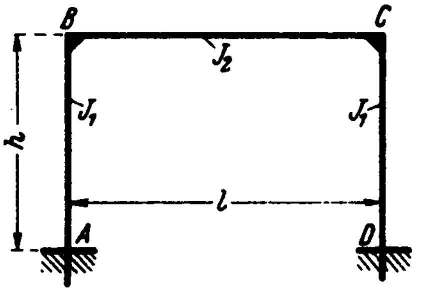
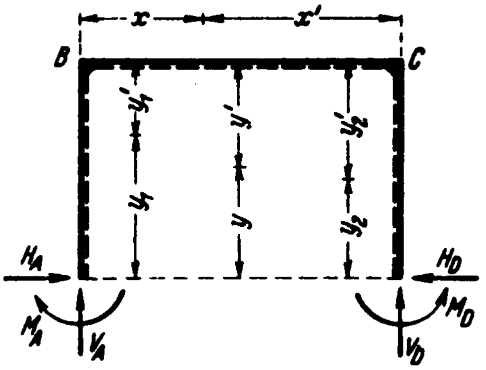
Telaio 41

Portale rettangolare simmetrico incastrato



Caratteristiche geometriche e relativi simboli.

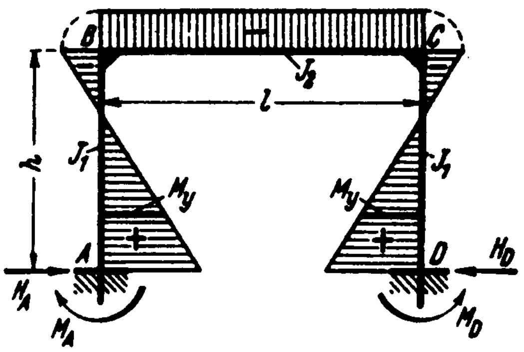


Versi positivi per le reazioni vincolari e coordinate per le sezioni delle aste. Si adottano y e y' nei casi di simmetria. Momento flettente positivo quando genera trazione dalla parte del tratteggio.

Costanti:

$$k = \frac{J_2}{J_1} \cdot \frac{h}{l} \qquad D_1 = k + 2 \qquad D_2 = 6k + 1 .$$

Caso 41/1 - Aumento uniforme di temperatura in tutto il telaio [1]



E = modulo elastico
α_T = coeff. dilat. termica
t = variazione termica in gradi

Grandezza ausiliaria: $T = \dfrac{3\,E\,J_2\,\alpha_T\,t}{h\,D_1}$.

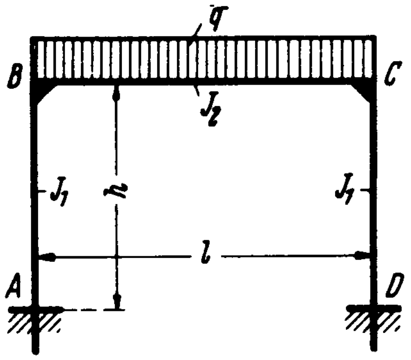
$$M_A = M_D = +\,T \cdot \frac{k+1}{k} \qquad M_B = M_C = -\,T$$

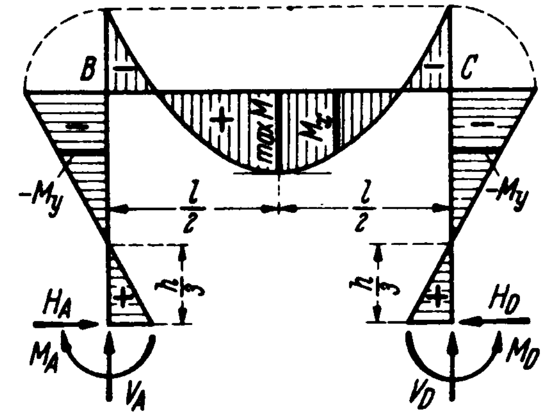
$$M_y = M_A - H_A\,y\,; \qquad H_A = H_D = \frac{T}{h} \cdot \frac{2k+1}{k}$$

N.B. Se si ha diminuzione di temperatura tutte le forze cambiano verso e i momenti invertono il segno.

[1] Solo una variazione termica nel traverso genera uno stato di sforzo. Per una diminuzione di lunghezza $\Delta l = \varepsilon l$ porre ε in luogo di α_T nelle formule per diminuzione di temperatura. Per il caso di variazione termica antisimmetrica ($+t$ nel ritto sinistro, $-t$ nel destro) occorre introdurre nella nota a pag. 159 $C_s = 12\,EJ_2\,h\,\alpha_T\,t/l^2$ e $S_d = 0$

Caso 41/2 - Carico uniformemente distribuito sul traverso



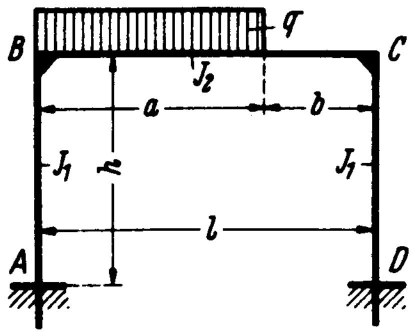


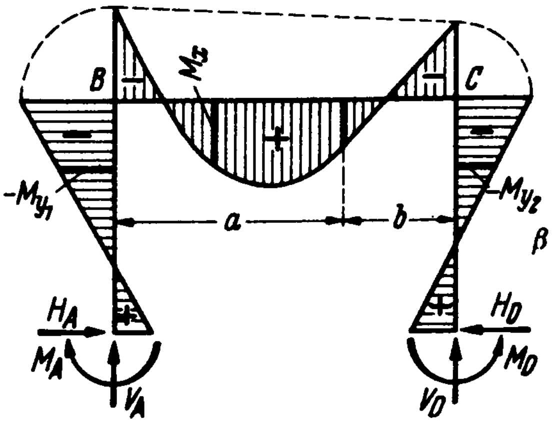
$$M_A = M_D = +\frac{q\,l^2}{12D_1} \qquad M_B = M_C = -\frac{q\,l^2}{6\,D_1} = -2\,M_A$$

