



U.S. Department  
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Federal Highway  
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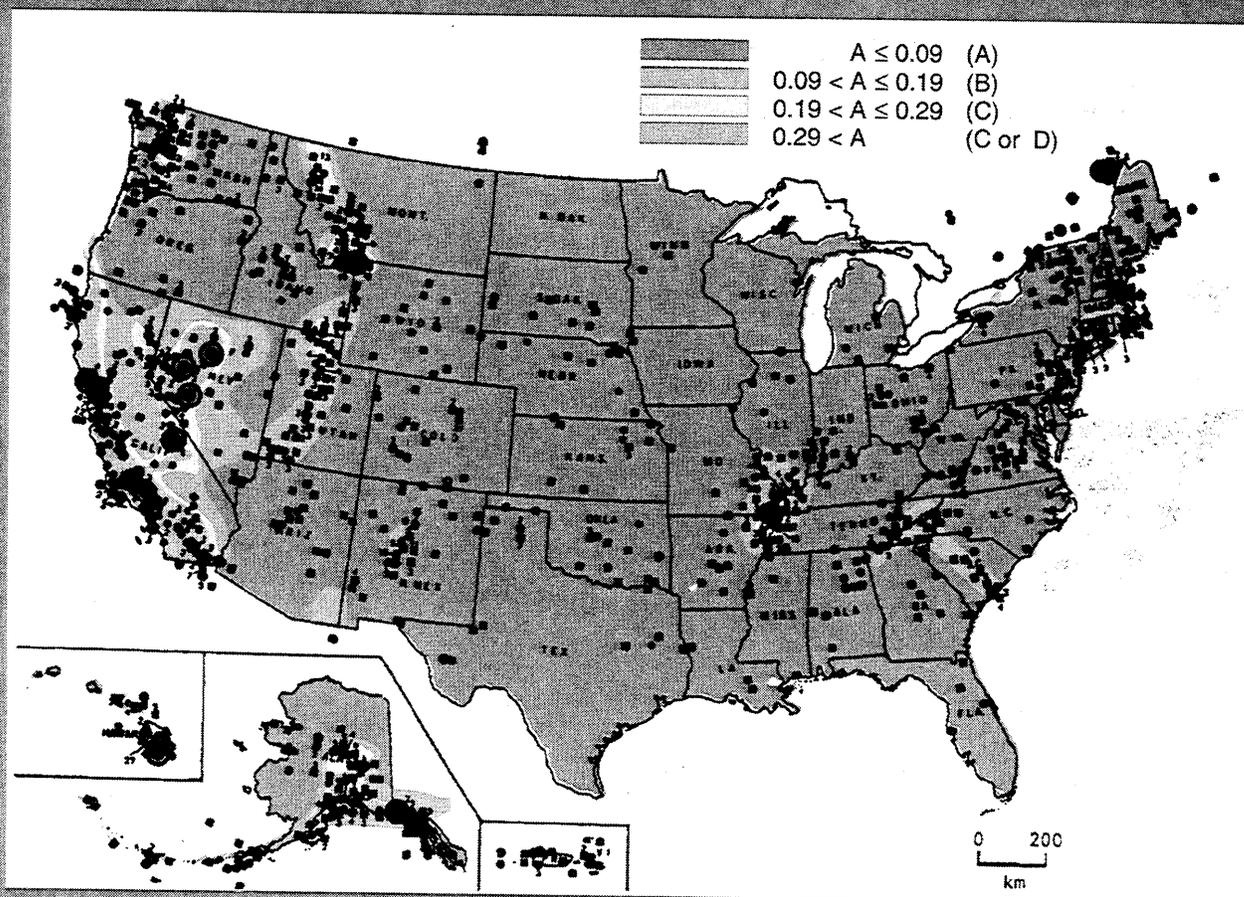
MATARAZO

October 1996

# *Seismic Design of Bridges*

## *Design Example No. 3*

### *Single Span AASHTO Precast Girder Bridge*



Publication No. FHWA-SA-97-008



Technical Report Documentation Page

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16. Abstract This document describes one of seven seismic design examples that illustrate "how" to apply AASHTO's seismic analysis and design requirements on actual different bridge types across the United States. Each provides a complete set of "designer's notes" covering the seismic analysis, design, and details for that particular bridge including flow charts, references to applicable AASHTO requirements, and thorough commentary that explains each step. In addition, each example highlights separate issues (skew effects, wall piers, elastomeric bearings, pile foundations, etc.). The <b>first example</b> is a 242' reinforced concrete box girder two span overcrossing with spread footing foundations, SPC-C & A = 0.28g. The <b>second example</b> is a 400' 3-span skewed steel plate girder bridge over a river in New England with spread footing foundations, SPC-B & A = 0.15g. The <b>third example</b> is a skewed 70' single span prestressed concrete girder bridge with tall-closed seat-type abutments on spread footings, SPC-C & A = 0.36g. The <b>fourth example</b> is a 320' reinforced concrete box girder 3-span skewed bridge in the western United States with spread footing foundations, SPC-C & A = 0.30g. The <b>fifth example</b> is a 1488' steel plate girder bridge in the inland Pacific Northwest with pile foundations, SPC-B & A = 0.15g. It has nine spans and consists of two units: a four-span tangent (Unit 1) and a five-span with a 1300-foot radius curve (Unit 2). The <b>sixth example</b> is a 290' sharply curved (104 degrees) 3-span concrete box girder bridge in the Northwestern United States with pile abutment foundations and drilled shaft pier foundations, SPC-C & A = 0.20g. The <b>seventh example</b> is a 717' 10-span prestressed girder bridge with open pile bents, SPC-B & A = 0.10g. The superstructure consists of three continuous span units arranged in a 3-4-3 span series.					
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# **Seismic Design Course**

## **Design Example No. 3**

**Prepared for**

**U.S. Department of Transportation  
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**PLEASE NOTE**

Data, specifications, suggested practices, and drawings presented herein are based on the best available information, are delineated in accord with recognized professional engineering principles and practices, and are provided for general information only. None of the procedures suggested or discussed should be used without first securing competent advice regarding their suitability for any given application.

This document was prepared with the help and advice of FHWA, State, academic, and private engineers. The intent of this document is to aid practicing engineers in the application of the AASHTO seismic design specification. BERGER/ABAM and the United States Government assume no liability for its contents or use thereof.

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**Section I**  
**Introduction**

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**PURPOSE  
OF DESIGN  
EXAMPLE**

This is the third in a series of seismic design examples developed for the FHWA. A different bridge configuration is used in each example. The bridges are in either Seismic Performance Category B or C sites. Each example emphasizes different features that must be considered in the seismic analysis and design process. The matrix below is a summary of the features of the first seven examples.

DESIGN EXAMPLE NO.	DESIGN EXAMPLE DESCRIPTION	SEISMIC CATEGORY	PLAN GEOMETRY	SUPER-STRUCTURE TYPE	PIER TYPE	ABUTMENT TYPE	FOUNDATION TYPE	CONNECTIONS AND JOINTS
1	Two-Span Continuous	SPC - C	Tangent Square	CIP Concrete Box	Three-Column Integral Bent	Seat Stub Base	Spread Footings	Monolithic Joint at Pier Expansion Bearing at Abutment
2	Three-Span Continuous	SPC - B	Tangent Skewed	Steel Girder	Wall Type Pier	Tall Seat	Spread Footings	Elastomeric Bearing Pads (Piers and Abutments)
3	Single-Span	SPC - C	Tangent Square	AASHTO Precast Concrete Girders	(N/A)	Tall Seat (Closed-In)	Spread Footings	Elastomeric Bearing Pads
4	Three-Span Continuous	SPC - C	Tangent Skewed	CIP Concrete	Two-Column Integral Bent	Seat	Spread Footings	Monolithic at Col. Tops Pinned Column at Base Expansion Bearings at Abutments
5	Nine-Span Viaduct with Four-Span and Five-Span Continuous Struc.	SPC - B	Curved Square	Steel Girder	Single-Column (Variable Heights)	Seat	Steel H-Piles	Conventional Steel Pins and PTFE Sliding Bearings
6	Three-Span Continuous	SPC - C	Sharply-Curved Square	CIP Concrete Box	Single Column	Monolithic	Drilled Shaft at Piers, Steel Piles at Abutments	Monolithic Concrete Joints
7	12-Span Viaduct with (3) Four-Span Structures	SPC - B	Tangent Square	AASHTO Precast Concrete Girders	Pile Bents (Battered and Plumb)	Seat	Concrete Piles and Steel Piles	Pinned and Expansion Bearings

**REFERENCE  
AASHTO  
SPECIFICATIONS**

The examples conform to the following specifications.

**AASHTO Division I (herein referred to as "Division I")**

*Standard Specifications for Highway Bridges*, American Association of State Highway and Transportation Officials, Inc., 15th Edition, as amended by the Interim Specifications-Bridges-1993 through 1995.

**AASHTO Division I-A (herein referred to as "Division I-A" or the "Specification")**

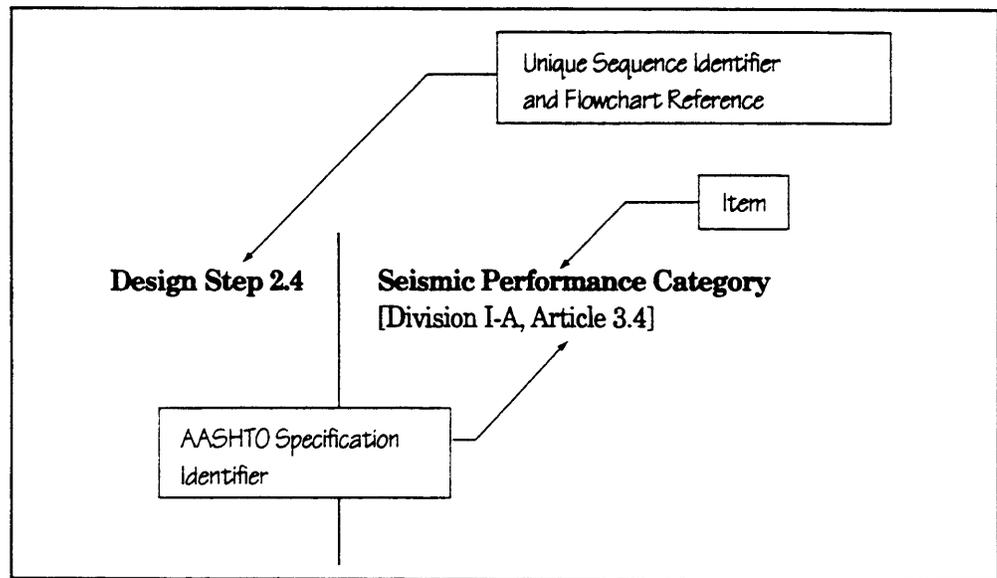
*Standard Specifications for Highway Bridges, Division I-A, Seismic Design*, American Association of State Highway and Transportation Officials, Inc., 15th Edition, as amended by the Interim Specifications-Bridges-1995.

**FLOWCHARTS  
AND  
DESIGN STEPS**

This third example follows the outline given in detailed flowcharts presented in Section II, Flowcharts. The flowcharts include a main chart, which generally follows the one currently used in AASHTO Division I-A, and several subcharts that detail the operations that occur for each Design Step.

The purpose of Design Steps is to present the information covered by the example in a logical and sequential manner that allows for easy referencing within the example itself. Each Design Step has a unique number in the left margin of the calculation document. The title is located to the right of the Design Step number. Where appropriate, a reference to either Division I or Division I-A of the AASHTO Specification follows the title.

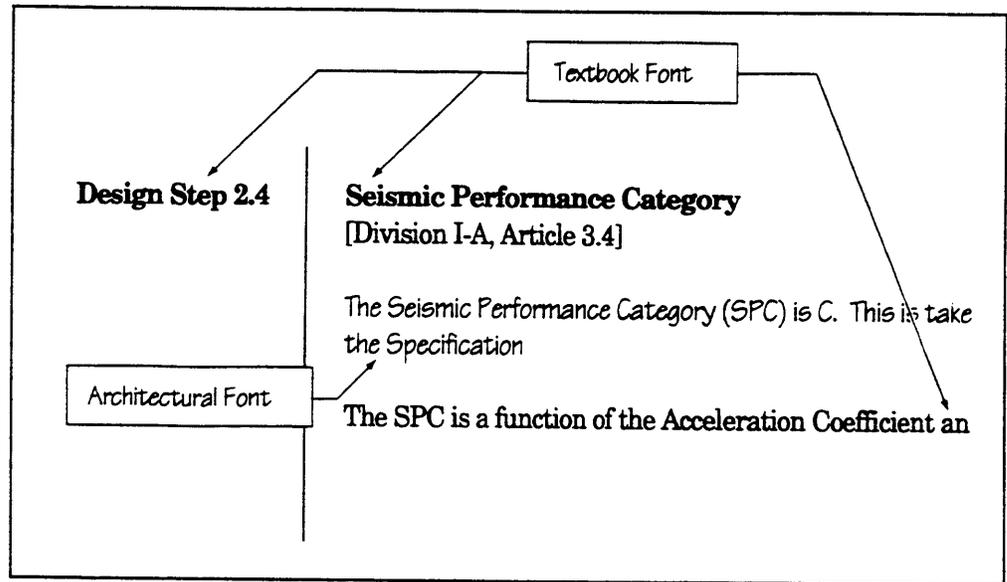
An example is shown below.



**USE OF  
DIFFERENT  
TYPE FONTS**

In the example, two primary type fonts have been used. One font, similar to the type used for textbooks, is used for all section headings and for commentary. The other, an architectural font that appears hand printed, is used for all primary calculations. The material in the architectural font is the essential calculation material and essential results.

An example of the use of the fonts is shown below.

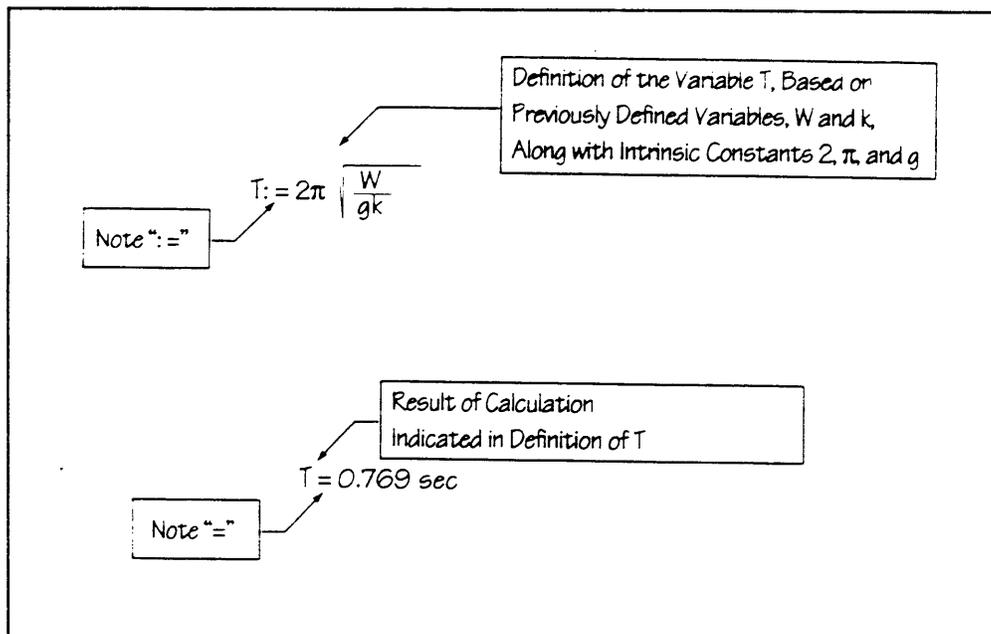


**USE OF  
MATHCAD®**

To provide consistent results and quality control, all calculations have been performed using the program Mathcad®.

The variables used in equations calculated by the program are defined before the equation, and the **definition** of either a variable or an equation is distinguished by a ':=' symbol. The **echo** of a variable or the result of a calculation is distinguished by a '=' symbol, i.e., no colon is used.

An example is shown below.



Note that Mathcad® carries the full precision of the variables throughout the calculations, even though the listed result of a calculation is rounded off. Thus, hand-calculated checks made using intermediate rounded results may not yield the same result as the number being checked.

Also, Mathcad® does not allow the superscript “^” to be used in a variable name. Therefore, the specified compressive strength of concrete is defined as  $f_c$  in this example (not  $f^c$ ).

