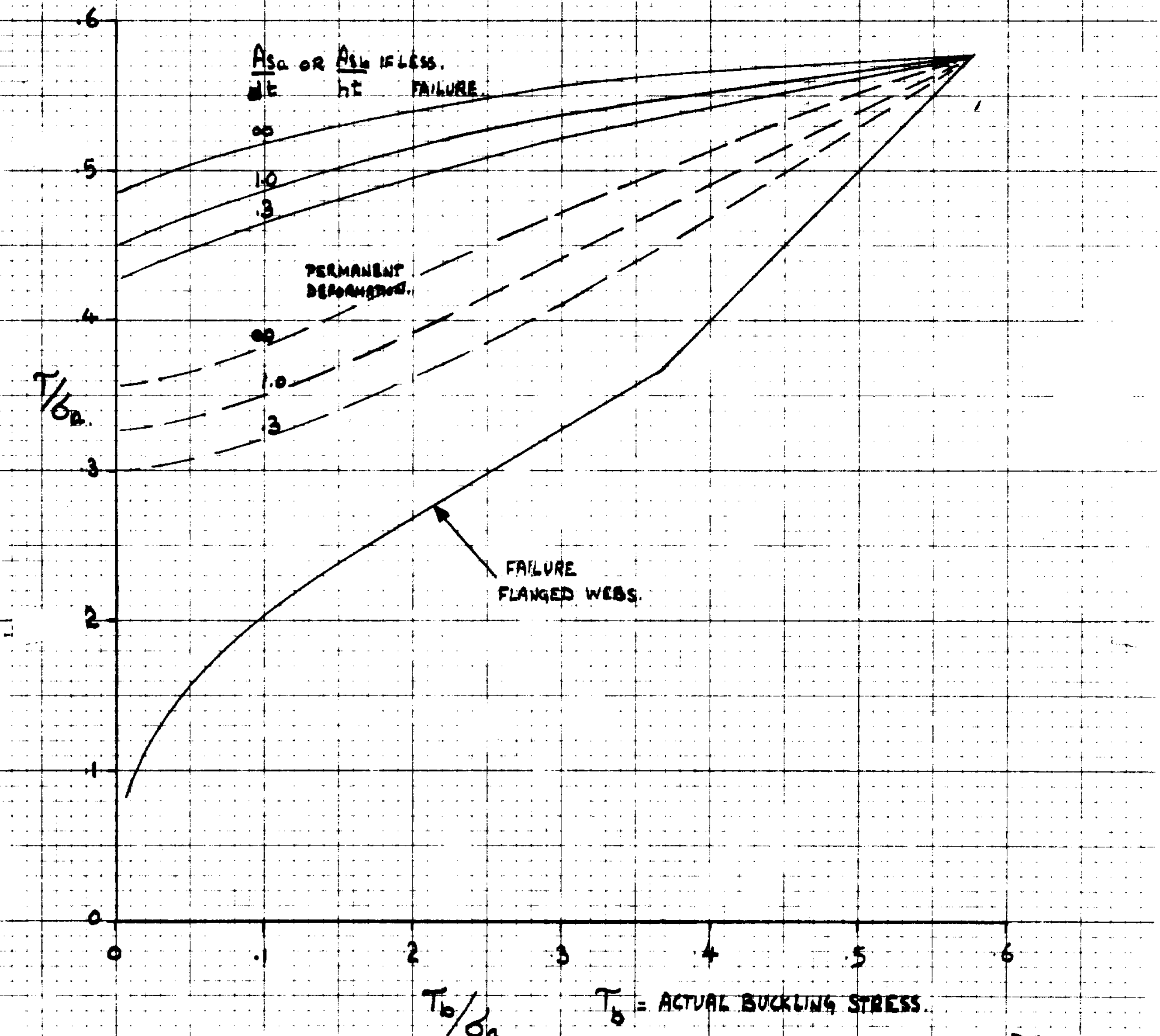
(b) FAILING STRESS FOR FLANGED PANELS WHERE ATTACHMENT TO EDGE MEMBER IS MADE THROUGH FLANGE OR FLANGE FORMS THE EDGE MEMBER. τ_{FAIL}.

GENERALISED CURVES.

(a) FOR PERMISSIBLE WEB STRESS ; $\sigma_a = \sigma_{ULT}$, $\tau = \tau_{all}$.

FOR PERMANENT BUCKLING. ; $\sigma_a = \sigma_1$ (.1% P.S.), $\tau = \tau_{plo}'$.

b) FOR FAILING STRESS. ; $\sigma_a = \sigma_{ULT}$, $\tau = \tau_{FAIL}$.



τ_b = ACTUAL BUCKLING STRESS.

(TO CONVERT ELASTIC BUCKLING STRESS, τ_{bo} TO ACTUAL BUCKLING STRESS, τ_b, USE:-

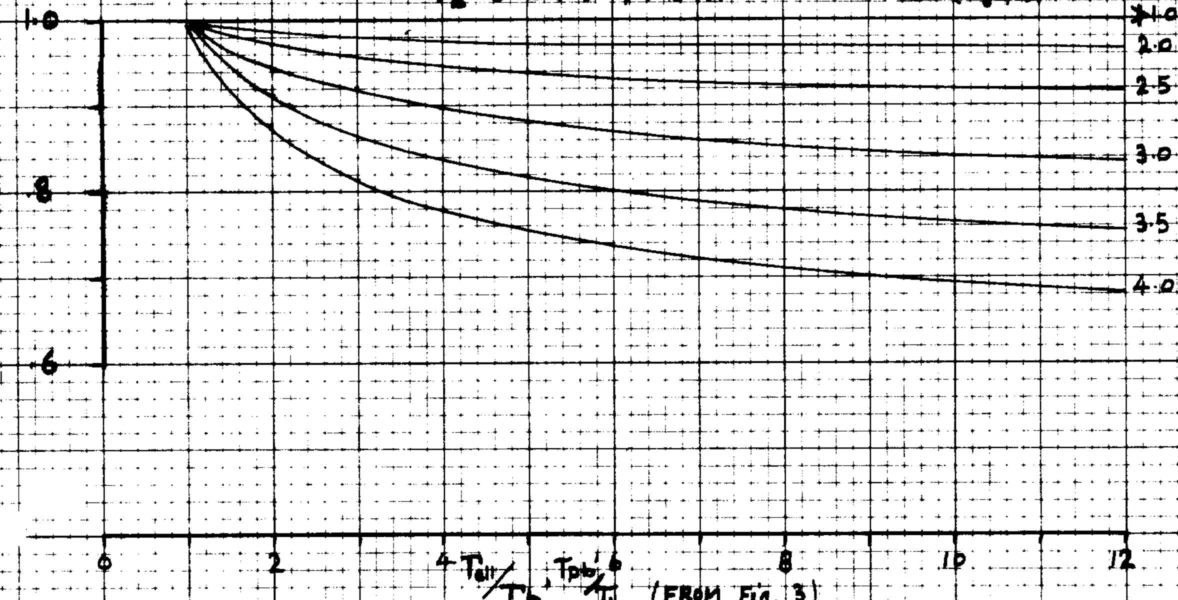
$\tau_b = \frac{G_{sec}}{G_s} \tau_{bo}$ WHERE G_{sec} IS THE SECANT SHEAR MODULUS AT SHEAR STRESS τ_b.

$\frac{G_{sec}}{G}$ AT SHEAR STRESS, τ_b, $\cong \frac{E_{sec}}{E}$ AT DIRECT STRESS $\sigma = \sqrt{3}\,\tau_{bo}$)

Fig. 4a. EFFECT OF EDGE MEMBER FLEXIBILITY ON FAILING SHEAR STRESS AND PERMANENT BUCKLING STRESS.

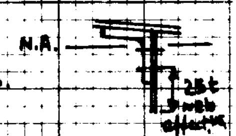
(Ref 1. $T_{MAX} = T(1 + kC_2)$).

T_{FAIL} = ALLOWABLE PANEL SHEAR STRESS lb/in²
T_{all} = BASIC ALLOWABLE PANEL STRESS. lb/in² (Fig 3)
T_{pb} = PERMANENT BUCKLING STRESS. lb/in²
T_{pb}' = BASIC PERMANENT BUCKLING STRESS lb/in² (Fig 3)
T_b = BUCKLING STRESS. lb/in² (Fig 2, 3)



use $\omega h = .7h\left[\frac{t}{d(I_T+I_C)}\right]^{1/4}$ IF STIFFENERS S_a are edge members,

or $\omega d = .7d\left[\frac{t}{h(I_T+I_C)}\right]^{1/4}$ IF STIFFENERS S_b are edge members



Where I_T, I_C are centroidal moments of inertia of edge members plus 25t of web.

IF EDGE MEMBER IS FORMED SOLELY BY FLANGING THE WEB, T_{FAIL} IS OBTAINED DIRECTLY FROM Fig. 3e.

NOTE. IF STIFFENER PITCH GREATLY EXCEEDS DISTANCE BETWEEN EDGE MEMBERS, ω SHOULD BE MULTIPLIED BY 1.25.

Fig 4b. ALLOWABLE PANEL SHEAR STRESS BASED ON STRENGTH OF JOINT ATTACHMENTS.

T_{all} lb/in² (FROM 50.03.52 SHEET II, $F = Tt(1 + .414k)$.)

F = ALLOWABLE JOINT LOADING. (FOR RIVETED OR BOLTED JOINTS TAKE MINIMUM VALUE GIVEN BY SHEAR OR BEARING). lb/in.

FOR PANEL-EDGE MEMBER JOINTS, CURVE MAY BE USED DIRECTLY. (i.e. $T_{all} = T_{FAIL}$).

FOR JOINT IN PANEL, USE T_{all} FROM THIS CURVE WITH Fig. 4a ABOVE TO OBTAIN T_{FAIL}.

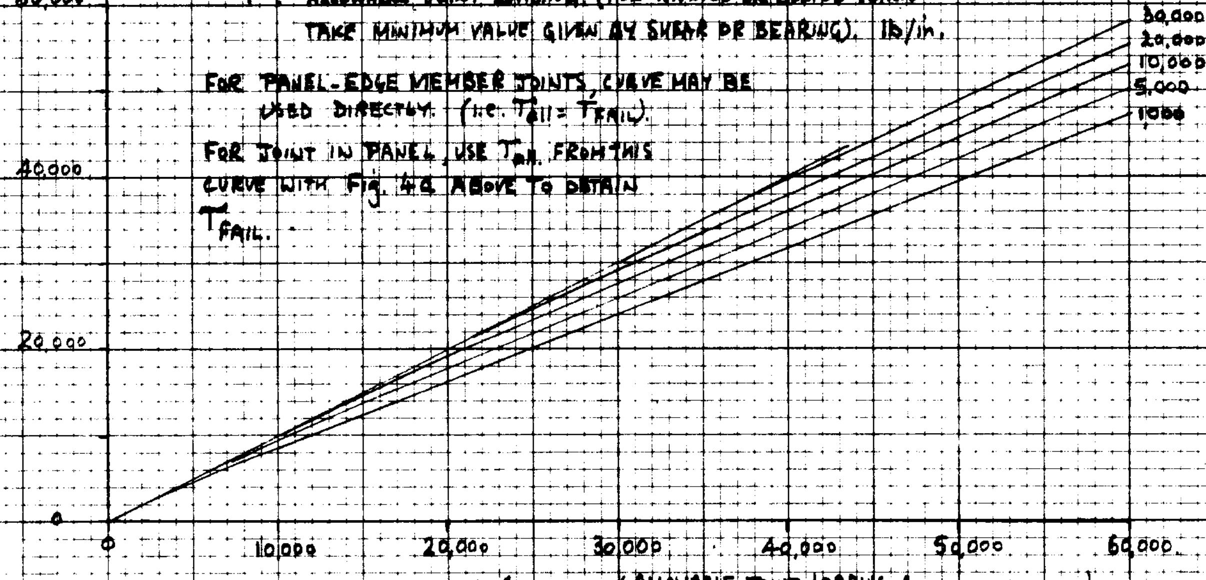


Fig 5. COMBINED SHEAR AND DIRECT STRESS AT WEB TO EDGE MEMBER RIVET LINE.

Refs:- R.Ae.S. 02.03.04. D.H. Report A.R. No 20., B.A. Noble.
(From these the expression $\left(\frac{\sigma_{FAIL}}{\sigma_{all.}}\right)^2 + \left(\frac{T'_{FAIL}}{T_{FAIL}}\right)^2 = 1$ was derived.)

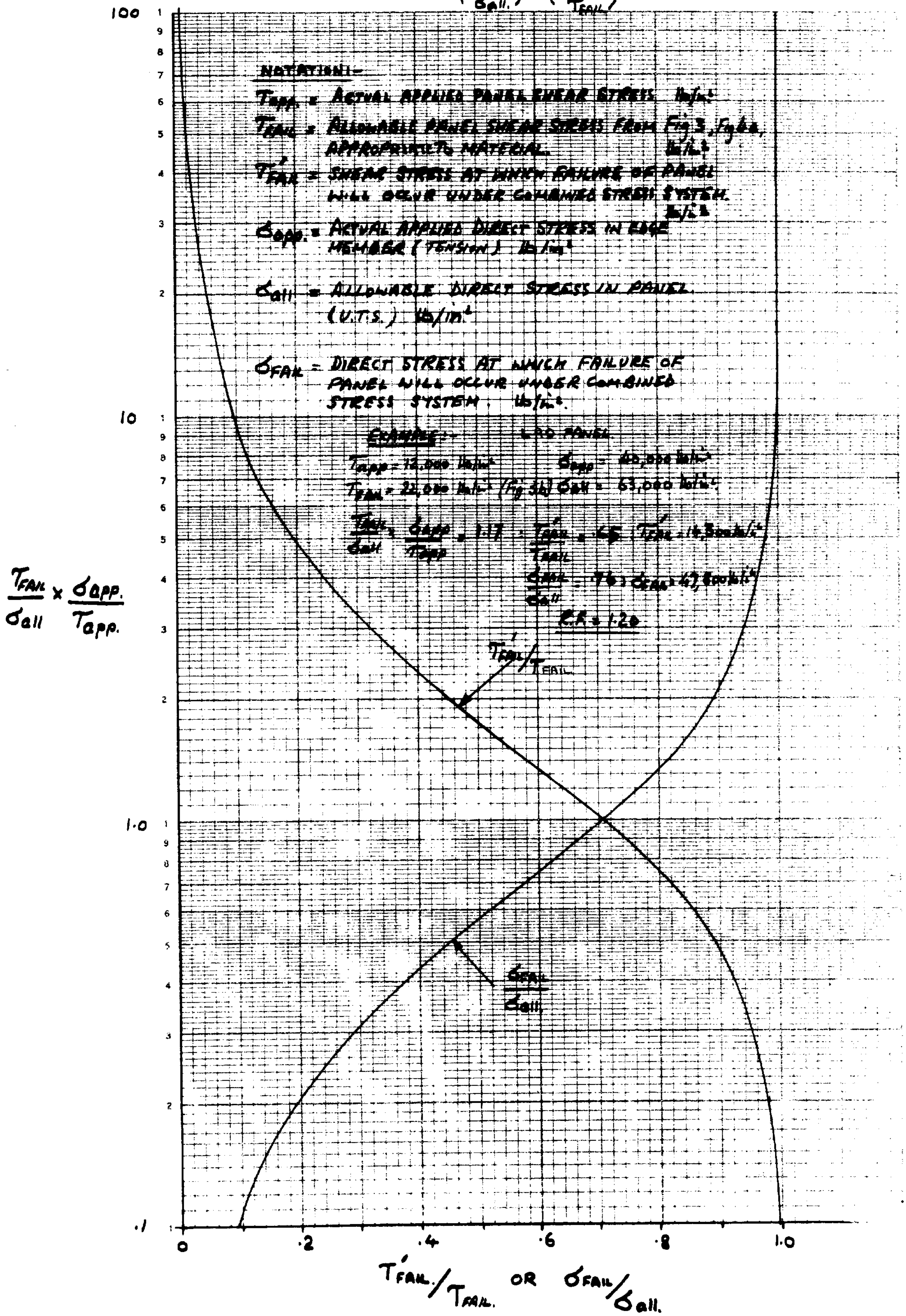
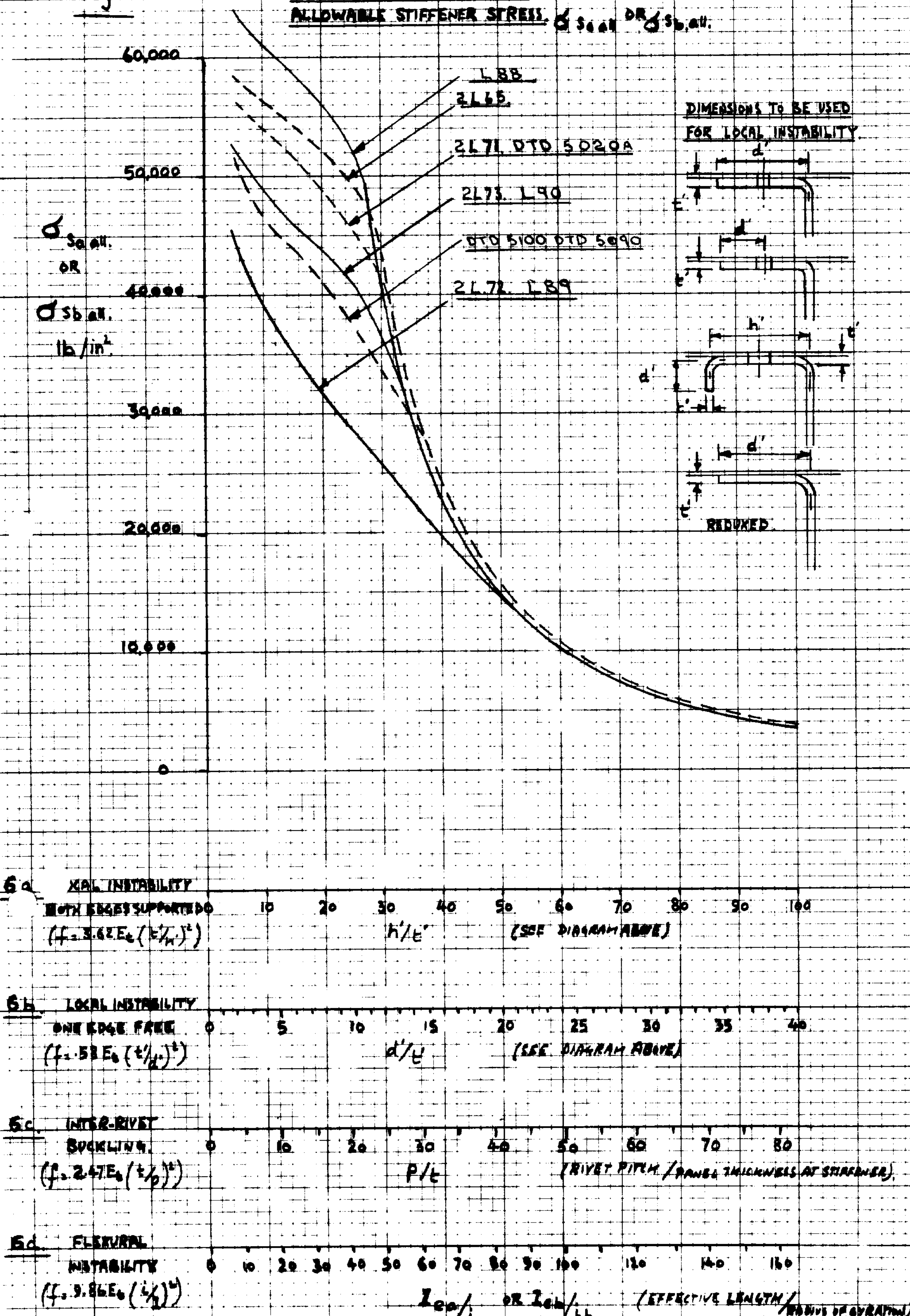


Fig 6

SHEAR PANEL STIFFENERS.
ALLOWABLE STIFFENER STRESS. $\sigma_{Sa\,all}$ OR $\sigma_{Sb\,all}$



$(f = 3.62 E_t (t'/h')^2)$

$(f = .58 E_t (t'/d')^2)$

$(f = 2.47 E_t (t/p)^2)$

$(f = 9.86 E_t (i/l)^2)$

Fig 7 AVERAGE STIFFENER STRESS COEFFICIENT C_S.

(AVERAGE OVER LENGTH OF STIFF^R).

FOR STIFFENER S_a. $\sigma_{Sa_{AV.}} = C_S(T - T_0) \Big/ \left(\frac{A_{Sa\,eff}}{b't} + \frac{S}{b'}\right)$. $\left.\begin{matrix} S = 40t,\ 40t/b' < .667 \\ S = .667b',\ 40t/b' > .667 \end{matrix}\right\}$

FOR STIFFENER S_b. $\sigma_{Sb_{AV.}} = C_S(T - T_0) \Big/ \left(\frac{A_{Sb\,eff}}{a't} + \frac{S}{a'}\right)$. $\left.\begin{matrix} S = 40t,\ 40t/a' < .667 \\ S = .667a',\ 40t/a' > .667. \end{matrix}\right\}$

Ref. SECTION 3.2.

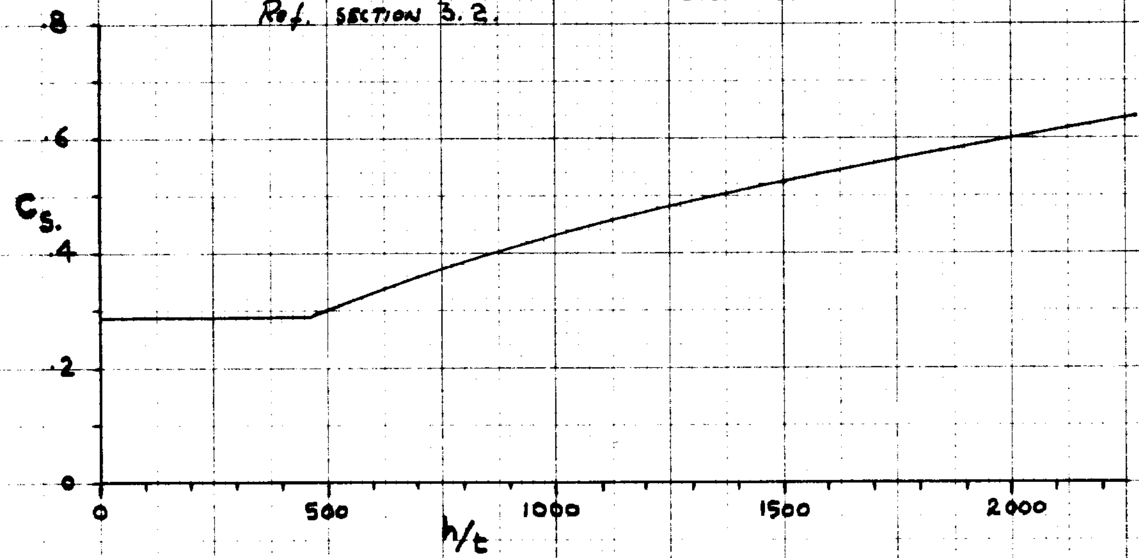


Fig 8 MAXIMUM STIFFENER STRESS. $\sigma_{S\,MAX}$.

a). CONTINUOUS SINGLE STIFFENERS, AWAY FROM END BAY, AND CONTINUOUS OR SINGLE BAY DOUBLE STIFFENERS. Ref. SECT. 3.2.

$$\sigma_{S\,MAX} = \sigma_{S_{AV.}}(1 + .15\,h/d) \quad , h/d \not< 1.0.$$
$$= 1.15\,\sigma_{S_{AV}} \quad , h/d < 1.0.$$

b). SINGLE STIFFENERS.

SINGLE BAY AND END BAY OF CONTINUOUS STIFFENERS.

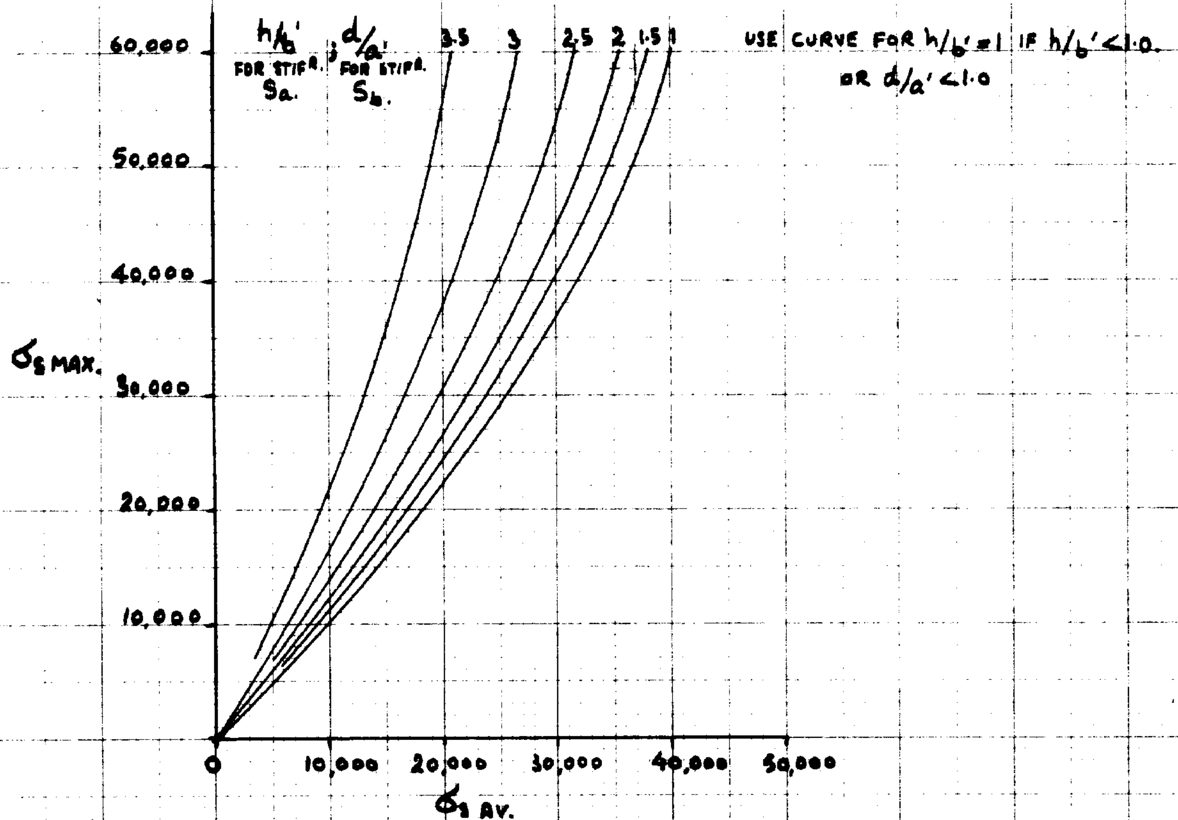
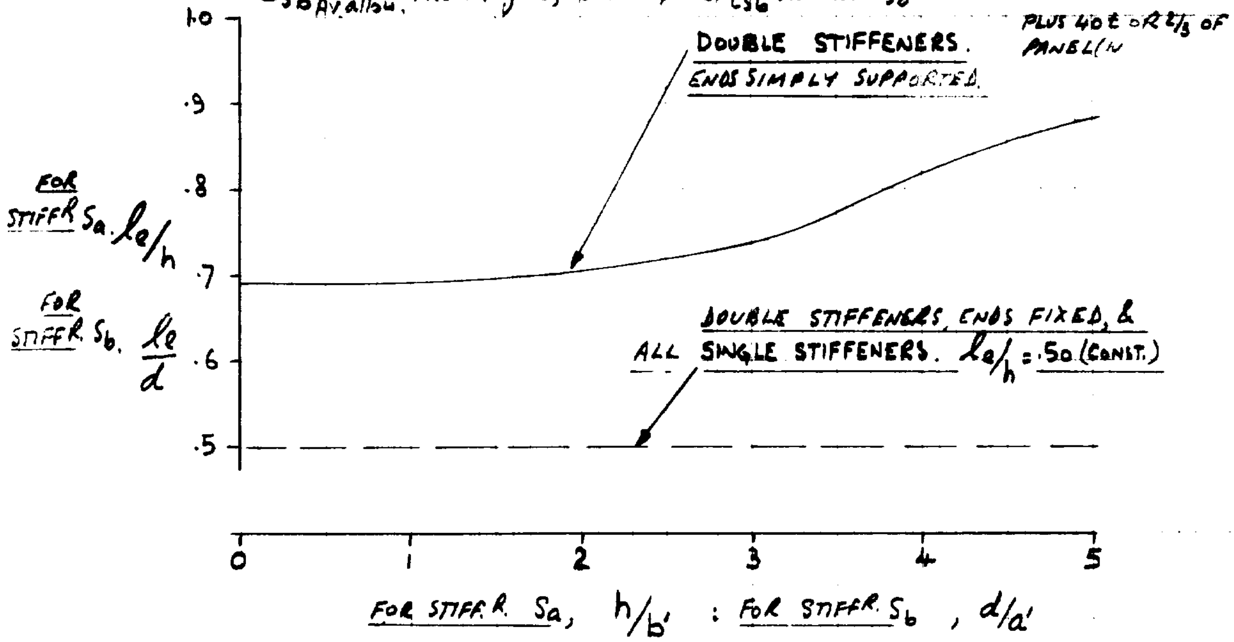


Fig. 9. EFFECTIVE STRUT LENGTH l_e FOR FLEXURAL INSTABILITY.