$$\max M = \frac{q\,l^2}{8} + M_B \qquad V_A = V_D = \frac{q\,l}{2} \qquad H_A = H_D = \frac{3\,M_A}{h}$$

$$M_x = \frac{q\,x\,x'}{2} + M_B \qquad M_y = M_A - H_A\,y\,.$$

Caso 41/3 - Carico uniformemente distribuito sulla parte sinistra del traverso





$$\beta = \frac{b}{l}$$

$$\left.\begin{matrix} M_A \\ M_D \end{matrix}\right\rangle = +\frac{q\,a^2}{2}\left[\frac{1+2\,\beta}{6D_1} \mp \frac{\beta^2}{2\,D_2}\right] \qquad \left.\begin{matrix} M_B \\ M_C \end{matrix}\right\rangle = -\frac{q\,a^2}{2}\left[\frac{1+2\,\beta}{3D_1} \pm \frac{\beta^2}{2D_2}\right]$$

$$V_D = \frac{q\,a^2}{2\,l}\left(1 - \frac{\beta^2}{D_2}\right) \qquad V_A = q\,a - V_D \qquad H_A = H_D = \frac{q\,a^2\,(1+2\,\beta)}{4\,h\,D_1}$$

Nel tratto a:

$$M_x = \left(V_A - \frac{q\,x}{2}\right)x + M_B$$

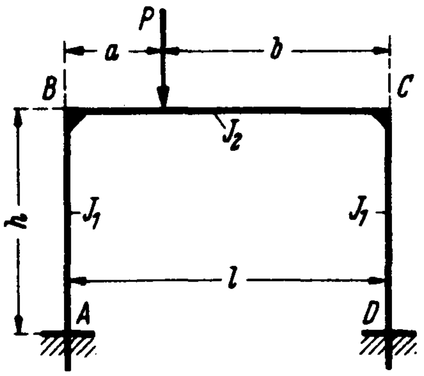
$$M_{y1} = M_A - H_A\,y_1$$

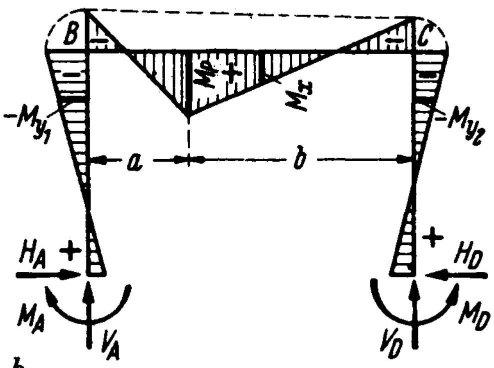
Nel tratto b:

$$M_x = V_D\,x' + M_C$$

$$M_{y2} = M_D - H_D\,y_2\,.$$

Caso 41/4 - Carico concentrato in posizione generica sul traverso





$$\alpha = \frac{a}{l} \qquad \beta = \frac{b}{l} \qquad (\alpha + \beta = 1)$$

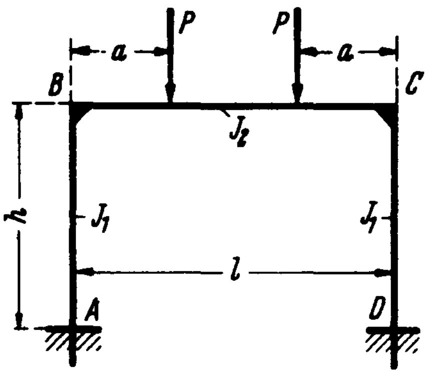
$$\left.\begin{matrix} M_A \\ M_D \end{matrix}\right\rangle = + \frac{P\,a\,b}{l}\left[\frac{1}{2\,D_1} \mp \frac{\beta - \alpha}{2 D_2}\right] \qquad \left.\begin{matrix} M_B \\ M_C \end{matrix}\right\rangle = - \frac{P\,a\,b}{l}\left[\frac{1}{D_1} \pm \frac{\beta - \alpha}{2\,D_2}\right]$$

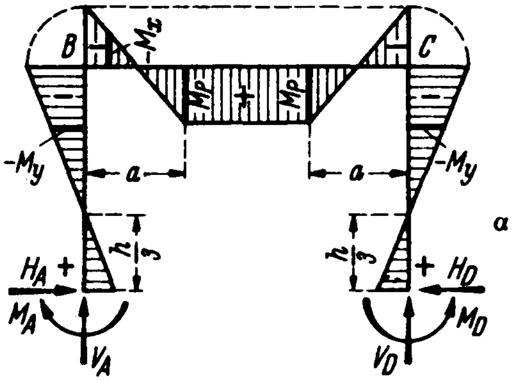
$$V_A = P\beta\left[1 + \frac{\alpha(\beta - \alpha)}{D_2}\right] \qquad V_D = P - V_A \qquad H_A = H_D = \frac{3\,P\,a\,b}{2\,l\,h D_1}$$

$$M_P = \frac{P\,a\,b}{l} + \beta\,M_B + \alpha\,M_C \qquad M_{y1} = M_A - H_A\,y_1 \qquad M_{y2} = M_D - H_D\,y_2$$

Nel tratto a: $M_x = V_A\,x + M_B$; Nel tratto b: $M_x = V_D\,x' + M_C$.