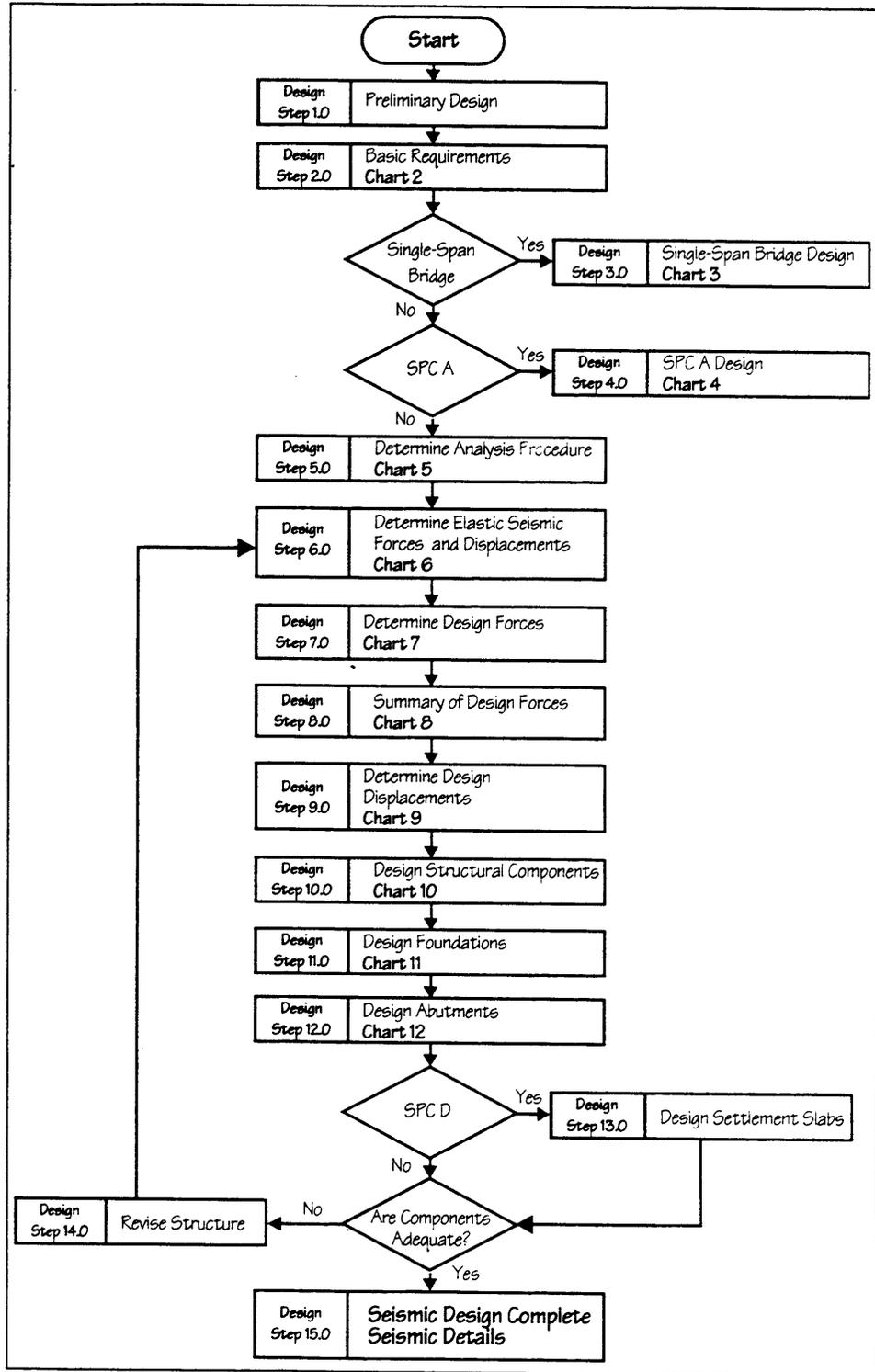


**Section II**  
**Flowcharts**

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FLOWCHARTS



Main Flowchart — Seismic Design AASHTO Division I-A

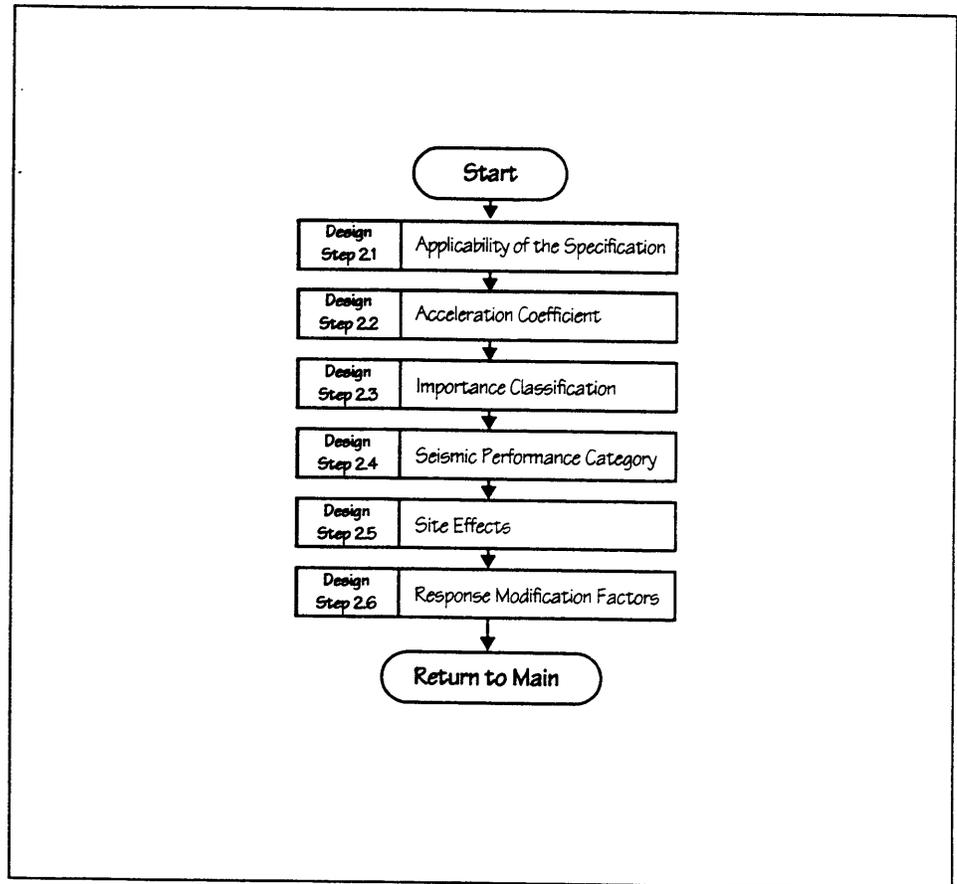
**FLOWCHARTS**  
(continued)

**Key to Detailed Flowcharts**

- Design Step 1.0 — Not Focused on in Example No. 3
- Design Step 2.0 — Page 2-3
- Design Step 3.0 — Page 2-4
- Design Step 4.0 — Not Applicable for Example No. 3
- Design Step 5.0 — Not Required for Example No. 3
- Design Step 6.0 — Not Applicable for Example No. 3
- Design Step 7.0 — Page 2-5
- Design Step 8.0 — Not Required for Example No. 3
- Design Step 9.0 — Page 2-6
- Design Step 10.0 — Not Applicable for Example No. 3
- Design Step 11.0 — Not Applicable for Example No. 3
- Design Step 12.0 — Page 2-7
- Design Step 13.0 — Not Applicable for Example No. 3

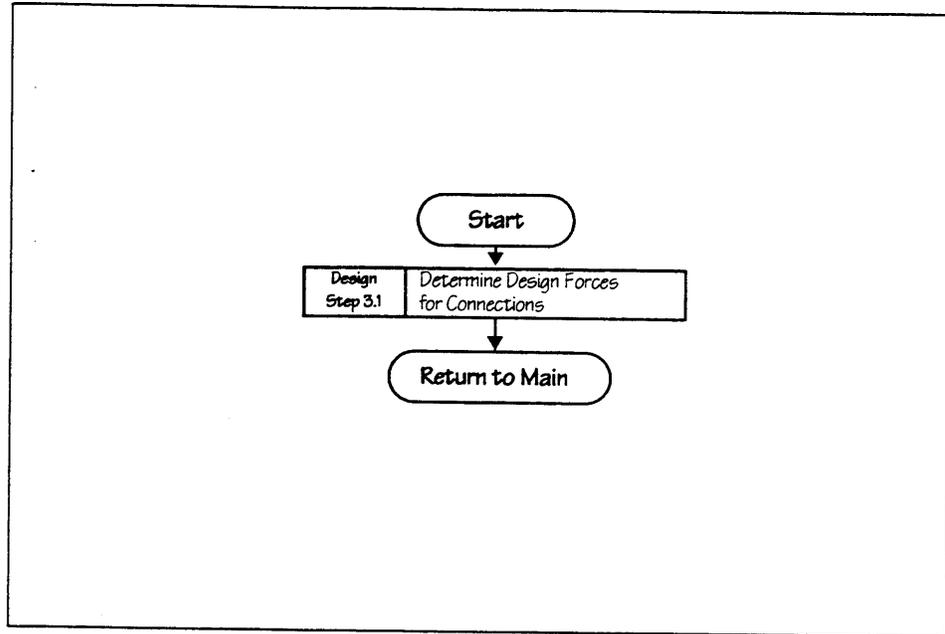
**Chart 1 — Preliminary Design**

**FLOWCHARTS**  
(continued)



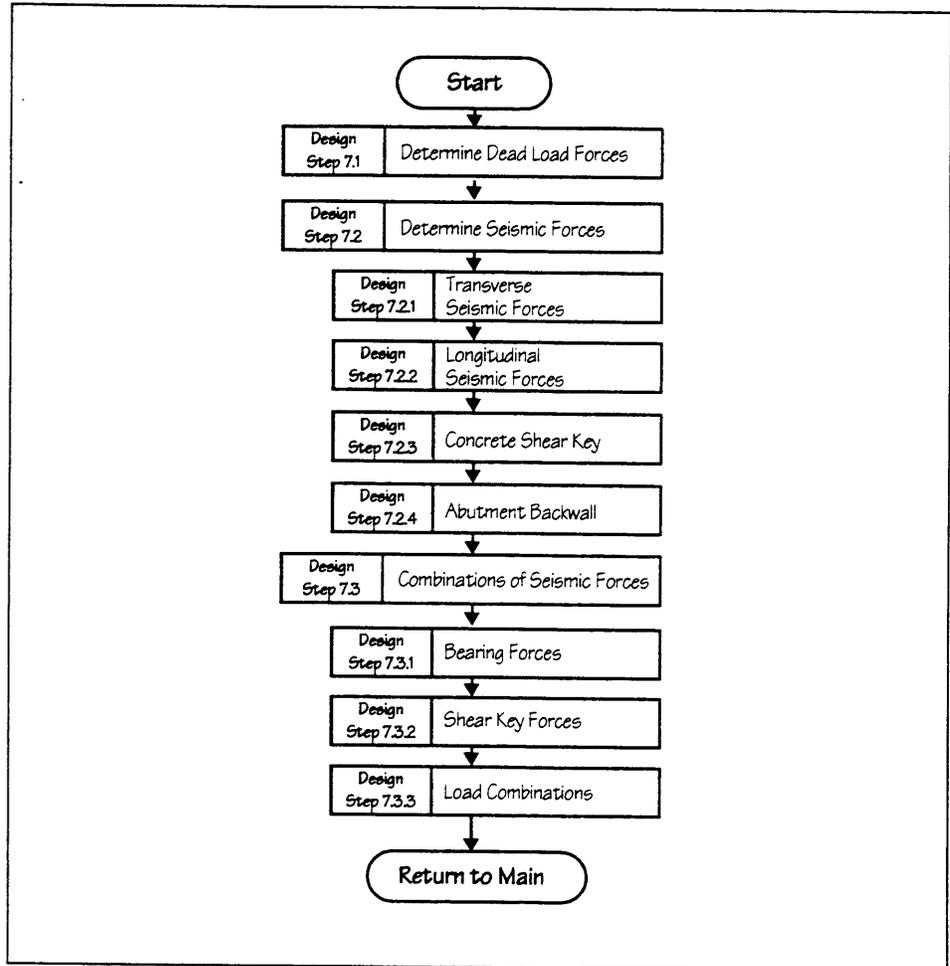
**Chart 2 — Basic Requirements**

**FLOWCHARTS**  
(continued)



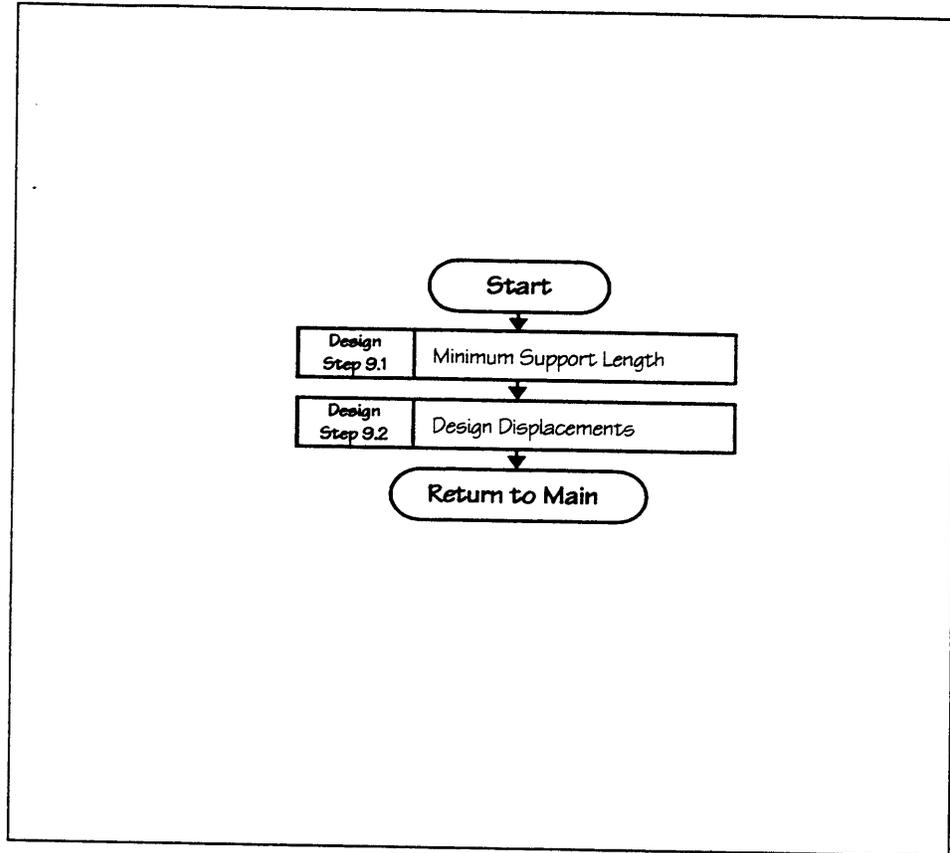
**Chart 3 — Single-Span Bridge Design**

**FLOWCHARTS**  
(continued)



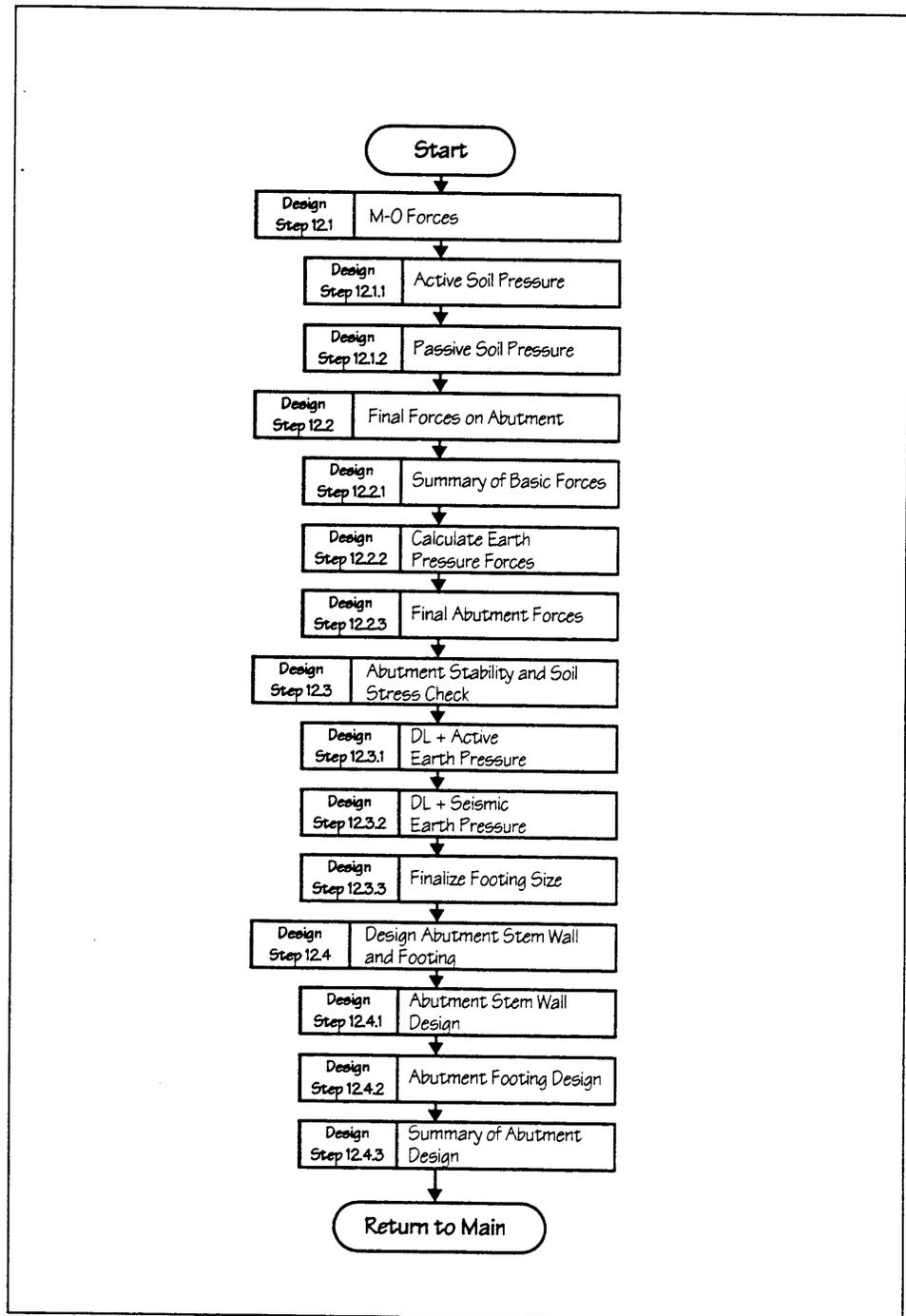
**Chart 7 — Determine Design Forces (SPC C)**

**FLOWCHARTS**  
(continued)



**Chart 9 — Determine Design Displacements**

**FLOWCHARTS**  
(continued)



**Chart 12 — Design Abutments**



**Section III**  
**Analysis and Design**

---



**SECTION III****ANALYSIS AND DESIGN****DATA**

The bridge is to be built in the Mississippi Valley in a seismic zone with an acceleration coefficient of 0.36g.