$\sigma_{Sa\,Av\,allow}$. FROM Fig. 10, 11 USING l_e/i_{sa}, WHERE i_{sa} IS BASED ON STIFFENER SECTION PLUS $40t$ OR $2/3$ OF PANEL (WHICHEVER IS LESS).

$\sigma_{Sb\,Av\,allow}$. FROM Fig 10, 11 USING l_e/i_{sb}, WHERE i_{sb} IS BASED ON STIFFR. SECTION PLUS $40t$ OR $2/3$ OF PANEL (W



STIFFENERS MUST ALSO BE CHECKED FOR FORCED CRIPPLING OF STIFFENER FLANGE AND TORSIONAL INSTABILITY. SECTION 3.4.c & d.

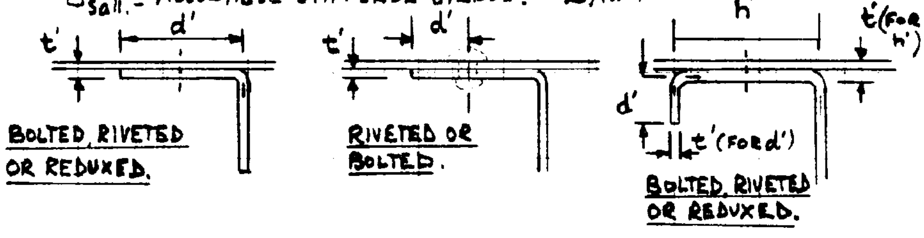
TABLE 10 ALLOWABLE LOCAL AND FLEXURAL INSTABILITY STRESSES FOR STIFFENERS AND PANEL INTER-RIVET BUCKLING STRESS.
THIS TABLE GIVES COEFFICIENTS FOR USE WITH Fig 11 ON FOLLOWING SHEET.

No.	CONDITION OF	FAILURE.	K_c	ζ	$\sigma_{s\,all.}$
1	LOCAL INSTABILITY OF FLANGE ADJACENT TO PANEL. (SEE BELOW)	FREE EDGE.	.58	d'/t' (SEE BELOW)	σ_{SB}
2		SUPPORTED EDGES.	3.62	h'/t' (" ")	σ_{SB}.
3	INTER-RIVET BUCKLING.	BOLTED ATTMT.	3.29	p/t (BOLT, RIVET OR SPOTWELD PITCH DIVIDED BY PANEL THICKNESS).	SEE. SECT. 3.4. a.
4		SPOTWELDS.	2.88		
5		MUSH. HD. RIVETS.	2.47		
6		CSK OR DIMPLED RIVETS.	1.23		
7	FLEXURAL INSTABILITY		9.86	l_e/i_s SECT. 3.4.b.	σ_{SB}

$$\sigma_{SB} = \frac{K_c E_t}{\zeta^2}$$ WHERE:–

E_t = TANGENT MODULUS FOR MATL UNDER CONSIDERATION. lb/in²
K_c = COMPRESSIVE BUCKLING STRESS COEFFICIENT.
ζ = RATIO OF LENGTH TO THICKNESS ETC. (ABOVE TABLE).
σ_{SB} = STRESS AT WHICH BUCKLING WILL OCCUR. lb/in².
$\sigma_{s\,all.}$ = ALLOWABLE STIFFENER STRESS. lb/in².

LOCAL INSTABILITY.
h', d' AND t'



SHEAR PANEL STIFFENERS.

Fig. 11. STIFFENER AND PANEL BUCKLING STRESS. σ_{SB}.

FOR VALUES OF K AND ξ SEE Fig. AND TABLE 10

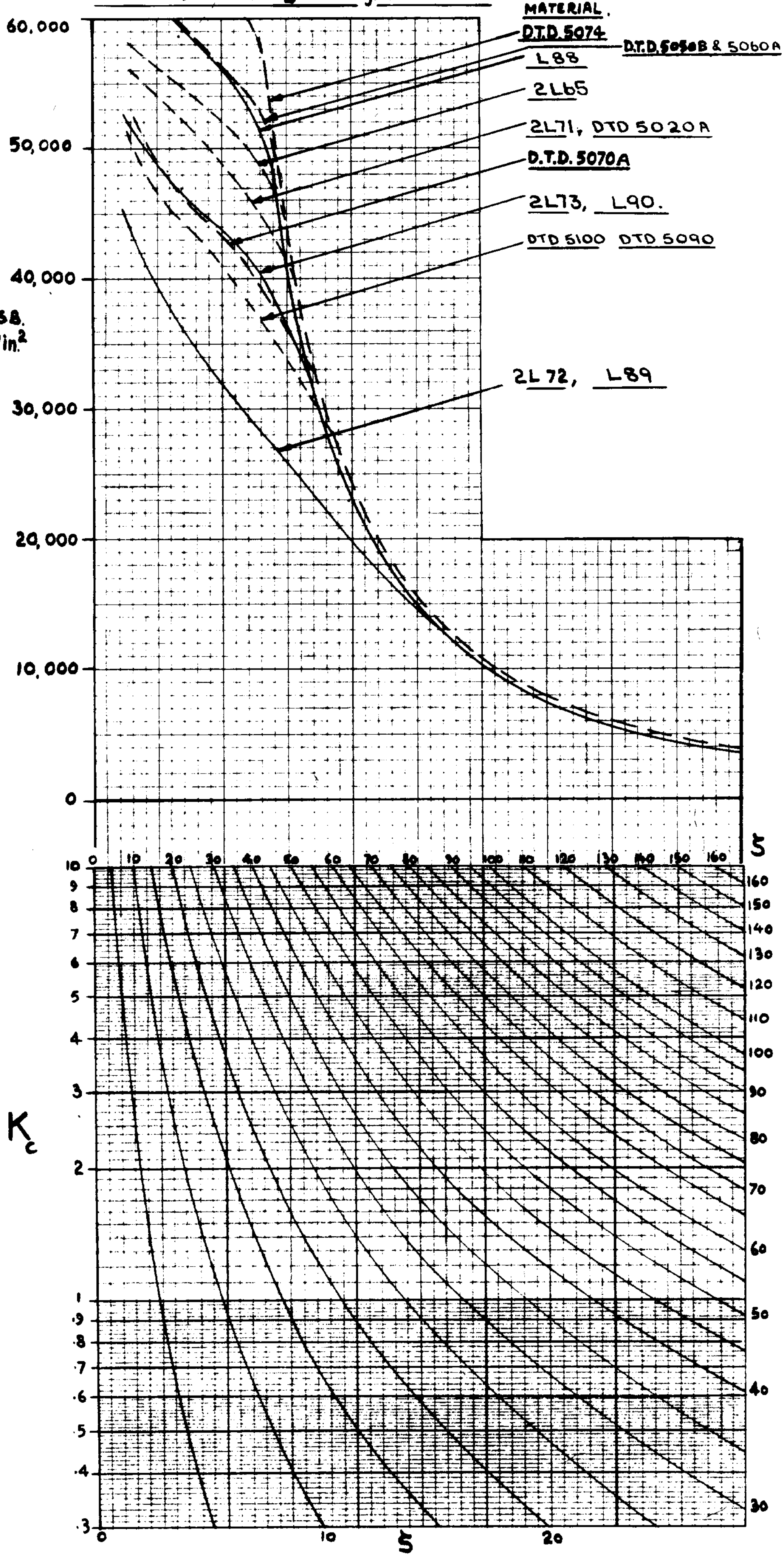


Fig 12. FORCED CRIPPLING FAILURE OF STIFFENER FLANGE.

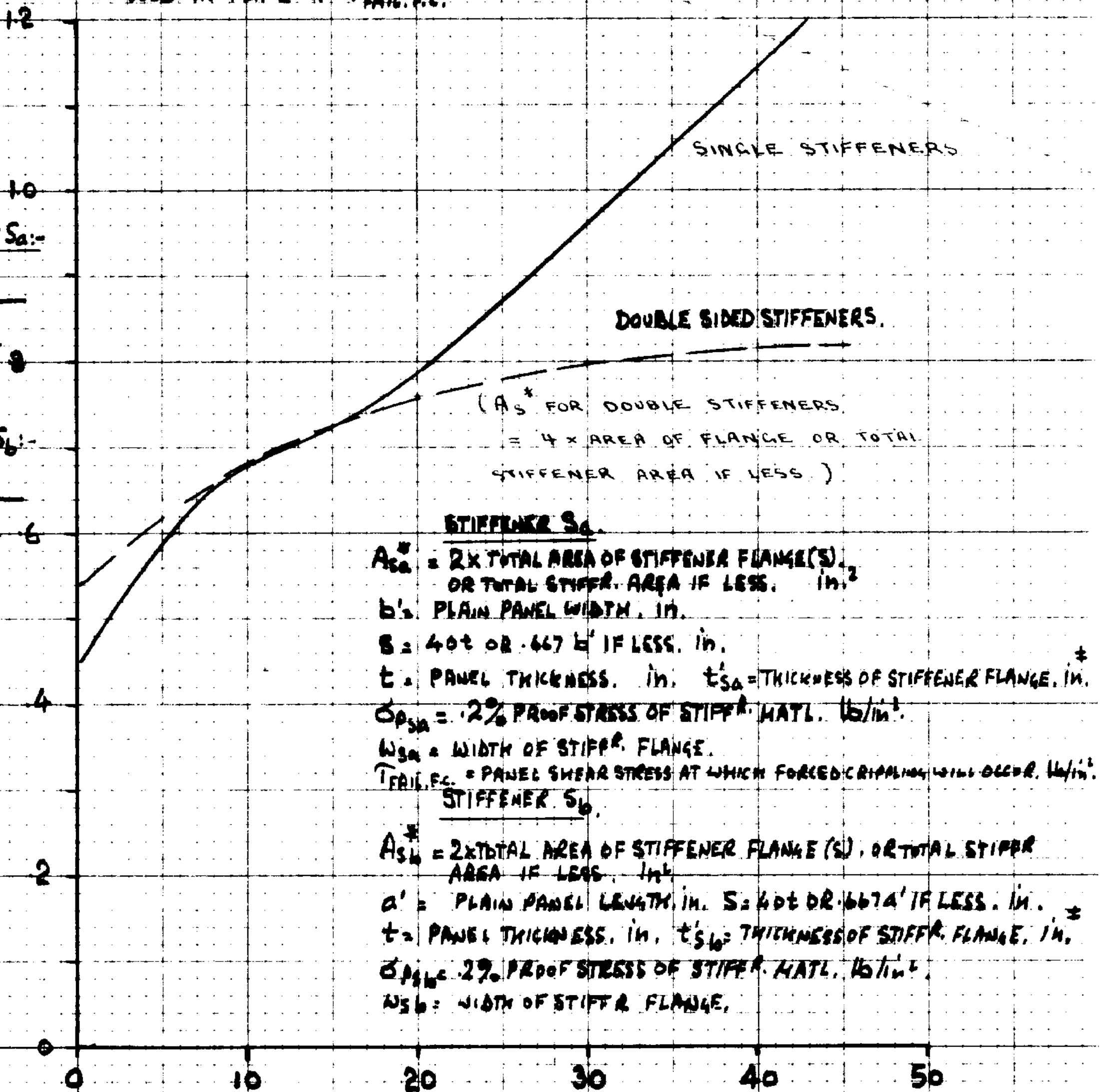
NOTE:- IF $\tau_{FAIL.F.C.}$, THE PANEL SHEAR STRESS AT WHICH FORCED CRIPPLING OF THE STIFFENER FLANGE WILL OCCUR, IS LESS THAN τ_b, THE BUCKLING STRESS, THAN FORCED CRIPPLING WILL OCCUR AT BUCKLING AND τ_b IS USED IN PLACE OF $\tau_{FAIL.F.C.}$

FOR STIFFENER S_a:-

$$\frac{\tau_{FAIL.F.C.}}{\sigma_{P_{Sa}}} \times \frac{1}{\frac{A_{Sa}^*}{b't} + \frac{S}{b'}}$$

FOR STIFFENER S_b:-

$$\frac{\tau_{FAIL.F.C.}}{\sigma_{P_{Sb}}} \times \frac{1}{\frac{A_{Sb}^*}{a't} + \frac{S}{a'}}$$



FOR STIFFR S_a:- $\dfrac{b' t'_{Sa}}{w_{Sa} t}$ $\left(\dfrac{\text{PANEL WIDTH} \times \text{FLANGE THICKNESS}}{\text{FLANGE WIDTH} \times \text{PANEL THICKNESS}}\right)$

FOR STIFFR S_b:- $\dfrac{a' t_{Sb}}{w_{Sb} t}$

‡ FOR REDUCED FLANGES, USE $t'_s = t_{FLANGE} + t_{PANEL}$, AND $S = 40t + w$.

FOR LIPPED FLANGES, DEPTH OF LIP $\not< 5t'_s$, MULTIPLY $\tau_{FAIL.F.C.}$ BY 1.17.

FOR CLOSED SECTION STIFFENERS, w = WIDTH OF EACH FLANGE AND MULTIPLY $\tau_{FAIL.F.C.}$ BY 2.0.

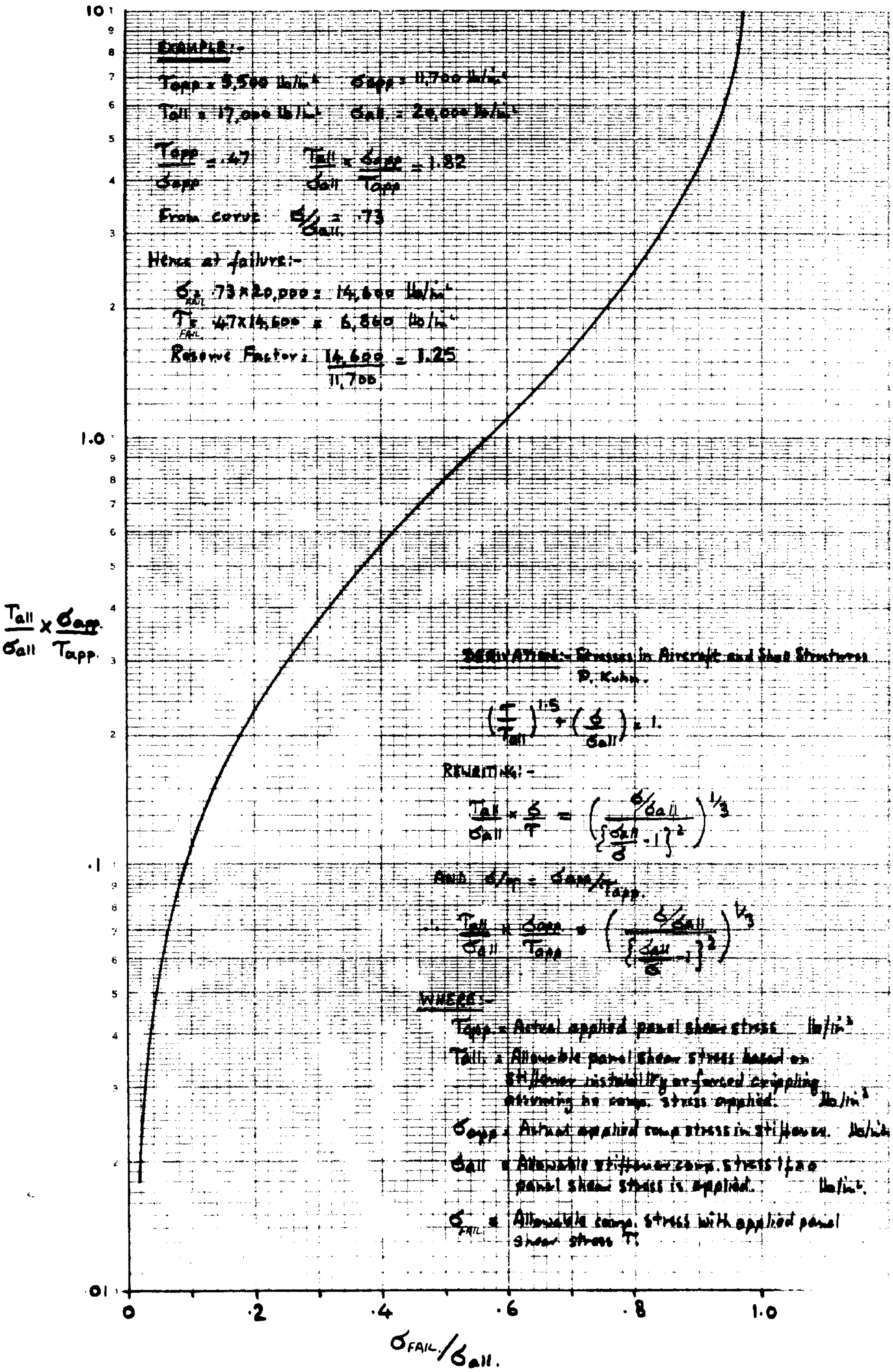
VALUES OF σ_p FOR VARIOUS MATERIALS.

MATL.		σ_p lb/in^2.
L89	2L72.	35,800
L90	≮16G	53,700
"	" 17-19G	51,500
"	" 20-24G.	49,300
2L73		49.300
L88	≮20G.	62,600
"	" <20G.	60,500

MAT'L		σ_p lb/in^2.
2L65	≯½	55,000
"	≯1"	59,400
"	≯2"	63,900
"	≯3"	61,600
"	≯4"	59,400
"	≯6"	55,000

Fig 13. **SHEAR PANEL STIFFENERS.**

EFFECT OF COMBINED DIRECT COMPRESSION STRESS IN STIFFENER AND PANEL SHEAR STRESS ON ALLOWABLE STIFFENER STRESS.



SHEAR PANEL EDGE MEMBERS.

EFFECT OF COMBINATION OF DIRECT LOAD AND TENSION FIELD ON FAILING SHEAR AND COMPRESSIVE STRESSES.

FROM SECTION 4.

Fig. 14a

$$\alpha = \frac{R}{\frac{2A_{sa}}{b't} + .5(1-R)} \quad \text{(SEE SECT. 4)} \quad \text{FOR STIFFENER } S_a \text{ AS EDGE MEMBER}$$

$$\alpha = \frac{R}{\frac{2A_{sb}}{a't} + .5(1-R)} \quad \text{FOR STIFFENER } S_b \text{ AS EDGE MEMBER}$$

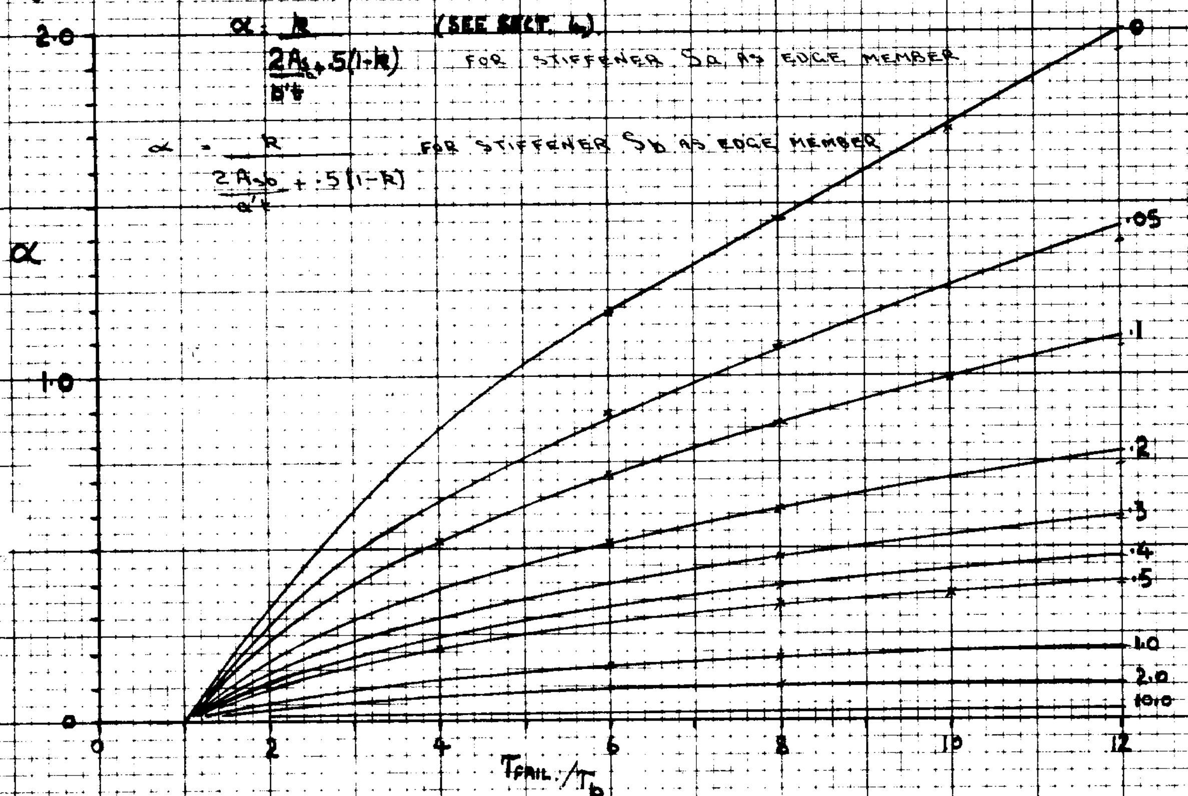
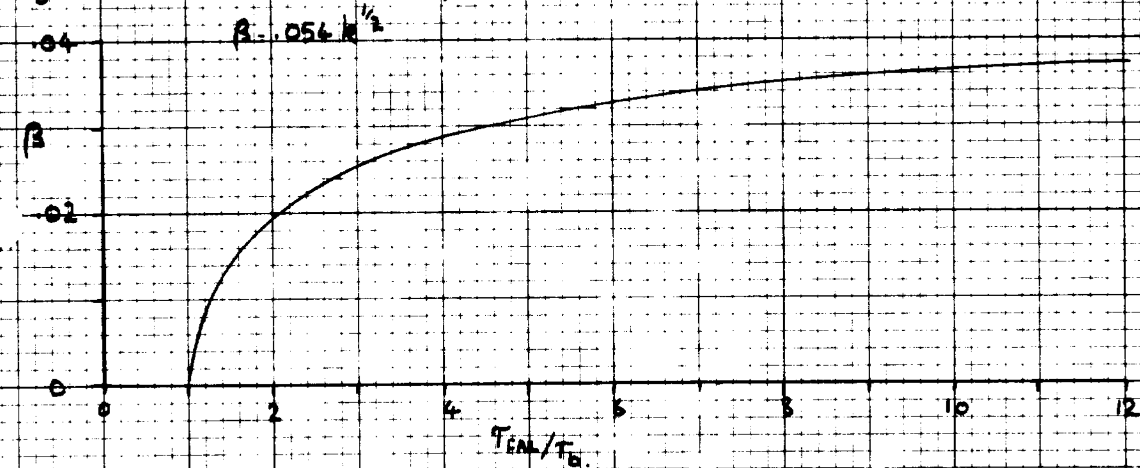


Fig. 14b

$\beta = .056\,k^{1/2}$



FOR STIFFENER S_a AS EDGE MEMBER:-

$$\frac{\tau_{FAIL}}{\sigma_{all.}} = \frac{1}{\frac{\sigma_{app}}{\tau_{app}} + \left(\alpha + \frac{\beta t h^2}{F Z_{sa\,min.}}\right)}$$

FOR STIFFENER S_b AS EDGE MEMBER:-

$$\frac{\tau_{FAIL}}{\sigma_{all.}} = \frac{1}{\frac{\sigma_{app}}{\tau_{app}} + \left(\alpha + \frac{\beta t d^2}{F Z_{sb\,min.}}\right)}$$

WHERE:-

t = PANEL THICKNESS in^2. $\quad$ F = FORM FACTOR. R.Ae.S. D.S. 01.06.02 OR 01.06.03.

d = PITCH OF STIFFENERS S_a in. $\quad$ FOR [illegible] WITH R.Ae.S. DATA SHEETS $\tau_{s\,max.}$ SHOULD

h = PITCH OF STIFFENERS S_b in. $\quad$ BE USED IN CONJUNCTION WITH F

$Z_{sa\,min.}$, $Z_{sb\,min.}$ = MINIMUM SECTION MODULI OF EDGE MEMBER ABOUT AN AXIS THRO CENTROID NORMAL TO PANEL. in^3.

$\tau_{app.}$ = APPLIED SHEAR STRESS IN PANEL. lb/in^2.

$\tau_{FAIL.}$ = SHEAR STRESS AT WHICH EDGE MEMBER WILL FAIL IN COMP. lb/in^2. } OBTAINED BY METHOD OF APPROXIMATION.

τ_b = PANEL BUCKLING STRESS. lb/in^2

$\sigma_{app.}$ = APPLIED COMP. STRESS IN EDGE MEMBER. lb/in^2.

$\sigma_{all.}$ = ALLOWABLE COMP. STRESS IN EDGE MEMBER WITH NO TENSION FIELD EFFECTS. lb/in^2

EFFECTIVE SHEAR MODULUS FOR PANEL.