Caso 41/5 - Due carichi concentrati in posizione generica ma simmetrici sul traverso





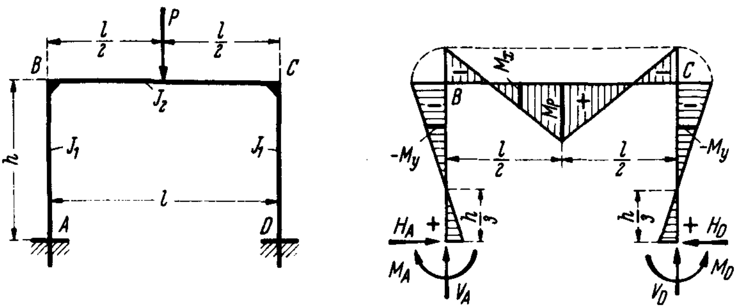
$$\alpha = \frac{a}{l}$$

$$M_A = M_D = + \frac{Pa\,(1 - \alpha)}{D_1} \qquad M_B = M_C = -2\,M_A \qquad M_P = P\,a + M_B$$

$$V_A = V_D = P \qquad H_A = H_D = \frac{3\,M_A}{h}$$

Nel tratto a: $M_x = P\,x + M_B \qquad M_y = M_A - H_A\,y$.

Caso 41/6 - Carico concentrato in mezzeria del traverso (Caso simmetrico)

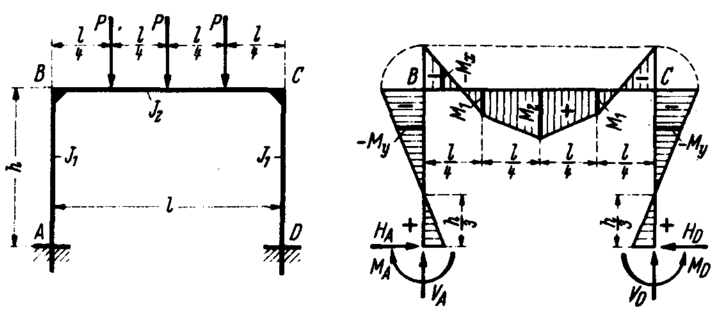


$$M_A = M_D = +\frac{P\,l}{8D_1} \qquad M_B = M_C = -2\,M_A$$

$$V_A = V_D = \frac{P}{2} \qquad H_A = H_D = \frac{3\,M_A}{h} \qquad M_P = \frac{P\,l}{4} + M_B$$

$$M_x = \frac{P\,x}{2} + M_B \qquad M_y = M_A - H_A\,y\,.$$

Caso 41/7 - Tre carichi concentrati sul traverso (Caso simmetrico)



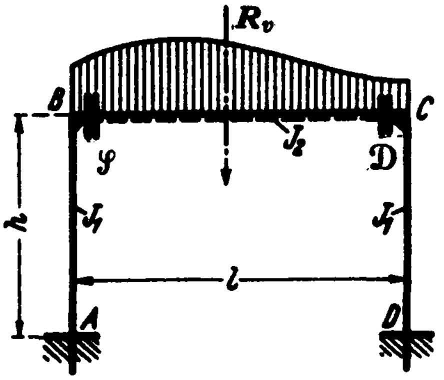
$$M_A = M_D = +\frac{5\,P\,l}{16D_1} \qquad M_B = M_C = -2\,M_A \qquad H_A = H_D = \frac{3\,M_A}{h}$$

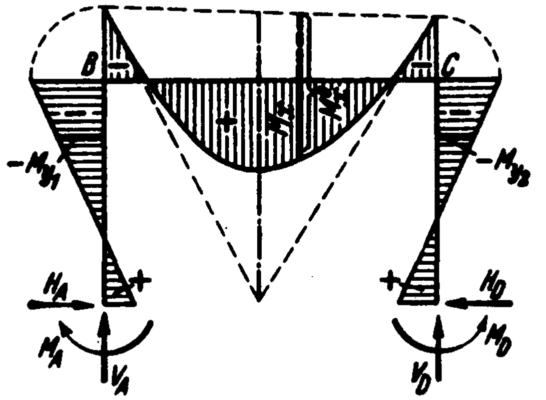
$$M_1 = \frac{3\,P\,l}{8} + M_B \qquad M_2 = \frac{P\,l}{2} + M_B \qquad V_A = V_D = \frac{3\,P}{2}$$

Nel tratto $B-1$: $\qquad M_x = V_A\,x + M_B \qquad M_y = M_A - H_A\,y\,.$

V. il paragrafo *Termini di carico*

Caso 41/8 - Carico verticale generico sul traverso [1]





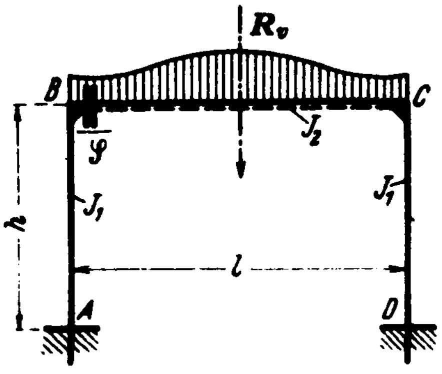
Grandezze ausiliarie: $X_1 = \dfrac{(C_s + C_d)}{6D_1}$ $\qquad X_3 = \dfrac{(C_s - C_d)}{2D_3}$.