The configuration of the bridge consists of a single-span superstructure using precast concrete AASHTO girders and a CIP concrete deck. The substructure consists of tall-closed seat-type abutments with retaining walls parallel to the abutment with expansion joints provided at both ends of the bridge superstructure. Figure 1 (a to e) provides details of the bridge configuration.

It should be noted that single-span bridges sometimes use integral abutment superstructure details without expansion joints. Integral abutment details restrain movement of the superstructure and also the top of the substructure. The level of restraint depends on the bearing detail. These bridges may develop higher seismic earth pressures behind the abutment wall, than in this example.

The alignment of the roadway on the bridge is straight and there is no vertical curve. The bridge has a 28 degree skew to the roadway below, thus the ends of the bridge, including abutments, are skewed also.

**REQUIRED**

Design the bridge for seismic loading using the *Standard Specifications for Highway Bridges, Division I-A, Seismic Design*, American Association of State Highway and Transportation Officials, Inc., 15th Edition, as amended by the Interim Specifications-Bridges-1995.

**FEATURES****ISSUES EMPHASIZED IN THIS EXAMPLE**

- Single-Span Bridge
- Tall-Closed Seat-Type Abutments (not fixed to superstructure)
- SPC C Design
- Skew Effects on Abutment
- Seismic Earth Pressure Forces on the Abutment Walls as Developed from the Mononobe-Okabe Equations

**BRIDGE DATA**  
(continued)

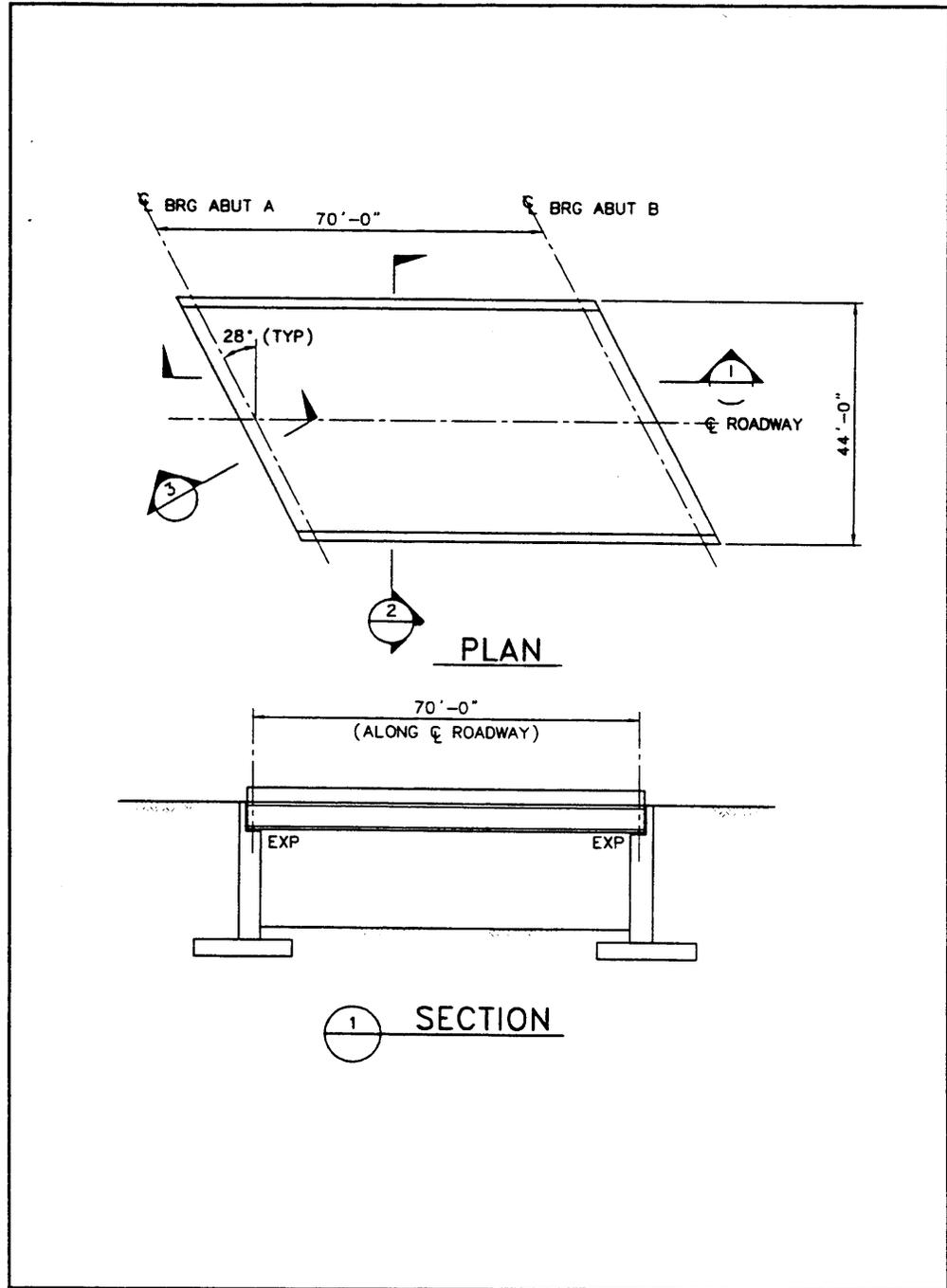
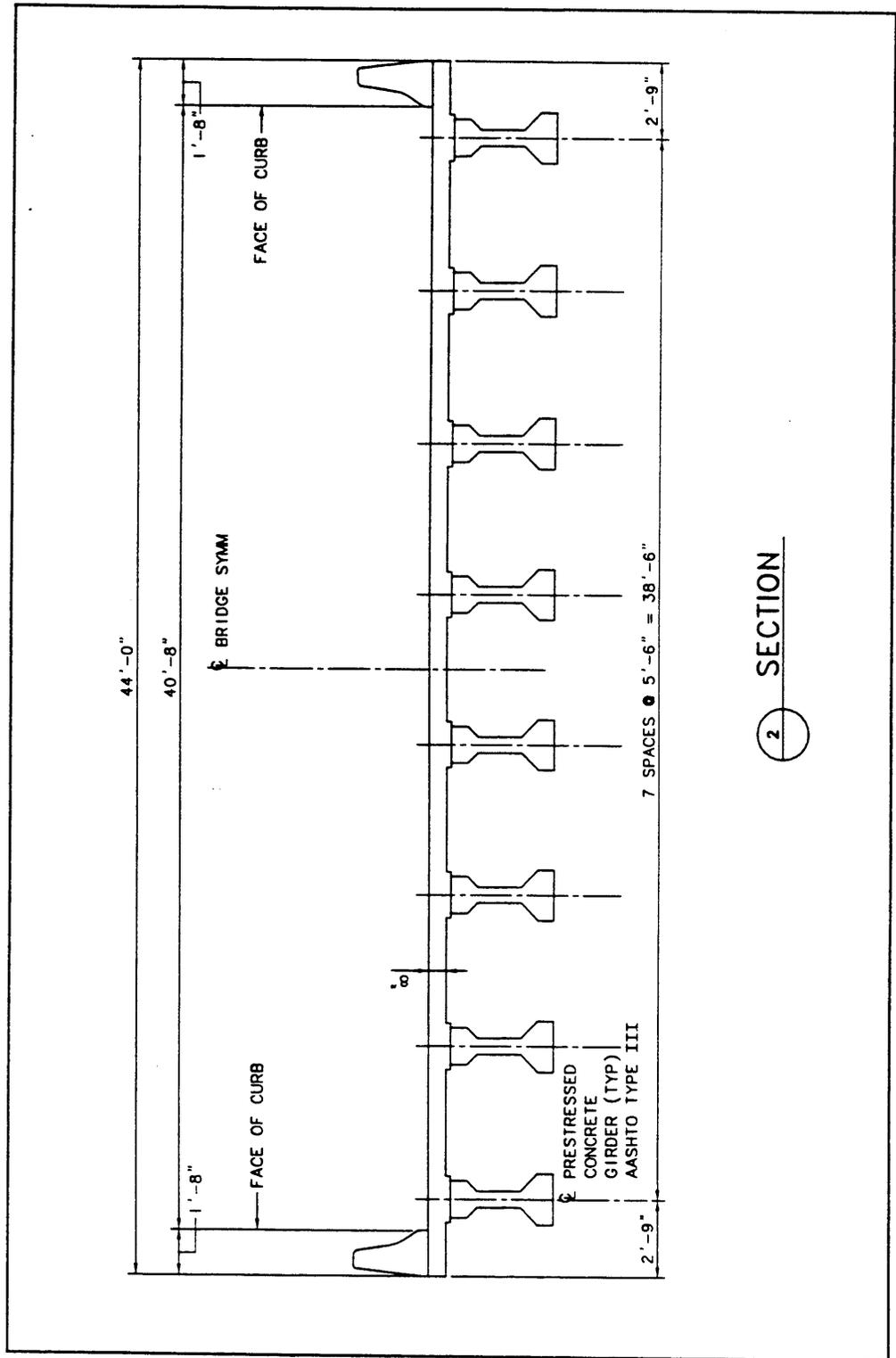