Fig 15a ELASTIC SHEAR MODULUS, INCLUDING EFFECT OF INCOMPLETE DIAGONAL TENSION. G_{idt}. lb/in²

G = ELASTIC SHEAR MODULUS FOR PANEL MATERIAL. lb/in²

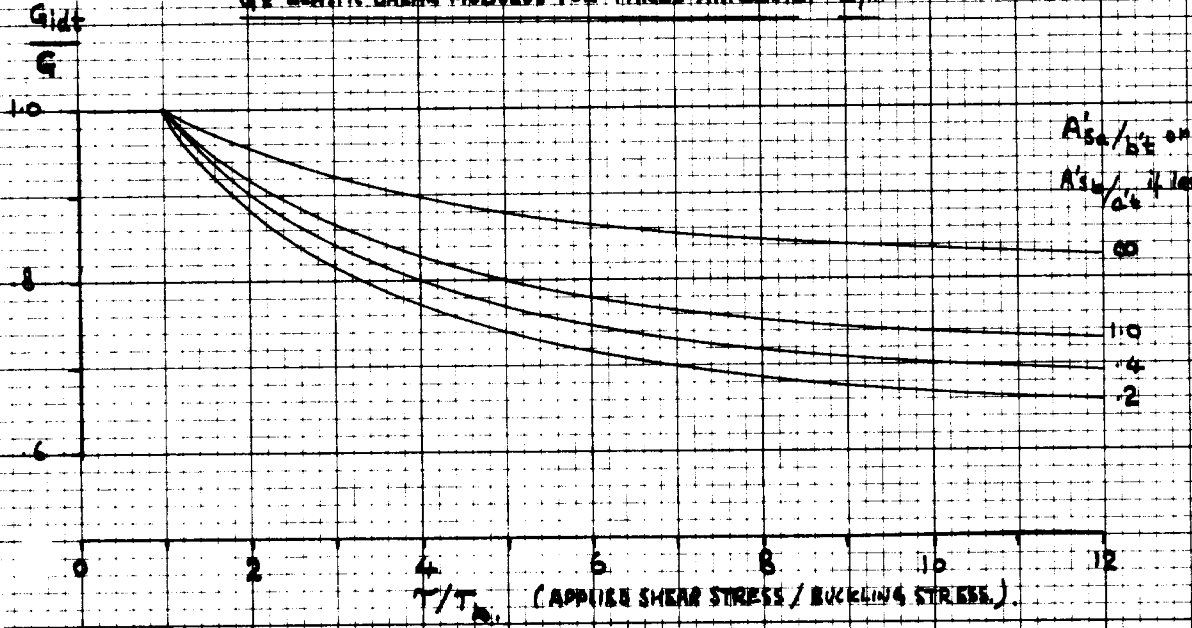
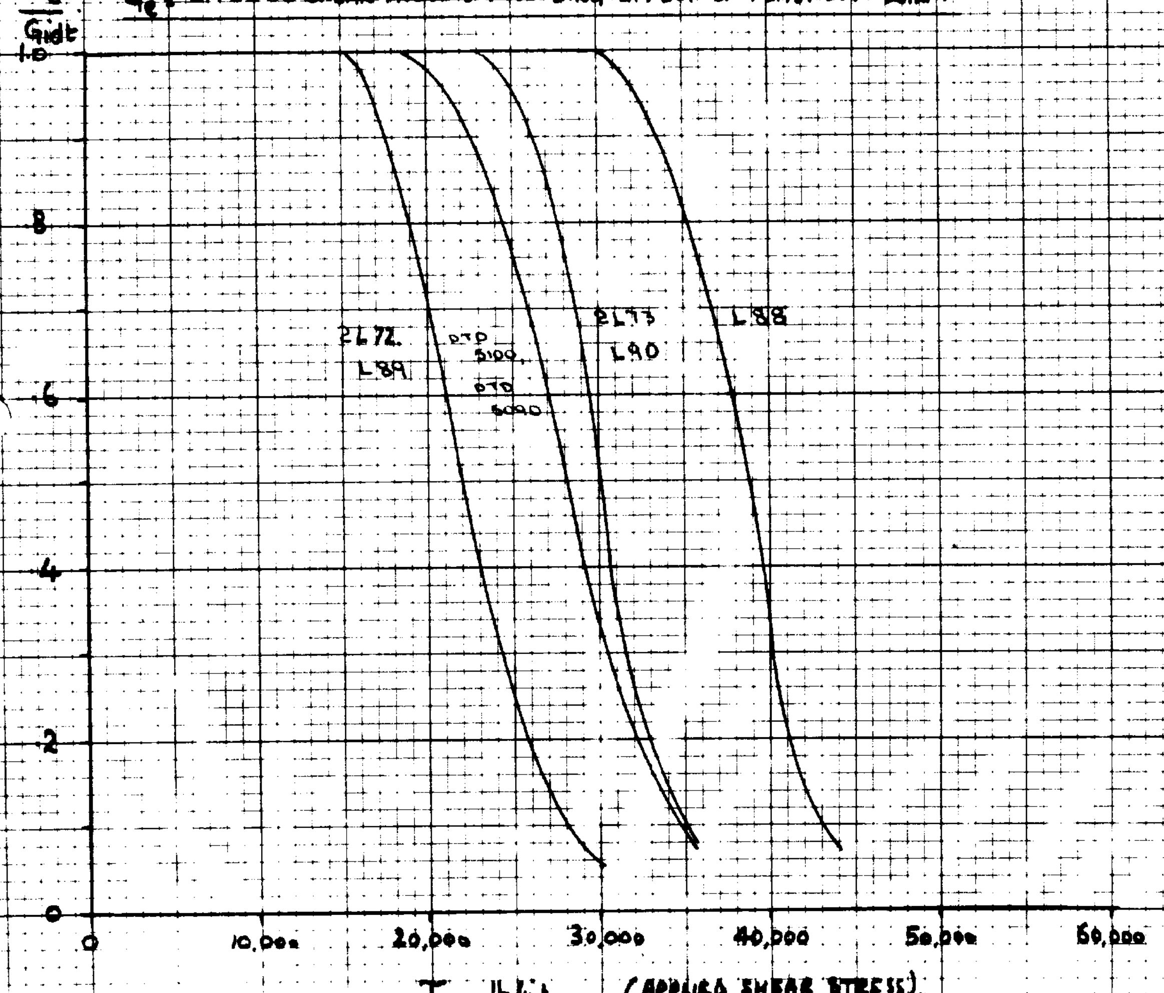


Fig. 15b. EFFECT OF PLASTICITY.

G_e = EFFECTIVE SHEAR MODULUS INCLUDING EFFECT OF PLASTICITY lb/in².



COMPARISON BETWEEN TEST RESULTS AND PREDICTIONS.

TEST RESULTS REFS. 3 AND 23.

ACTUAL FAILURE:- PANEL. ○
COLUMN. □
FORCED CRIPPLING OR LOCAL INSTABILITY, DOUBLE STIFFRS. △
SINGLE STIFFRS. ×

Fig. 16. a. USING Ref. 1. N.A.C.A. T.N. 2661.

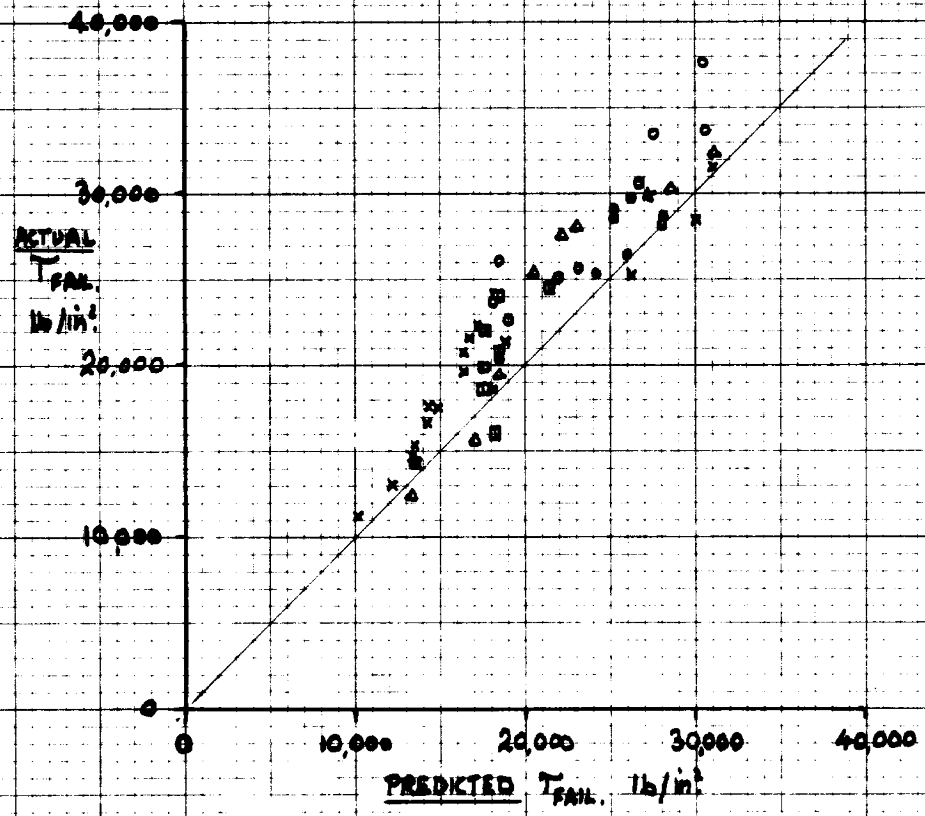
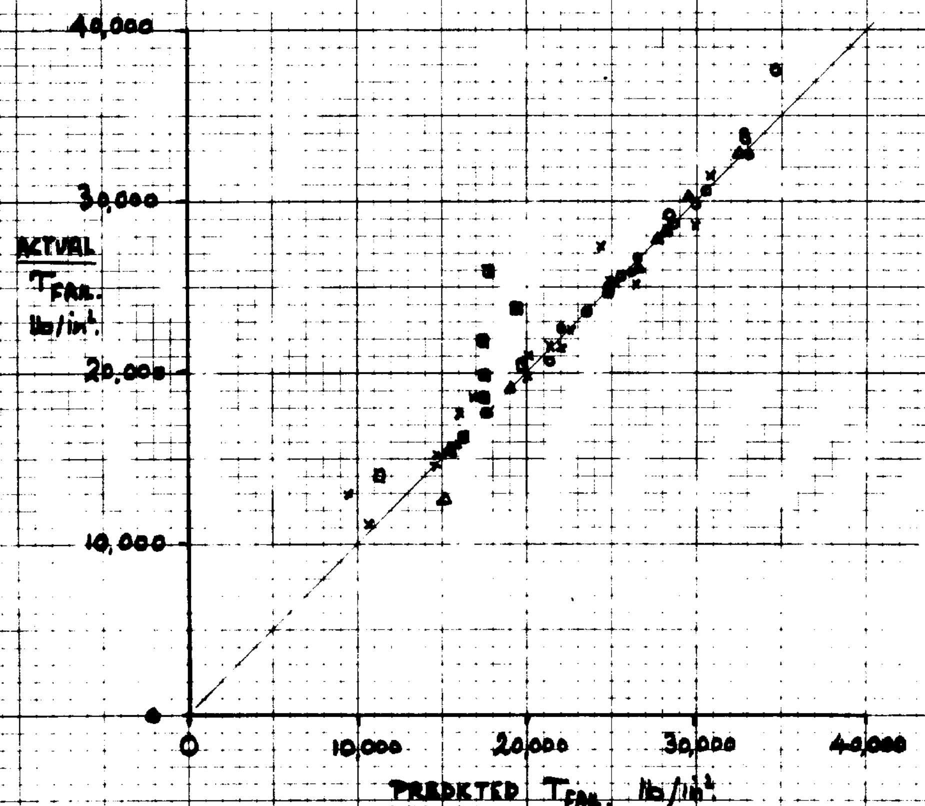


Fig. 16. b. USING S.O.R. N° 61



The post-buckling stress distribution in a flat shear panel and supporting stiffeners.

The two differential equations governing the post buckling behaviour of a shear panel are, (Ref. 13), :-

$$\nabla^4\phi = \frac{\partial^4\phi}{\partial x^4} + \frac{2\partial^4\phi}{\partial x^2\partial y^2} + \frac{\partial^4\phi}{\partial y^4} = E\left\{\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right\}. \qquad (1)$$

$$\nabla^4 w = \frac{\partial^4 w}{\partial x^4} + \frac{2\partial^4 w}{\partial x^2\partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{t}{D}\left\{\frac{\partial^2\phi}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2\phi}{\partial y^2}\frac{\partial^2 w}{\partial x^2} - 2\frac{\partial^2\phi}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y}\right\} \qquad (2)$$

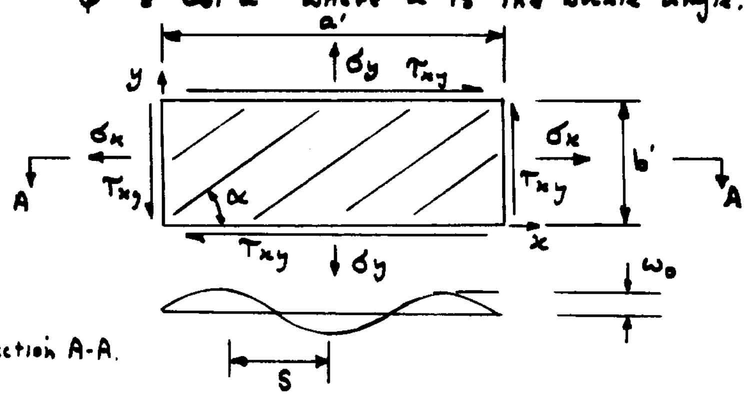
Where w is the panel deflection and ϕ is Airy's stress function; D = Flexural rigidity of panel = $Et^3/12(1-\nu^2)$; t = panel thickness.

A reasonably representative form for the deflection of a buckled shear panel may be taken as:-

$$w = w_0 \operatorname{Cos}\frac{\pi}{S}(x - \psi y) \qquad (3)$$

Where:- S is the half-wave length. in.
w_0 is the maximum deflection. in.
ψ = Cot α where α is the buckle angle. } Fig. 1.

Fig. 1.



Section A-A.

Expression (3) does not satisfy the boundary conditions but is representative of the form of buckle noted on test, where a large part of the crests and throughs is uniform in displacement.

Substitution of the derivatives of (3) in equation (1) yields :-

$$\nabla^4\phi = E w_0^2\left\{\left(\frac{\psi\pi^2}{S^2}\operatorname{Cos}\frac{\pi}{S}(x-\psi y)\right)^2 - \frac{\pi^4\psi^2}{S^4}\operatorname{Cos}^2\frac{\pi}{S}(x-\psi y)\right\} = 0 \qquad (4)$$

Substitution of the derivatives of (3) in equation (2) yields :-

$$\psi^2\frac{\partial^2\phi}{\partial x^2} + 2\psi\frac{\partial^2\phi}{\partial x\partial y} + \frac{\partial^2\phi}{\partial y^2} = -\frac{D}{t}\frac{\pi^2}{S^2}(1+\psi^2)^2 \qquad (5)$$

Considering now the conditions at buckling; $\sigma_x = \frac{\partial^2\phi}{\partial y^2} = 0$, $\sigma_y = \frac{\partial^2\phi}{\partial x^2} = 0$ and $\tau_{xy} = -\frac{\partial^2\phi}{\partial x\partial y} = \tau_b$.

Hence from (5), $$\tau_b = \frac{\pi^2 D}{2tS^2\psi}(1+\psi^2)^2 \qquad (6)$$

Taking $\alpha = 45°$ and $S = b/2$, representing approximately the conditions of buckle angle and wave length of a square panel, then equation (6) gives $\tau_b = 7.25\,E\,(t/b)^2$ which compares well with the expression $\tau_b = 8.32\,E\,(t/b')^2$ obtained by Stein and Neff, Ref. 11.

Substituting $2\psi\tau_b$ for $\frac{D}{t}\frac{\pi^2}{S^2}(1+\psi^2)^2$ from (6) in (5) :-

$$\psi^2 \frac{\partial^2\phi}{\partial x^2} + 2\psi \frac{\partial^2\phi}{\partial x \partial y} + \frac{\partial^2\phi}{\partial y^2} = -2\psi\tau_b \tag{7}$$

This equation is satisfied by :-

$$\phi = \frac{Ax^2}{2} + \frac{By^2}{2} + Cxy$$ where A, B and C are constants.

Substituting back in (7)

$$\psi^2 A + 2\psi C + B = -2\psi\tau_b \tag{8}$$

Now $\tau_{xy} = \tau = -\frac{\partial^2\phi}{\partial x \partial y}$, $\therefore\ C = -\tau$

$\therefore\ \psi^2 A + B = 2\psi(\tau - \tau_b)$, hence

$$A = \frac{1}{\psi}(\tau - \tau_b) \quad ; \quad B = \psi(\tau - \tau_b)$$

Then :-

$$\left.\begin{aligned} \sigma_x &= \frac{\partial^2\phi}{\partial y^2} = \psi(\tau - \tau_b) \\ \sigma_y &= \frac{\partial^2\phi}{\partial x^2} = \frac{1}{\psi}(\tau - \tau_b) \\ \tau_{xy} &= -\frac{\partial^2\phi}{\partial x \partial y} = \tau \end{aligned}\right\} \tag{9}$$

ψ may be taken as 1.0 for flat panels ($\alpha \simeq 45°$), and the above stresses, σ_x and σ_y, may be assumed to be working effectively over some proportion C_s of the actual panel. It may also be assumed that 40t width of panel is working with the stiffeners in reacting the panel load.

Then load to stiffener Sa. // x axis is given by:-

$$P_{Sa} = C_s\, b' t\, (\tau - \tau_b) \tag{10}$$

Using $A_{sa\,eff}$ to denote the effective area of the stiffener, the average stiffener stress is given by:-

$$\sigma_{sa_{AV.}} = C_s(\tau - \tau_b) \Big/ \left(\frac{A_{sa\,eff}}{b't} + 40\frac{t}{b'}\right) \tag{11}$$

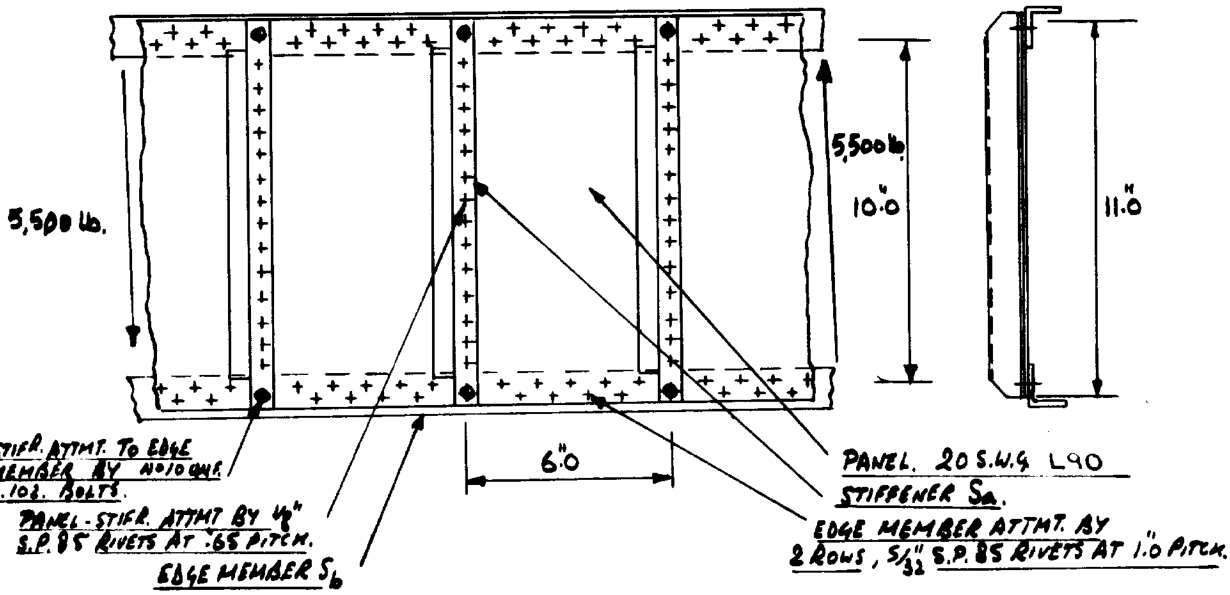
Similarly, the average stress in stiffener S_b, // y axis, $\sigma_{sb_{AV}}$ is:-

$$\sigma_{sb_{AV.}} = C_s(\tau - \tau_b) \Big/ \left(\frac{A_{sb\,eff}}{a't} + 40\frac{t}{a'}\right) \tag{12}$$

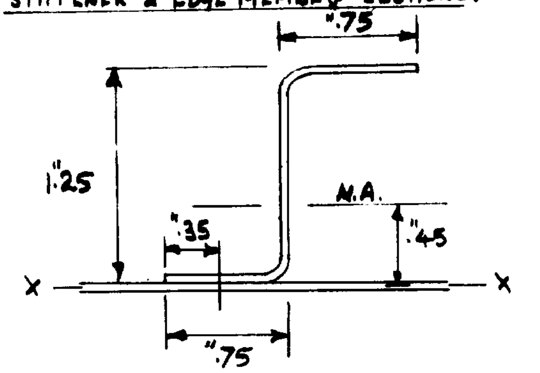
These expressions were used to obtain the value of C_s given in Fig 7. of this report. C_s was found to vary with h/t only.

NUMERICAL EXAMPLE OF USE OF S.O.R. No. 51 PART 1 FOR SPAR WEB AND STIFFENER ANALYSIS.

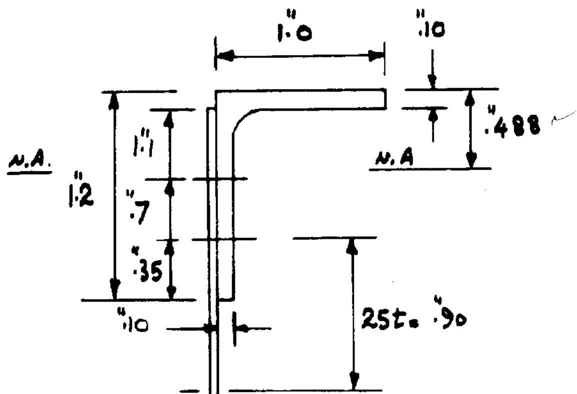
SPAR WEB GEOMETRY.



STIFFENER & EDGE MEMBER SECTIONS.



STIFFENER S_a. 18S.W.G. L90



EDGE MEMBERS, S_b, 2L65 EXTRUSION.

Ref. PART 1, SECT. 3.3.

$A_{sa} = \cdot 1223\ in^2$; $st = 40 \times \cdot 036^2 = \cdot 0518\ in^2$

$b't/2 = 3 \times \cdot 036 = \cdot 108\ in^2$.

$(Ay)_{sa} = \cdot 0782\ in^3$. ; $(I_x)_{sa} = \cdot 0798\ in^4$.

$\bar{y}$ at $.450"$ above x-x

$(I_{sa}) = \cdot 0446\ in^4$, $I'_{sa} = \cdot 0532\ in^4$.

$A_{sa}' = \cdot 0457\ in^2$, $i_{sa} = \cdot 506$

Ref. PART 1, SECT. 4.

$A_{sb} = \cdot 210\ in^2$, $St = 25 \times \cdot 036^2 + \cdot 75 \times \cdot 036 = \cdot 0593\ in^2$

$I_c = I_T = \cdot 0583\ in^4$ About N. Axis

$\omega d = \cdot 7 \times 6 \left[\dfrac{\cdot 036}{11 \times 2 \times \cdot 0583}\right]^{1/4} = 1.71$. (Fig. 4a).

$Z_{Sb} = \dfrac{\cdot 0583}{1 \cdot 262} = \cdot 0462\ in^3$

PANEL ANALYSIS. PART 1, SECTION 2.

ITEM	VALUE	REF.
t in.	.036"	Fig. 1b.
a = a' in.	10.0"	
b = b' = d in.	6.0"	
t_{sa}' in.	.048"	
t_{sb}' in.	.100"	
t_{sa}'/t	1.33	
t_{sb}'/t	2.76	
a_e/a	.79	
b_e/b	1.0	
a_e in.	7.9	
b_e in.	6.0	

PANEL BUCKLING STRESS. SECT. 2.1.

ITEM	VALUE	REF.
h in.	11.0"	Fig. 1c.
I'_{sa}/dt^3	190	
b_e/a_e	.76	
b_e'/b_e	1.0	
b_e' in.	6.0"	
$a_e (= a_e)$	7.9	
b_e'/a_e'	.76	Fig. 2.
K	7.0	
T_{bo} lb/in²	2,520	$KE(t/b_e')^2$
T_b lb/in²	2,520	Fig. 3b.

PANEL STRESSES AT FAILURE & PERMANENT DEFORMATION. PART I, SECT. 2.2.

ITEM	VALUE	REF.
A_{sb}/dt in.	·53	Fig. 3b.
T_{all} lb/in²	27,800	
T_{pb}' "	15,300	
T_{all}/T_b	11.0	Fig. 4a.
ωd	1.71	
T_{FAIL}/T_{all}	·98	
T_{FAIL} lb/in²	27,200	
T_{pb}'/T_b	6·10	Fig. 4a.
T_{pb}/T_{pb}'	·98	
T_{pb} lb/in²	15,000	
$1.5 \times T_{pb}$ lb/in²	22,500	

MAXIMUM PANEL ALLOWABLE SHEAR STRESS
= 22,500 lb/in²
CRITERION IS PERMANENT BUCKLING AT PROOF.

PANEL ATTACHMENT TO EDGE MEMBERS. SECT. 2.3.1. & Fig. 4b.

ATTACHMENT BY 5/32" DIA. S.P. 85 RIVETS AT 1·0 PITCH EACH ROW.
ALLOWABLE LOAD / RIVET = 460 lb. ∴ r = 920 lb/in.
$r/t = \frac{920}{\cdot 036} = 25{,}500$ lb/in². FROM Fig. 4b, T_{all} = 21,500 lb/in².

STIFFENER LOADS AND STRESSES. PART I, SECT 3.4.

ALLOWABLE STRESSES:-

a) LOCAL INSTABILITY AND PANEL INTER-RIVET BUCKLING. 3.4.a Figs 8, 10, & 11.
FLANGE + PANEL. $d' = \cdot 726$, $t' = \cdot 084$, $d'/t' = 8.67 = \xi$
FLANGE ONLY $d' = \cdot 35$, $t' = \cdot 048$, $d'/t' = 7.30 = \xi$
THE FORMER IS CRITICAL AND FROM TABLE 10. $K_c = .58$, AND FROM Fig 11, $\sigma_{SA_{ALL}} = 43{,}000$ lb/in².
INTER-RIVET BUCKLING OF PANEL. $p = \cdot 65$", $t = \cdot 036$", $p/t = 18 = \xi$
FROM TABLE 10, $K_c = 2.47$ FOR MUSH. HD. RIVETS, AND FROM Fig 11, $\sigma_{SA_{ALL}} = 41{,}000$ lb/in².

AS THE INTER-RIVET BUCKLING STRESS IS LOWER THAN THE MINIMUM LOCAL INSTABILITY STRESS, EXPRESSION (3.8) MUST BE USED TO OBTAIN THE MEAN ALLOWABLE MAX. STRESS.

$\frac{A_{sa\,eff}}{b't} = \frac{A_{sa}'}{b't} = \frac{\cdot 0457}{6 \times \cdot 036} = \cdot 211$; $S/b' = \frac{40 \times \cdot 036}{6} = \cdot 240$.

FROM (3.8) EFF. $\sigma_{SA\,MAX.} = \frac{\cdot 211 \times 43{,}000 + \cdot 240 \times 41{,}000}{\cdot 451} = 42{,}000$ lb/in².

FROM Fig. 8 AT $h/b' = 11/6 = 1.83$ AND $\sigma_{S\,MAX} = 42{,}000$ lb/in², $\sigma_{SAV.}$ = 29,000 lb/in².

σ_{SAV} BASED ON FLANGE STRESS AT SKIN OF 29,000 lb/in² IS CRITICAL.

b). FLEXURAL INSTABILITY. 3.4(b), Figs. 9, 10, 11.
FROM Fig. 9 $l_e/h = \cdot 5$; $l_e = 5.5$. $i_{sa} = \cdot 506$. $l_e/i_{sa} = 10.9 = \xi$
FROM TABLE 10 $K_c = 9.86$, AND FROM Fig 11, $\sigma_B = \sigma_{SAV} = 51{,}500$ lb/in².