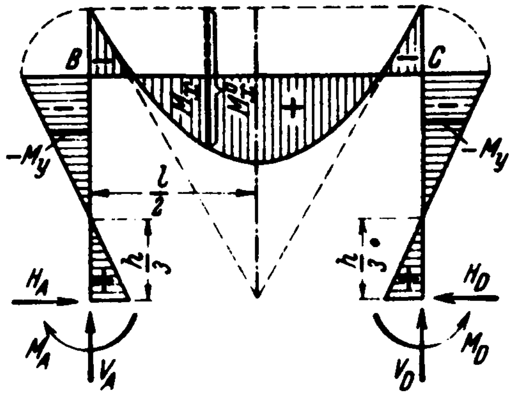
$$\left.\begin{matrix} M_A \\ M_D \end{matrix}\right\rangle = +X_1 \mp X_3 \qquad \left.\begin{matrix} M_B \\ M_C \end{matrix}\right\rangle = -2X_1 \mp X_3\,;$$

$$V_A = \frac{S_d + 2X_3}{l} \qquad V_D = R_v - V_A \qquad H_A = H_D = \frac{3X_1}{h};$$

$$M_{y1} = M_A - H_A y_1 \qquad M_x = M_x^0 + \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = M_D - H_D y_2\,.$$

Caso 41/9 - Carico verticale generico, ma simmetrico, sul traverso





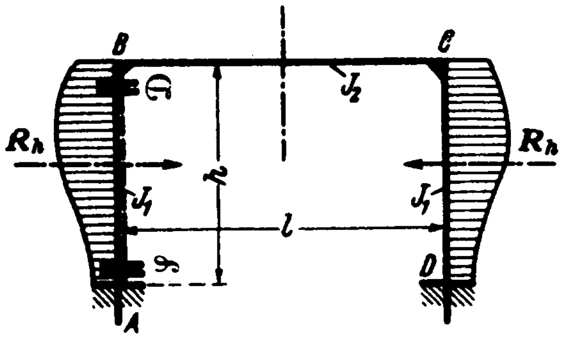
$$M_A = M_D = +\frac{C_s}{3D_1}; \qquad H_A = H_D = \frac{3M_A}{h} \qquad V_A = V_D = \frac{R_v}{2};$$

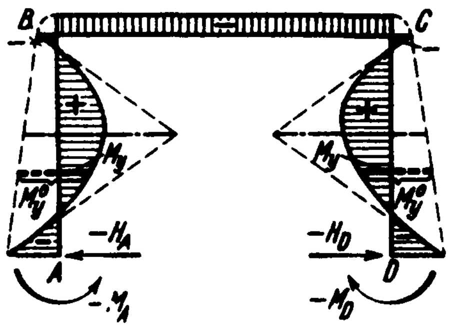
$$M_B = M_C = -2M_A \qquad M_x = M_x^0 + M_B \qquad M_y = M_A - H_A y\,.$$

[1] Con carico antisimmetrico ($C_s = -C_d$) si ha $X_1 = 0$ e $X_3 = C_s/D_3$; quindi $M_D = M_C = -M_A = -M_B = C_s/D_3$ e $H_A = H_D = 0$.

V. il paragrafo *Termini di carico*

Caso 41/10 - Carichi orizzontali generici ma uguali agenti verso l'interno sui ritti (Caso simmetrico)[1]

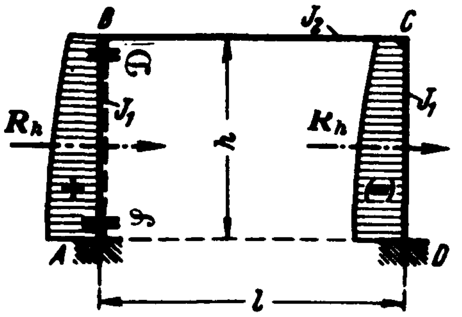


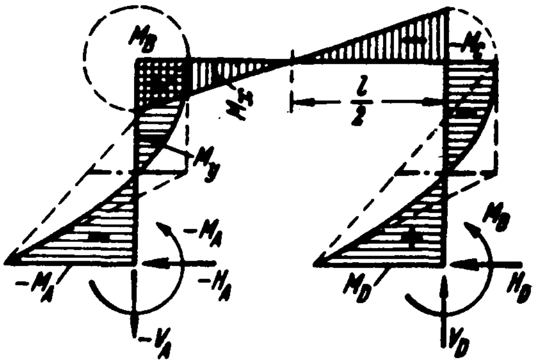


$$M_A = M_D = -\frac{C_s\,(2k+3) - C_d\,k}{3D_1} \qquad M_B = M_C = -\frac{(2C_d - C_s)\,k}{3D_1};$$

$$H_A = H_D = -\frac{S_d - M_A + M_B}{h}; \qquad M_y = M_y^0 + \frac{y'}{h}M_A + \frac{y}{h}M_B.$$

Caso 41/11 - Carichi orizzontali generici ma uguali agenti verso destra sui ritti (Caso antisimmetrico)[1]





$$M_B = -M_C = [3S_s - (C_s + C_d)]\,\frac{k}{D_2} \qquad M_D = -M_A = S_s - M_B; \qquad H_D = -H_A = R_h$$

$$V_D = -V_A = \frac{2M_B}{l} \qquad M_y = M_y^0 + \frac{y'}{h}M_A + \frac{y}{h}M_B \qquad M_x = \frac{x' - x}{l}\cdot M_B$$