Figure 1a — Bridge No. 3 - Plan and Section

**BRIDGE DATA**  
(continued)



SECTION 2

Figure 1b — Bridge No. 3 - Typical Cross Section

**BRIDGE DATA**  
(continued)

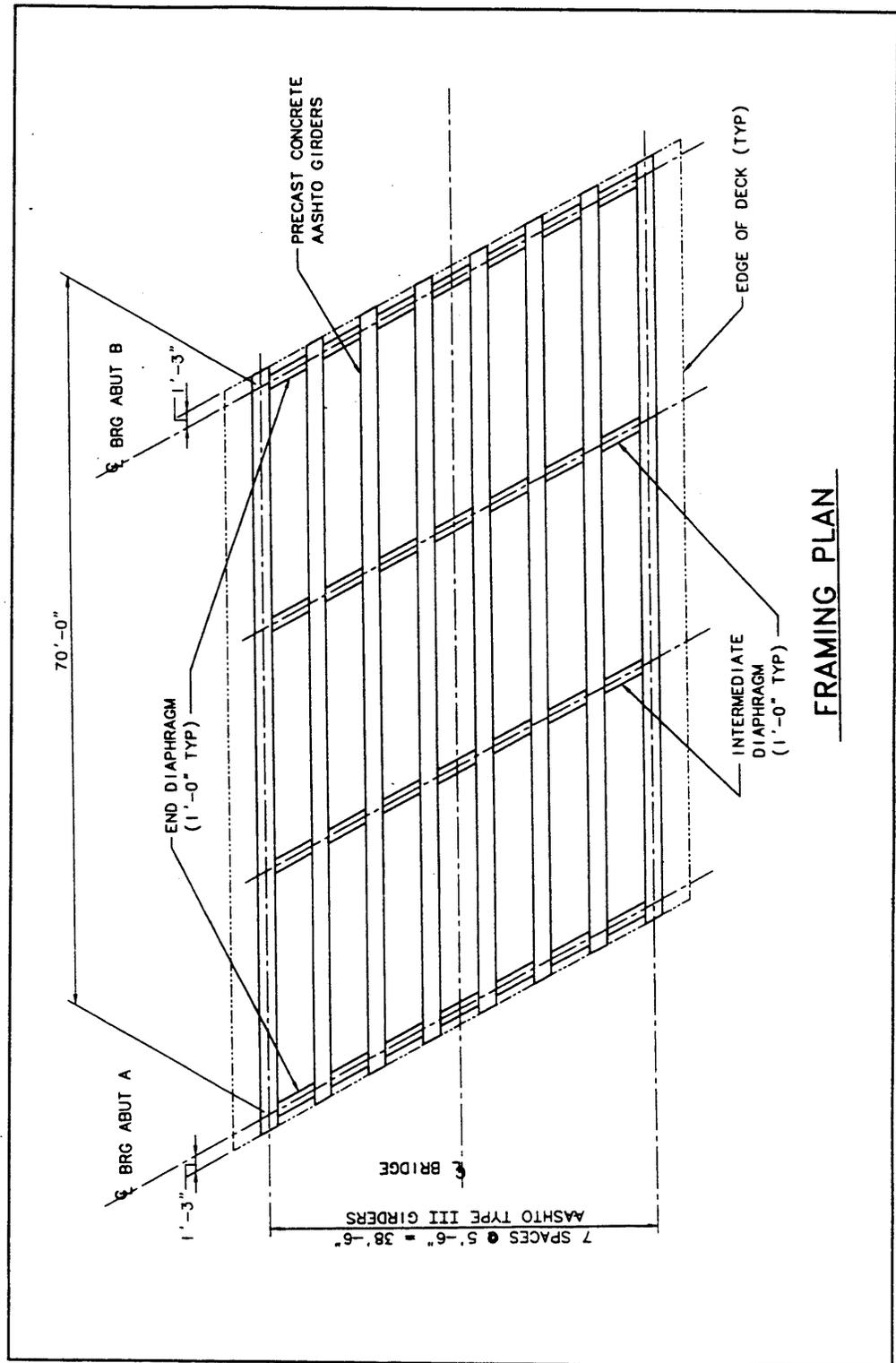


Figure 1c — Bridge No. 3 - Framing Plan

**BRIDGE DATA**  
(continued)

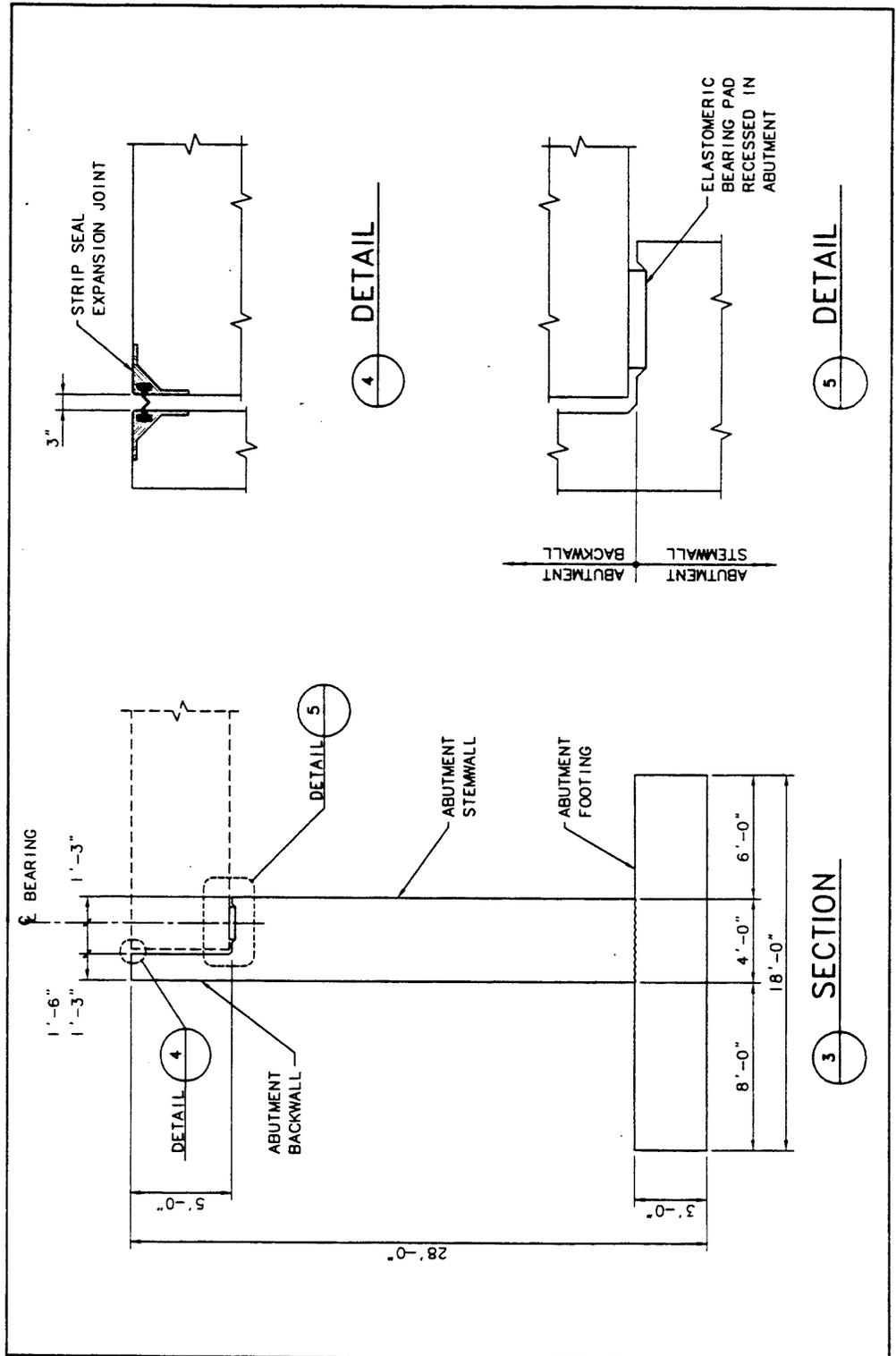


Figure 1d – Bridge No. 3 - Abutment Section and Details

**BRIDGE DATA**  
(continued)

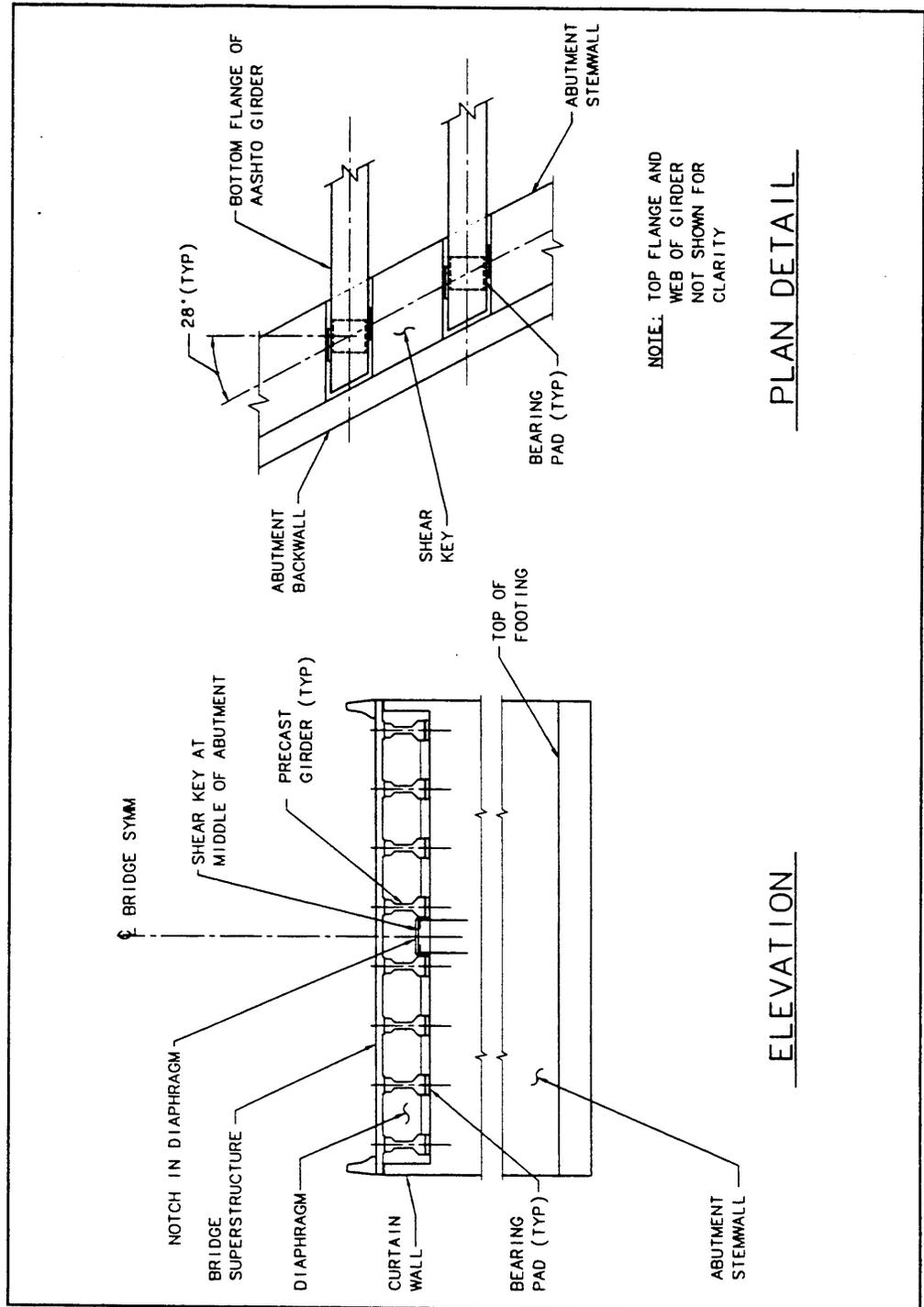


Figure 1e – Bridge No. 3 - Abutment Elevation and Detail

**SOLUTION****DESIGN STEP 1****PRELIMINARY DESIGN**

The preliminary seismic design of the bridge has been completed. The results are shown in this section.

Some confusion may arise from the statement in Division I-A, Article 3.11, “The detailed analysis and design requirements of Sections 4, 5, 6, and 7 are not required for a single-span bridge.” Some designers may have been incorrectly interpreting this statement to mean “no” analysis or design is required. AASHTO Division I-A, Article 7.4.3(A), paragraph two, states that “The seismic design of free-standing abutments should take into account ... the transfer of seismic forces from the bridge deck through bearing supports which do not slide freely (e.g., elastomeric bearings).” Figure 38 of the Division I-A Commentary shows the bearing forces that must be designed for in a sketch.

Connection forces will be transferred from the superstructure to the abutment through the bearings and shear key (also known as seismic key or earthquake buffer). These forces must be carried through the abutment into the soil. These connection forces are in addition to the lateral soil and inertia forces on the abutment.

In the global transverse direction, the abutments are assumed to provide restraint to the superstructure through the shear keys. This restraint is considered a “connection” and is designed to withstand the full connection force.

In the global longitudinal direction, the abutments are assumed to be free to deflect at the top under lateral earth pressures, and are considered to be “free-standing” abutments per the code. In addition, the bearings transfer a sliding force to the top of the abutments as the superstructure moves in the longitudinal direction. Both of these effects must be accounted for when checking for sliding and overturning. Division I-A, Article 7.4.3(A) states that when checking for stability, it is desirable to have the abutment slide before it overturns.

For a single-span bridge with free-standing abutments, it can be argued that the effect of overturning is not important because the superstructure will act as a strut between the abutment walls. Therefore, the abutment walls may move, but they cannot tip over. In a multispan bridge, where the superstructure does not restrain the abutments, the concept of

**DESIGN STEP 1**  
(continued)

overturning is very important. Therefore, in order to “emphasize” the above concepts in this example, both overturning and sliding will be checked.

If integral abutments are used instead of free-standing abutments, overturning will not be a concern because strut action of the superstructure restrains the top of the abutment. Because the abutment wall will not be allowed to deflect outward, it must be designed for the larger at-rest earth pressure under normal loads. Sliding potential will have to be considered, so that the abutment will not rotate about the top and tend to “kick-out” at the bottom of the footing.

The skew effect of the abutments will also be discussed. The global forces are resolved into local components relative to the abutment wall.

The design of the abutment must account for the lateral earth pressure during a seismic event. The active earth pressure increases and the passive pressure decreases, as the magnitude of the earthquake increases. In addition, the inertia effect of the abutment itself, and the soil supported on top of the abutment footing, must be accounted for in the lateral loads. The Mononobe-Okabe equations will be used to calculate these seismic effects.

**Superstructure**

*See Design Step 7.*

**Substructure**

*See Design Step 12.*

**DESIGN STEP 2****BASIC REQUIREMENTS****Design Step  
2.1****Applicability of Specification**

[Division I-A, Article 3.1]

The bridge consists of a single-span. Division I-A, Article 3.11 applies. The requirements for minimum seat length and connection forces are given in Division I-A, Article 3.10.

**Design Step  
2.2****Acceleration Coefficient**

[Division I-A, Article 3.2]

The bridge is located in an area where the Acceleration Coefficient  $A$  is 0.36.

$$A = 0.36$$

**Design Step  
2.3****Importance Classification**

[Division I-A, Article 3.3]

The Importance Classification (IC) of this bridge is taken to be II. It is assumed not to be essential for use following an earthquake. However, this type of bridge is likely to remain functional, providing the soil embankment at each end survives, to provide access to the bridge.

$$IC = II$$

**Design Step  
2.4****Seismic Performance Category**

[Division I-A, Article 3.4]

The Seismic Performance Category (SPC) is C. This is taken from Table 1 of the Specification.

$$SPC = C$$

**Design Step  
2.5****Site Effects**

[Division I-A, Article 3.5]

The site conditions affect the design as reflected by a coefficient based on the soil profile. At this site, rock is located at a depth of about 35 feet below the underpass roadway, and the soils overlaying rock are stable deposits of stiff clay. The conditions correspond to SOIL PROFILE TYPE I.

The Site Coefficient ( $S$ ) for this type of soil is 1.0, per Table 2 of the Specification.

$$S = 1.0$$

**Design Step  
2.6**

**Response Modification Factors**  
[Division I-A, Article 3.7]

Since this bridge is classified as SPC C, appropriate Response Modification Factors (R Factors) must be selected for later use in establishing appropriate design force levels.

In this case, Table 3 of the Specification gives the following R Factors.

$$R = 0.8 \quad \text{For the connection of the superstructure to the abutment}$$

This factor will be used to ensure that the shear key is designed to resist the probable forces delivered by the superstructure without incurring any damage.

**DESIGN STEP 3**

**SINGLE-SPAN BRIDGE DESIGN**  
[Division I-A, Article 3.11]

This section applies to single-span bridges. There is no detailed analysis required, but connection forces must be designed for and seat lengths per Division I-A, Article 3.10 must be provided.

**Design Step  
3.1**

**Determine Design Forces for Connections**

For this example, these forces are calculated in Design Step 7.2.

**DESIGN STEP 4**

**SEISMIC PERFORMANCE CATEGORY A DESIGN**

Not applicable.

**DESIGN STEP 5**

**DETERMINE ANALYSIS PROCEDURE**

Not required for a single-span bridge.

**DESIGN STEP 6**

**DETERMINE ELASTIC SEISMIC FORCES AND DISPLACEMENTS**

Not required for a single-span bridge.

**DESIGN STEP 7****DETERMINE DESIGN FORCES****Design Step  
7.1****Determine Dead Load Forces**

The superstructure dead load consists of eight precast concrete AASHTO girders, a CIP concrete deck, four diaphragms, and a traffic barrier on each side of the bridge. See Figure 1 (a to e) for details.

Assumptions used in the weight calculations are

$$\gamma_c := 150 \cdot \text{pcf} \quad \text{Unit weight of the concrete}$$

$$L_s := 70.0 \cdot \text{ft} \quad \text{Span length of bridge from centerline of bearings}$$

$$L_o := 72.5 \cdot \text{ft} \quad \text{Overall length of bridge superstructure}$$

$$\text{Width} := 44.0 \cdot \text{ft} \quad \text{Overall width of bridge superstructure}$$

Weight of AASHTO Type III girders

$$A_g := 560 \cdot \text{in}^2 \quad \text{Area of each girder}$$

$$w := \left( A_g \cdot \frac{\text{ft}^2}{144 \cdot \text{in}^2} \right) \cdot \gamma_c$$

$$w = 0.583 \cdot \frac{\text{kip}}{\text{ft}} \quad \text{Weight of each girder}$$

Therefore, the weight of all eight girders in the bridge is

$$L_o = 72.5 \cdot \text{ft}$$

$$W_g := 8 \cdot w \cdot L_o \quad W_g = 338.3 \cdot \text{kip}$$

**Design Step**  
**7.1**  
(continued)

For calculating the weight of the deck slab, assume average thickness of 8 inches (includes weight of girder pads).

$$W_s := (72.5 \text{ ft}) \cdot (44.0 \text{ ft}) \cdot \left(\frac{8}{12} \text{ ft}\right) \cdot \gamma_c \quad W_s = 319 \cdot \text{kip}$$

For calculating the weight of the two traffic barriers, assume 0.40 kip per foot each.

$$W_b := 2 \cdot \left(0.40 \frac{\text{kip}}{\text{ft}}\right) \cdot (72.5 \text{ ft}) \quad W_b = 58.0 \cdot \text{kip}$$

For calculating the weight of four diaphragms, conservatively assume each to be 12 inches wide and the full depth of the girders.

$$W_d := 4 \cdot (34 \text{ ft}) \cdot (3.75 \text{ ft}) \cdot (1.0 \text{ ft}) \cdot \gamma_c \quad W_d = 76.5 \cdot \text{kip}$$

Therefore, the total weight of bridge superstructure is  $W$ .

$$W := W_g + W_s + W_b + W_d$$

$$W_g = 338.3 \cdot \text{kip} \quad \text{Weight of AASHTO girders}$$

$$W_s = 319.0 \cdot \text{kip} \quad \text{Weight of deck slab}$$

$$W_b = 58.0 \cdot \text{kip} \quad \text{Weight of traffic barriers}$$

$$W_d = 76.5 \cdot \text{kip} \quad \text{Weight of all diaphragms}$$

Therefore, the total weight of the superstructure is

$$W = 791.8 \cdot \text{kip}$$

The reaction at each end of the bridge is the total bearing load at each end.

$$P_{\text{brg}} := \frac{W}{2} \quad P_{\text{brg}} = 395.9 \cdot \text{kip}$$

**Design Step**  
**7.1**  
(continued)

The reaction per unit length along the abutment is calculated for later use in Design Step 12.2.1.

Find the unit weight along the length of the abutment,  $L$ .

$$\lambda := 28 \cdot \text{deg} \quad \text{Skew angle of abutment}$$

Length of abutment along the skew is

$$L := \frac{\text{Width}}{\cos(\lambda)} \quad L = 49.8 \cdot \text{ft}$$

$$w_R := \frac{P_{\text{brg}}}{L} \quad \text{Reaction per unit length}$$

$$w_R = 7.94 \cdot \frac{\text{kip}}{\text{ft}}$$

**Design Step**  
**7.2****Determine Seismic Forces**  
[Division I-A, Article 3.11]

Article 3.11 states that the detailed analysis requirements of Sections 4, 5, 6, and 7 are not required for single-span bridges. But the connection between the abutment and the superstructure must be designed to resist the tributary weight at the abutment multiplied by  $A \times S$  in each horizontally restrained direction. In the unrestrained direction, the connection must be designed to withstand the sliding friction force in the bearings.

**Design Step**  
**7.2.1****Transverse Seismic Forces**  
(Force on Concrete Shear Key)

For movement in the transverse direction, perpendicular to the centerline of the bridge, a concrete shear key is provided to resist the movement of the superstructure. The force is  $A \times S$  multiplied by the reaction at the abutment. See Figure 2.

$$A := 0.36 \quad \text{Acceleration coefficient}$$

$$S := 1.0 \quad \text{Site coefficient}$$

$$A \cdot S = 0.36$$

Design Step  
7.2.1  
(continued)

$P_{brg} = 395.9 \cdot \text{kip}$  Reaction force at each abutment

The connection force in the restrained direction of a single-span bridge, which in this example is the transverse direction, is as follows.

$V_{eq,t} := A \cdot S \cdot P_{brg}$   $V_{eq,t} = 142.5 \cdot \text{kip}$

Design shear load to the shear key is

$R := 0.8$  For shear key, per AASHTO Division 1-A, Table 3

$V_{sk} := \frac{V_{eq,t}}{R}$   $V_{sk} = 178.2 \cdot \text{kip}$

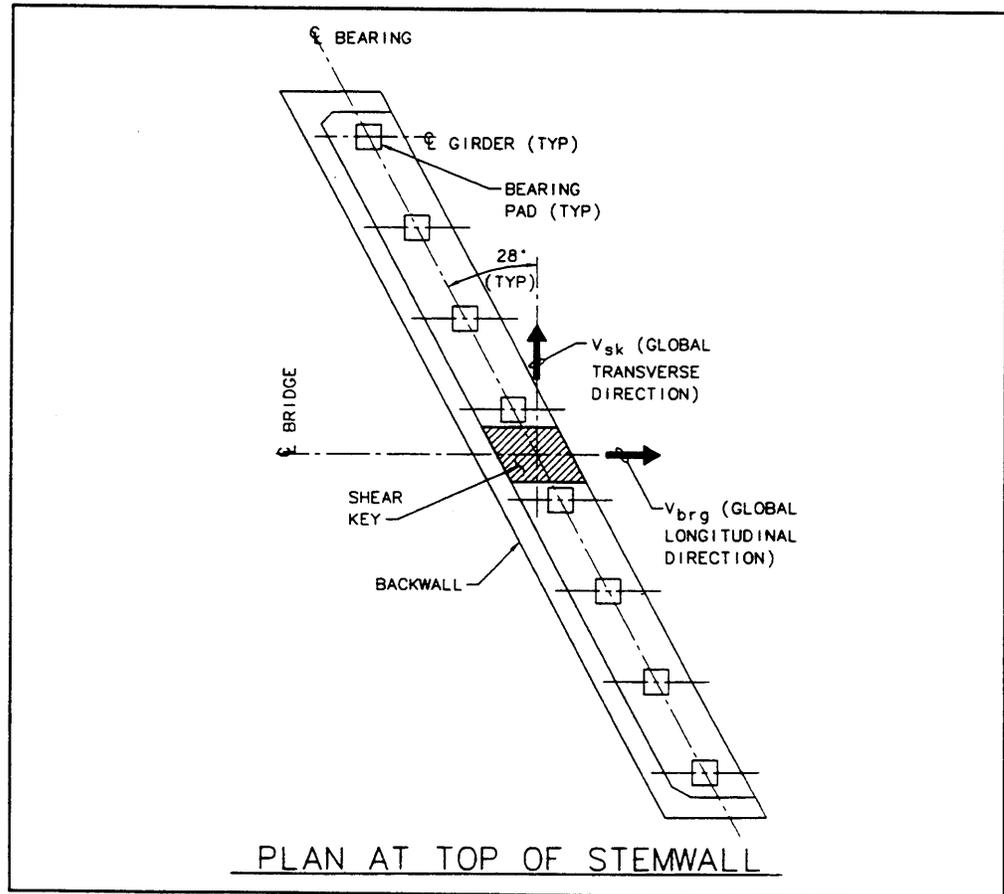


Figure 2 — Forces at Top of Abutment Seat

Design Step  
7.2.2Longitudinal Seismic Forces  
(Force Transferred Through the Bearings)

In the longitudinal direction, parallel to the centerline of the bridge, the superstructure is not rigidly restrained, but free to slide on elastomeric bearings. There is a sliding friction force transferred through the bearings to the abutment equal to the friction force on the bearing pad. For an elastomeric bearing, it is assumed in this example that the friction coefficient is 0.2.

$$\mu := 0.20 \quad \text{Sliding friction of the bearing pad}$$

$$P_{brg} = 395.9 \cdot \text{kip} \quad \text{Reaction force at each abutment}$$

The sliding friction force from all the bearings at one abutment is

$$V_{brg} := \mu \cdot P_{brg} \quad V_{brg} = 79.2 \cdot \text{kip}$$

The sliding friction force must be considered additive to earthquake effects (seismic earth pressure, inertia of abutment wall, etc.). While it is true that these forces will sometimes act in opposite directions, it would be unconservative to assume that these effects will always be out-of-phase with each other.