THE MAXIMUM ALLOWABLE AVERAGE STRESS IS 29,000 lb/in², FLANGE LOCAL INSTABILITY AT SKIN, USING NOW Fig. 7 TO OBTAIN THE PANEL SHEAR STRESS AT WHICH THIS AVERAGE STRESS OCCURS:-

$C_S = \cdot 290$ AS $h/t = 11/\cdot 036 = 306$. $\frac{A_{sa\,eff}}{b't} + \frac{S}{b'} = \cdot 451$ (ABOVE)

THEN, FROM EXPRESSION 3.1.

$$\sigma_{SAV} = 29{,}000 = \frac{C_S(T - T_b)}{\frac{A_{sa\,eff}}{b't} + S/b'} = \cdot 642(T - T_b)$$

$T_b = 2{,}520$ lb/in² ∴ $T_{FAIL\,L.I.} = \frac{29{,}000}{\cdot 642} + 2{,}520 = 47{,}600$ lb/in².

c) FORCED CRIPPLING. 3.4.(c), Fig 12

$\frac{A_{sa}^*}{b't} = \frac{2 \times .75 \times .048}{6 \times .036} = .334$; $S/_b = \frac{40 \times .036}{6} = .24$ ∴ $\frac{A_{sa}^*}{b't} + \frac{S}{b} = .574$. $\frac{b't_{sa}}{w_{sa}t} = \frac{6 \times .048}{.75 \times .036} = 10.62$.

FROM Fig. 12 $\frac{\tau_{FAIL\ F.C.}}{\sigma_p} \times \frac{1}{\frac{A_{sa}^*}{b't} + S/_b} = .685$. $\sigma_p = 51,500$ lb/in^2 ∴ $\tau_{FAIL\ F.C.} = .685 \times .574 \times 51,500$
$= 20,200$ lb/in^2

d). TORSIONAL INSTABILITY. 3.4.(d) AND EXPRESSION (3.10).

$J = \frac{A_{sa} t_{sa}'^2}{3} = \frac{.1223 \times .048^2}{3} = .94 \times 10^{-4}$ in^4 ; $I_p = (I_x)_{sa} + (I_y)_{sa} = .1095$ WHERE $(I_y)_{sa}$ IS ABOUT THE RIVET LINE.

$\Gamma = .0032$ FROM R.Ae.S. 00.07.02. ($\Gamma / \frac{t' h^3}{12} = .26$). FROM (3.10), $\sigma_{AV_{all}} = \frac{GJ}{I_p} + \frac{\pi^2 E_t \Gamma}{I_p h^2}$

$\frac{GJ}{I_p} = 3350$ lb/in^2 AND $\frac{\pi^2 E_t \Gamma}{I_p h^2}$ IS EVALUATED BY USE OF Fig. 11

$K_c = 9.86$, $\zeta = h\sqrt{\frac{I_p}{\Gamma}} = 64$, HENCE FROM Fig. 11 ∴ $\frac{\pi^2 E_t \Gamma}{I_p h^2} = 24,500$ lb/in^2.

∴ $\sigma_{AV_{all}} = 3350 + 24,500 = 27,850$ lb/in^2.

ATTACHMENT OF STIFFENER TO EDGE MEMBER. 3.5 AND EXPRESSION (3.11).

ATTACHMENT BY No 10 U.N.F. A 102 BOLT BEARING IN 18 S.W.G. L90. STIFFR AND .102 L 65 BOOM. ALLOWABLE LOAD = 91 × 10.3 = 936 lb.

FROM (3.11) $\sigma_{sa\ AV_{all}} = P_{sa}/A_{sa\ eff} = \frac{936}{.0457} = 20,500$ lb/in^2.

FROM (3.7). $\tau_{FAIL\ ATT.} = \frac{20,500}{.642} + 2,520 = 34,600$ lb/in^2.

ATTACHMENT OF PANEL TO STIFFENER. 3.6.

REQUD. TENSILE STRENGTH OF RIVETS /in. RUN = .075t σ_{ult}.

$\sigma_{ult} = 60,500$ lb/in^2 (WEB U.T.S.); t = .036"; RIVETS AT .65" PITCH.

HENCE REQUD. TENSILE STRENGTH /RIVET = .075 × .036 × 60,500 × .65 = 106 lb.
ACTUAL STRENGTH OF 1/8" S.P. 85 RIVET = 320/2 = 160 lb.. ATTACHMENT SATISFACTORY.

EDGE MEMBER STRESSES AND STRENGTH. PART 1, 4.2.

FROM EXP. 4.1. $\sigma_{TOTAL} = \sigma_{APP} + \tau_{APP}\left[\alpha + \frac{\beta t d^2}{Z_{sb\ min.}}\right]$

THEN DUE TO τ_{APP} ONLY , $\sigma_T/\tau_{APP} = \left[\alpha + \frac{\beta t d^2}{Z_{sb\ min.}}\right]$

FROM EXP 4.2. $\frac{\tau_{FAIL}}{\sigma_{ALL}} = \frac{1}{\frac{\sigma_{APP}}{\tau_{APP}} + \left[\alpha + \frac{\beta t d^2}{Z_{sb\ min}}\right]} = \frac{1}{\frac{\sigma_{APP}}{\tau_{APP}} + \frac{\sigma_T}{\tau_{APP}}}$

α IS OBTAINED FROM Fig. 14a, β FROM Fig. 14b.

$\tau_b = 2,520$ lb/in^2 ; $A_{sb}/d't = \frac{0.2693}{10 \times .036} = .75$, $\frac{td^2}{Z_{sb\ min.}} = \frac{.036 \times 6^2}{.0462} = 28.0$

$\tau_{APP.} = 13,850$ lb/in^2. $\sigma_{APP.} = 30,000$ lb/in^2 (COMP) AND $\sigma_{ALL} = 52,000$ lb/in^2 (SAY)

THEN $\tau_{FAIL} = \frac{52,000}{2.17 + \sigma_T/\tau_{APP.}}$

① $\tau_{APP.}$	② $\tau_{APP.}/\tau_b$	③ α Fig. 14a	④ β Fig. 14b.	⑤ 28 β	⑥ $\sigma_T/\tau_{APP.}$ ③+⑤	⑦ 2.17+⑥	⑧ $\tau_{FAIL} = \frac{52,000}{⑦}$
10,000	3.96	.15	.029	.812	.962	3.132	16,600
15,000	5.95	.20	.033	.925	1.125	3.295	15,800
20,000	7.95	.24	.035	.953	1.193	3.363	15,500
15,700	6.2	.21	.033	.924	1.134	3.304	15,700

HENCE $\tau_{FAIL} = 15,700$ lb/in^2 AND $\sigma_{FAIL} = 30,000 \times \frac{15,700}{13,850} = 34,000$ lb/in^2.

PANEL FAILURE DUE TO COMBINED STRESS AT RIVET LINE. PART 1, Fig. 5

$\tau_{APP.} = 13{,}850$ lb/in², $\tau_{FAIL} = 27{,}200$ lb/in²

$\sigma_{APP.} = 29{,}000$ lb/in², $\sigma_{ALL.} = 60{,}500$ lb/in²

$$\frac{\tau_{FAIL}}{\sigma_{ALL.}} \times \frac{\sigma_{APP.}}{\tau_{APP.}} = \cdot 94$$

FROM Fig. 5. $\tau_{FAIL}'/\tau_{FAIL} = \cdot 73$. $\therefore \tau_{FAIL}' =$ 19,800 lb/in².

SUMMARY OF ALLOWABLE PANEL SHEAR STRESSES AND RESERVE FACTORS.

NOTE :— ALL RESERVE FACTORS ARE TRUE I.E. $\dfrac{\text{ALLOWABLE LOAD OR STRESS}}{\text{APPLIED LOAD OR STRESS}}$.

APPLIED SHEAR STRESS = 13,850 lb/in². $\left(\dfrac{5{,}500}{11 \times \cdot 036}\right)$.

APPLIED COMPRESSION STRESS IN EDGE MEMBER = 30,000 lb/in². (TOP MEMBER)
(GIVING 29,000 lb/in² AT LOWER RIVET LINE).

CRITERION.		τ_{all} lb/in²	R.F.	PAGE
PANEL	FAILURE, ZERO EDGE MEMBER STRESS	27,200	1.97	2
	FAILURE, 30,000 lb/in² E.M. STRESS	19,800	1.43	4.
	PERMANENT BUCKLING AT PROOF LOAD	22,500	1.62	2
ATTACHMENT OF PANEL TO EDGE MEMBER, RIVET SHEAR		21,500	1.55	2
STIFFENER	LOCAL INSTABILITY. (FLANGE CRIT.)	47,600	O.K.	2
	FLEXURAL INSTABILITY	NOT CRIT.	O.K.	2
	FORCED CRIPPLING	20,200	1.46	3
	TORSIONAL INSTABILITY	27,850	2.02	3
	ATTMT. TO EDGE MEMBERS. BOLT BEARING	34,600	O.K.	3
ATTACHMENT OF PANEL TO STIFFENER.		SATISFACTORY		3
EDGE MEMBER UNDER COMBINED LOADS.		33,800 *	1.13	3

* $\sigma_{ALL} = \sigma_{FAIL}$ QUOTED, NOT τ.

November, 1965

Iss 2. JAN 1968

STRESS OFFICE REPORT No.51

THE ANALYSIS OF SHEAR PANELS

PART 2 THE ANALYSIS OF CURVED SHEAR PANELS

CONTENTS

Item	Section		Figure	
	Section No.	Page	Fig. No.	Page
Introduction	2.1	1		
Panel Analysis	2.2	1	2.1.a, 2.1.b	16, 17
Panel buckling, Introduction	2.2.1	1		
Buckling under pure shear	2.2.2	1, 2	2.2.a	18
Buckling under shear and internal pressure	2.2.3	2, 3	2.2.b, 2.2.d	19
Buckling under axial stress	2.2.4	3	2.2.e	20
Buckling under axial stress and internal pressure	2.2.5	3	2.2.f	21
Buckling under combined shear, axial stress and internal pressure	2.2.6	4	2.2.g	21
Conversion from elastic to true buckling stress	2.2.7	4	2.2.h	22
Post buckling behaviour	2.3	4, 5	2.3.a, 2.3.b 2.3.c, 2.3.d	23 24
Panel failure	2.3.1	6, 7	2.3.e	25
Permanent buckling	2.3.2	7, 8	"	"
Panel joints	2.3.3	8		
Stiffener analysis	2.4	8, 9		
Stringer stress	2.4.1	8, 9		
Frame stress	2.4.2	10		
Stringer failure	2.4.3	10, 12		
Frame failure	2.4.4	12, 13		
Panel shear stiffness and shear strain	2.5	13, 14		
APPENDIX				
Comparative buckling stresses, shear only			2.A.1	26
Comparative buckle angles, shear only			2.A.2	26
Comparative buckling stresses, combined loads			2.A.3	27
Comparative buckle angles, combined loads			2.A.4	27
Comparative failing stresses			2.A.5	28
Comparative permanent buckling stresses			2.A.6	28
Comparative average stringer stresses, shear only			2.A.7	29
Comparative average stringer stresses, combined loads			2.A.8	30
Comparative angle of twist			2.A.9	31

STRESS OFFICE REPORT NO.51

THE ANALYSIS OF SHEAR PANELS

PART 2: CURVED SHEAR PANELS

2.1 Introduction

This report has been written with the following objectives:-

a) To simplify existing methods of analysis.

b) To achieve better agreement between theory and test results.

c) To devise a method whereby true reserve factors for panel, stringers and frames may be obtained and the effects of internal pressure and axial compression may be determined.

This method of analysis is partly original and partly based on existing published methods. Reference is made to existing methods and theoretical and test results where applicable.

2.2 Panel Analysis

2.2.1 Introduction on panel buckling

The effective simply supported panel size concept, presented in section 2.1 of part 1 of this report, has been found to be applicable to curved panels in shear and axial compression. This has been used in conjunction with methods presented by Timoshenko, Ref. 13, for compression and Melcron and Ensrud, Ref.18 for shear, whereby the average compression and shear buckling stresses are given as the addition of the flat panel buckling stress and the buckling stress for cylinders without stringers of equal length to the panel under consideration. These methods have been developed for the estimation of buckling stresses for curved panels under internal pressure with and without longitudinal tension. The expressions for buckling stress, given in this report, are for the average buckling stress under fixed stress rather than fixed deflection conditions. (Described in Ref. 5, p.p. 207 - 214).

Comparative calculated and test buckling stresses are shown in Figs. 2.A.1 of the Appendix to Part 2 of the report.

2.2.2 Buckling stress under pure shear

The effective panel size is obtained from Fig. 2.1.a. The panel width will be modified if stringers are of insufficient stiffness using Fig. 2.1.b, which is also used to check that the frame stiffness is satisfactory. Fig.2.1.b is derived from Refs. 2, 8, 26 and 27.

The average elastic shear buckling stress is obtained from Fig.2.2.a and is given by the expression:-

$$\tau_{b_o} = \tau_{b_o}(1) + \tau_{b_o}(2). \tag{2.1}$$

Where $\tau_{b_o}(1) = \dfrac{.785\,E\,(t/R)}{\left(a_e^{2}/Rt\right)^{1/4}}$; $\tau_{b_o}(2) = 4.84\,E\left(t/b_e'\right)^{2}.$ (2.2)

The expression for $\tau_{b_o}(1)$ in (2.2) is the expression given by Melcron and Ensrud, Ref.18. This represents the incremental buckling stress due to panel curvature and length, and is similar to expressions derived by Donnell, Ref.24, Batdorf, Ref.38 and Pogorelov, Ref.30.

The expression for $\tau_{b_o}(2)$ in (2.2) is the buckling stress for a flat panel of width b_e' and infinite length, derived by Southwell. (See Ref.13, p.360).

Comparative test and calculated buckling stresses for various panels are compared in fig.2.A.1 of the Appendix.

2.2.3 Buckling under combined shear and internal pressure

A theoretical expression derived from the solution of the equations obtained by Hopkins and Brown, Ref.39, has been found to give good agreement with the test results for unstiffened and stiffened cylinders from Ref.37 and unstiffened cylinders from Ref.31. This expression gives closer agreement with test results than the empirical interaction expression presented by Crate, Batdorf and Baab, Ref. 31 and has the advantage of being usable for stiffened panels.

The average elastic buckling stress under combined shear and internal pressure is given by:-

$$\tau_{b_o}(p) = \tau_{b_o}(1)_{(p)} + \tau_{b_o}(2) + \tau_{b_o}(3)_{(p)} + \sigma_x \tag{2.3}$$

Where $\tau_{b_o}(p)$ is the shear buckling stress with internal pressure p lb/in^2

$\tau_{b_o}(1)_{(p)}$ is a reduced cylindrical buckling stress given by:-

$$\tau_{b_o}(1)_{(p)} = \frac{\tau_{b_o}(1)}{\left(1 + \frac{\sigma_\theta}{E}\frac{R}{t}\left(\frac{a_e^{2}}{Rt}\right)^{1/2}\right)}$$ and may be obtained from Fig.2.2.b

$\tau_{b_o}(1)$ and $\tau_{b_o}(2)$ are obtained from (2.2.) above or Fig.2.2.a

$$\tau_{b_o}(3)_{(p)} = .76\,F'\left(\sigma_\theta E \frac{t}{R}\right)^{1/2}$$ where $F' = 1.0$, $R/t \ngtr 400$

$F' = \dfrac{1}{\left(1 + .0082\,(R/t - 400)^{1/2}\right)}$ $R/t > 400.$

and is obtained from Fig.2.2.c and Fig.2.2.d

45

σ_θ is the average hoop stress and σ_x is the average longitudinal stress in the skin (See Fig. 2.2.b)

E is the modulus of elasticity for the skin

Test and calculated buckling stresses for pressurised cylinders in torsion are compared on Fig.2.A.3 of the Appendix.

2.2.4 Buckling under axial stress

The average elastic axial stress at buckling is obtained from Fig.2.2.e and is given by the expression:-

$$\sigma_{xb_0} = \sigma_{xb_0}(1) + \sigma_{xb_0}(2) \qquad (2.5)$$

Where $\sigma_{xb_0}(1) = \dfrac{\cdot 55\,E(t/R)}{\left(1 + \cdot 001\frac{R}{t}\right)}$; $\sigma_{xb_0}(2) = 3.62\,E\left(\dfrac{t}{b_e'}\right)^2$. (2.6)

The expression for $\sigma_{xb_0}(1)$ in (2.6) gives a close approximation to the mean of the test results from bending of cylinders given by Suer, Harris and Skene in Ref.4.1, and the expression for $\sigma_{xb_0}(2)$ is the classical buckling stress for a long flat plate of width b_e'. The effective width b_e' to be used with this expression is the same value as obtained from Fig.2.1.a for the shear buckling stress. Considerable scatter is present on test results on unstiffened cylinders but it has been found that with stiffened cylinders expression (2.5) gives reasonably good agreement to test results.

2.2.5 Buckling under internal pressure and axial stress

The buckling stress under internal pressure and axial stress is given by the expression:-

$$\sigma_{xb_0}(P) = \sigma_{xb_0} + \sigma_{xb_0}(3)_p + \sigma_x \qquad (2.7)$$

Where $\sigma_{xb_0}(P)$ is the axial buckling stress with internal pressure.

σ_{xb_0} is obtained from (2.5) and σ_θ and σ_x are the panel hoop and longitudinal stresses due to internal pressure p.

The second term in expression (2.7) $\sigma_{xb_0}(3)_p$ is the average incremental stresses due to hoop tension and pressure and was obtained from the test results in Ref.4.1, from which the expression $\sigma_{xb_0}(3)_p = \cdot 45\,\sigma_\theta/(\text{[illegible]}\,R/t)^{3/4}$ was derived. This may be obtained from Fig.2.2.f.

46

2.2.6 Buckling stress under combined shear and axial stress including the effect of internal pressure

The interaction expression derived from tests on cylinders by Bridget, Jerome and Vosseler (See Ref.13, p.p.489 - 490) for buckling under combined shear and axial stress, and substantiated by Peterson, Ref.16, has been found to be applicable to pressurised cylinders by Harris, Sner and Shene, Ref.40. This expression is considered applicable to stiffened panels and is given by:-

$$\frac{\sigma_{x\,APP.}}{\sigma_{x\,b_o}(P)} + \left(\frac{\tau_{APP.}}{\tau_{b_o}(P)}\right)^2 = 1 \qquad (2.8)$$

Where $\sigma_{x\,APP.}$ and $\tau_{APP.}$ are the applied axial stress and shear stress respectively, $\sigma_{x\,b_o}(P)$ and $\tau_{b_o}(P)$ are the elastic axial and shear buckling stresses, with appropriate modification if the panel is pressurised. (Sections 2.2.2. to 2.2.5).

The elastic shear buckling stress for non-pressurised panels under combined shear and axial stress, $\tau_{b_o}(c)$ is obtained directly from Fig.2.2.g.

For pressurised panels expression (2.8) must be used to obtain the shear buckling stress under combined shear and axial stresses using a method of successive approximations.

2.2.7 Conversion from elastic to true buckling stress

The elastic buckling stress τ_{b_o} is plotted against the true buckling stress τ_b for various materials on Fig.2.2.h. This is equally applicable for conversion of $\tau_{b_o}(P)$ to $\tau_b(P)$ and $\tau_{b_o}(c)$ to $\tau_b(c)$.

For other materials, curves may be obtained by use of the tensile stress-strain curve, where the strain is multiplied by $E/\sqrt{3}$ to become the elastic buckling stress and this is plotted against the tensile stress divided by $\sqrt{3}$ which becomes the true buckling shear stress (see Ref.5, p.91).

2.3 Post buckling panel behaviour

Visual observations, Ref.15 and photographs, Ref.10 28 and 34 of tests on stiffened curved shear panels and cylinders, have shown that the number and angle of buckles are dependant on panel geometry and for the majority of panels remain unchanged between initial buckling and failure. The initial buckling pattern is dependent on geometry only (Refs. 12, 13, 24 & 31) provided frames and stringers are of adequate stiffness, (Fig.2.1.b), and it seems that curvature has the effect of holding this pattern in the post buckling range.

2.3 (cont'd.)

For known panel geometry, the number of buckles is obtained from Fig.2.3.a, which is derived from test data. The angle of buckle is then obtained by further geometrical considerations. For an axial buckle pattern simple geometrical expressions have been evolved to obtain the buckle angle, and a curve, Fig.2.3.b is provided to obtain the angle for circumferential buckle patterns.

Comparative test and calculated buckle angles are given on Fig.2.A.2 of the Appendix.

This buckle angle is used to predict stringer stresses, frame stresses and panel failures and also overall panel deflections and is taken as fixed throughout the initial buckling to failure range.

Axial compression and hoop tension stresses in the skin increase the buckle angle, whereas axial tension reduces the buckle angle. Little test information is available except for Ref.37, from which test buckle angles under combined shear and internal pressure are summarised.

The curves obtained for low hoop tension stresses from theoretical calculations by Hopkins and Brown for the buckle angle for combined shear and hoop tension, Ref.39, are closely given by the following approximate expression, which has been extended to give reasonably good agreement with test results from Ref.37 which are outside the range of the curves in Ref.39.

This expression is:-

$$\operatorname{Tan}\alpha_{(p)} = \operatorname{Tan}\alpha\left(3.6 + .038\Psi - \frac{2.6}{(1+.008\Psi^2)}\right) \qquad (2.9)$$

Where $\alpha_{(p)}$ is the buckle angle under combined shear and internal pressure, α is the buckle angle under shear alone.

$$\Psi = \frac{(\sigma_\theta - \sigma_x)}{E}\,\frac{R}{t}\left(\frac{a_e^2}{Rt}\right)^{1/2}$$

Where σ_θ is the hoop tensile stress and σ_x is the longitudinal tensile stress.

Comparative test and calculated buckle angles are given on Fig.2.A.4 of the Appendix, and except for specimen 95, Ref.37 are in good agreement.

As there is lack of information on buckle angle under combined compression and shear, it is considered that expression (2.9) be used, replacing $\alpha_{(p)}$ by $\alpha_{(c)}$ and using $\Psi = \frac{\sigma_{x\,APP}}{E}\,\frac{R}{t}\left(\frac{a_e^2}{Rt}\right)^{1/2}$ where $\sigma_{x\,APP.}$ is the applied axial stress in the panel.

For panels with an axial buckle pattern (Fig.2.3.a) as will occur in most stiffened panels, modification should be made by taking the closest angle to the angle obtained by (2.9) using expressions below Fig.2.3.a.

(48)

2.3.1 Panel Failure

Nine panels, Refs. 9, 10 and 15, failed by tensile failure of the panel, and these are predicted with reasonable accuracy using Von Misès criterion for failure. (Ref. 2, Sheet 02.00.00 and Timoshenko, Theory of Elasticity, p.149), taking the longitudinal stress and the hoop stress in the panel respectively as $2(\tau-\tau_b)\operatorname{Cot}\alpha$ and $2(\tau-\tau_b)\operatorname{Tan}\alpha$. These stresses are twice the average stresses in the panel centre (See S.O.R.No.51, Part 1, Section 3.), which is reasonable as the stress along the buckle crests in a curved panel are considerably higher than the stress along the buckle troughs, which may be zero or even compressive.

Von Misès criterion is written:-

$$\sigma_x^2 + \sigma_\theta^2 - \sigma_x \sigma_\theta + 3\tau_{x\theta}^2 = \sigma_{ULT.}^2$$

Where $\sigma_x = 2(\tau-\tau_b)\operatorname{Cot}\alpha$; $\sigma_\theta = 2(\tau-\tau_b)\operatorname{Tan}\alpha$; $\tau_{x\theta} = \tau$.

Substituting for σ_x, σ_θ, $\tau_{x\theta}$ and substituting C_α

for $\frac{1}{4}\left(\frac{1}{\operatorname{Cot}^2 2\alpha + 7/16}\right)^{1/2}$

the expression for τ_{FAIL}, the failing shear stress, becomes:-

$$\tau_{FAIL} = \tau_b\left(1-3C_\alpha^2\right) + C_\alpha\sigma_{ULT.}\left(1-3\left(\frac{\tau_b}{\sigma_{ULT.}}\right)^2\left(1-3C_\alpha^2\right)\right)^{1/2} \qquad (2.10).$$

This expression gives predicted failing shear stresses within ± 10% of test results, but is complex, and it was found that simplified expressions for average and minimum expected failing shear stresses could be achieved with little or no loss of accuracy.

These simplified expressions are:-

$$\tau_{FAIL.AV.} = \tau_b + \left(.39\sigma_{ULT.} - .676\,\tau_b\right)\operatorname{Sin}2\alpha. \qquad (2.11)$$

For average failing stress $\tau_{FAIL.AV.}$. This expression gives stresses within ± 8% of test results and comparisons between test and estimated stresses are made on Fig.2.A.5 of the Appendix.

To cover all test results, the simplified expression is:-

$$\tau_{FAIL.} = \tau_b + \left(.36\sigma_{ULT.} - .614\,\tau_b\right)\operatorname{Sin}2\alpha. \qquad (2.12)$$

Where $\tau_{FAIL.}$ is the minimum expected failing stress. Expression (2.12) should be used for general stressing.

Expressions for failure under combined stresses using Von Misis criterion are very complex and investigations have been made which show that the general interaction expression given on Fig.5, S.O.R. No.51, Part 1, is applicable with reasonable accuracy at both the unbuckled condition and the condition where $\tau_b/\tau \rightarrow 0$.

This expression is written:-

$$\left(\frac{\tau_{FAIL}'}{\tau_{FAIL}}\right)^2 + \left(\frac{(\sigma_x - \sigma_\theta)^2 + \sigma_x \sigma_\theta}{\sigma_{ULT}^2}\right) = 1 \qquad (2.13)$$

Where τ_{FAIL}' is the shear stress at failure under combined shear and direct stresses; τ_{FAIL} is the failing shear stress under shear alone, and should be estimated from (2.12) or Fig. 2.3.e using the buckle angle and buckling stress for the combined stress system (Sec. 2.3 and Figs. 2.3.c and 2.3.d) σ_x and σ_θ are the nett area tensile axial and circumferential applied stresses.

Fig.5, p.23, S.O.R. No.51, Part 1 may be used to obtain τ_{FAIL}' directly by substitution of $\left(\frac{(\sigma_x - \sigma_\theta)^2 + \sigma_x \sigma_\theta}{\sigma_{ULT}^2}\right)$ for $\left(\frac{\sigma_{FAIL}}{\sigma_{all.}}\right)^2$. {NOT REQUIRED IF σ_x AND σ_θ ARE CONSTANT AS (2.13) MAY BE USED DIRECTLY.}

2.3.2 Permanent Buckling

The results of 18 curved panel tests from Ref.15 show that permanent buckling may occur at stresses very much lower than predicted by use of Ref.2, sheet 02.03.03. High aspect ratio panels may buckle and permanently buckle coincidentally. No information other than the results from Ref.15 are available so it is suggested that Fig.2.3 which was derived from these tests and Ref.2 be used until further information becomes available.

Comparative test and estimated permanent buckling stresses using Fig.2.3.e are given on Fig.2.A.6 of the Appendix.

It is further suggested that expression (2.13), suitably modified, be used for panels under combined stresses.

This expression becomes:-

$$\left(\frac{\tau_{pb.}'}{\tau_{pb.}}\right)^2 + \left(\frac{(\sigma_x - \sigma_\theta)^2 + \sigma_x \sigma_\theta}{\sigma_2^2}\right) = 1 \qquad (2.14)$$

Where $\tau_{pb.}'$ is the stress at permanent buckling under combined shear and direct stresses. $\tau_{pb.}$ is the stress at permanent buckling under shear alone estimated from Fig.2.3.e but using the buckle angle and buckling stress for the combined stress system. σ_x and σ_θ are the gross area axial and hoop applied tensile stresses and σ_2 is the 2% proof stress for the panel material.