Caso particolare 41/11a: Carico uniformemente distribuito $R_h = q\,h$

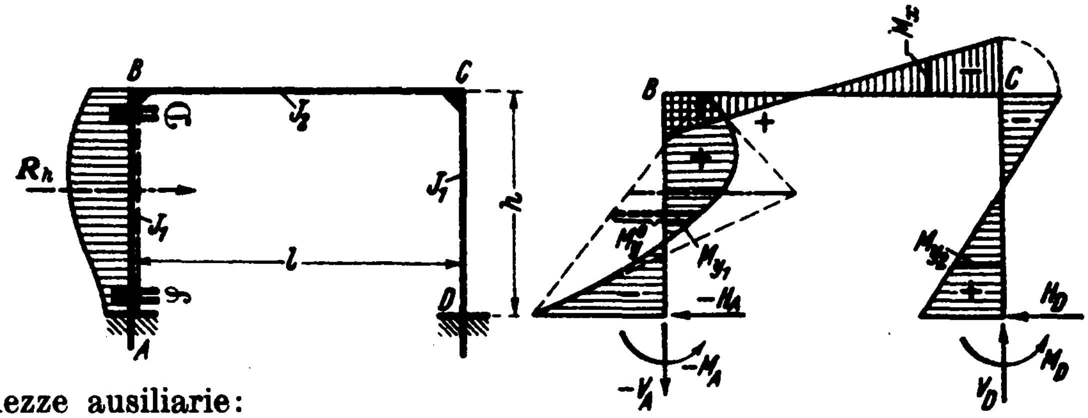
$$M_B = -M_C = q\,h^2\cdot\frac{k}{D_2} \qquad M_D = -M_A = \frac{q\,h^2}{2}\cdot\frac{4k+1}{D_2} \qquad M_y^0 = \frac{q\,y\,y'}{2}$$

Tutte le altre formule rimangono inalterate.

[1] Tutti i termini di carico sono riferiti al ritto sinistro.

Caso 41/12 - Carico orizzontale generico sul ritto sinistro

V. il paragrafo *Termini di carico*



Grandezze ausiliarie:

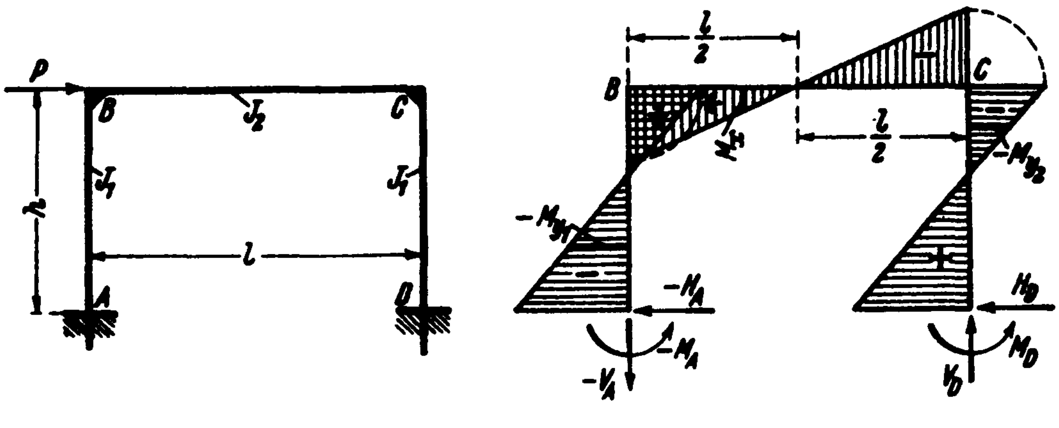
$$X_1 = \frac{C_s(2k+3) - C_d\,k}{6D_1} \qquad X_2 = \frac{(2C_d - C_s)\,k}{6D_1} \qquad X_3 = \frac{[3S_s - (C_s + C_d)]\,k}{2D_2}$$

$$\left.\begin{matrix} M_A \\ M_D \end{matrix}\right\rangle = -X_1 \mp \left(\frac{S_s}{2} - X_3\right) \qquad \left.\begin{matrix} M_B \\ M_C \end{matrix}\right\rangle = -X_2 \pm X_3;$$

$$H_D = \frac{S_s}{2h} - \frac{X_1 - X_2}{h} \qquad H_A = -(R_h - H_D) \qquad V_D = -V_A = \frac{2X_3}{l};$$

$$M_{y1} = M_y^0 + \frac{y_1'}{h} M_A + \frac{y_1}{h} M_B \qquad M_x = M_C + V_D\,x' \qquad M_{y2} = M_D - H_D\,y_2$$

Caso 41/13 - Carico concentrato orizzontale all'altezza del traverso

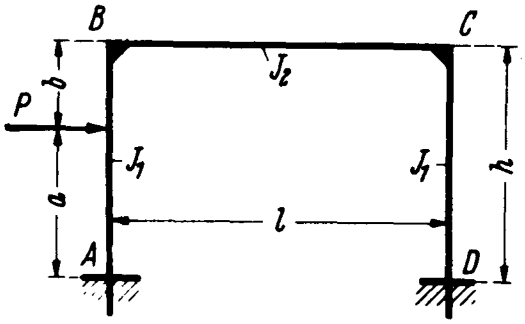


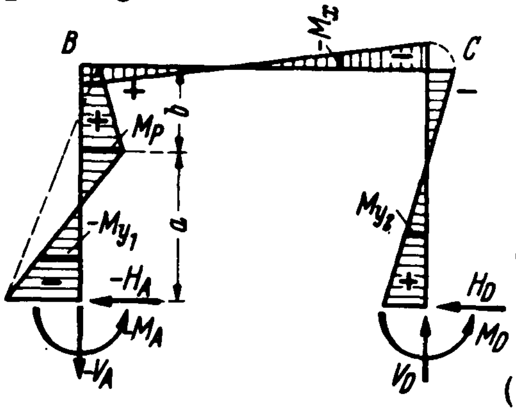
$$\left.\begin{matrix} M_A \\ M_D \end{matrix}\right\rangle = \mp \frac{Ph}{2} \cdot \frac{3k+1}{D_2} \qquad \left.\begin{matrix} M_B \\ M_C \end{matrix}\right\rangle = \pm \frac{Ph}{2} \cdot \frac{3k}{D_2};$$

$$H_D = -H_A = \frac{P}{2}; \qquad V_D = -V_A = \frac{2M_B}{l};$$

$$M_{y1} = M_A + \frac{P}{2} y_1 \qquad M_x = \frac{x' - x}{l} M_B \qquad M_{y2} = M_D - \frac{P}{2} y_2.$$