Now that the orthogonal seismic forces have been determined for the two major axis, the forces for the various components can also be determined. Design Step 7.3.3 covers the combination of orthogonal seismic forces.

Design Step  
7.2.3

## Concrete Shear Key

The concrete shear key is located at the centerline of the bridge, and is oriented parallel to the centerline of the roadway. The shear key is placed at the centerline to avoid unsymmetrical loading on the abutment. The diaphragm at the end of the superstructure should be designed to transfer the seismic force from the superstructure to the shear key.

Calculate the shear stress on the shear key.

$$V_{sk} = 178.2 \cdot \text{kip} \quad \text{Force on shear key}$$

Design Step  
7.2.3  
(continued)

Assuming a 40- by 30-inch shear key

$$A_{sk} := 1200 \cdot \text{in}^2 \quad \text{Area of shear key}$$

Per AASHTO Division I, Article 8.16.6.4.5, the shear stress must be less than 800 psi for shear friction.

$$v_u := \frac{V_{sk}}{A_{sk}}$$

$$v_u = 148 \cdot \text{psi} \quad \text{Less than 800 psi, okay}$$

Design Step  
7.2.4

Abutment Backwall

During a significant earthquake, the superstructure will engage the abutment backwall after overcoming the sliding friction force in the bearings. Therefore, the abutment backwall should be designed to resist the full earthquake shear in the longitudinal direction, and not break off.

Treating the longitudinal as a “restrained” direction, design the backwall to take a shear equal to  $A \times S$  multiplied by the weight of the superstructure, divided by “R” for a connection.

Calculate the shear stress on the backwall.

$$A \cdot S = 0.36 \quad \text{Acceleration multiplied by site coefficient}$$

$$R = 0.8 \quad \text{For shear key, per AASHTO Division I-A, Table 3}$$

$$W = 791.8 \cdot \text{kip} \quad \text{Weight of superstructure}$$

$$V_{bw} := \frac{A \cdot S}{R} \cdot W \quad \text{Design shear force on the backwall}$$

$$V_{bw} = 356.3 \cdot \text{kip}$$

Design Step  
7.2.4  
(continued)

The shear area of the backwall that must resist the longitudinal force is  $A_{bw}$ .

$\lambda = 28.0 \cdot \text{deg}$  Skew angle of abutment

$L = 49.8 \cdot \text{ft}$  Length of abutment along skew

$A_{bw} := L \cdot 1.25 \cdot \text{ft}$   $A_{bw} = 8970 \cdot \text{in}^2$

The shear stress on the backwall is

$v_u := \frac{V_{bw}}{A_{bw}}$   $v_u = 40 \cdot \text{psi} < 800 \text{ psi, okay}$

Design Step  
7.3**Combinations of Seismic Forces**

[Division I-A, Article 3.9]

Because of the 28-degree skew, forces on the abutment need to be broken down into orthogonal forces about the local axes of the abutment. See Figures 3 and 4. Forces in the local transverse direction of the abutment will determine the design for sliding and overturning stability. Forces in the local longitudinal direction of the abutment are calculated, but not used in this example. Because the abutment footing is much longer than it is wide in the transverse direction (50 feet versus 18 feet), overturning stability in the long direction is not as critical, though it should still be checked for a tall abutment.

Before the criteria of Article 3.9 are applied, the local components of the two global forces (longitudinal friction in the bearings and transverse shear on the shear key) must be calculated. It is assumed that these two forces are both acting in a direction that tends to pull the abutment away from the soil, creating sliding and overturning effects.

Design Step  
7.3.1**Local Forces on Abutment Due to Friction on Bearings**

See Figure 3. Given the basic conditions

$$V_{brg} = 79.2 \cdot \text{kip} \quad \text{Global shear on bearings at each abutment}$$

$$\lambda := 28 \cdot \text{deg} \quad \text{Skew angle of abutment}$$

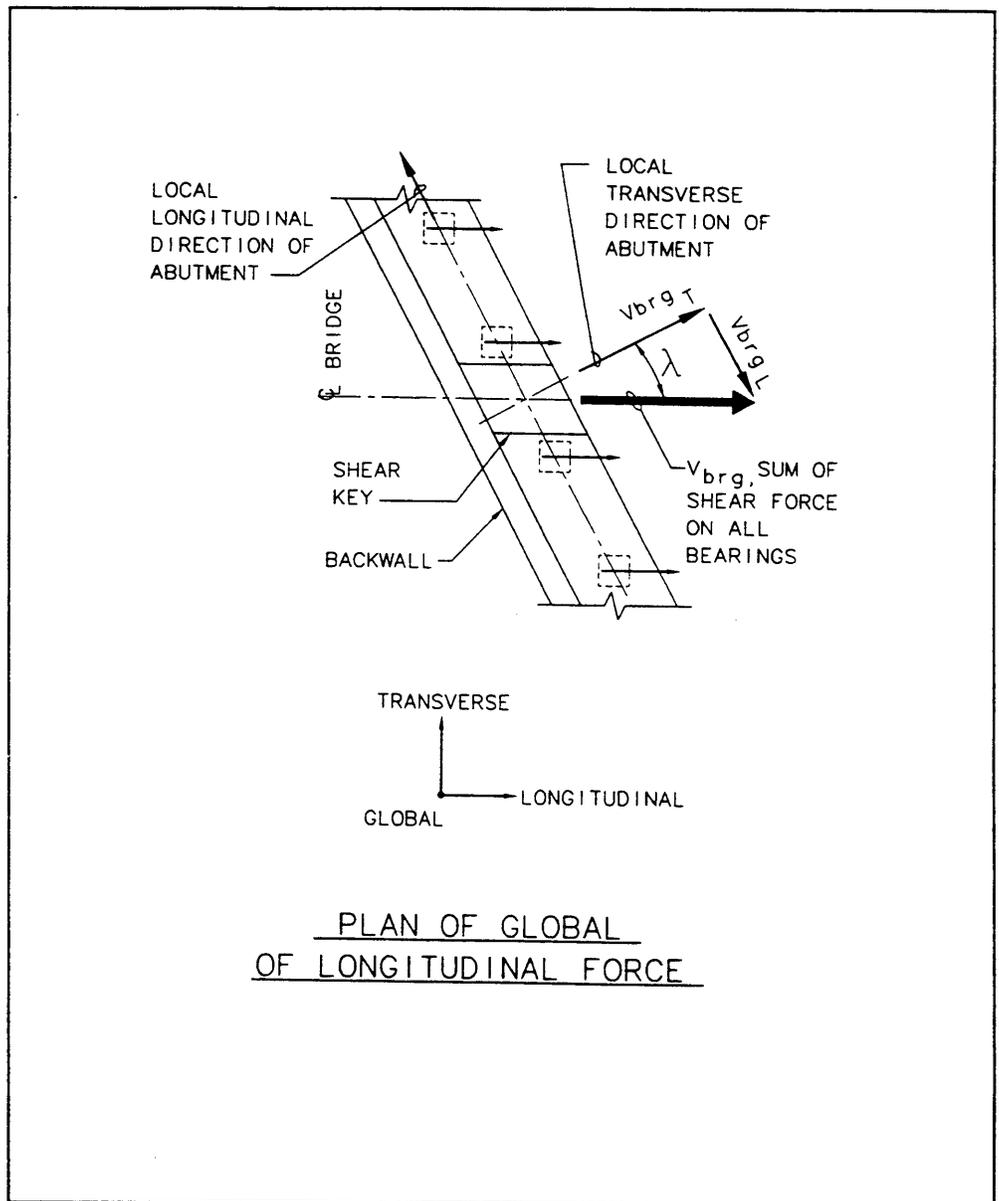
*a) Local Transverse Shear on Abutment*

$$V_{brg_T} := V_{brg} \cdot \cos(\lambda) \quad V_{brg_T} = 69.9 \cdot \text{kip}$$

*b) Local Longitudinal Shear on Abutment*

$$V_{brg_L} := V_{brg} \cdot \sin(\lambda) \quad V_{brg_L} = 37.2 \cdot \text{kip}$$

Design Step  
7.3.1  
(continued)



Figures 3 — Horizontal Component of Bearing Forces

Design Step  
7.3.2

Local Forces on Abutment Due to Shear Key Forces

See Figure 4. Given the basic conditions

$V_{sk} = 178.2 \cdot \text{kip}$       Global shear on the shear key at each abutment

$\lambda := 28 \cdot \text{deg}$       Skew angle of abutment

a) Local Transverse Shear on Abutment

$V_{sk_T} := V_{sk} \cdot \sin(\lambda)$        $V_{sk_T} = 83.6 \cdot \text{kip}$

b) Local Longitudinal Shear on Abutment

$V_{sk_L} := V_{sk} \cdot \cos(\lambda)$        $V_{sk_L} = 157.3 \cdot \text{kip}$

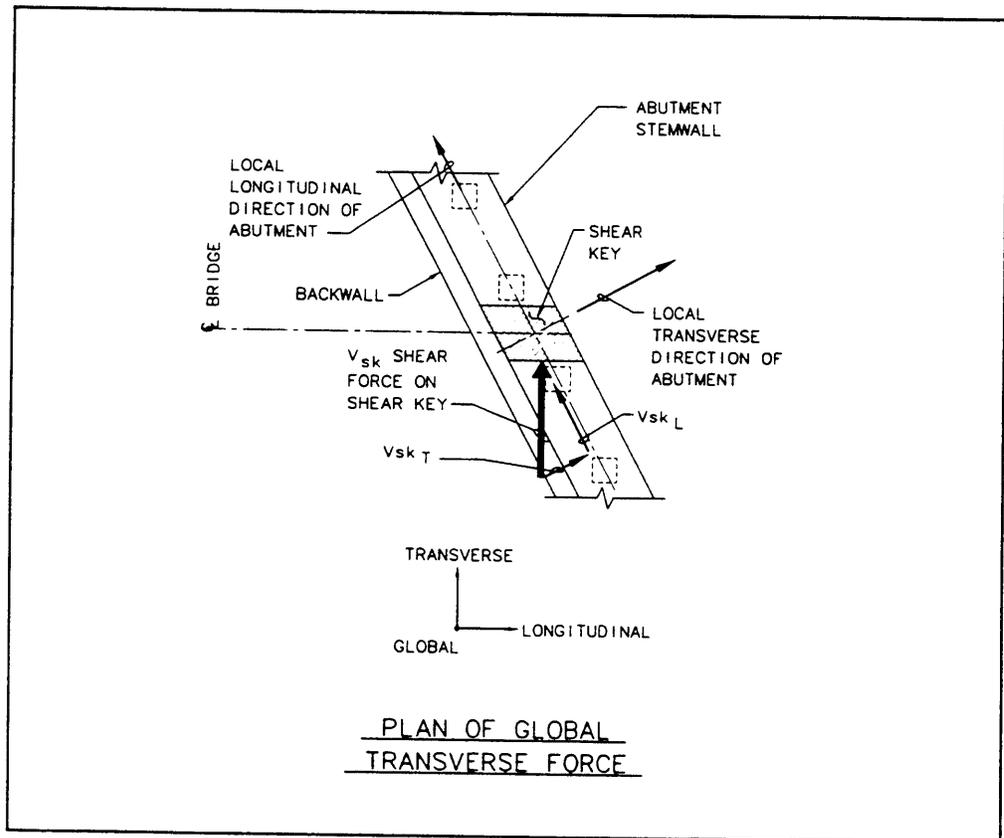


Figure 4 — Horizontal Component of Shear Key Forces

Design Step  
7.3.2  
(continued)

The criteria in Article 3.9 pertain to the combination of orthogonal forces. Even though this is a single-span bridge, each of the global forces has an effect on the local axes of the abutment. When 100 percent of one global force is combined with 30 percent of the other global force, it is a bookkeeping exercise to calculate the resulting forces correctly. All the seismic forces are absolute values, i.e., forces occur together in the direction of the worst condition. Below is a summary of the basic local forces.

Forces from the bearings at each abutment are

$$\begin{aligned} V_{brg_T} &= 69.9 \cdot \text{kip} && \text{Local transverse shear on abutment} \\ V_{brg_L} &= 37.2 \cdot \text{kip} && \text{Local longitudinal shear on abutment} \end{aligned}$$

Forces from the shear key at each abutment are

$$\begin{aligned} V_{sk_T} &= 83.6 \cdot \text{kip} && \text{Local transverse shear on abutment} \\ V_{sk_L} &= 157.3 \cdot \text{kip} && \text{Local longitudinal shear on abutment} \end{aligned}$$

Design Step  
7.3.3

**Load Combinations**

a) *LC1 (100 Percent Global Longitudinal Plus 30 Percent Transverse)*

Resulting transverse force on the abutment is

$$V_T := 1.0 \cdot (V_{brg_T}) + 0.3 \cdot (V_{sk_T}) \qquad V_T = 95.0 \cdot \text{kip}$$

The unit transverse shear on the abutment is

$$\begin{aligned} L &= 49.8 \cdot \text{ft} && \text{Length of abutment} \\ v_T &:= \frac{V_T}{L} && v_T = 1.91 \cdot \frac{\text{kip}}{\text{ft}} \end{aligned}$$

Resulting longitudinal force on abutment is

$$V_L := 1.0 \cdot (V_{brg_L}) + 0.3 \cdot (V_{sk_L}) \qquad V_L = 84.4 \cdot \text{kip}$$

Design Step  
7.3.3  
(continued)

The unit longitudinal shear on the abutment is

$$v_L := \frac{V_L}{L} \qquad v_L = 1.69 \cdot \frac{\text{kip}}{\text{ft}}$$

b) *LC2 (100 Percent Global Transverse Plus 30 Percent Global Longitudinal)*

Resulting transverse force on the abutment is

$$V_T := 1.0 \cdot (V_{sk_T}) + 0.3 \cdot (V_{brg_T}) \qquad V_T = 104.6 \cdot \text{kip}$$

The unit transverse shear on the abutment is

$$v_T := \frac{V_T}{L} \qquad v_T = 2.10 \cdot \frac{\text{kip}}{\text{ft}} \quad \leftarrow \text{Controls}$$

Resulting longitudinal force on the abutment is

$$V_L := 1.0 \cdot (V_{sk_L}) + 0.3 \cdot (V_{brg_L}) \qquad V_L = 168.5 \cdot \text{kip}$$

The unit longitudinal shear on the abutment is

$$v_L := \frac{V_L}{L} \qquad v_L = 3.38 \cdot \frac{\text{kip}}{\text{ft}}$$

c) *The Controlling Load Case for the Largest Force in the Local Transverse Direction of the Abutment is LC2 with  $V_T = 2.10$  Kip Per Foot*

**DESIGN STEP 8**

**SUMMARY OF DESIGN FORCES**

Not applicable.

**DESIGN STEP 9****DETERMINE DESIGN DISPLACEMENTS**

[AASHTO Division I-A, Article 7.3]

**Design Step  
9.1****Minimum Support Length**

[AASHTO Division I-A, Article 7.3.1]

Calculate the minimum seat length required for the top of the abutment stemwall to support the single-span bridge superstructure.

$L := 72.5 \text{ ft}$                       Length between abutments

$H := 0 \text{ ft}$                               For single-span bridges

$S := 28$                                   Skew angle in degrees

Per Division I-A, Equation 7-3A

$$N := \left( 12 \cdot \text{in} + 0.03 \cdot L \cdot \frac{\text{in}}{\text{ft}} + 0.12 \cdot H \cdot \frac{\text{in}}{\text{ft}} \right) \cdot \left( 1 + 0.000125 \cdot S^2 \right)$$

$$\left( 1 + 0.000125 \cdot S^2 \right) = 1.098 \quad \text{Skew effect}$$

$$N = 15.6 \cdot \text{in} \quad \text{Minimum seat length}$$

Refer to Figure 1d. The seat length provided is 2 feet 6 inches, okay.

**Design Step  
9.2****Design Displacements**

Because no detailed analysis is required for the single-span bridge superstructure, check the gap between the abutment backwall and the end of the bridge superstructure for temperature loads, and for ability of the abutment to slide under an earthquake loading. Refer to Figure 1d. The gap provided is 3 inches, okay.

*a) Check the Gap for Temperature Movement*

$\Delta T := 30$                                   Change in temperature

$\gamma := 5.5 \cdot 10^{-6}$                               Temperature coefficient for concrete

**Design Step**  
**9.2**  
(continued)

$$\Delta := \gamma \cdot \Delta T \cdot \left(\frac{L}{2}\right) \quad \text{Deflection due to temperature}$$

$$\Delta = 0.07 \cdot \text{in} \quad \text{Deflection is small, no problem}$$

*b) Check the Gap to Allow the Abutment to Move a Distance of 10 x A*  
[AASHTO Division I-A, Article 7.4.3(A)]

$$A := 0.36 \quad \text{Acceleration Coefficient}$$

$$10 \cdot A \cdot \text{in} = 3.6 \cdot \text{in} \quad \text{Maximum expected abutment movement}$$

As long as there is a gap of 3.6 inches between the backwall and the end of the superstructure, there is theoretically enough space to allow the abutment to displace laterally. Note that a 3-inch gap is shown in the Figure 1d. Because this is a single-span bridge, chances are that both abutments will not displace outward 3.6 inches. Therefore, two gaps of 3 inches or 6 inches of total gap is sufficient.

**DESIGN STEP 10**

**DESIGN STRUCTURAL COMPONENTS**

Not applicable.