Fig.5, S.O.R. No.51, Part 1, p.23 may be used to obtain τ_{pb}', by substitution of $\left(\frac{\tau_{pb}'}{\tau_{pb}}\right)$ for $\left(\frac{\tau_{FAIL}'}{\tau_{FAIL}}\right)$

and $\left(\frac{(\sigma_x - \sigma_\theta)^2 + \sigma_x \sigma_\theta}{\sigma_e^{\,2}}\right)$ for $\left(\frac{\sigma_{FAIL}}{\sigma_{FAIL}}\right)^2$. {NOT REQUIRED IF σ_x AND σ_θ ARE CONSTANT AS (2.14) MAY BE USED DIRECTLY.}

2.3.3 Panel Joints

The resultant load per inch to joint attachments, r, is given by the expressions

$$r_c = t\left[\tau^2 + \left(\sigma_x + (\tau - \tau_{b(c)})\,\mathrm{Cosec}\,2\alpha_c\right)^2\right]^{1/2} \qquad (2.15)$$

for circumferential joints

$$r_A = t\left[\tau^2 + \left(\sigma_\theta + (\tau - \tau_{b(c)})\,\mathrm{Cosec}\,2\alpha_c\right)^2\right]^{1/2} \qquad (2.16)$$

for longitudinal joints

These expressions are extensions of expression (11), Ref.18, where η is replaced by $\mathrm{Cosec}\,2\alpha$, σ_x and σ_θ are the axial and hoop applied tensile stresses. Buckling stress $\tau_{b(c)}$ and buckle angle α_c are those applicable for the combined stress spectrum.

It should be noted that the buckling stress for a panel with a joint made by a butt strap between stiffeners or frames should be based on the full size of the panel as the strap is normally of insufficient thickness to change the overall buckle pattern and hence the buckling stress.

2.4. Stiffener Analysis

2.4.1 Axial Stiffener (Stringer) Stress

Various methods and expressions have been proposed for the estimation of stiffener stresses. The most complex (Refs. 1 and 2) currently in use, underestimate the measured stresses on the cylinders in Refs. 10 and 34, though reasonable agreement is obtained for the cylinders in Ref.9. The method devised by Melcron and Ensrud Ref.18 gives close agreement with all test results but was considered too complex for general use. This expression is partly empirical. It was found that satisfactory agreement with all test results from Refs. 9, 10, 16 and 34 could be obtained by modification of the simple expression (3) from Ref.34. This expression is similar to expressions (1.1) and (1.2) of S.O.R. No.51, Part 1, Sect 3, and is written:-

$$\sigma_{s\,AV.} = -\frac{b't}{A_s}\left[\tau - \left(\tau_{b(1)}\,\mathrm{Tan}\,\alpha + \tau_{b(2)}\right)\right] \qquad (2.17)$$

Where $\sigma_{S\,AV.}$ is the average compressive stress in the stiffener.

τ is the applied shear stress

$\tau_b(\ell), \tau_b(b)$, are the incremental buckling shear stresses due to panel length and radius, and panel width respectively (See Sect. 2.2.2)

α is the buckle angle, b' the plain panel width, t the panel thickness and A_s is the stiffener area. For closed section stringers an additional area of skin of $t(d-b')$ is added to the area of the stringer to form A_s.

Comparison between test and predicted stresses for 24 specimens are given on Fig.2.A.1 of the Appendix.

For combined axial and shear stresses, the average stiffener stress is given by the expression:-

$$\sigma_{S\,AV.} = \sigma_{ST\,AV.} + \sigma_{SA\,AV.} \tag{2.18}$$

Where $\sigma_{ST\,AV.}$ is the average stiffener stress due to shear, given by

$$\sigma_{ST\,AV.} = -\frac{b't}{A_s}\left[\tau - \tau_b(c)\right] \tag{2.19A}$$

and $\tau_b(c)$ is the buckling stress under the combined stress system.

$$\text{and} \quad \sigma_{SA\,AV.} = \frac{P_x}{A_s + \eta\, b't} \tag{2.19B}$$

Where $\sigma_{SA\,AV.}$ is the average stiffener stress due to P_x, the load applied to stiffener plus width $b't$ of skin.

η is the panel efficiency factor, using the Karhran-Sechler expression where $\eta = .89\,\frac{\sigma_{x_b}(c)}{\sigma_{x_b}}\sqrt{\frac{\sigma_{x_b}(c)}{\sigma_{SA\,AV.}}}$, and $\sigma_{x_b}(c)$ is the buckling axial compressive stress, under the combined stress system.

σ_{x_b} is the buckling stress under axial stress system only when P_x is compressive (negative) and η is 1.0 when P_x is tensile (positive).

Comparative test and calculated stresses using expression (2.18) are given on Fig.2.A.8. of the Appendix.

52

2.4.2 Frame (Ring) Stress

An empirical expression for effective frame stress at frame failure has been developed from the results of 21 tests from Ref.15. Further 'negative' checks have been made by applying the expression to 24 test cylinders, Refs. 9, 10, 34, to determine whether frame failure could be predicted (none of these cylinders had frame failures), and in all cases, the shear stress for predicted frame failure was greater than the actual failing stress.

This expression for frame stress is written:-

$$\sigma_{F_{AV.}} = -\cdot 5\,\frac{ht}{A_F}\left[\tau - \tau_b + \frac{b' A_F}{a\,A_F^*}\,\tau_b(2)\right] \tag{2.20}$$

Where $\sigma_{F_{AV.}}$ is the effective average frame stress,

τ is the applied shear stress in the panel.

$\tau_b(2)$ is the incremental buckling stress from panel width.

τ_b is the buckling stress.

A_F is the total frame area between stringers

A_F^* is an effective frame area given by twice the frame flange area.

b' is the plain panel width, h is the frame pitch, a is the axial length of the panel and t is the panel thickness. It should be noted that . expression (2.20) is applicable only when buckles are fully developed and gives an over-estimation of frame stress when used for shear stresses of same order as buckling stresses, $(\tau < 1.3\tau_b)$.

The term in $\tau_b(2)$ in expression (2.20) represents the frame stress due to the effect of the frame in dividing the panels. For panels where the frame is unattached to the skin or attached through stringer flanges only, this term is taken as zero (i.e. $a = \infty$).

2.4.3 Stiffener failure

The four modes of failure, local instability, flexural instability, forced crippling and torsional instability, described in S.O.R. No.51, Part 1, Sect. 3. Paras 3.4. to 3. 44. must be investigated for stiffener failure. For curved panels some modifications are however made. These are as follows:-

2.4.3.a. Local instability of stiffener flanges and inter-rivet buckling

Expression (1.7) of Appendix 1 is re-written, from (2.17) Section 2.4.1:-

$$\tau_{FAIL} = \sigma_{S_{AV\,all.}}\,\frac{A_S}{b't} + \tau_b(1)\,\tan\alpha + \tau_b(2) \tag{2.21}$$

A_s is taken as the total stringer area and is not reduced as for flat panels, and $\sigma_{S\,AV.all.}$ is taken as the minimum local instability stress for inner or outer flange or web. For open section stiffeners no local skin is included in A_s so it appears that inter-rivet buckling of the panel is unimportant, however rivets should be pitched such that the inter-rivet buckling stress for the panel is as high as the minimum stiffener instability stress. For closed section riveted stiffeners skin area ct is taken as working with the stringer area to give A_s . If the panel inter-rivet buckling stress is less than the local instability stress, the allowable stress $\sigma_{S\,AV.all.}$ is taken as:-

$$\sigma_{S\,AV_{all.}} = \frac{\sigma_{L.I.}\,(A_s - ct) + \sigma_{I.R.}\,ct}{A_s} \qquad (2.22)$$

Where $\sigma_{L.I.}$ is the minimum local instability stress and

$\sigma_{I.R.}$ is the inter-rivet buckling stress for the panel.

Panel instability under a closed section stiffener and at the intersection of perpendicular open section stiffeners as described in Pt.1 is applicable to curved panels.

2.4.3.b <u>Flexural Instability</u>

The effective column length of axial stiffeners (stringers) should be taken as half the frame pitch, The allowable stress $\sigma_{S\,AV.all.}$ to be used in expression (2.21.) is obtained from Table 10 and Fig.11 of Part 1..

2.4.3.c <u>Forced Crippling</u>

Section 3.4.c of Part 1, and Fig.12 of Part 1 is applicable with the following modifications to panel width s .

For riveted open section stiffeners, $s = 0$

For reduxed flanges $s = w$, $t'_s = t_{FLANGE} + t_{PANEL.}$

For lipped flanges, depth of lip $\not< 5t'_s$ multiply $T_{FAIL_{F.C.}}$ by 1.17

For closed section stiffeners $s = d - b'$ and multiply $T_{FAIL_{F.C.}}$ by 2.0

2.4.3.d <u>Torsional Instability</u>

Section 3.4.d of Part 1 is applicable for closed section stiffeners and open section cleated stiffeners. For open section stiffeners attached by one flange to panel and frame, expression (3.10) should be modified, becoming:-

$$\sigma_{S\,AV\,all.} = \frac{GJ}{I_P} + \frac{\pi^2 E_t \Gamma}{I_P h^2} \qquad (2.23)$$

(4)

The second term in this expression is obtained from Fig.1 1, using $K = 9.86$ and $\zeta = h\sqrt{\frac{I_p}{T}}$

The value of $\sigma_{S\,AV.\,all.}$ is now substituted in expression 2.21.to obtain $T_{FAIL.}$

2.4.3.e <u>Failure under action of combined shear and axial stress</u>

Expression (2.18) for average stiffener stress may be used to obtain T_{FAIL} by equating the average stiffener stress to the allowable stiffener stress. This process involves successive approximation or graphical methods and for general purposes the inter-action expression given in Ref.16 is considered acceptable.

This expression is written:-

$$\left(\frac{T}{T_{all.}}\right)^{3/2} + \left(\frac{\sigma_x}{\sigma_{x\,all.}}\right) = 1 \qquad (2.24)$$

Where T is the applied stress at failure. $T_{all.}$ is the shear stress under shear alone at which stiffener failure due local or torsional instability or forced crippling will occur. σ_x is the nominal applied compressive stress $P_x/(A_s + b't)$ and $\sigma_{x\,all.}$ is the nominal compression stress at failure due local or torsional instability $P_{x\,all.}/(A_s+b't)$. This expression is not applicable to failure in the flexural instability mode as the wave length used for flexural instability is $h/2$ for stresses from shear against h for direct compression.

Fig.13 of Part 1 to S.O.R. No.51 may be used to obtain $T_{all.}$ directly. This Fig. is based on equation (2.24)

2.4.4 <u>Frame Failure</u>

The shear stress at which frame failure occurs may usually be estimated from expression (2.20) which is written:-

$$T_{FAIL.} = 2\sigma_{F\,all.}\frac{A_F}{ht} + T_b - \frac{b' A_F}{a A_F^{*}} T_b(2). \qquad (2.25)$$

Where $\sigma_{F\,all.}$ is the allowable frame stress at failure given by $(\sigma_b \sigma_p)^{1/2}$, where σ_b is the buckling stress from R.Ae.S. D.S. 01.01.09 or 01.01.18. σ_p is the 2% proof stress for the frame. This expression is used to estimate the panel shear stress at which forced crippling of the frame flange occurs when the flange is riveted to the panel; the panel shear stress at which frame instability occurs, neglecting the term in $T_b(2)$ for frames attached through stringer flanges only; and covers frame failure between stringers when the normal section frames are used. (⅃ or ⅂)

Expression (2.25) is also used to predict the panel shear stress at which failure of the frame at a stringer cut-out should occur. For this form of failure $\sigma_{F_{all.}} A_F$ in the first term of the expression is replaced by $P_{all.}$ where $P_{all.}$ is the sum of the compression strength of the frame at the cut-out and the strength of the panel under the stringer (S.O.R. No.51, Part 1, Sect. 3 p.7, expression (3.9)), or if less the strength of stringer-frame attachments. For frames riveted to panel between stringers, the strength of the stringer-frame attachment is taken as the strength of the rivet through the frame flange plus the strength of the first adjacent rivet through the panel. Comparative test and predicted shear stresses at stringer and frame failure are given on Fig.2.A.5 of the Appendix.

2.5. Panel Shear Stiffness and Shear Strain

The expression for the shear modulus under pure diagonal tension given by Kuhn, Ref.1, expression (4.6), has been found to give very good agreement with the measured values from 17 specimens, Refs. 9, 10, 15, 28 and 34. The buckle angle α used with this expression is the calculated angle using section 2.3. in place of the diagonal tension field angle as used in Ref.1.

Denoting the post buckling shear modulus by G_b and re-writing in the notation of this report, for 'Z' or channel section stringers and frames, G_b is given by:-

$$\frac{E}{G_b} = 4\,\mathrm{Cosec}^2 2\alpha + \frac{dt}{A_s}\,\mathrm{Cot}^2\alpha + \frac{ht}{A_F}\,\mathrm{Tan}^2\alpha . \tag{2.26}$$

Where E is the modulus of elasticity of the panel.

α is the buckle angle, d is the stringer pitch,

h is the frame pitch, t is the panel thickness

and A_s and A_F are the cross-section areas of stringer and frame respectively.

This expression is modified for closed section stringers, becoming.

$$\frac{E}{G_b} = 4\frac{b'}{d}\,\mathrm{Cosec}^2 2\alpha + \frac{E}{G}\left(1-\frac{b'}{d}\right) + \frac{b't}{A_s}\,\mathrm{Cot}^2\alpha + \frac{ht}{A_F}\,\mathrm{Tan}^2\alpha . \tag{2.27}$$

Where b' is the plain panel width. G is the panel shear modulus. Frame area A_s includes the local skin area under the stringer, $(d-b')t$

The overall panel shear strain is composed of three elements; the pre-buckling strain, an incremental strain at buckling dependent upon loading conditions and finally the strain in the buckled condition.

.../14

Denoting total shear strain by γ_T and the incremental strain, based on fixed stress conditions, by $\delta\gamma$, γ_T is given by:-

$$\left.\begin{aligned} \gamma_T &= \tau/G &&: \tau < \tau_b \\ \gamma_T &= \tau_b/G + \delta\gamma &&: \tau = \tau_b \\ \gamma_T &= \tau_b/G + \delta\gamma + (\tau-\tau_b)/G_b &&: \tau > \tau_b \end{aligned}\right\} \quad (2.28)$$

The value of the incremental strain $\delta\gamma$ is dependent on the wave form, and is given by:-

$$\delta\gamma = \frac{1}{48}\left(\frac{b'}{R}\right)^2 \mathrm{Tan}\,\alpha \quad \text{for an axial wave pattern } (N_A \text{ buckles})$$

$$\delta\gamma = \frac{1}{48}\left(\frac{a}{R}\right)^2 \mathrm{Tan}^3\alpha \quad \text{for a circumferential wave pattern } (N_C \text{ buckles}) \qquad (2.29)$$

The wave pattern is determined from Fig.2.3.a.

Plasticity correction to the panel term in expressions (2.26) and (2.27), when the effective average panel tensile stress is greater than the stress at the elastic limit of the panel material, is necessary.
These expressions are re-written:-

$$\frac{E}{G_b} = 4\frac{E}{E_s}\,\mathrm{Cosec}^2 2\alpha + \frac{d't}{A_s}\,\mathrm{Cot}^2\alpha + \frac{ht}{A_F}\,\mathrm{Tan}^2\alpha . \qquad (2.30)$$

for Z or channel section stringers

and $$\frac{E}{G_b} = 4\frac{E}{E_s}\frac{b'}{d}\,\mathrm{Cosec}^2 2\alpha + \frac{E}{G}\left(1-\frac{b'}{d}\right) + \frac{b't}{A_s}\,\mathrm{Cot}^2\alpha + \frac{ht}{A_F}\,\mathrm{Tan}^2\alpha \qquad (2.31)$$

for closed section stringers.

Where E_s is the secant modulus at stress $\sigma_{45} = 3(\tau-\tau_b)\,\mathrm{Cosec}\,2\alpha$.

Comparative test and calculated deflections are given on Figs.2.A.9 and 2.A.10 of the Appendix.

THE ANALYSIS OF CURVED SHEAR PANELS

NOTATION ADDITIONAL TO p.p. 12-14 S.O.R. No.51 PART 1

A_F =	Cross section of area of frame	in^2	Sect.2.4
A_F^* =	2 x Cross section area of frame flange	in^2	"
A_S =	Cross section area of stringer (including effective skin unless otherwise stated)	in^2	"
A_S^* =	2 x Cross section area of stringer flange or area of stringer if less	in^2	"
$(EI)_F$ =	Frame stiffness	$lb.in^2$	Fig.2.2.a
$(EI)_S$ =	Stringer stiffness	$lb.in^2$	"
F =	Suffix denoting frame		
F' =	Reduction factor for buckling of pressurised panels		Fig.2.2.d
G_{pb} =	Post buckling shear modulus	lb/in^2	Sect.2.5
i_S =	Radius of gyration of stringer	in.	Sect.2.4
P_x =	Applied tensile load to skin plus stringer over stringer pitch d	lb.	"
$P_{F_{all.}}$ =	Allowable frame compressive load	lb.	Sec.2.4.4
R =	Panel radius	in.	Fig.2.1.a
t_F' =	Frame flange thickness	in.	Fig.2.1.a
t_S' =	Stringer flange thickness	in.	"
α =	Buckle angle	Rad.	Figs.2.3.a, 2.3.b
Γ =	Warping constant	in^6	Sect.2.4.3.d
σ_x =	Longitudinal tensile stress	lb/in^2	Sects.2.3, 2.4
σ_θ =	Hoop tensile stress	lb/in^2	"
σ_2 =	.2% Proof stress	lb/in^2	"
$\sigma_{ULT.}$ =	Ultimate tensile stress	lb/in^2	"
$\sigma_{F_{AV.}}$ =	Average frame stress	lb/in^2	Sect. 2.4.2
$\sigma_{F_{AV\,all.}}$ =	Allowable average frame stress	lb/in^2	Sect. 2.4.4
$\sigma_{S_{AV.}}$ =	Average stringer stress	lb/in^2	Sect. 2.4.1.
$\sigma_{S_{AV\,all.}}$ =	Allowable average stringer stress	lb/in^2	Sect. 2.4.3
Ψ =	Coefficient for buckling of pressurised panels		Fig.2.3.c

58

Fig 2.1a. EFFECTIVE SIZE OF CURVED PANELS FOR DETERMINATION OF BUCKLING STRESSES.

a AND b ARE AXIAL LENGTH AND CIRCUM^{AL} LENGTH TAKEN AS MIN. DISTANCE BETWEEN RIVET LINES OR, IF REDUCED, AS DISTANCE BETWEEN MID-FLANGES. (SEE BELOW).

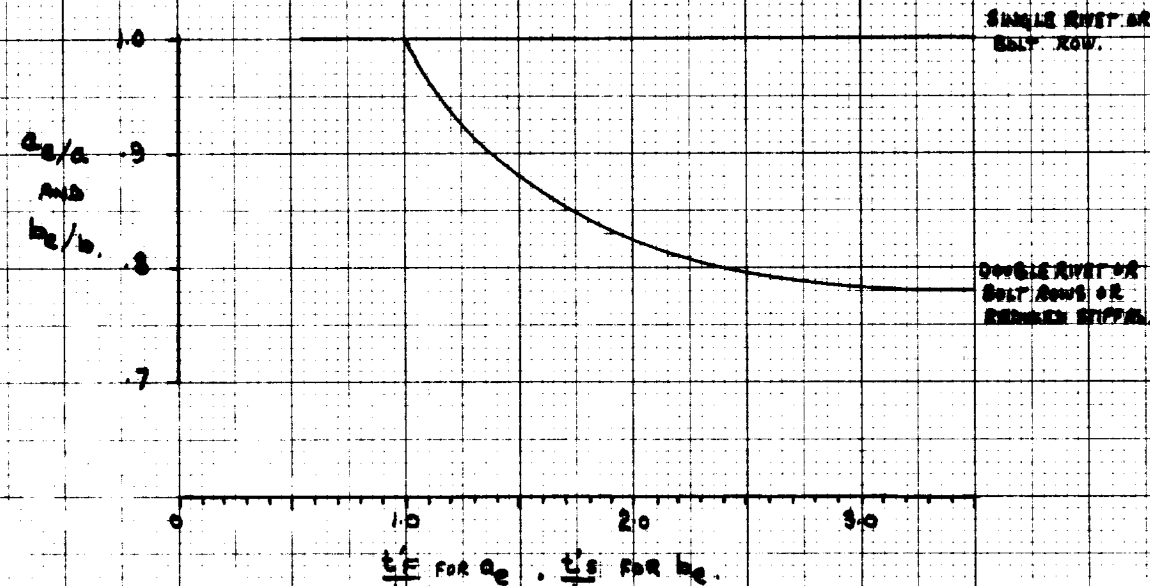
a_e = EFFECTIVE AXIAL LENGTH OF PANEL. in.

b_e = EFFECTIVE CIRCUMFERENTIAL LENGTH OF PANEL. in.

t = PANEL THICKNESS. in.

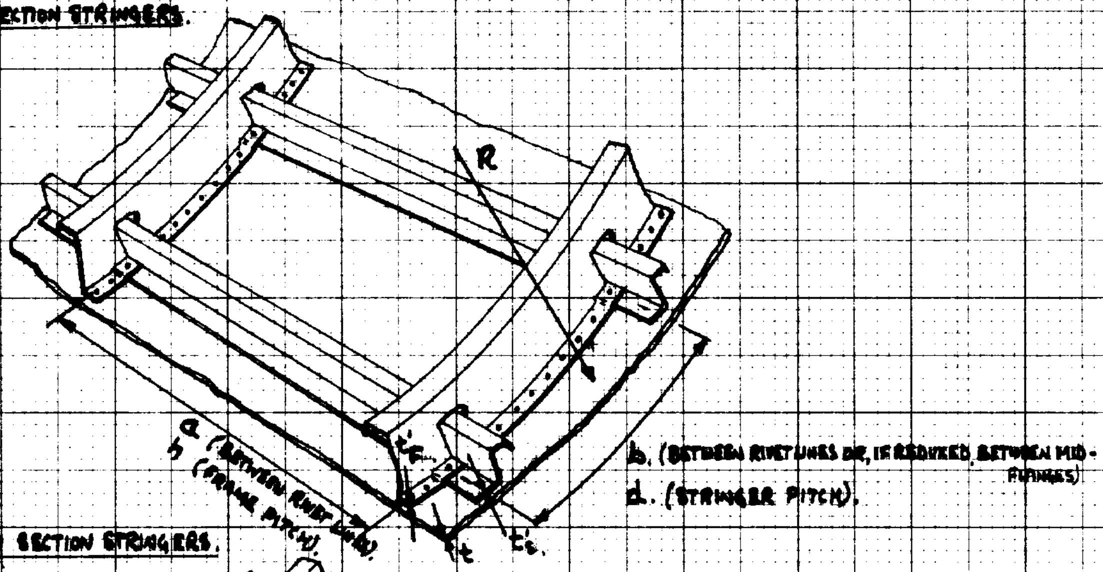
t'_F = FRAME THICKNESS in.

t'_s = STRINGER THICKNESS in.



PANEL GEOMETRY.

OPEN SECTION STRINGERS.



CLOSED SECTION STRINGERS.

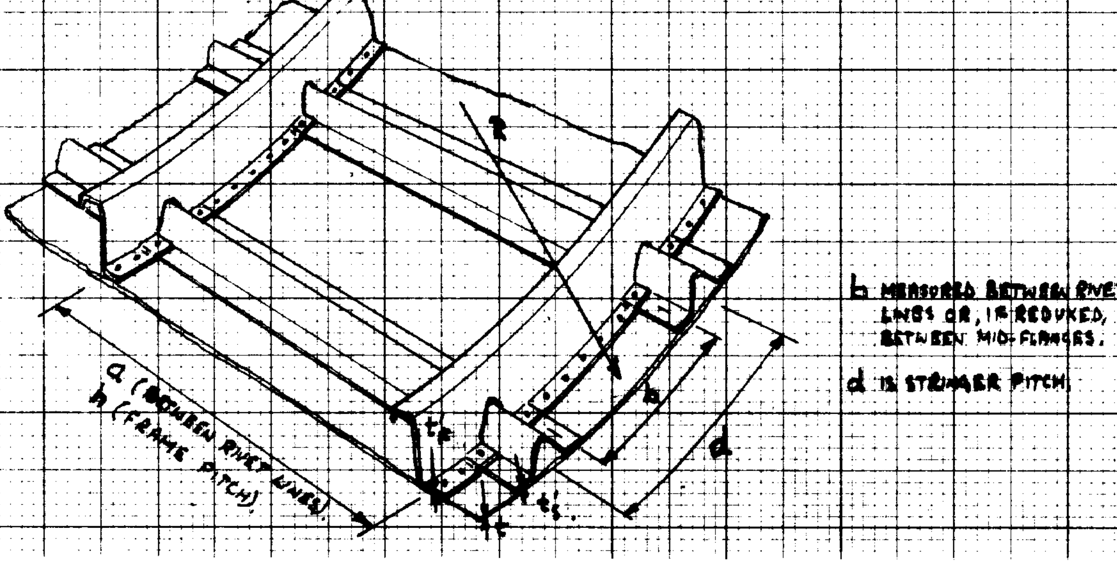
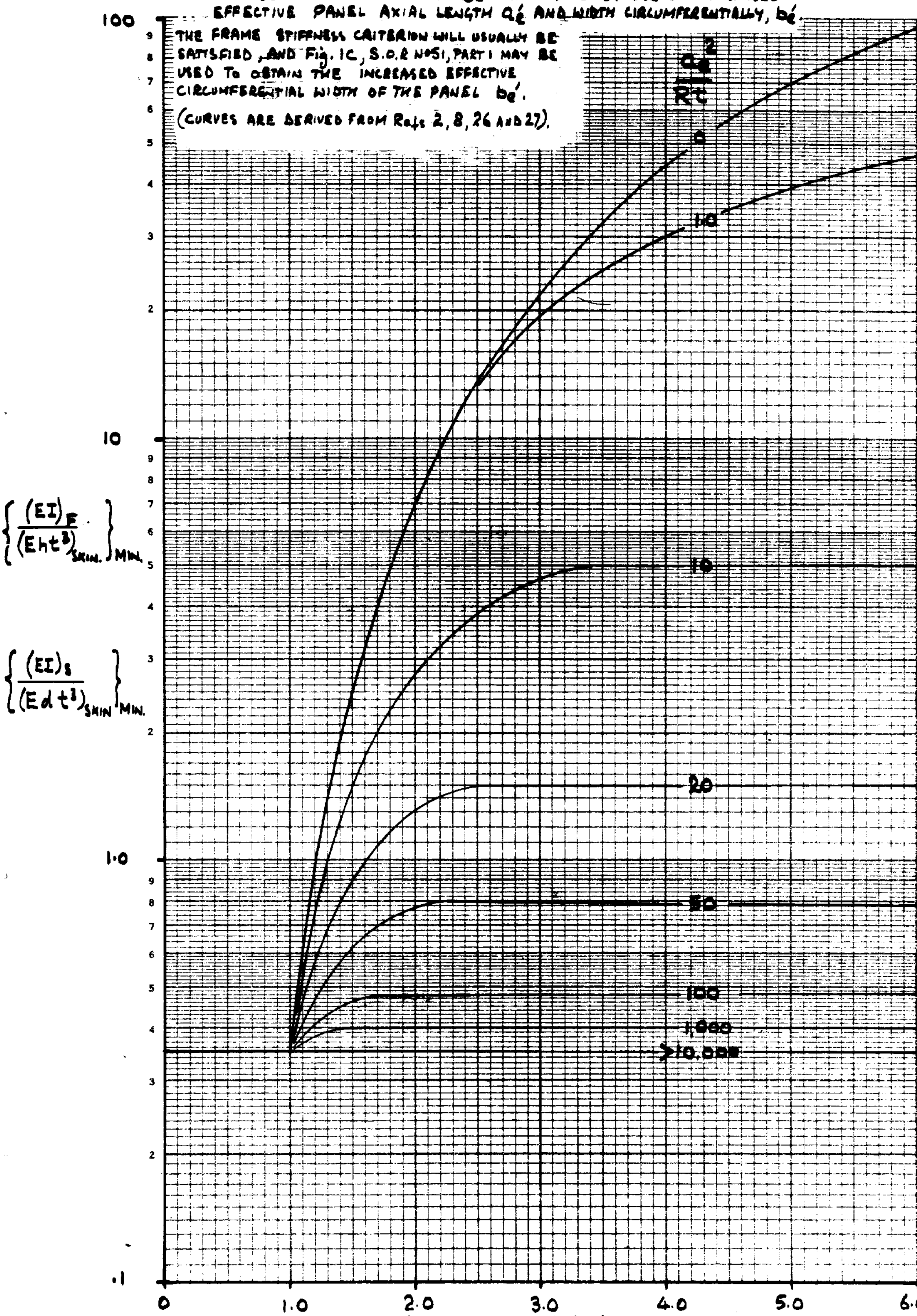


Fig. 2.1(b) MINIMUM FRAME AND STRINGER STIFFNESS REQUIRED TO PREVENT BUCKLING ACROSS FRAMES OR STRINGERS.