Caso 41/14 - Carico concentrato in un punto generico del ritto sinistro





$\alpha = \frac{a}{h}$

$\beta = \frac{b}{h}$

Grandezze ausiliarie: $(\alpha + \beta$

$$X_1 = \frac{Pab}{h} \cdot \frac{1+\beta+\beta k}{2D_1}, \qquad X_2 = \frac{Pab}{h} \cdot \frac{\alpha k}{2D_1}, \qquad X_3 = \frac{3Pa\alpha k}{2D_2}$$

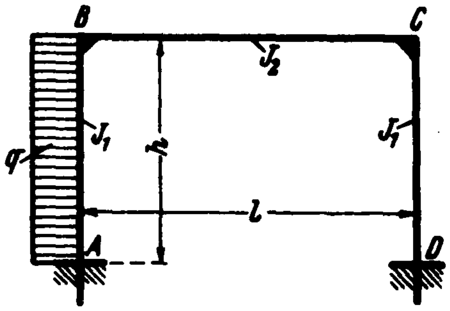
$$\left.\begin{matrix} M_A \\ M_D \end{matrix}\right\rangle = -X_1 \mp \left(\frac{Pa}{2} - X_3\right) \qquad \left.\begin{matrix} M_B \\ M_C \end{matrix}\right\rangle = -X_2 \pm X_3$$

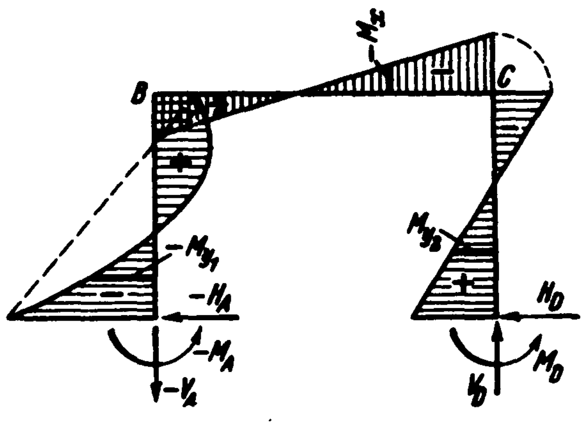
$$H_D = \frac{Pa}{2h} - \frac{X_1 - X_2}{h} \qquad H_A = -(P - H_D) \qquad V_A = -V_D = -\frac{2X_3}{l}$$

$$M_P = \frac{Pab}{h} + \beta M_A + \alpha M_B \qquad M_x = M_C + V_D x' \qquad M_{y2} = M_D - H_D y_2$$

Nel tratto a: $M_{y1} = M_A - H_A y_1$; Nel tratto b: $M_{y1} = M_B + H_D y_1'$.

Caso 41/15 - Carico uniformemente distribuito sul ritto sinistro





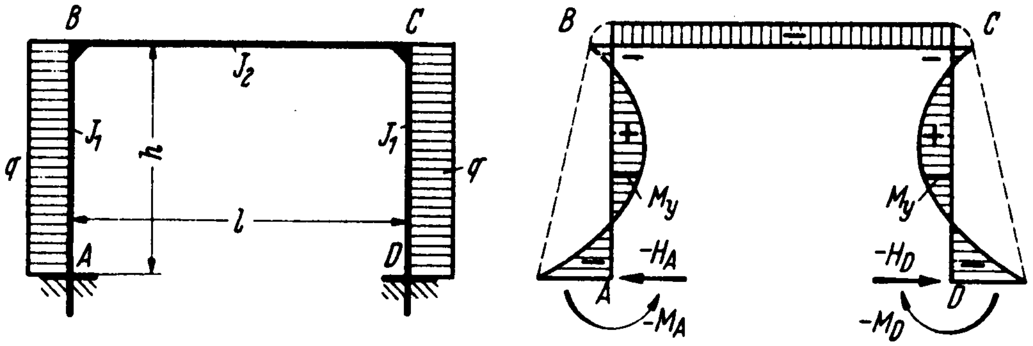
$$\left.\begin{matrix} M_A \\ M_D \end{matrix}\right\rangle = \frac{qh^2}{4}\left[-\frac{k+3}{6D_1} \mp \frac{4k+1}{D_2}\right] \qquad \left.\begin{matrix} M_B \\ M_C \end{matrix}\right\rangle = \frac{qh^2}{4}\left[-\frac{k}{6D_1} \pm \frac{2k}{D_2}\right];$$

$$H_D = \frac{qh(2k+3)}{8D_1} \qquad H_A = -(qh - H_D); \qquad V_D = -V_A = \frac{qh^2 k}{lD_2};$$