**DESIGN STEP 11**

**DESIGN FOUNDATIONS**

Not applicable.

**DESIGN STEP 12****DESIGN ABUTMENTS****Design Step  
12.1****Mononobe-Okabe (M-O) Forces**

*Lateral Soil Pressures during Seismic Loading (M-O) Analysis*  
AASHTO Division I-A, Articles 7.4.3(A) and C7.4.3(A)

Article 7.4.3 in AASHTO I-A, along with the Commentary, outlines the recommended analysis procedures. These include applying the M-O Method of analysis for the lateral earth overpressure, and accounting for the seismic inertia forces of both the abutment self-weight and the soil resting on the abutment footing. These effects are especially critical for tall abutments in high seismic zones.

The M-O Method of analysis is recommended for calculating the lateral earth overpressures during a seismic event. The Specification Commentary includes a good discussion on the use of this analysis method. Although the equations are lengthy and complex, they are basically modified “Coulomb Theory” equations that take into account the horizontal and vertical acceleration effects of an earthquake.

Whereas the basic “Rankine Theory” of lateral soil pressure is based on the assumption that there is no friction between the abutment and the soil, the Coulomb Theory takes into account the effects of  $\delta$ , the angle of friction between the soil and the abutment. The M-O analysis is an extension of the Coulomb Theory, taking into account horizontal and vertical inertia forces acting on the soil as a result of an earthquake.

In general, an earthquake increases the active soil pressure and decreases the passive soil pressure. In fact, the M-O equations reduce to the Coulomb equations when the earthquake effects ( $k_h$  and  $k_v$ ) are set equal to zero.

As noted in Article 7.4.3(A), the value of the horizontal acceleration coefficient depends on whether the abutment wall is free to deflect or is restrained. For free standing abutments, where the wall is free to deflect outward without significant restraint, the recommended value of the seismic coefficient is  $k_h = 0.5 \times A$ , where “A” is the Acceleration Coefficient. Lateral abutment displacements up to  $10 \times A$  inches should be accommodated. See Figure 5.

Design Step  
12.1  
(continued)

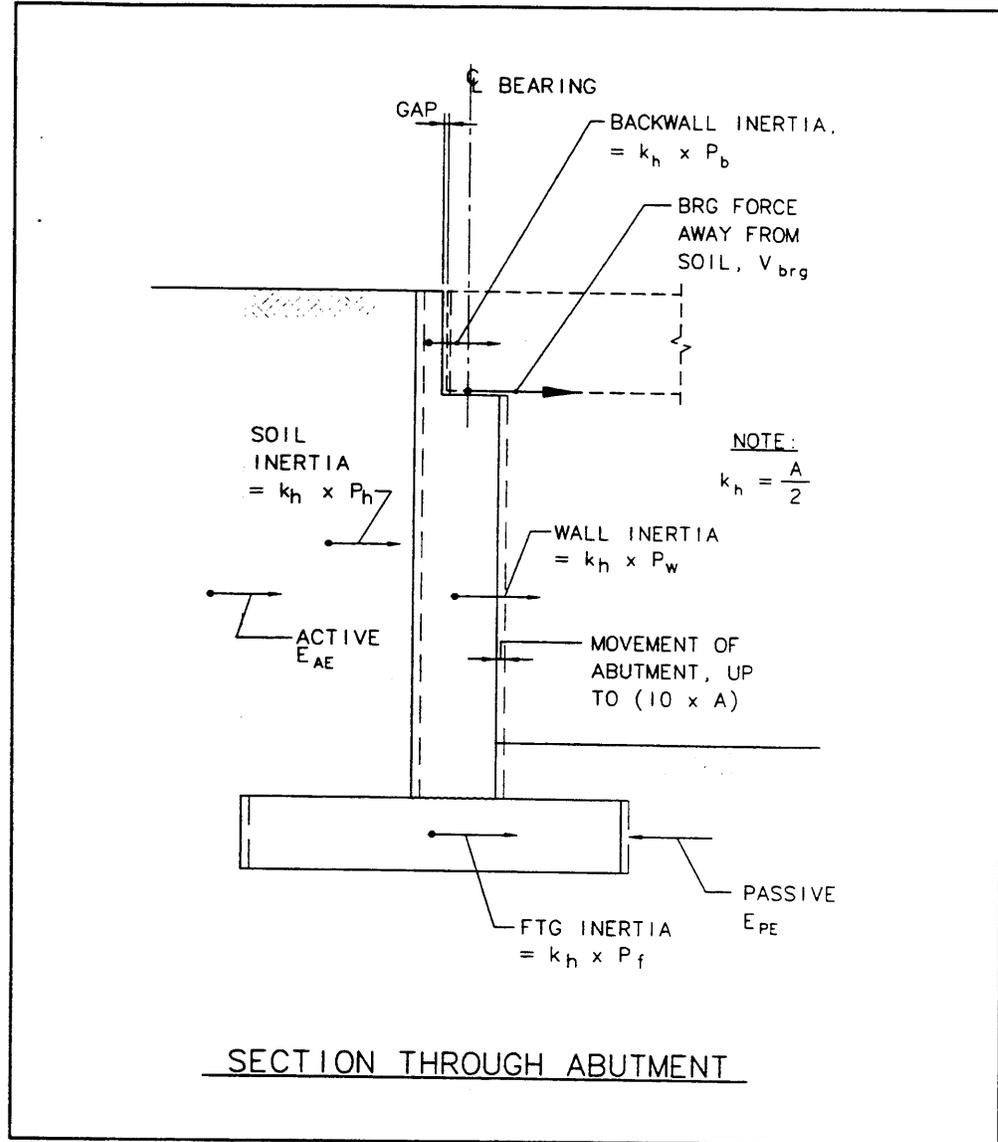


Figure 5 — Lateral Forces on Abutment Footing

For free-standing abutments that are restrained at the top, or are prevented from moving by batter piles or tie-backs, the static pressure behind the wall is higher, similar to the at-rest condition. The resulting earthquake forces will also be higher. The first approximation of the seismic coefficient for this case is  $k_h = 1.5 \times A$ .

In this example, the superstructure is supported by elastomeric bearing pads on the top of the abutment wall. Under seismic loading, when the superstructure moves away from the abutment, the friction force in the

**Design Step**  
**12.1**  
(continued)

bearings then tends to pull the wall away from the soil. It is assumed that this effect does not change the seismic coefficient significantly; and, therefore,  $k_h = 0.5 \times A$  is used. It is anticipated that the soil pressure, together with the friction force in the bearings pulling the wall away from the soil, will be the critical abutment sliding and overturning loads. They will be used to design the flexural reinforcement in the inside face of the abutment stemwall.

In the other case, when the superstructure moves toward the abutment at the same time the soil is pushing the wall outward, the bearings tend to restrain the top of the abutment wall. This partial restraint will tend to increase the value of  $k_h$ , although the actual increase in  $k_h$  will probably be small with the use of elastomeric bearings. While this restraint may cause a slight increase in the value of  $k_h$ , it will not cause an increase large enough to produce larger passive pressures behind the wall. Because this case controls neither the sliding nor overturning, it will not be calculated in this example. This load would be used to design the flexural reinforcement in the outside face of the wall.

The basic seismic parameters are

$$A := 0.36 \quad \text{Acceleration coefficient}$$

Calculate the seismic coefficient,  $k_h$ .

$$k_h := 0.5 \cdot A \quad k_h = 0.18$$

The vertical acceleration is assumed to be zero.

$$k_v := 0.0$$

The basic soil parameters are

$$\phi := 35 \cdot \text{deg} \quad \text{Angle of friction of the soil}$$

$$\beta := 0 \quad \text{Slope of soil face (batter angle of wall)}$$

$$i := 0 \quad \text{Backfill slope angle}$$

$$\gamma := 130 \cdot \text{pcf} \quad \text{Unit weight of soil}$$

**Design Step  
12.1**  
(continued)

The angle of friction between the soil and the abutment is  $\delta$ .

$$\delta := \frac{\phi}{2} \qquad \delta = 17.5 \cdot \text{deg}$$

The resulting seismic inertia angle is  $\theta$ .

$$\theta := \text{atan}\left(\frac{k_h}{1 - k_v}\right) \qquad \theta = 10.2 \cdot \text{deg}$$

**Design Step  
12.1.1**

**Active Soil Pressure**

a) Calculate the Seismic Active Earth Pressure Coefficient,  $K_{AE}$

The basic M-O equations are broken into two parts for clarity.

$$\Gamma := \left( 1 + \frac{\sin(\phi + \delta) \cdot \sin(\phi - \theta - i)}{\sqrt{\cos(\delta + \beta + \theta) \cdot \cos(i - \beta)}} \right)^2 \qquad \text{Coefficient in the denominator of Equation (CB-4)}$$

$$K_{AE} := \left[ \frac{(\cos(\phi - \theta - \beta))^2}{\Gamma \cdot \cos(\theta) \cdot (\cos(\beta))^2 \cdot \cos(\delta + \beta + \theta)} \right] \qquad \text{AASHTO 1-A, Equation (CB-4)}$$

With  $\beta = i = 0$ , the equations reduce to

$$\Gamma := \left( 1 + \frac{\sin(\phi + \delta) \cdot \sin(\phi - \theta)}{\cos(\delta + \theta)} \right)^2 \qquad \Gamma = 2.602$$

$$K_{AE} := \left[ \frac{(\cos(\phi - \theta))^2}{\Gamma \cdot \cos(\theta) \cdot \cos(\delta + \theta)} \right] \qquad K_{AE} = 0.364$$

Therefore, the seismic active pressure coefficient is

$$K_{AE} = 0.364$$

For purposes of comparison, the results are plotted in Figure 6 using  $k_h = 0.18$  and  $\phi = 35$  degrees. In both case, the results are the same:  $K_{AE} = 0.36$ .

Design Step  
12.1.1  
(continued)

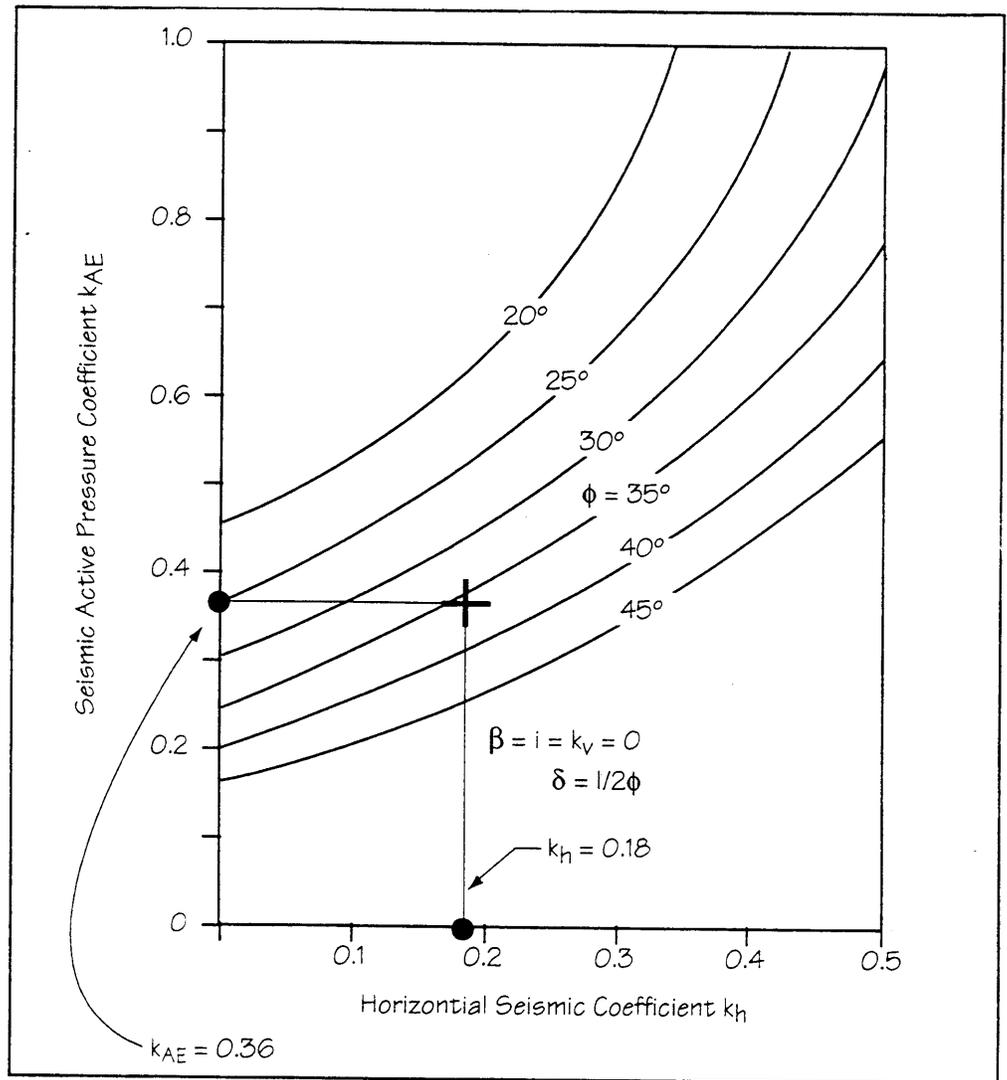


Figure 6 — Verification of M-O Coefficient

As was mentioned previously, the M-O equations are simply a derivative of Coulomb's equation. As an aid to the designer in gaining confidence in their use, the following calculations demonstrate how the M-O equations reduce down to the Coulomb and Rankine equations.

*b) Simplification of M-O Equation into Coulomb's Equation*

When there is no seismic acceleration,  $\theta = 0$  (as well as  $\beta = i = 0$ ), the M-O equation defaults to Coulomb's equation.

$$\theta = 0 \cdot \text{deg}$$

Design Step  
12.1.1  
(continued)

$$\Gamma := \left( 1 + \sqrt{\frac{\sin(\phi + \delta) \cdot \sin(\phi)}{\cos(\delta)}} \right)^2 \quad \Gamma = 2.859$$

$$K_A := \left[ \frac{(\cos(\phi))^2}{\Gamma \cdot \cos(\delta)} \right] \quad K_A = 0.25$$

The coefficient, assuming  $k_h = 0.0$  for a nonseismic event, gives the same result as using the Coulomb Theory,  $K_A = 0.25$ . Note that the active force exerted on the abutment by the soil during a seismic event is larger than for a normal active pressure (0.36 versus 0.25).

*c) Simplification of M-O Equation into Rankine's Equation*

Further, if the angle of friction between the soil and abutment,  $\delta$ , is ignored (set equal to zero), the equation reduces to Rankine.

$$\delta := 0 \cdot \text{deg}$$

$$\Gamma := (1 + \sin(\phi))^2$$

$$K_A := \left[ \frac{(\cos(\phi))^2}{\Gamma} \right] \quad K_A = 0.27$$

The Rankine equation is usually written in either of the two forms shown below, both of which result in the same answer as calculated above.

$$K_A := \frac{1 - \sin(\phi)}{1 + \sin(\phi)} \quad K_A = 0.27$$

$$K_A := \tan\left(45 \cdot \text{deg} - \frac{\phi}{2}\right)^2 \quad K_A = 0.27$$

In summary, in calculating the active earth pressure, the Rankine equation does not consider  $\delta$ ; therefore, it is more conservative than Coulomb's equation for calculating nonseismic loads (0.27 versus 0.25). When Coulomb's equation is modified to account for seismic forces in the soil, the soil pressure goes up as the acceleration increases (0.25 versus 0.36).

Design Step  
12.1.1  
(continued)

The Specification Commentary also states that the inertia force due to the mass of the abutment itself should be accounted for. Calculation of this inertia force should also take into account the soil mass above the footing. This additional seismic force is calculated in Table 3 of Design Step 12.2.3.

Design Step  
12.1.2

Passive Soil Pressure

a) Calculate the Seismic Passive Pressure Coefficient,  $K_{PE}$

In summary, the basic parameters used in the equations below are

$\phi := 35 \cdot \text{deg}$  Angle of friction of the soil

$\beta := 0$  Slope of soil face (batter angle of foundation element)

$i := 0$  Backfill slope angle

The angle of friction between the soil and the abutment is  $\delta$ .

$$\delta := \frac{\phi}{2} \qquad \delta = 17.5 \cdot \text{deg}$$

The resulting seismic inertia angle is  $\theta$ .

$$\theta := \text{atan}\left(\frac{k_h}{1 - k_v}\right) \qquad \theta = 10.2 \cdot \text{deg}$$

The basic equations are

$$\Gamma := \left(1 - \frac{\sin(\phi + \delta) \cdot \sin(\phi - \theta + i)}{\cos(\delta - \beta + \theta) \cdot \cos(i - \beta)}\right)^2$$

Coefficient in the denominator of Equation (CB-6)

$$K_{PE} := \left[ \frac{(\cos(\phi - \theta + \beta))^2}{\Gamma \cdot \cos(\theta) \cdot (\cos(\beta))^2 \cdot \cos(\delta - \beta + \theta)} \right]$$

AASHTO 1-A, Equation (CB-6)

Design Step  
12.1.2  
(continued)

With  $\beta = i = 0$ , the equations reduce to

$$\Gamma := \left( 1 - \frac{\sin(\phi + \delta) \cdot \sin(\phi - \theta)}{\cos(\delta + \theta)} \right)^2 \quad \Gamma = 0.15$$

$$K_{PE} := \left[ \frac{(\cos(\phi - \theta))^2}{\Gamma \cdot \cos(\theta) \cdot \cos(\delta + \theta)} \right] \quad K_{PE} = 6.32$$

Therefore, the seismic passive pressure coefficient is

$$K_{PE} = 6.32$$

*b) Simplification of M-O Equation into Coulomb's Equation*

With  $\theta = 0$ , the equations reduce to Coulomb's equation.

$$\theta = 0 \text{ deg}$$

$$\Gamma := \left( 1 - \frac{\sin(\phi + \delta) \cdot \sin(\phi)}{\cos(\delta)} \right)^2 \quad \Gamma = 0.096$$

$$K_P := \left[ \frac{(\cos(\phi))^2}{\Gamma \cdot \cos(\delta)} \right] \quad K_P = 7.36$$

The Commentary does not have a figure that plots various values of  $K_{PE}$  in the same fashion as for  $K_{AE}$ . However, the M-O equation, assuming  $k_h = 0.0$  for a nonseismic event, gives the same result as using Coulomb's equation,  $K_{PE} = 7.36$ . Note that the passive resistance coefficient during a seismic event is smaller (6.32 versus 7.36). The passive pressure will be used to calculate the resistance when the abutment footing moves toward the soil in front of the toe of the footing.