IF THESE REQUIREMENTS ARE NOT SATISFIED, THE BUCKLING STRESS WILL BE REDUCED. THIS MAY BE CALCULATED BY USE OF INCREASED EFFECTIVE PANEL AXIAL LENGTH a_e' AND WIDTH CIRCUMFERENTIALLY, b_e'.



b_e/h FOR $\left\{\frac{(EI)_F}{(Eht^3)_{SKIN}}\right\}_{MIN.}$ USING APPROPRIATE VALUE OF a^2/Rt

h/b_e FOR $\left\{\frac{(EI)_S}{(Edt^3)_{SKIN}}\right\}_{MIN}$ USING CURVE FOR $a^2/Rt = 0$.

Fig. 22a AVERAGE ELASTIC BUCKLING STRESS FOR CURVED PANELS IN SHEAR.

$$\tau_{b_0} = \tau_{b_0}(1) + \tau_{b_0}(2) \quad \text{WHERE:-} \quad \tau_{b_0}(1) = \frac{.785Et}{\left(\frac{a_e^2}{Rt}\right)^{1/4} R} \; ; \; \tau_{b_0}(2) = 4.84E\left(\frac{t}{b_e'}\right)^2 \quad (\nu = .3).$$

τ_{b_0} = ELASTIC BUCKLING STRESS. lb/in².

a_e = EFFECTIVE AXIAL LENGTH OF PANEL (Fig 2.1.a). in.

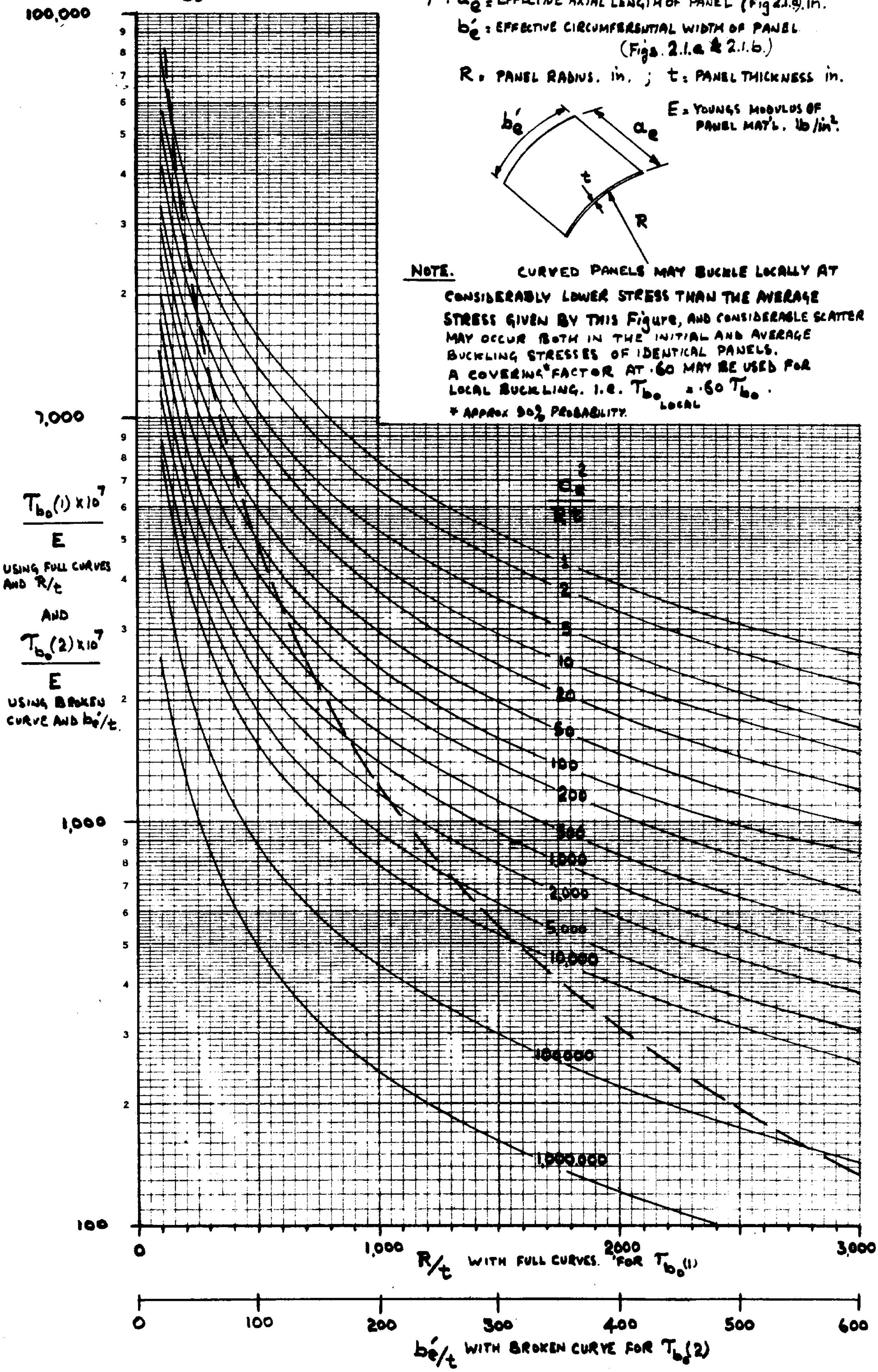
b_e' = EFFECTIVE CIRCUMFERENTIAL WIDTH OF PANEL (Figs. 2.1.a & 2.1.b.)

R = PANEL RADIUS. in. ; t = PANEL THICKNESS in.

E = YOUNGS MODULUS OF PANEL MAT'L. lb/in².

NOTE. CURVED PANELS MAY BUCKLE LOCALLY AT CONSIDERABLY LOWER STRESS THAN THE AVERAGE STRESS GIVEN BY THIS Figure, AND CONSIDERABLE SCATTER MAY OCCUR BOTH IN THE INITIAL AND AVERAGE BUCKLING STRESSES OF IDENTICAL PANELS. A COVERING* FACTOR AT .60 MAY BE USED FOR LOCAL BUCKLING. i.e. $\tau_{b_{0\,LOCAL}} = .60\,\tau_{b_0}$.

* APPROX 90% PROBABILITY.



$$\tau_{b_0(p)} = \tau_{b_0}(1)_{(p)} + \tau_{b_0}(2) + \tau_{b_0}(3)_{(p)} + \sigma_x \, ;$$

$\tau_{b_0}(1)_{(p)}$ FROM Fig. 2.2.b. , $\tau_{b_0}(1)$ FROM Fig. 2.2.a.

$\tau_{b_0}(2)$ FROM Fig. 2.2.a.

$\tau_{b_0}(3)_{(p)}$ FROM Figs. 2.2.c & 2.2.d.

$$\sigma_\theta = \text{AV. HOOP STRESS} = \frac{pR}{t}\left\{\frac{1 + A_s/b't + .15\,A_F/ht}{1 + A_s/b't + A_F/ht + .91\,\frac{A_s}{b't}\cdot\frac{A_F}{ht}}\right\}$$

$$\sigma_x = \text{AV. LONGIT}^{L}\text{ STRESS} = \frac{pR}{2t}\left\{\frac{1 + A_F/ht + .5\,A_s/b't}{1 + A_s/b't + A_F/ht + .91\,\frac{A_s}{b't}\,\frac{A_F}{ht}}\right\}$$

p = INTERNAL PRESSURE, p.s.i. E = MOD. OF ELASTICITY OF PANEL.

Fig. 2.2.b. VALUE OF $\tau_{b_0}(1)_{(p)}/\tau_{b_0}(1)$

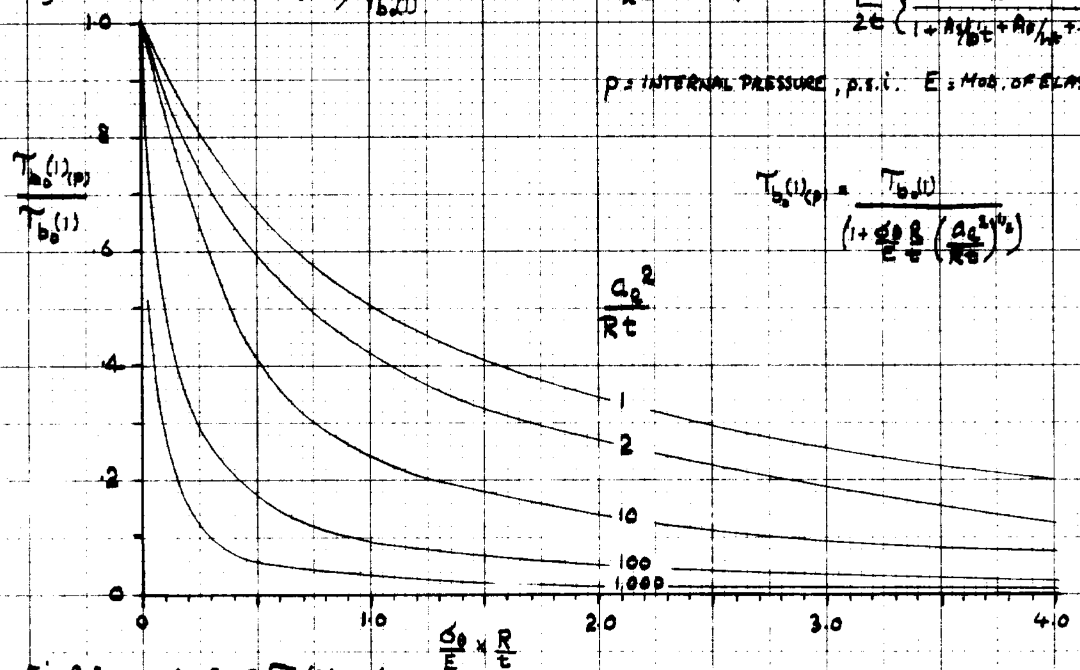


Fig. 2.2.c. VALUE OF $\tau_{b_0}(3)_{(p)}/\sigma_\theta F'$

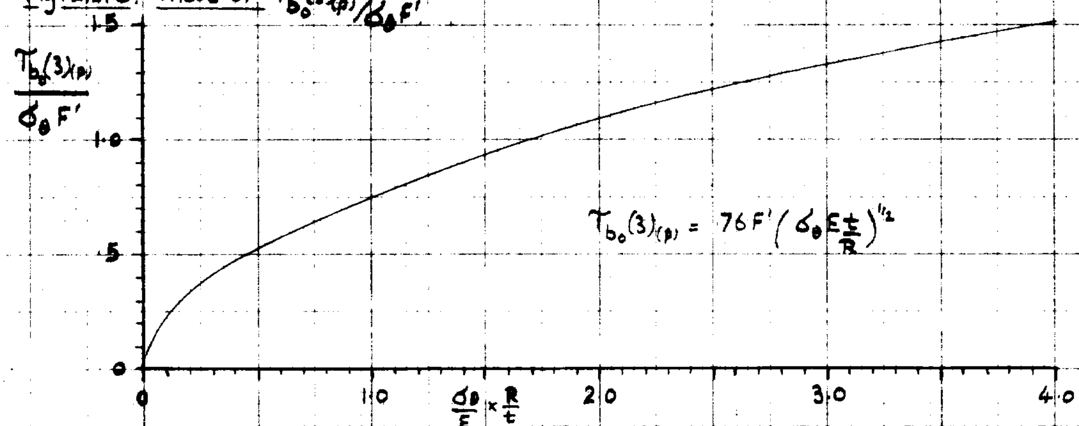


Fig. 2.2.d. REDUCTION FACTOR F' FOR $\tau_{b_0}(3)_{(p)}$

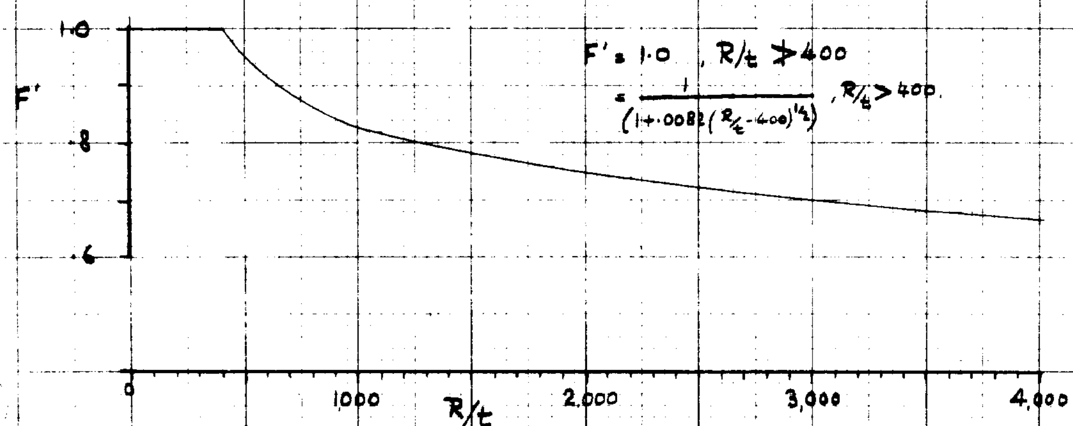


Fig 2.2.e. AVERAGE ELASTIC AXIAL BUCKLING STRESS FOR CURVED PANELS.

$\sigma_{xb_o} = \sigma_{xb_o}(1) + \sigma_{xb_o}(2)$ WHERE:- σ_{xb_o} = AV. AXIAL ELASTIC BUCKLING STRESS lb/in²

a_e = EFFECTIVE AXIAL LENGTH OF PANEL. in.
b_e' = EFFECTIVE CIRCUMFER^{L}. WIDTH OF PANEL in. } Fig. 2.1.a.

R = PANEL RADIUS. in. t = PANEL THICKNESS in.

E = PANEL MODULUS OF ELASTICITY. lb/in².

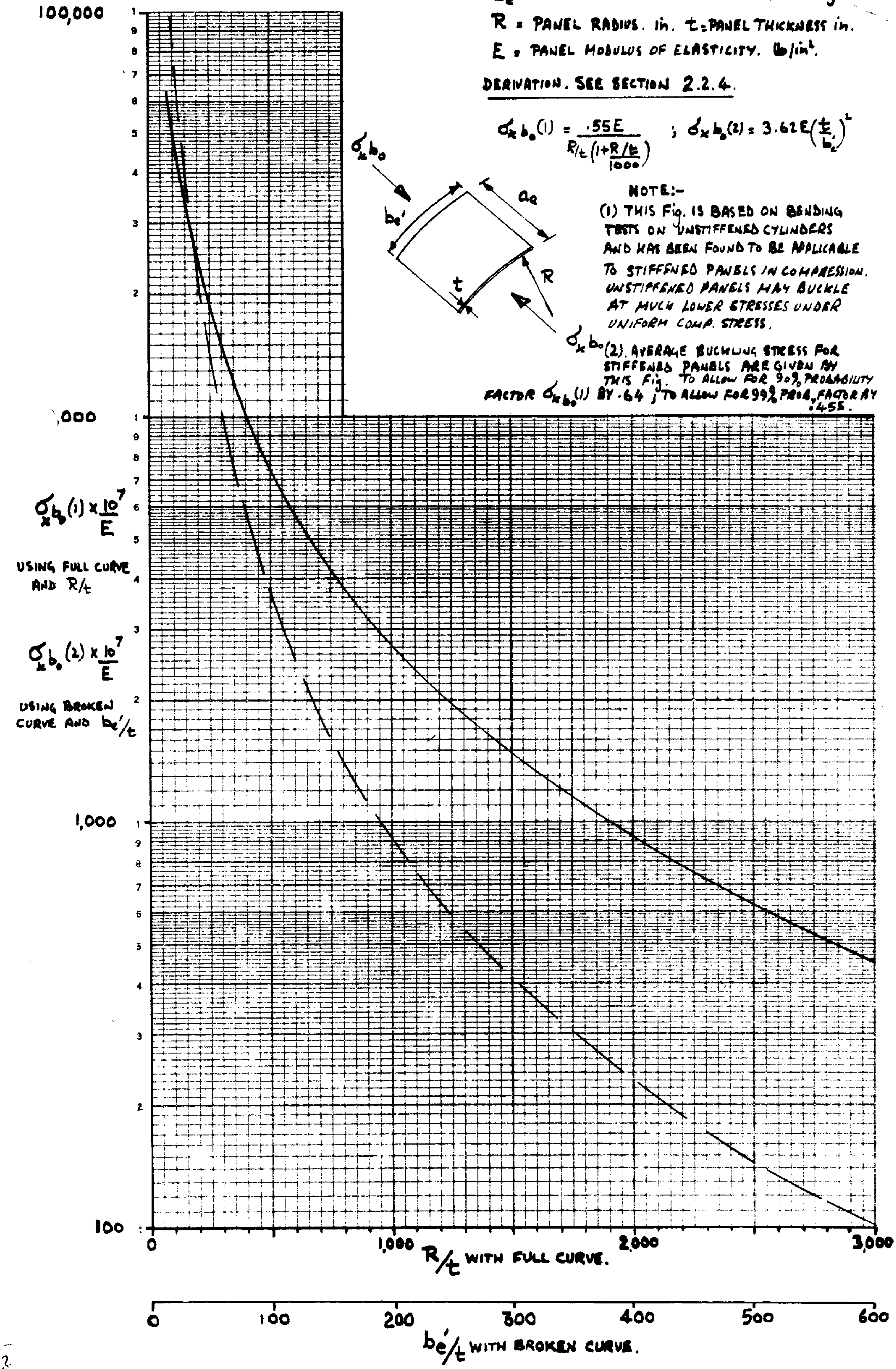
DERIVATION. SEE SECTION 2.2.4.

$$\sigma_{xb_o}(1) = \frac{.55E}{R/t\left(1+\frac{R/t}{1000}\right)} \quad ; \quad \sigma_{xb_o}(2) = 3.62E\left(\frac{t}{b_e'}\right)^2$$

NOTE:-

(1) THIS Fig. IS BASED ON BENDING TESTS ON UNSTIFFENED CYLINDERS AND HAS BEEN FOUND TO BE APPLICABLE TO STIFFENED PANELS IN COMPRESSION. UNSTIFFENED PANELS MAY BUCKLE AT MUCH LOWER STRESSES UNDER UNIFORM COMP. STRESS.

(2) AVERAGE BUCKLING STRESS FOR STIFFENED PANELS ARE GIVEN BY THIS Fig. TO ALLOW FOR 90% PROBABILITY FACTOR $\sigma_{xb_o}(1)$ BY .64 ; TO ALLOW FOR 99% PROB. FACTOR BY .455.



63

Fig. 2.2.f INCREMENTAL AXIAL BUCKLING STRESS DUE TO HOOP TENSION. Ref. SECTION 2.2.5

$$\sigma_{x b_o}(3) = \frac{4.5\,\sigma_\theta}{\left(\frac{\sigma_\theta}{E}\frac{R}{t}\right)^{3/4}}$$

WHERE σ_θ = AV. HOOP STRESS SEE Fig. 2.2.b.

BUCKLING STRESS UNDER COMBINED AXIAL STRESS & INTERNAL PRESSURE IS GIVEN BY:- $\sigma_{x b_o}(p) = \sigma_{x b_o} + \sigma_{x b_o}(3) + \sigma_x$

$\sigma_{x b_o}$ FROM Fig. 2.2.e.

σ_x = LONGITUDINAL STRESS, SEE Fig. 2.2.b.

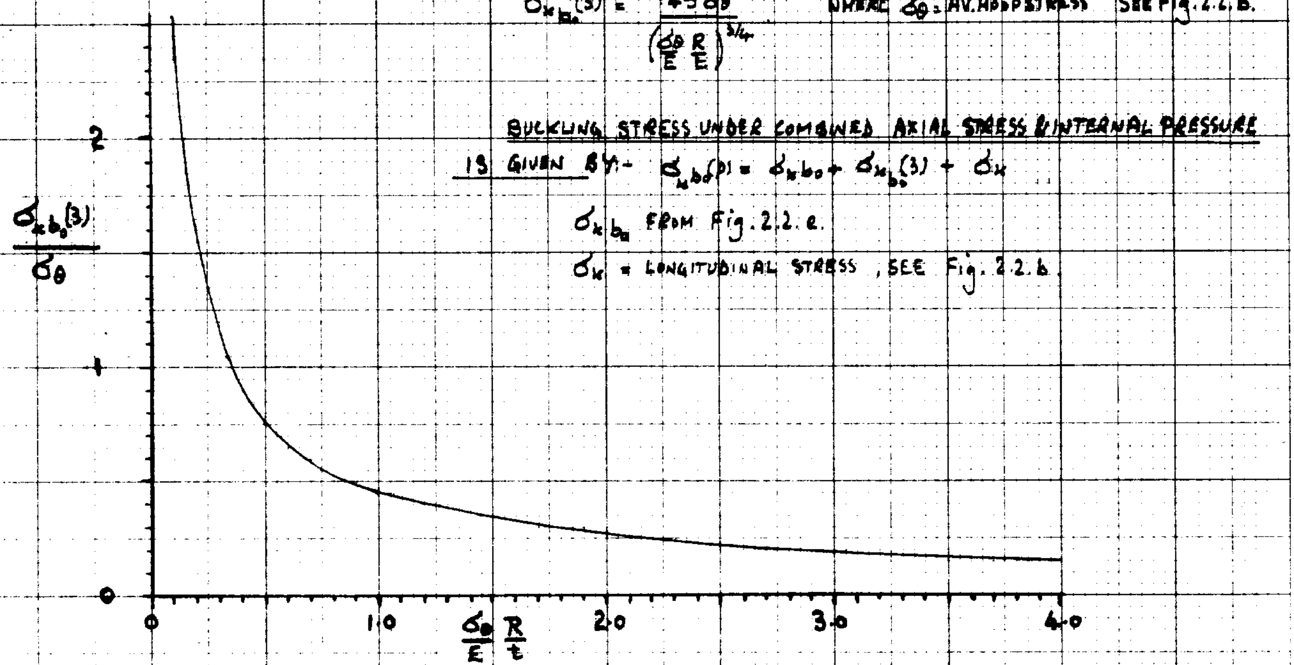


Fig. 2.2.g. BUCKLING STRESS UNDER COMBINED SHEAR AND AXIAL STRESSES. Ref. SECTION 2.1.6.

FROM EXPRESSION $\frac{\sigma_{x APP.}}{\sigma_{x b_o}} + \left(\frac{\tau_{APP.}}{\tau_{b_o}}\right)^2 = 1$ WHICH IS REWRITTEN

$$\frac{\tau_{b_o}(c)}{\tau_{b_o}} = \frac{1}{2}\left[-\frac{\sigma_{x APP.}}{\tau_{APP.}}\times\frac{\tau_{b_o}}{\sigma_{x b_o}} \pm \sqrt{\left(\frac{\sigma_{x APP.}}{\tau_{APP.}}\times\frac{\tau_{b_o}}{\sigma_{x b_o}}\right)^2 + 4}\right]$$

$\tau_{b_o}(c)$ = ELASTIC BUCKLING SHEAR STRESS UNDER COMBINED STRESS SYSTEM.		lb/in^2
τ_{b_o} = ELASTIC SHEAR BUCKLING STRESS.	Fig. 2.2.a	lb/in^2
$\sigma_{x b_o}$ = ELASTIC AXIAL BUCKLING STRESS.	Fig. 2.2.e.	lb/in^2.
$\sigma_{x APP.}$ = APPLIED AXIAL STRESS IN PANEL		lb/in^2.
$\tau_{APP.}$ = APPLIED SHEAR STRESS IN PANEL.		lb/in^2.

WITHOUT INTERNAL PRESSURE.

WITH INTERNAL PRESSURE THE EXPRESSION FOR $\frac{\tau_{b_o}(c)}{\tau_{b_o}}$ ABOVE CANNOT BE USED AS $\frac{\tau_{b_o}(c)}{\sigma_{x b_o}(c)} \neq \frac{\tau_{APP.}}{\sigma_{x APP.}}$

AND INSTEAD THE EQUATION $\frac{(\sigma_{x APP.} - \sigma_x)}{\sigma_{x b_o}(p)} + \left(\frac{\tau_{APP.}}{\tau_{b_o}(p)}\right)^2 = 1$

MUST BE SOLVED BY SUCCESSIVE APPROXIMATION.

σ_x = LONGITUDINAL STRESS, SEE Fig. 2.2.b.		lb/in^2.
$\sigma_{x b_o}(p)$ = ELASTIC BUCKLING STRESS FROM Fig. 2.2.e, Fig. 2.2.f.		lb/in^2
$\tau_{b_o}(p)$ = ELASTIC SHEAR BUCKLING STRESS FROM Figs. 2.2.a to Fig. 2.2.d.		lb/in^2.

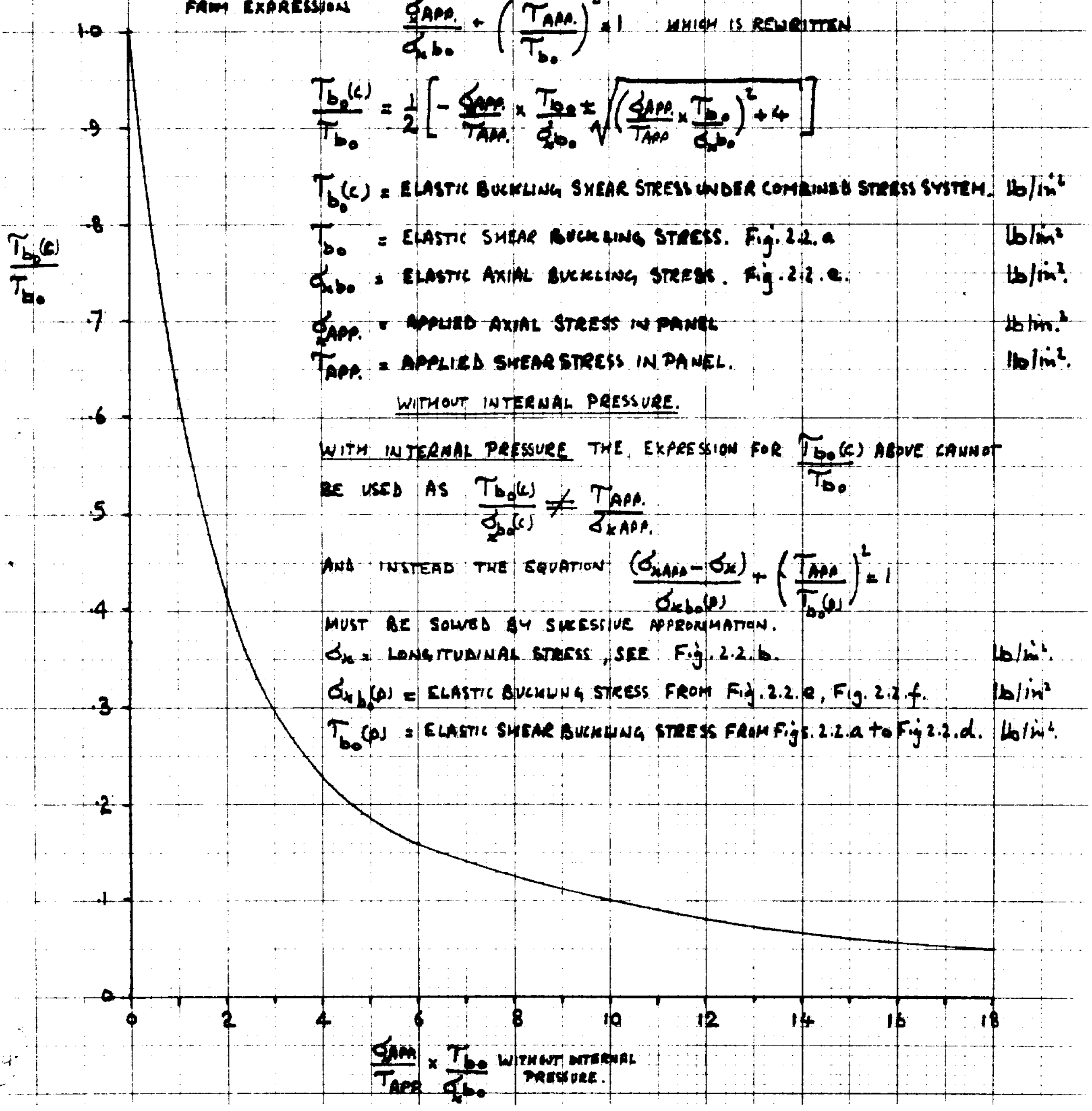


Fig. 2.2.h. ELASTIC BUCKLING STRESS TO TRUE BUCKLING STRESS CONVERSION FOR VARIOUS MATERIALS. (SEE SECTION 2.2.7).