$$M_{y1} = \frac{q y_1 y_1'}{2} + \frac{y_1'}{h} M_A + \frac{y_1}{h} M_B$$

$$M_x = M_C + V_D x' \qquad M_{y2} = M_D - H_D y_2.$$

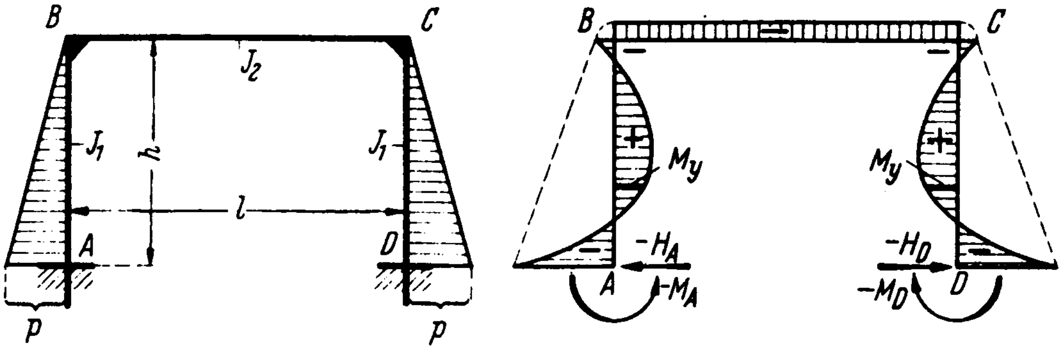
Caso 41/16 - Carico uniformemente distribuito su entrambi i ritti (Caso simmetrico)



$$M_A = M_D = -\frac{q\,h^2}{12} \cdot \frac{k+3}{D_1} \qquad M_B = M_C = -\frac{q\,h^2}{12} \cdot \frac{k}{D_1}$$

$$H_A = H_D = -\frac{q\,h}{4} \cdot \frac{2k+5}{D_1} \qquad M_y = \frac{q\,y\,y'}{2} + \frac{y'}{h} M_A + \frac{y}{h} M_B\,.$$

Caso 41/17 - Carico con distribuzione triangolare su entrambi i ritti (Caso simmetrico)

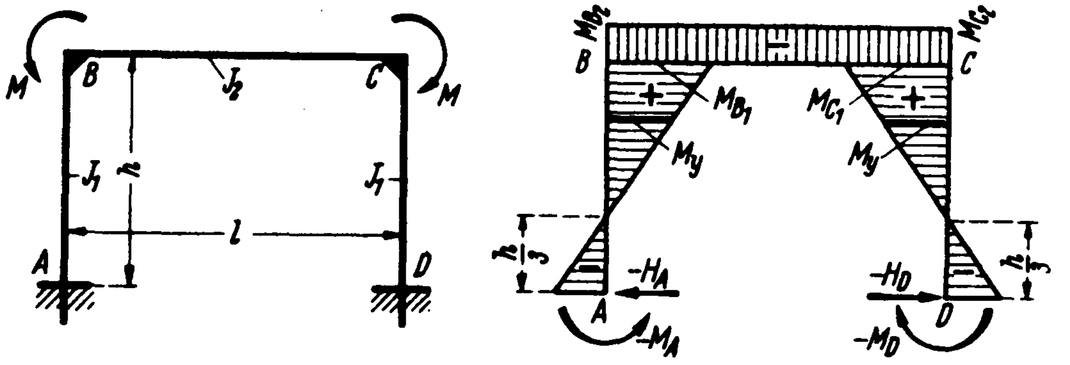


$$M_A = M_D = -\frac{p\,h^2}{60} \cdot \frac{3k+8}{D_1} \qquad M_B = M_C = -\frac{p\,h^2\,k}{30 D_1}$$

$$H_A = H_D = -\frac{p\,h}{20} \cdot \frac{7k+16}{D_1}$$

$$M_y = \frac{p\,h^2}{6} \cdot \omega_D' + \frac{y'}{h} M_A + \frac{y}{h} M_B \qquad \text{con} \qquad \omega_D' = \frac{y'}{h} - \left(\frac{y'}{h}\right)^3$$

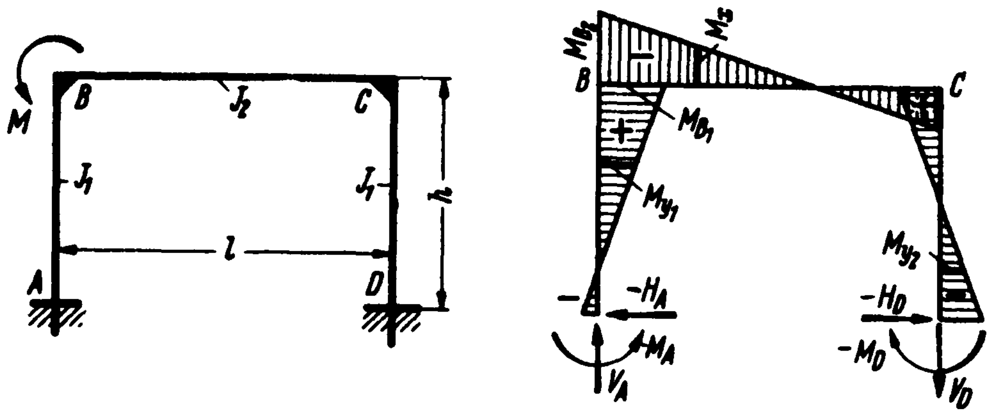
Caso 41/18 - Coppie applicate ai nodi B e C in modo simmetrico



$$M_A = M_D = -\frac{M}{D_1} \qquad M_{B1} = M_{C1} = +\frac{2\,M}{D_1} \qquad M_{B2} = M_{C2} = -\frac{M\,k}{D_1}$$

$$(M_{B1} - M_{B2} = M) \qquad H_A = H_D = -\frac{3\,M}{h\,D_1} \qquad M_y = M_A - H_A\,y.$$