*c) Simplification of Coulomb's Equation into Rankine's Equation*

With  $\delta = 0$ , the equations reduce to Rankine's equation.

$$\delta := 0$$

Design Step  
12.1.2  
(continued)

$$\Gamma := (1 - \sin(\phi))^2$$

$$K_p := \left[ \frac{(\cos(\phi))^2}{\Gamma} \right]$$

$$K_p = 3.69$$

The Rankine equation is usually seen in either of the two forms shown below, both of which result in the same answer as calculated above.

$$K_p := \frac{1 + \sin(\phi)}{1 - \sin(\phi)}$$

$$K_p = 3.69$$

$$K_p := \tan\left(45 \cdot \text{deg} + \frac{\phi}{2}\right)^2$$

$$K_p = 3.69$$

In summary, for the passive earth pressure, the Rankine formula does not consider  $\delta$ ; therefore, it is more conservative than Coulomb's equation for nonseismic loads (3.69 verses 7.36). However, the Coulomb equation is known to become unconservative when  $\delta$  exceeds 15 degrees (as compared to log spiral methods where a nonlinear failure surface is used; NAVFAC Design Manual 7.02, 1986). Therefore,  $\delta$  should be limited to about 15 degrees (in this case, the value of  $\delta = \phi/2$  only slightly exceeds this recommendation and is considered satisfactory). When Coulomb's equation is modified to account for seismic forces in the soil, the passive pressure decreases as the acceleration increases (7.36 verses 6.32).

**Design Step  
12.2**

**Final Forces on Abutment**

**Design Step  
12.2.1**

**Summary of Basic Forces**

The abutment footing will be sized for the following effects: sliding, overturning, and maximum soil pressure. Any and all of these should control the size of the footing. Also, the footing should be proportioned to slide before it overturns. [AASHTO Division I-A, Article 7.4.3(A)]

The basic dimensions of the abutment are shown in Table 1 and Figure 7.

$L := 1.0 \cdot \text{ft}$                       Unit length of abutment

$H := 28.0 \cdot \text{ft}$                       Height of soil face

$B := 18.0 \cdot \text{ft}$                       Footing width

The distance from the bottom of the footing to the centroid of the active soil pressure is

$$H_A := \frac{H}{3} \qquad H_A = 9.33 \cdot \text{ft}$$

The distance from the bottom of the footing to the centroid of the additional seismic overpressure force is approximated by

$$\Delta H_{AE} := 0.6 \cdot H \qquad \Delta H_{AE} = 16.80 \cdot \text{ft}$$

The distance from the bottom of the footing to the centroid of the passive soil resistance at the front face of the footing (assuming no passive pressure from the soil above the top of footing) is

$$H_{PE} := 1.29 \cdot \text{ft} \qquad \text{Calculated from centroid of a trapezoid}$$

Design Step  
12.2.1  
(continued)

**Table 1**  
**Overall Dimensions of Abutment**

Dimensions of Abutment (ft)		Description
L =	1.0	Unit Length Along Abutment
H =	28.0	Total Height of Abutment
H <sub>w</sub> =	20.0	Stemwall Height
T <sub>s</sub> =	4.0	Stemwall Thickness
X <sub>brg</sub> =	1.25	Distance, Bearing CL to Front of Stemwall
H <sub>b</sub> =	5.0	Backwall Height
T <sub>b</sub> =	1.25	Backwall Thickness
B =	18.0	Footing Width
D =	3.0	Footing Thickness
L <sub>h</sub> =	8.0	Heel Length
L <sub>t</sub> =	6.0	Toe Length
H <sub>t</sub> =	2.0	Soil Depth Above Toe of Footing



Design Step  
12.2.1  
(continued)

The information provided by the geotechnical engineer is summarized below.

$$\gamma_s := 0.13 \cdot \text{kcf} \quad \text{Unit weight of soil}$$

$$\gamma_c := 0.15 \cdot \text{kcf} \quad \text{Unit weight of concrete}$$

$$\mu_{\text{static}} := 0.37 \quad \text{Sliding friction coefficient under working loads}$$

$$\mu_{\text{eq}} := 0.55 \quad \text{Ultimate sliding friction coefficient, seismic}$$

$$q_{\text{static}} := 6 \cdot \frac{\text{kip}}{\text{ft}^2} \quad \text{Soil pressure under static loads} \\ \text{(controlled by settlement)}$$

$$q_{\text{eq}} := 24 \cdot \frac{\text{kip}}{\text{ft}^2} \quad \text{Ultimate soil pressure under seismic loads}$$

The following data were previously calculated in Design Step 12.1.1.

$$K_h := 0.18 \quad \text{Seismic coefficient}$$

$$K_A := 0.25 \quad \text{Coulomb active pressure coefficient}$$

$$K_{AE} := 0.364 \quad \text{M-O seismic active pressure coefficient}$$

$$K_P := 7.36 \quad \text{Coulomb passive pressure coefficient}$$

$$K_{PE} := 6.32 \quad \text{M-O seismic passive pressure coefficient}$$

The data below were calculated in Design Steps 7.2.2 and 7.3.3b. These forces are the basic vertical and lateral external forces exerted by the superstructure on the top of the abutment seat. Note that live load surcharge loads are not included in this example and should be added for Group I loads.

$$P_{\text{brg}} := 7.94 \cdot \text{kip} \quad \text{Vertical reaction per foot of wall}$$

$$V_{\text{brg}} := 2.10 \cdot \text{kip} \quad \text{Shear force in bearings per foot of wall}$$

Design Step  
12.2.2

## Calculate Earth Pressure Forces

The soil pressure forces are now calculated for the basic load conditions. The vertical component of the active force, due to wall friction acting on the inside face of the abutment wall, is not included in the calculations. This is a conservative approach. All forces are calculated based on a 1-foot strip of abutment wall.

a) *Active Earth Pressure Force (Static Effect Only),  $E_A$* 

This force is due to the normal active earth pressure of the wall under static conditions. This force is triangular shaped, with the resultant acting at  $H/3$  above the top of footing. See Figure 7 in Design Step 12.2.1.

$$\gamma_s = 0.130 \cdot \text{kcf} \quad \text{Unit weight of soil}$$

$$H = 28.0 \cdot \text{ft} \quad \text{Height of soil face}$$

$$K_A = 0.250 \quad \text{Coulomb active pressure coefficient}$$

$$E_A := \frac{1}{2} \cdot \gamma_s \cdot H^2 \cdot K_A \quad E_A = 12.74 \cdot \frac{\text{kip}}{\text{ft}}$$

b) *Total Active Earth Pressure Force (Static + Seismic Effects),  $E_{AE}$* 

This force is the total active earth pressure that occurs during a seismic event. Note that this force includes both static active pressure and additional seismic overpressure effects.

$$K_{AE} = 0.36 \quad \text{M-0 seismic active pressure coefficient}$$

$$E_{AE} := \frac{1}{2} \cdot \gamma_s \cdot H^2 \cdot K_{AE} \quad \text{AASHTO I-A} \\ \text{Eqn (CB-3)}$$

$$E_{AE} = 18.55 \cdot \frac{\text{kip}}{\text{ft}}$$

Design Step  
12.2.2  
(continued)

*c) Additional Dynamic Earth Pressure Force (Seismic Effect Only)*

This is the seismic overpressure force. It includes only the “additional” seismic pressure that occurs during an earthquake. It does not include the original active pressure present under static conditions. The Specification Commentary states that the centroid of this additional force acts from 0.5H to 0.6H above the top of footing (0.6H was used in the example). The shape of this stress block can be approximated as a trapezoid acting over the height of the wall. See Figure 7 in Design Step 12.2.1.

$$E_{AE} = 18.55 \cdot \frac{\text{kip}}{\text{ft}} \quad \text{Total active earth pressure force, including seismic}$$

$$E_A = 12.74 \cdot \frac{\text{kip}}{\text{ft}} \quad \text{Active earth pressure force}$$

$$\Delta E_{AE} := E_{AE} - E_A \quad \text{Earthquake overpressure force}$$

$$\Delta E_{AE} = 5.8 \cdot \frac{\text{kip}}{\text{ft}}$$

*d) Passive Earth Pressure (Static Effect Only)*

The normal passive earth pressure force acts on the front face of the footing. This passive pressure develops only when the front face of the footing moves toward the soil. Although the magnitude of the force is calculated below, it is conservative to ignore this effect when checking for sliding under dead load because the passive earth pressure does not develop until significant wall movement occurs. This is because it is not desirable to have the footing slide in order to develop this force under dead load conditions. In this example, the pressure is in the shape of a trapezoid because the upper 2 feet have been ignored. See Figure 7.

$$\gamma_s = 0.130 \cdot \text{kcf} \quad \text{Unit weight of soil}$$

$$D := 3.0 \cdot \text{ft} \quad \text{Footing depth}$$

$$H_t := 2.0 \cdot \text{ft} \quad \text{Toe soil depth}$$

$$K_p = 7.4 \quad \text{Coulomb passive pressure coefficient}$$

Design Step  
12.2.2  
(continued)

The passive pressure force on the face of the footing toe is

$$E_P := \frac{1}{2} \cdot \gamma_s \cdot \left[ (D + H_t)^2 - H_t^2 \right] \cdot K_P \qquad E_P = 10.0 \cdot \frac{\text{kip}}{\text{ft}}$$

e) *Total Passive Earth Pressure (Static + Seismic Effects)*

This force is the total passive earth pressure, acting on the front face of the footing, that occurs during a seismic event. Note that this force includes the effect of the static passive pressure. As was mentioned in Design Step 12.1.2, it is assumed that as the magnitude of the earthquake increases, the magnitude of the passive pressure resisting the movement decreases.

Note here that there is no experimental justification for applying the M-O equations to the passive case. It should also be noted that the original M-O work was limited to the case where the water table is below the back of the wall. The bridge designer should discuss consideration of passive pressure with the project geotechnical engineer.

$$\gamma_s = 0.130 \cdot \text{pcf} \qquad \text{Unit weight of soil}$$

$$D := 3.0 \cdot \text{ft} \qquad \text{Footing depth}$$

$$H_t := 2.0 \cdot \text{ft} \qquad \text{Toe soil depth}$$

M-O seismic passive pressure coefficient is

$$K_{PE} = 6.3$$

$$E_{PE} := \frac{1}{2} \cdot \gamma_s \cdot \left[ (D + H_t)^2 - H_t^2 \right] \cdot K_{PE} \qquad E_{PE} = 8.63 \cdot \frac{\text{kip}}{\text{ft}}$$

Design Step  
12.2.3

## Final Abutment Forces

*a) Calculate Vertical Force and Resisting Overturning (OT) Moments in Footing*

The vertical dead load and resisting moments are calculated in Table 2, and summarized as follows.

$$P_v := 56.5 \cdot \frac{\text{kip}}{\text{ft}} \quad \text{Total vertical load on footing}$$

Resisting OT moment about the toe of the footing (without passive) is

$$M_{\text{toe}_r} := -603.9 \cdot \text{kip} \cdot \frac{\text{ft}}{\text{ft}}$$

Resisting OT moment about the centerline of the footing (without passive) is

$$M_{\text{cl}_r} := -95.1 \cdot \text{kip} \cdot \frac{\text{ft}}{\text{ft}}$$

Design Step  
12.2.3  
(continued)

**Table 2**  
**Vertical Loads and Resisting OT Moments on Abutment**

Vertical Loads (Per Unit Length of Abutment Footing)					
Force	Width (ft)	Depth (ft)	Area (ft <sup>2</sup> )	Unit Wt (kip/ft)	P <sub>v</sub> (kip)
P <sub>brg</sub>	-	-	-	-	7.9
P <sub>w</sub>	4.0	20.0	80.0	0.15	12.0
P <sub>f</sub>	18.0	3.0	54.0	0.15	8.1
P <sub>b</sub>	1.25	5.0	6.3	0.15	0.9
P <sub>h</sub>	8.0	25.0	200.0	0.13	26.0
P <sub>t</sub>	6.0	2.0	12.0	0.13	1.6
Sum Σ =					56.5

Resisting Overturning Moment (Not Including Passive Pressure)					
Force	Wt P <sub>v</sub> (kip)	About Toe of Ftg		About CL Ftg	
		Arm x (ft)	M <sub>toe,r</sub> = P <sub>v</sub> * x (k-ft)	Arm x (ft)	M <sub>cl,r</sub> = P <sub>v</sub> * x (k-ft)
P <sub>brg</sub>	7.9	-7.25	-57.6	1.75	13.9
P <sub>w</sub>	12.0	-8.0	-96.0	1.0	12.0
P <sub>f</sub>	8.1	-9.0	-72.9	0.0	0.0
P <sub>b</sub>	0.9	-9.38	-8.8	-0.38	-0.4
P <sub>h</sub>	26.0	-14.0	-364.0	-5.0	-130.0
P <sub>t</sub>	1.6	-3.0	-4.7	6.0	9.4
Sum Σ =	56.5		-603.9		-95.1

Note: See Figure 7 for location of forces.

Design Step  
12.2.3  
(continued)

b) The Horizontal Forces and Overturning Moments Are Calculated in Table 3

**Table 3**  
**Lateral Forces and Overturning Moments on Abutment**

Lateral Forces for (DL + E) Load Case			
Force	F <sub>hA</sub> (kip)	y (ft)	Mot <sub>A</sub> = F <sub>hA</sub> * y (k-ft)
V <sub>brg</sub>	-	23.0	-
V <sub>w</sub>	-	13.0	-
V <sub>f</sub>	-	1.5	-
V <sub>b</sub>	-	25.5	-
V <sub>h</sub>	-	15.5	-
V <sub>t</sub>	-	4.0	-
E <sub>A</sub>	12.74	9.33	118.9
Sum Σ	12.74	-	118.9
E <sub>p</sub> force not considered in this load case			

Design Step  
12.2.3  
(continued)

**Table 3**  
**Lateral Forces and Overturning Moments on Abutment**  
**(continued)**

Lateral Forces for (DL+E+EQ) Load Case					
Force	Wt (kip)	k <sub>h</sub> Factor	F <sub>hAE</sub> = Wt*k <sub>h</sub> (kip)	y (ft)	Mot <sub>AE</sub> = F <sub>hAE</sub> * y (k-ft)
V <sub>brg</sub>	-	-	2.10	23.0	48.3
V <sub>w</sub>	12.0	0.18	2.16	13.0	28.1
V <sub>f</sub>	8.1	0.18	1.46	1.5	2.2
V <sub>b</sub>	0.9	0.18	0.17	25.5	4.3
V <sub>h</sub>	26.0	0.18	4.68	15.5	72.5
V <sub>t</sub>	1.6	0.00	0.00	4.0	0.0
E <sub>A</sub>	-	-	12.74	9.3	118.9
ΔE <sub>AE</sub>	-	-	5.81	16.80	97.6
Σ of Total Active Lateral Forces			29.12	-	371.9
Resisting Passive Forces E <sub>PE</sub> =			-8.63	1.29	-11.1

Note: See Figure 7 for locations of forces

Active Pressure Only DL + E

Horizontal force due to active pressure is

$$F_{hA} := 12.74 \cdot \frac{\text{kip}}{\text{ft}}$$

Overturning moment due to active pressure is

$$\text{Mot}_A := 118.9 \cdot \text{kip} \cdot \frac{\text{ft}}{\text{ft}}$$

Active + Seismic Pressure are DL + E + EQ

Design Step  
12.2.3  
(continued)

Net horizontal force due to total active pressure (including seismic) is

$$F_{h_{AE}} := 29.12 \frac{\text{kip}}{\text{ft}}$$

Net overturning moment due to total active pressure (including seismic) is

$$M_{ot_{AE}} := 371.9 \text{ kip} \frac{\text{ft}}{\text{ft}}$$

There are issues in Table 3 that need to be discussed in further detail. They are included below.

- Seismic overpressure  $\rho E_{AE}$  is shown separate from  $E_A$  to demonstrate that the earthquake creates an overpressure, and to show its effect relative to the active condition. The resultant of the overpressure is located at about 0.6 times the soil height above the bottom of the footing, per AASHTO Division I-A, C7.4.3(A).
- Note that passive pressure acting in front of the footing was ignored for the DL + E load condition, because in order to develop passive pressure the abutment must move laterally. Under normal D + E loading, the abutment is not expected to move enough to develop this load.
- AASHTO Division I-A, Article 7.4.3(A) requires that the inertia effect of the abutment itself be included in calculating the lateral load. Figure 29 of the AASHTO Division I-A Commentary shows both the soil supported above the footing heel, and the abutment self weight included in the inertia effect. It is conservative to use the value of  $k_h \times W$ , as shown in Figure 29, for the calculation of the inertia force.
- The inertia effect of soil on top of the footing toe is ignored, since its mass is not restrained by the abutment wall.