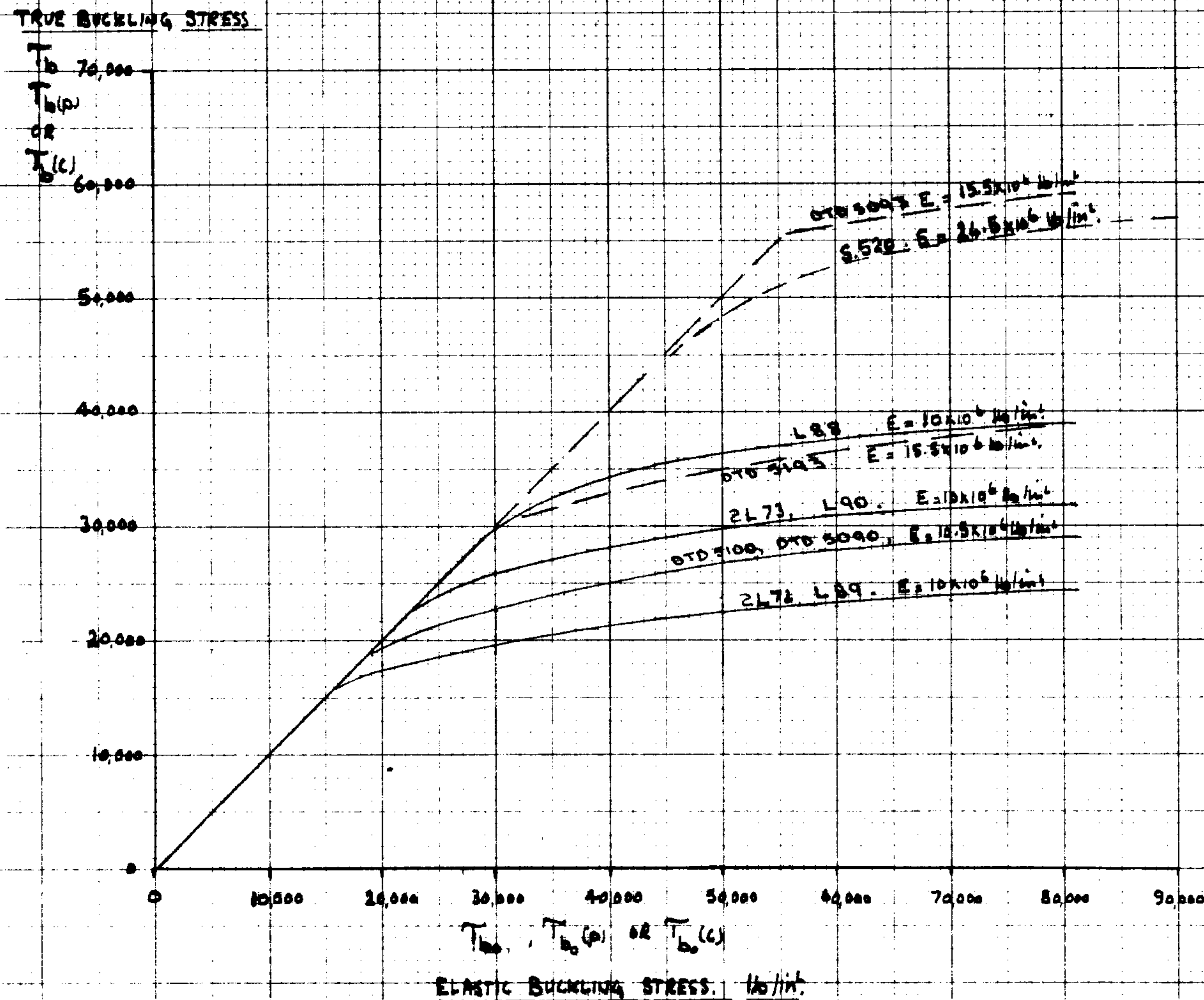
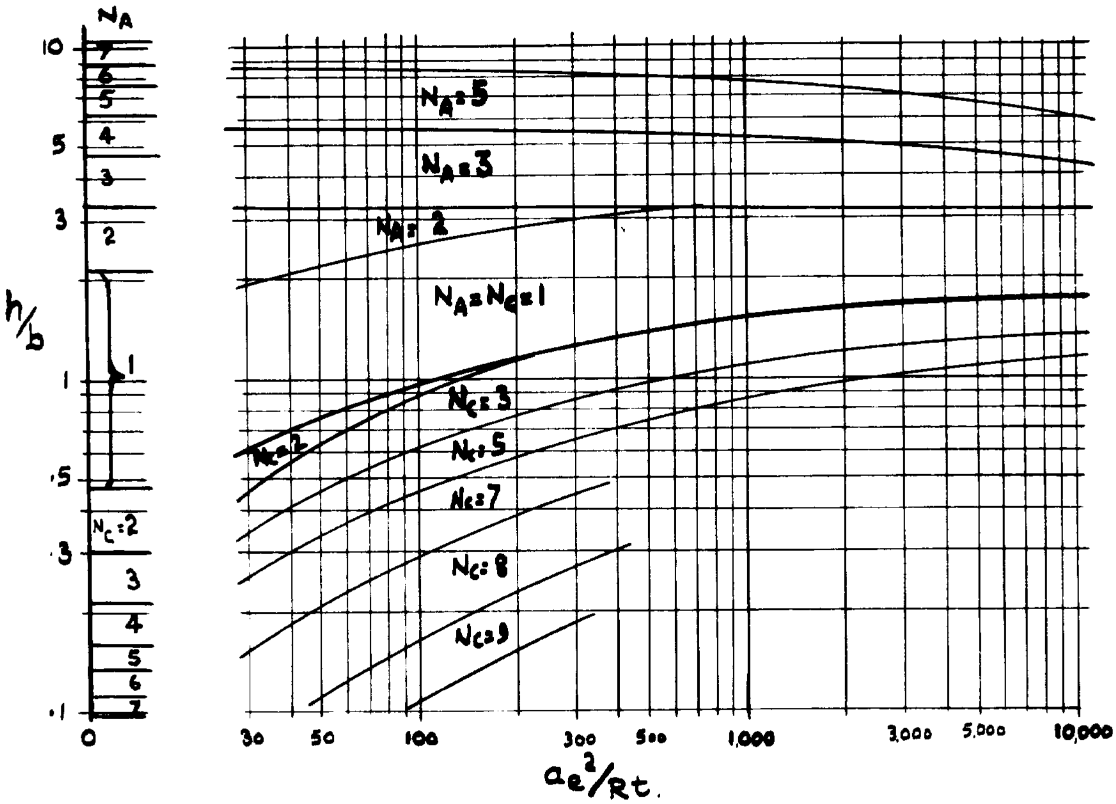


Fig. 2.3.a NUMBER OF AXIAL AND CIRCUMFERENTIAL FULL CRESTS AND TROUGHS IN BUCKLED CURVED SHEAR PANELS.

a_e = EFFECTIVE AXIAL LENGTH OF PANEL. in. (Fig 2.1.a).
b = CIRCUMFERENTIAL LENGTH OF PANEL. in. (Fig. 2.1.(a)).
h = FRAME PITCH. in. (Fig. 2.1.(a)).
R = PANEL RADIUS in. t = PANEL THICKNESS in.

N_A = No OF AXIAL CRESTS AND TROUGHS.
N_C = No OF CIRCUMFERENTIAL CRESTS AND TROUGHS.
α = ANGLE OF BUCKLE.



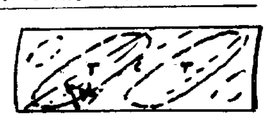
ANGLE OF BUCKLE, α, IS GIVEN BY THE FOLLOWING EXPRESSIONS WHEN $N = N_A$. (i.e. N AXIAL BUCKLES).

PANELS WITH 'Z' OR CHANNEL SECTION STRINGERS AT PITCH d, FRAMES AT PITCH h,

$$\tan\alpha = (N_A+1)\frac{d}{2h}.$$

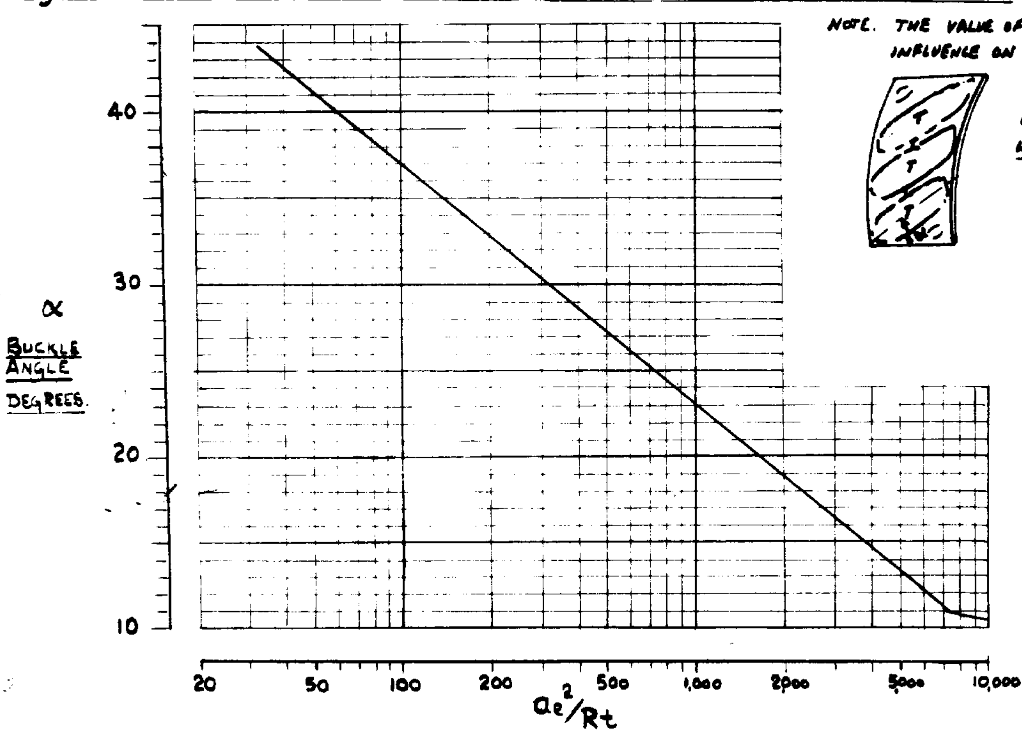
PANELS WITH CLOSED SECTN. STRINGERS AT PITCH d, PLAIN PANEL WIDTH b' AND FRAME PITCH h.

N_A	$\tan\alpha$	N_A	$\tan\alpha$
1	b'/h	5	$2d/h$
2	d/h	7	$(2d+b')/h$.
3	$(d+b'/2)/h$		



BUCKLED PANEL WHEN N_A = 3.

Fig. 2.3.b ANGLE OF BUCKLE, α, FOR PANELS WITH CIRCUMFERENTIAL BUCKLES, $N = N_C$.



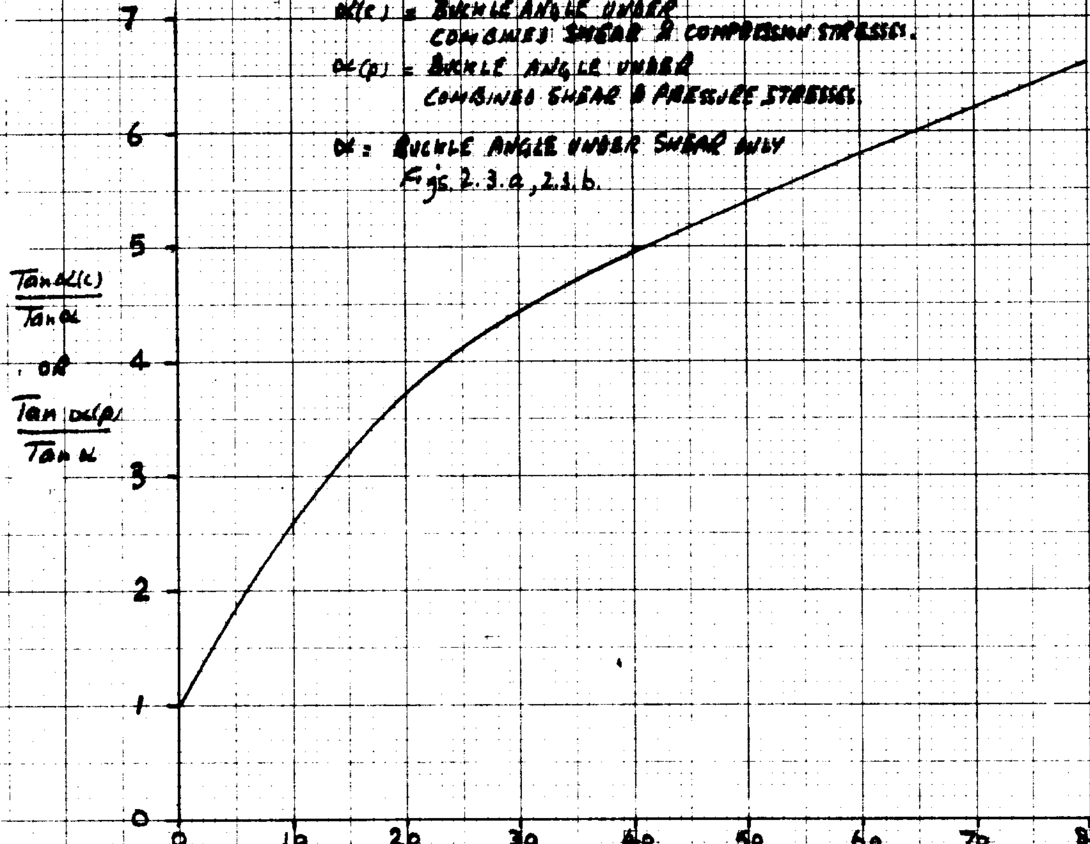
NOTE. THE VALUE OF N_C ($N_C \neq 1$) HAS LITTLE INFLUENCE ON BUCKLE ANGLE.

BUCKLED PANEL WHEN N_C = 5

Fig. 2.3.c **BUCKLE ANGLE FOR PANELS UNDER COMBINED SHEAR AND PRESSURE OR COMBINED SHEAR AND AXIAL STRESS.** (SECTION 2.3.)

FROM THE EXPRESSION:-

$$\tan\alpha_{(c)} = \tan\alpha_{(p)} = \tan\alpha\left(3.6 + .038\Psi - \frac{2.6}{\left(1+.8\left(\frac{\Psi}{10}\right)^2\right)}\right)$$



FOR PRESS + SHEAR $\Psi = \frac{(\sigma_\theta - \sigma_x)}{E}\frac{R}{t}\left(\frac{a_s^2}{Rt}\right)^{1/2}$ WHERE σ_θ = HOOP TENSION STRESS
σ_x = LONG. TENSION STRESS.

FOR AXIAL COMP. + SHEAR $\Psi = \frac{\sigma_{x}\,APP.}{E}\frac{R}{t}\left(\frac{a_s^2}{Rt}\right)^{1/2}$ WHERE $\sigma_{x\,APP}$ = APPLIED COMP. STRESS.

Fig. 2.3.d.

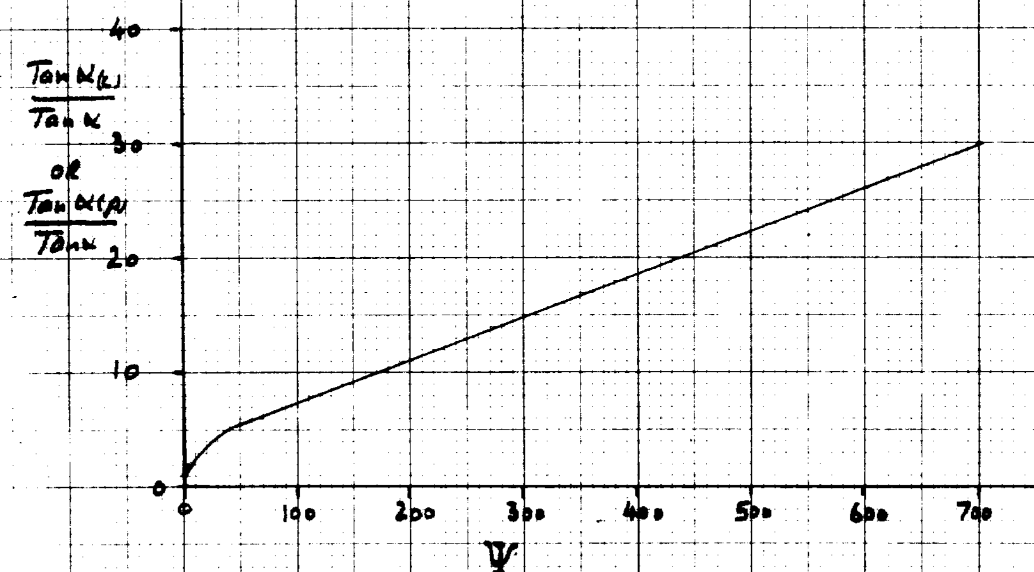


Fig. 2.3. e. FAILING SHEAR STRESS AND PERMANENT BUCKLING STRESS FOR CURVED SHEAR PANELS (SEE SECT 2.3.1 & 2.3.2)

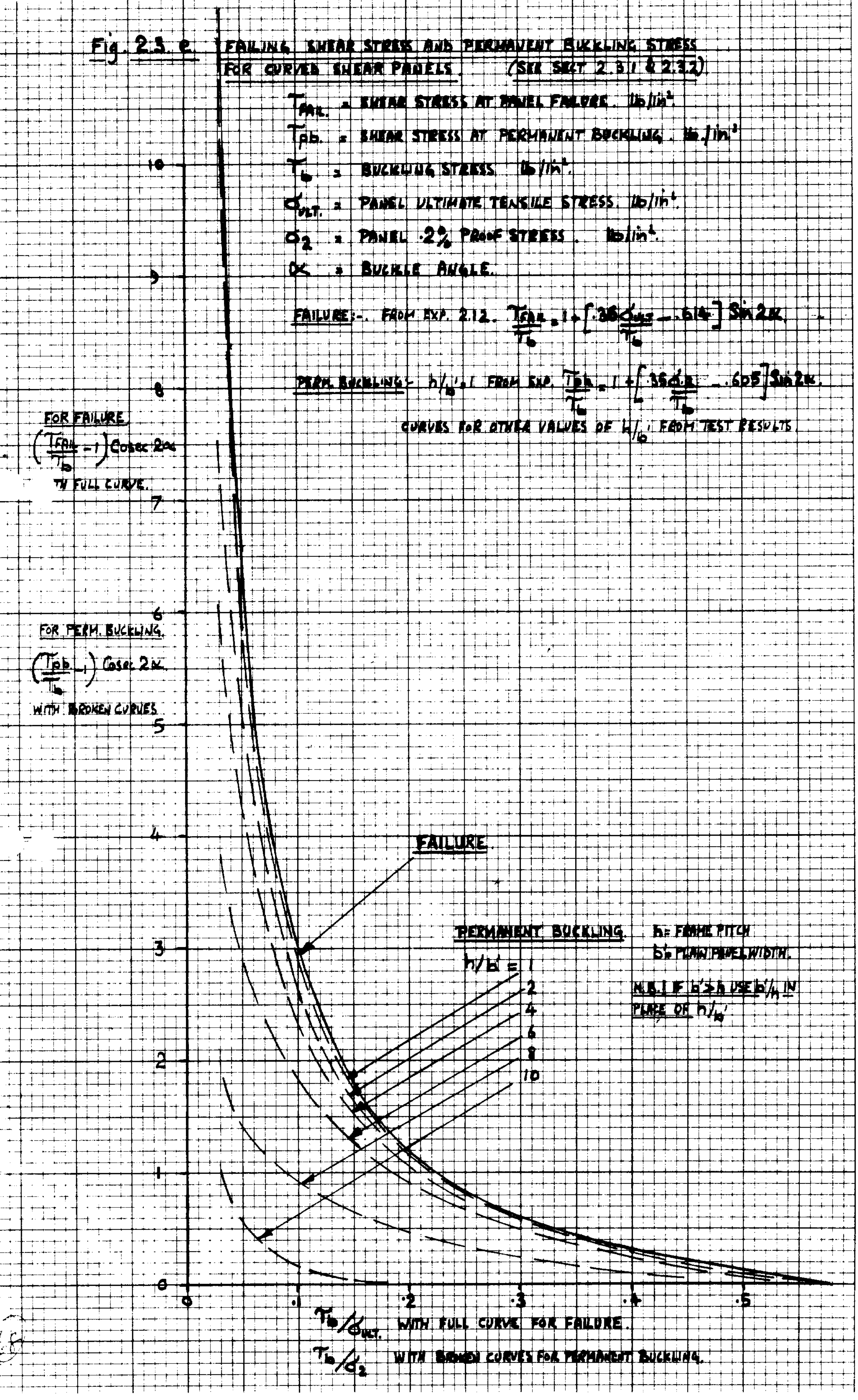


Fig. 2.A.1. COMPARISON BETWEEN CALCULATED AND TEST AVERAGE BUCKLING STRESS IN SHEAR ONLY.
STIFFENED CYLINDERS AND PANELS. (STRINGERS & FRAMES).

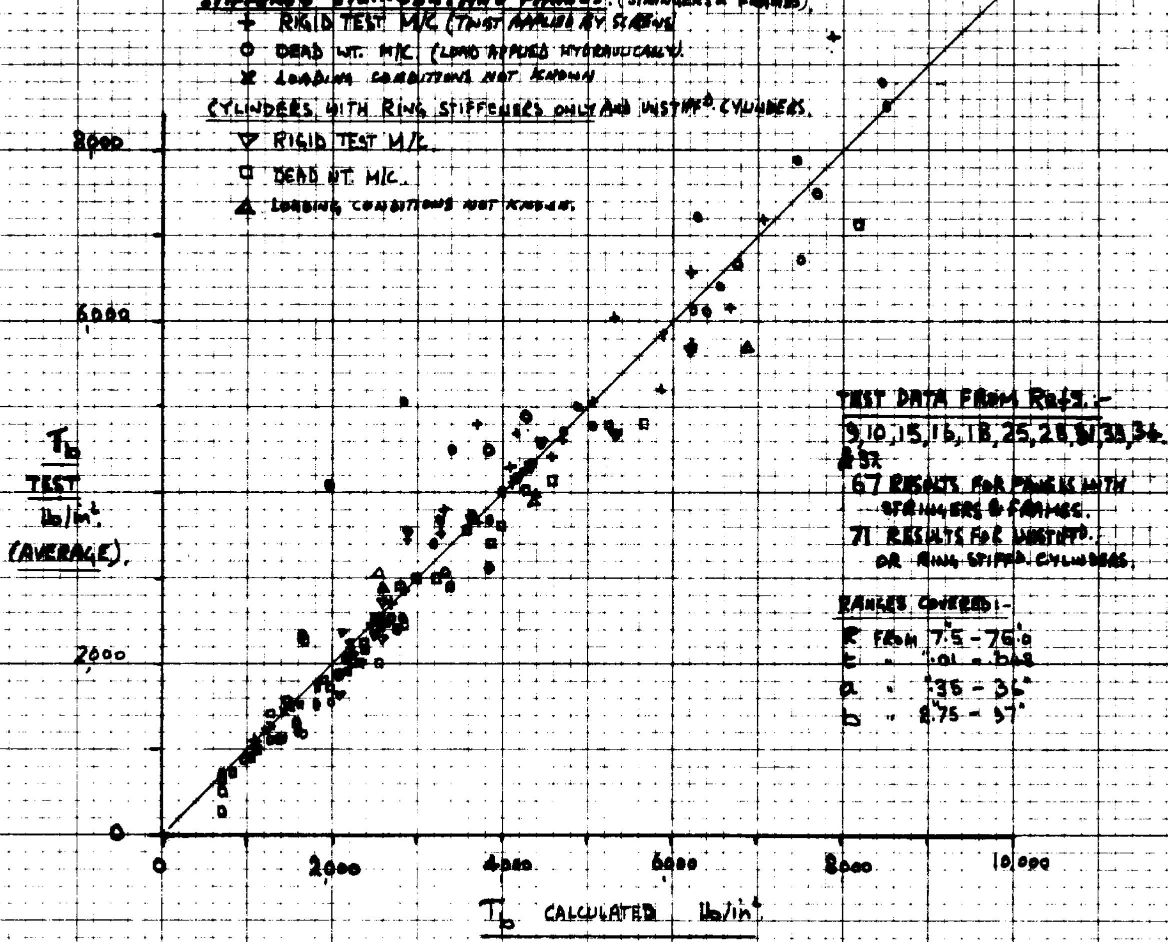


Fig. 2.A.2. COMPARATIVE ESTIMATED AND MEASURED BUCKLE ANGLE.
STIFFENED CYLINDERS AND PANELS. (STRINGERS AND FRAMES). +
CYLINDERS WITH RING STIFFENERS ONLY ▽.