Caso 41/19 - Coppia al nodo B

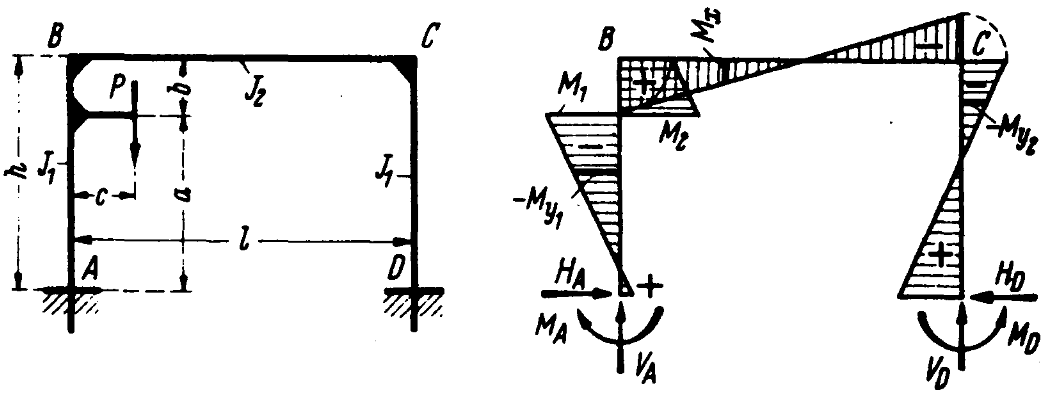


$$\left.\begin{matrix} M_A \\ M_D \end{matrix}\right\rangle = -\frac{M}{2D_1} \pm \frac{M}{2\,D_2} \qquad \left.\begin{matrix} M_{B1} \\ M_C \end{matrix}\right\rangle = +\frac{M}{D_1} \pm \frac{M}{2D_2}$$

$$M_{B2} = -(M_{B1} - M) \qquad H_A = H_D = -\frac{3\,M}{2\,hD_1} \qquad V_A = -V_D = \frac{6\,M\,k}{lD_2}$$

$$M_{y1} = M_A - H_A\,y_1 \qquad M_x = M_C + V_D\,x' \qquad M_{y2} = M_D - H_D\,y_2\,.$$

Caso 41/20 - Carico concentrato su una mensola del ritto sinistro



$$\alpha = \frac{a}{h} \qquad \beta = \frac{b}{h} \qquad (\alpha + \beta = 1)$$

Grandezze ausiliarie:

$$X_1 = \frac{Pc}{2D_1}[1 + 2\beta k - 3\beta^2(k+1)], \qquad X_2 = \frac{Pck\alpha(3\alpha - 2)}{2D_1},$$

$$X_3 = \frac{3Pck\alpha}{D_2}. \qquad \begin{matrix} M_A \\ M_D \end{matrix}\Big\rangle = +X_1 \mp \left(\frac{Pc}{2} - X_3\right) \qquad \begin{matrix} M_B \\ M_C \end{matrix}\Big\rangle = +X_2 \pm X_3$$

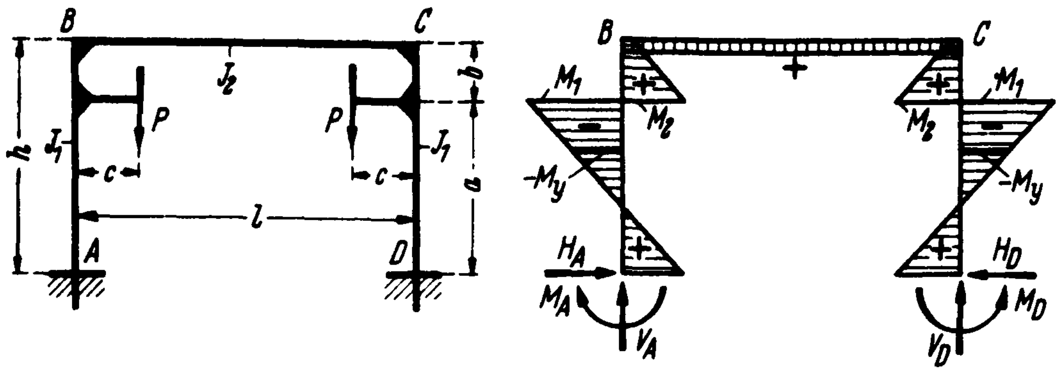
$$H_A = H_D = \frac{Pc}{2h} + \frac{X_1 - X_2}{h} \qquad V_D = \frac{2X_3}{l} \qquad V_A = P - V_D$$

$$M_1 = M_A - H_A a \qquad M_2 = M_B + H_D b \qquad (M_2 - M_1 = Pc)$$

Nel tratto a: $M_{y1} = M_A - H_A y_1$ $\qquad M_x = M_C + V_D x'$

Nel tratto b: $M_{y1} = M_B + H_D y_1'$ $\qquad M_{y2} = M_D - H_D y_2$.

Caso 41/21 - Carico concentrato su mensole di entrambi i ritti (Caso simmetrico)



$$\alpha = \frac{a}{h} \qquad \beta = \frac{b}{h} \qquad (\alpha + \beta = 1)$$

$$M_A = M_D = \frac{Pc}{D_1}[1 + 2\beta k - 3\beta^2(k+1)] = 2X_1\text{[1]} \qquad V_A = V_D = P$$

$$M_B = M_C = \frac{Pck\alpha(3\alpha - 2)}{D_1} = 2X_2\text{[1]} \qquad H_A = H_D = \frac{Pc + M_A - M_B}{h}$$

$$M_1 = M_A - H_A a \qquad M_2 = M_B + H_D b \qquad (M_2 - M_1 = Pc)$$

Nel tratto a: $M_y = M_A - H_A y$; Nel tratto b: $M_y = M_B + H_D y'$.

[1] X_1 e X_2 come a pag. 165.