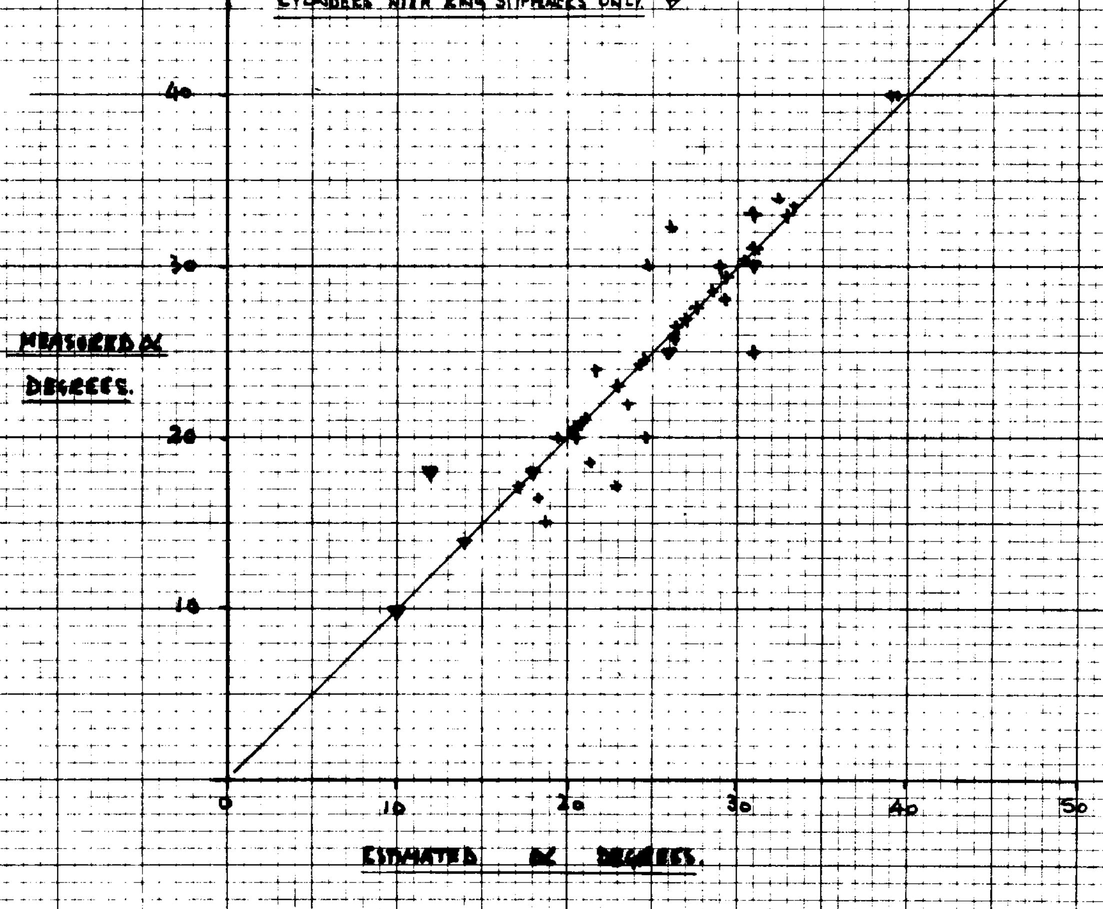


Fig. 2.A.3 COMPARATIVE CALCULATED AND TEST BUCKLING STRESS FOR COMBINED SHEAR AND PRESSURE AND SHEAR AND AXIAL COMPRESSION.

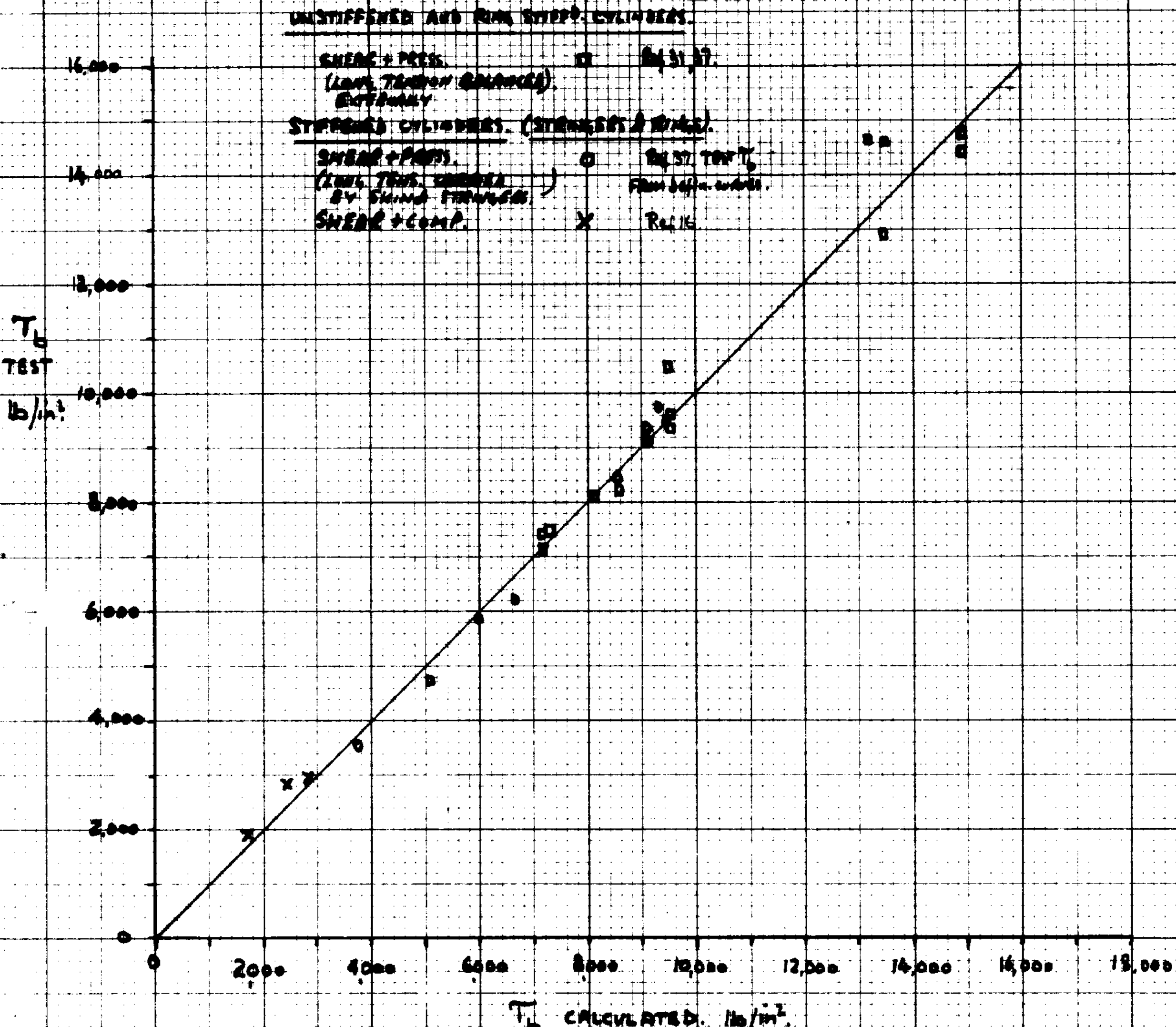


Fig. 2.A.4 COMPARATIVE CALCULATED AND TEST BUCKLE ANGLES. FOR PANELS UNDER COMBINED SHEAR & PRESSURE. NOTATION, AS ABOVE.

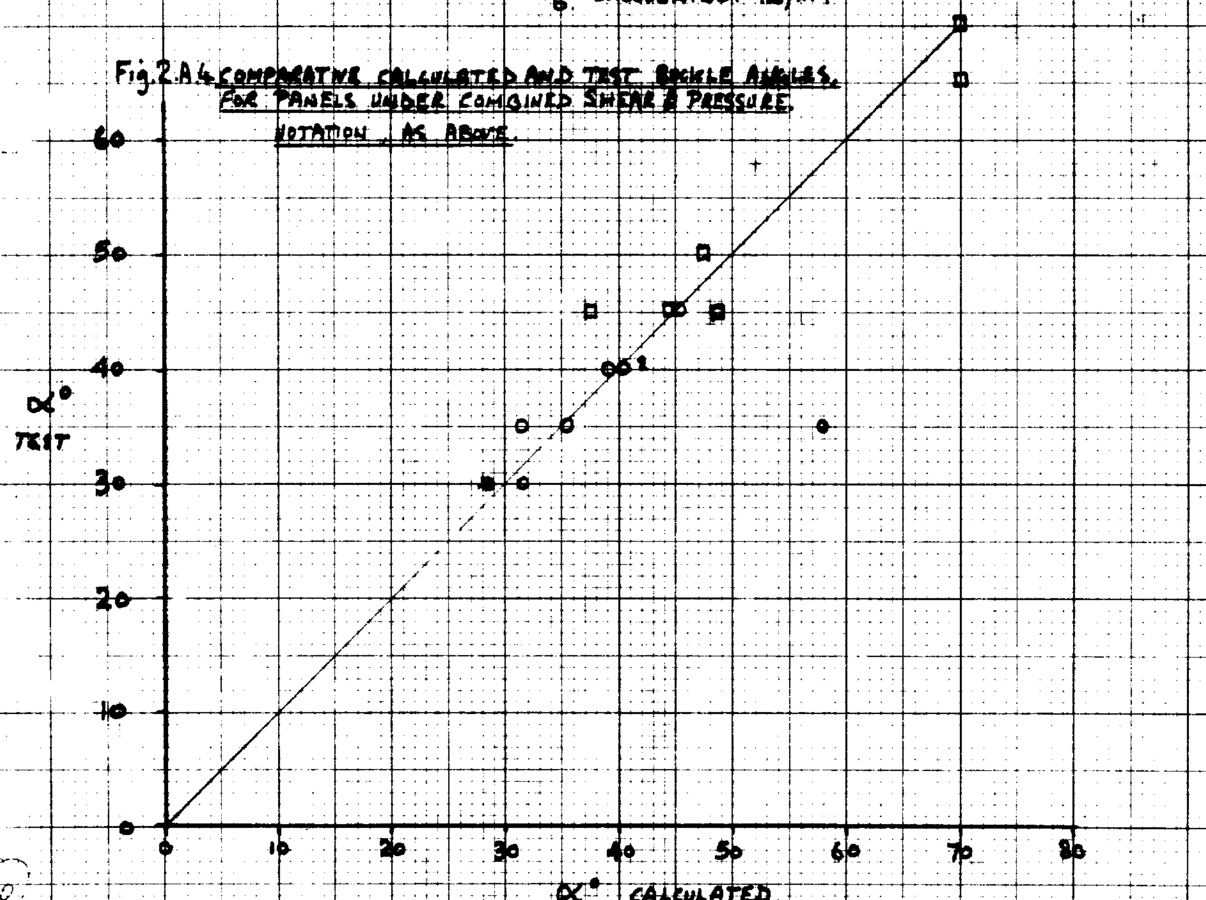


Fig. 2.A.5. COMPARATIVE TEST AND PREDICTED FAILING STRESSES, CURVED PANELS.

TEST RESULTS, Refs. 9, 10, 15, 16, 28, 34.

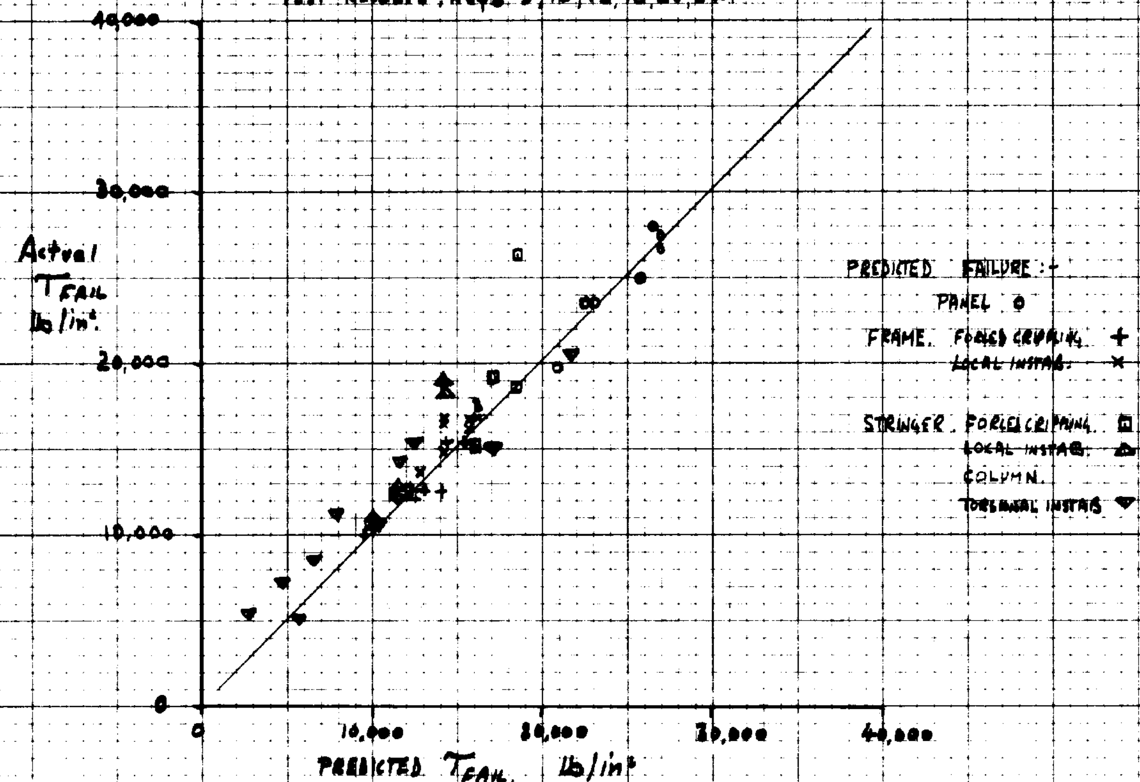


Fig. 2.A.6. COMPARATIVE TEST AND PREDICTED PERMANENT BUCKLING STRESSES, CURVED PANELS.

TEST RESULTS FROM Ref. 15. ○.

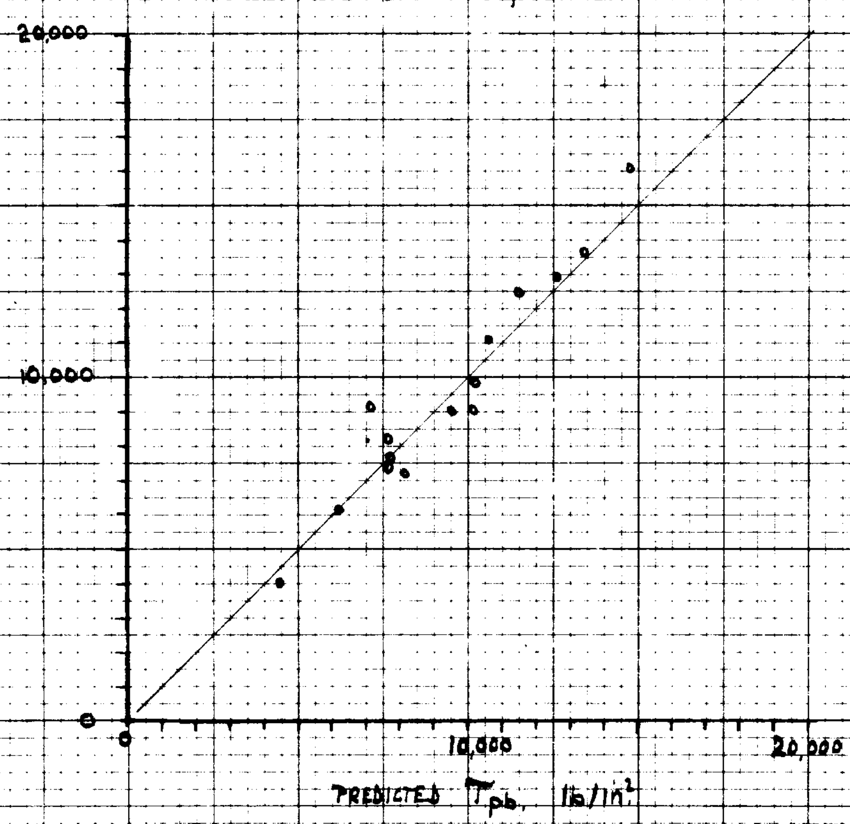
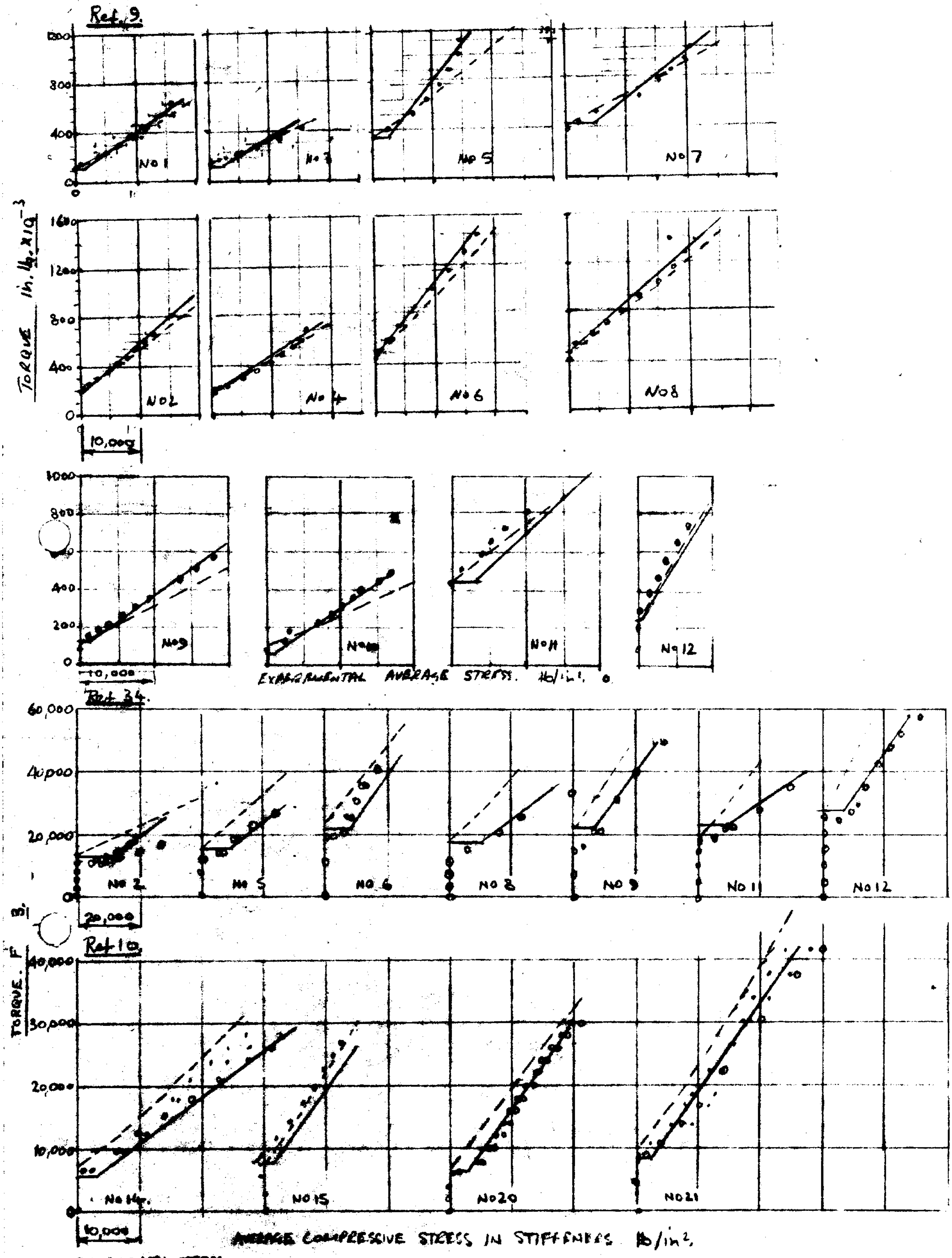


Fig 2.A.7 COMPARATIVE CALCULATED AND MEASURED AVERAGE STRINGER STRESSES.



EXPERIMENTAL STRESS
- ⊗ 10 in WHITTEMORE GAUGES
- • WIRE RESISTANCE GAUGES

CALCULATED AV. STRESS
- - - - KUHN & GRIFFITH Ref.
- —— EQUATION (2.4.1)

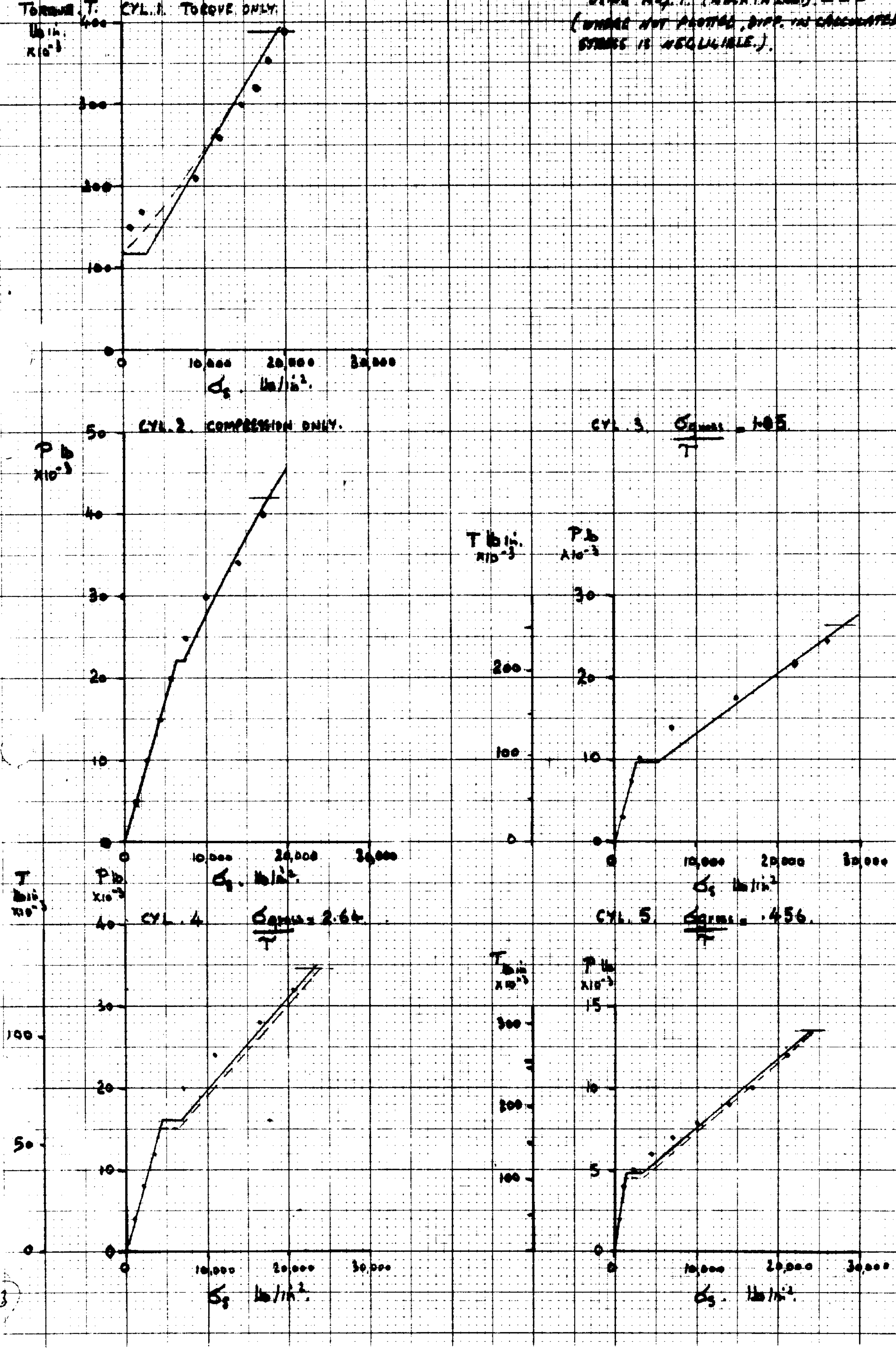
$$\sigma_{s_{av}} = -\frac{\left(T - \left(T_H(1)\right)\ldots + T_b(2)\right)}{(A_s/bt)}$$

Fig 2.A.8. COMPARATIVE CALCULATED & MEASURED AVERAGE STRINGER STRESSES.

Ref 16 CYLINDERS. COMBINED SHEAR & AXIAL COMPRESSION.

$T' = \frac{T}{35.4}$; $\sigma_{gross} = \frac{P}{3.454}$. R = 15", b = 7.37, h = 15", t = .025, A_s = .092, A_F = .255.

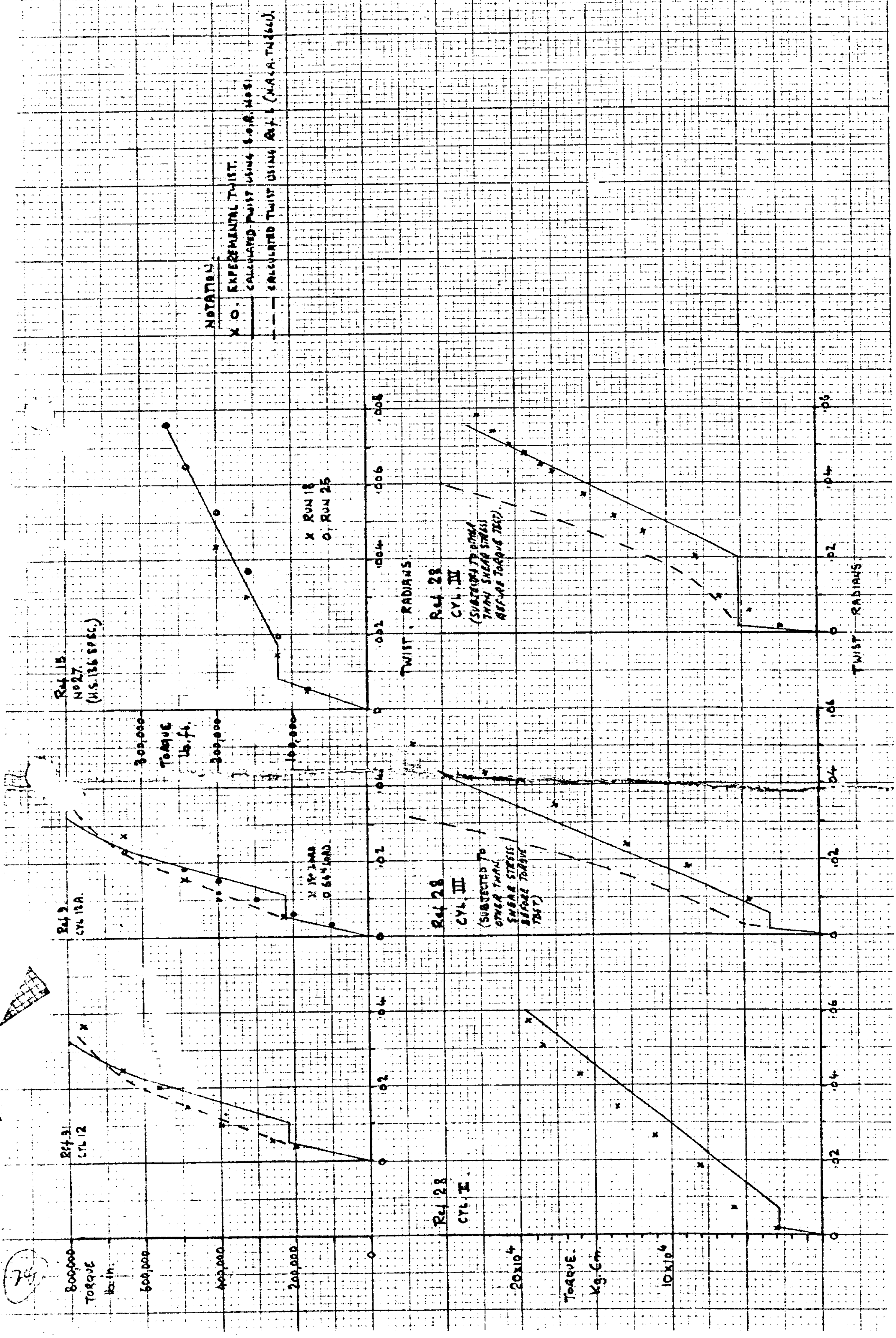
AVERAGE MEASURED STRESS, σ_s — •. CALCULATED, USING THIS REPORT ——
USING Ref. 1. (NACA TN 2661) – – –
(WHERE NOT PLOTTED, DIFF. IN CALCULATED STRESS IS NEGLIGIBLE.)



S.O.R. No 51. PART 2. APPENDIX.

Fig. 2.A9 COMPARATIVE EXPERIMENTAL AND CALCULATED TWIST OF TEST CYLINDERS.

REFS. 9, 15 & 28.



S.O.R. No 51. PART 2. APPENDIX.

Fig. 2.A.10. COMPARATIVE EXPERIMENTAL AND CALCULATED TWIST OF TEST CYLINDERS. REPS. 10 & 34.

P32