Design Step  
12.3

## Abutment Stability and Soil Stress Check

Design Step  
12.3.1Check for Dead Load Plus Active Earth Pressure Loads  
(But Not for Seismic Loads)

Note that for this condition, no live load surcharge load has been added. The objective here is to consider the potential long-term pressure under the footing. The goal is to have an approximately uniform load for differential settlement purposes.

*a) Factor of Safety (FS) Against the Abutment Sliding*

Table 4 presents a general guide for determining the ultimate values of the coefficient of the sliding friction,  $\mu$ , between the bottom of the footing and the soil, based on the soil type. The values in this table are approximate in nature. For this design example, a gravel-sand mixture was assumed, with the lower bound having an ultimate value of 0.55. Under working-level loads, a factor of safety of 1.5 is applied to the ultimate value to get the working load value of 0.37.

Note that the coefficient of friction, for a given type of soil, is the same under both static and seismic loads. But, for design purposes, the value is reduced by 1.5 for static working loads, to provide a higher factor of safety under this long-term loading. Also note that the passive pressure on the face of the footing is not included under working-level loads.

Ultimate sliding coefficient of friction is

$$\mu_{eq} = 0.55$$

Sliding coefficient of friction under normal active loads,  $= \mu_{eq} / 1.5$ , is

$$\mu_{static} = 0.37$$

Total vertical load on the footing is

$$P_v = 56.5 \cdot \frac{\text{kip}}{\text{ft}}$$

Design Step  
12.3.1  
(continued)

**Table 4**  
**Ultimate Values of Coefficient of Friction**  
**for Concrete Foundations on Rock/Soil**

Material	Relative Density/ Consistency	Coefficient of Friction <sup>1</sup>	Adhesion <sup>1</sup> (PSF) <sup>2</sup>
Clean, Sound Rock <sup>3</sup>	Not Applicable	0.70 - 0.80	---
Clean Gravel, Gravel-Sand Mixtures	Dense to Very Dense Medium Dense	0.55 - 0.70 0.55 - 0.65	--- ---
Clean to Slightly Silty/Clayey Sand with or without Gravel	Dense to Very Dense Medium Dense	0.45 - 0.60 0.45 - 0.55	--- ---
Silty/Clayey Sand and Sandy Silt with or without Gravel	Dense to Very Dense Medium Dense	0.40 - 0.55 0.35 - 0.50	--- ---
Silty Clay and Clayey Silt with or without Sand and Gravel (Low Plasticity) <sup>4</sup>	Very Stiff to Hard Medium Stiff to Stiff	0.40 - 0.50 0.30 - 0.45	1000 - 1500 500 - 1000

## Notes:

1. The lesser of (1) the coefficient of friction times the normal force acting on the base and (2) the adhesion times the base width should be used for design. Where only the coefficient of friction or the adhesion is shown, that parameter should be used.
2. PSF = pounds per square foot.
3. The sliding resistance of weathered and jointed rock may be controlled by the presence of joints. The determination of a coefficient of friction or adhesion value should be made by a geotechnical engineer.
4. The strength of high plasticity clay/silt (LL > 50) may be reduced significantly and should be evaluated by a geotechnical engineer.

From Potyondy, 1961; Goh and Donald, 1984; U.S. Department of the Navy, 1986

Design Step  
12.3.1  
(continued)

Horizontal force due to active pressure (without passive pressure) is

$$F_{hA} = 12.7 \cdot \frac{\text{kip}}{\text{ft}}$$

Per AASHTO Division I, Article 7.5.2.1, the minimum factor of safety against sliding is 1.5.

$$FS_{\text{sliding}} := \frac{\text{"Resisting" Sliding Force}}{\text{"Applied" Sliding Force}}$$

$$FS_{\text{sliding}} := \frac{\mu_{\text{static}} \cdot P_v}{F_{hA}} \qquad FS_{\text{sliding}} = 1.64 \qquad \text{okay}$$

***b) Factor of Safety Against the Abutment Overturning About the Toe of the Footing***

Resisting OT moment about the toe of the footing (without passive) is

$$M_{\text{toe}_r} = -603.9 \cdot \text{kip} \cdot \frac{\text{ft}}{\text{ft}}$$

Overturning moment due to active pressure is

$$M_{\text{ot}_A} = 118.9 \cdot \text{kip} \cdot \frac{\text{ft}}{\text{ft}}$$

Per AASHTO Division I, Article 7.5.2.1, the minimum factor of safety is 2.0.

$$FS_{\text{ot}} := \frac{\text{"Resisting" Overturning Moment}}{\text{"Applied" Overturning Moment}}$$

$$FS_{\text{ot}} := \frac{-M_{\text{toe}_r}}{M_{\text{ot}_A}} \qquad FS_{\text{ot}} = 5.1 \qquad \text{okay}$$

Note that the factor of safety against overturning is much greater than the factor of safety against sliding ( $5.1 > 1.64$ ); therefore, for long-term dead load only, the footing is proportioned properly so that it slides before it overturns.

Design Step  
12.3.1  
(continued)*c) Calculate the Eccentricity (e) of the Vertical Load About the Centerline*

Resisting OT moment about the centerline of the footing (without passive) is

$$M_{cl_r} = -95.1 \cdot \text{kip} \cdot \frac{\text{ft}}{\text{ft}}$$

$$P_v = 56.5 \cdot \frac{\text{kip}}{\text{ft}} \quad \text{Total vertical load on footing}$$

The resulting net eccentricity of the vertical load is

$$e := \frac{\text{"Net" Overturning Moment}}{\text{Total Vertical Load}}$$

$$e := \frac{M_{cl_r} + M_{ot_A}}{P_v} \quad e = 0.42 \cdot \text{ft}$$

The kern distance from the centerline of the footing, below which the footing experiences no uplift, is

$$B = 18.0 \cdot \text{ft} \quad \text{Footing width}$$

$$e_{\max} := \frac{B}{6} \quad e_{\max} = 3.0 \cdot \text{ft}$$

The actual eccentricity is nearly zero, therefore, the soil pressure under the footing, due to dead load plus normal earth pressure, is approximately uniform.

*d) Calculate the Soil Pressure Under the Footing for Dead and Normal Earth Loads*

The maximum soil pressure, assuming no uplift, is

$$q_{\max} := \frac{P_v}{B} \cdot \left( 1 + 6 \cdot \frac{e}{B} \right) \quad \text{Bowles, 3rd Edition Eqn. (8-13)}$$

$$q_{\max} = 3.58 \cdot \text{ksf}$$

Design Step  
12.3.1  
(continued)

The minimum soil pressure is

$$q_{\min} := \frac{P_v}{B} \cdot \left( 1 - 6 \cdot \frac{e}{B} \right) \qquad q_{\min} = 2.70 \cdot \text{ksf}$$

The soil pressure under the footing is nearly uniform, and should not cause unequal settlements.

The allowable static soil pressure under the footing is

$$q_{\text{static}} = 6.0 \cdot \text{ksf} \qquad \text{Therefore, the stress is okay}$$

Design Step  
12.3.2

**Check for Dead Load Plus Earth Pressure Plus Seismic Overpressure**

*a) Factor of Safety Against the Abutment Sliding*

The ultimate sliding coefficient of friction, used for seismic loads, is

$$\mu_{eq} = 0.55$$

Total vertical load on footing is

$$P_v = 56.5 \cdot \frac{\text{kip}}{\text{ft}}$$

Horizontal force due to total seismic active pressure is

$$F_{h_{AE}} = 29.1 \cdot \frac{\text{kip}}{\text{ft}}$$

The factor of safety against sliding is

$$FS_{\text{sliding}} := \frac{\text{"Resisting" Sliding Force}}{\text{"Applied" Sliding Force}}$$

With no passive pressure considered in front of the footing

Design Step  
12.3.2  
(continued)

$$FS_{\text{sliding}} := \frac{\mu_{eq} \cdot P_v}{F_{hAE}} \quad FS_{\text{sliding}} = 1.07 > 1.0 \text{ okay}$$

At this time, it is appropriate to check the flexibility of the abutment for the use of the lower bound value of  $k_h = 0.5 \times A$ . This lower bound value requires the abutment to move sufficiently to achieve the “active” soil state of stress. This movement can be an outward rotation of the abutment, an outward sliding, or a combination of the two. An outward movement at the top of the abutment equal to about 0.001 to 0.004 times the wall height is required for the range of soils usually placed behind abutments. The geotechnical engineer should be consulted when assessing these criteria and the design value for  $k_h$ .

With passive pressure considered in front of the footing

If the passive pressure on the face of the abutment footing is included in the calculation, the “net” resistance against sliding of the footing is increased.

$$E_{PE} = 8.6 \cdot \frac{\text{kip}}{\text{ft}} \quad \text{Total passive earth pressure}$$

$$FS_{\text{sliding}} := \frac{\mu_{eq} \cdot P_v + E_{PE}}{F_{hAE}} \quad FS_{\text{sliding}} = 1.36 > 1.0 \text{ okay}$$

The code implies that the factor of safety against sliding should be greater than 1.0. Some state DOTs recommend a 1.1 factor of safety.

*b) Factor of Safety against the Abutment Overturning About the Toe of the Footing (including passive pressure)*

Resisting OT moment about the toe of the footing (without passive) is

$$M_{\text{toe}_r} = -603.9 \cdot \text{kip} \cdot \frac{\text{ft}}{\text{ft}}$$

Resisting OT moment due to seismic passive pressure is

$$M_{\text{ot}_{PE}} := -11.1 \cdot \text{kip} \cdot \frac{\text{ft}}{\text{ft}}$$

Design Step  
12.3.2  
(continued)

Overturning moment due to total seismic active pressure is

$$Mot_{AE} = 371.9 \cdot \text{kip} \cdot \frac{\text{ft}}{\text{ft}}$$

The factor of safety against overturning is

$$FS_{ot} := \frac{\text{"Resisting" Overturning Moment}}{\text{"Applied" Overturning Moment}}$$

$$FS_{ot} := \frac{-(M_{toe_r} + Mot_{PE})}{Mot_{AE}} \qquad FS_{ot} = 1.65$$

Note that the factor of safety against overturning is greater than the factor of safety against sliding whether or not the soil passive pressure is considered (sliding factor of safety is 1.07 or 1.36, overturning factor of safety is 1.65). The abutment will, therefore, theoretically tend to slide before it overturns.

*c) Calculate the Eccentricity of the Vertical Load About the Centerline of the Footing*

In addition to checking for maximum soil stresses, footing uplift must be considered. Per Division I-A, Article 7.4.2(B), the footing can experience a separation of the soil up to one-half of the contact area of the foundation under seismic loading. This corresponds to an eccentricity of the footing of B/3 or less.

Resisting OT moment about the centerline of the footing (without passive pressure) is

$$M_{cl_r} = -95.1 \cdot \text{kip} \cdot \frac{\text{ft}}{\text{ft}}$$

Resisting OT moment due to seismic passive pressure is

$$Mot_{PE} := -11.1 \cdot \text{kip} \cdot \frac{\text{ft}}{\text{ft}}$$

Design Step  
12.3.2  
(continued)

Overturning moment due to total seismic active pressure is

$$Mot_{AE} = 371.9 \cdot \text{kip} \cdot \frac{\text{ft}}{\text{ft}}$$

Total vertical load on the footing is

$$P_v = 56.5 \cdot \frac{\text{kip}}{\text{ft}}$$

The resulting net eccentricity of the vertical load is

$$e := \frac{\text{"Net" Overturning Moment}}{\text{Total Vertical Load}}$$

$$e := \frac{(Mcl_r + Mot_{PE}) + Mot_{AE}}{P_v} \quad e = 4.70 \cdot \text{ft}$$

When the eccentricity of the footing is greater than the kern distance of  $B/6$ , the footing experiences partial uplift. Because in this use the eccentricity (4.70 feet) is greater than this limit (3.0 feet), there is uplift on the footing.

To ensure that there is no more than one-half uplift on the footing, the eccentricity,  $e$ , must be less than the limit of  $B/3$ .

$$B = 18.0 \cdot \text{ft} \quad \text{Footing width}$$

$$\frac{B}{3} = 6 \cdot \text{ft} \quad \text{Maximum eccentricity to not exceed one-half uplift}$$

Because  $B/3$  is greater than  $e_{\max}$ , ( $6.0 > 4.7$  ft) an 18-foot-square footing is large enough to resist half uplift.

*d) Calculate the Soil Pressure under the Footing for Group VII Loading*

In cases like this bridge, where the soil material has a large ultimate capacity, the maximum allowable soil stress limit will usually not control the footing design.

Design Step  
12.3.2  
(continued)

The maximum soil pressure at the toe of the footing, allowing for uplift of the footing, is  $q$ .

$$P_v = 56.5 \cdot \frac{\text{kip}}{\text{ft}} \quad \text{Total vertical load on footing}$$

$$B = 18.0 \cdot \text{ft} \quad \text{Footing width}$$

$$L = 1.0 \cdot \text{ft} \quad \text{Unit length of footing}$$

Eccentricity of vertical load about the centerline of footing is

$$e = 4.7 \cdot \text{ft}$$

Note that some DOTs use a rectangular stress block, and some codes use a rectangular stress block, for ultimate loads. The half-uplift criteria under seismic loads, when using a rectangular stress block, need improved definition.

In this example, a triangular stress block is assumed. The soil stress for this triangular stress block is

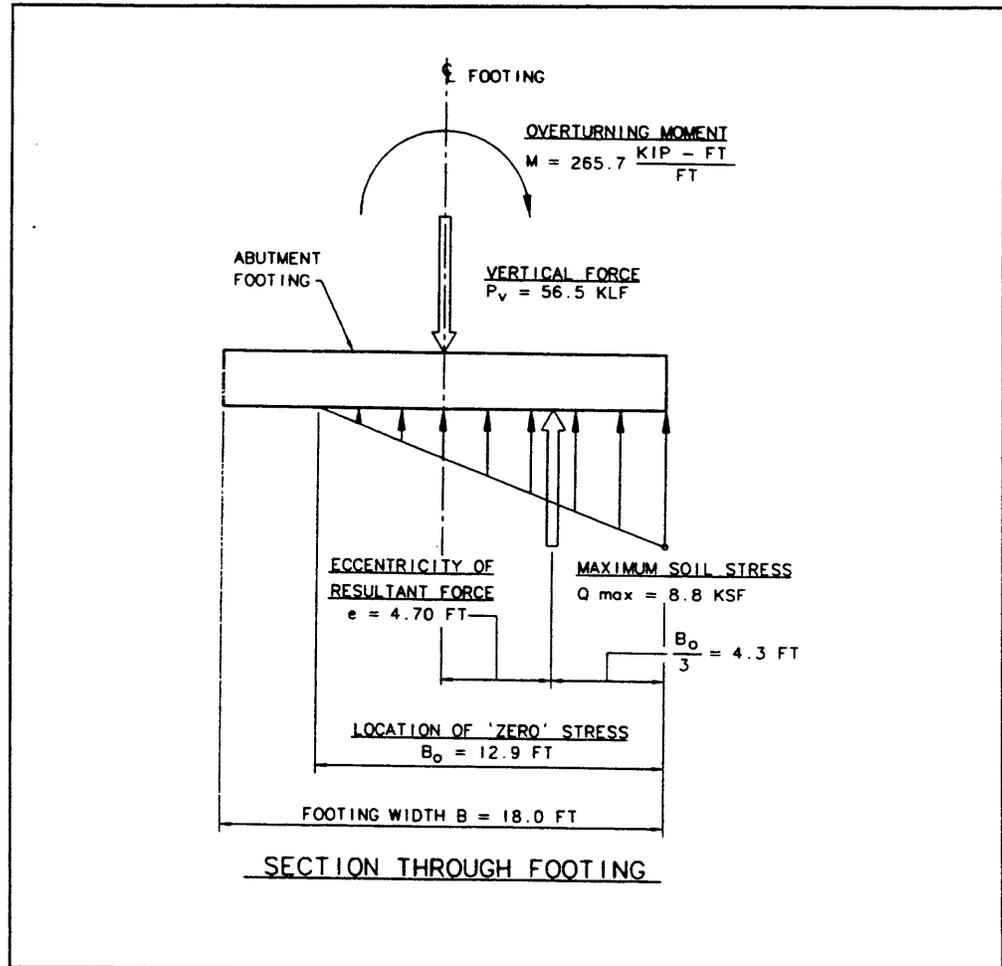
$$q := \frac{2 \cdot P_v}{3 \cdot \left( \frac{B}{2} - e \right)} \quad \begin{array}{l} \text{Bowles, 3rd Edition} \\ \text{Eqn (8-14)} \end{array}$$

$$q = 8.8 \cdot \text{ksf} \quad \text{Maximum soil pressure}$$

$$q_{eq} = 24.0 \cdot \text{ksf} \quad \begin{array}{l} \text{Ultimate soil pressure under} \\ \text{seismic loads} \end{array}$$

The soil must be able to resist this pressure under short-term load without failure. Commonly, the ultimate soil pressure is at least twice the allowable soil pressure. Because  $q$  is less than  $q_{eq}$ , ( $8.8 < 24.0$  ksf), the maximum soil pressure is not exceeded. See Figure 8. The bridge designer should consult with the project geotechnical engineer regarding ultimate soil pressure capacities.

Design Step  
12.3.2  
(continued)



**Figure 8 — Soil Stress Under Abutment Footing**

*e) Quick Check of Abutment Footing for Sliding and Soil Stresses (Longitudinal Loads Combined with Transverse Loads)*

In a typical abutment design, only the forces in the local transverse direction are used in the design checks for sliding and soil stresses. Typically, soil pressures under the abutment footing due to forces in the long direction are ignored because of the proportions of the footing. But, as the abutment wall gets taller and the bridge gets narrower, this effect becomes more important.

The following calculations include the longitudinal loads on the abutment in combination with the transverse loads. The loads will be taken from LC2 in Design Step 7.2.2.

Design Step  
12.3.2  
(continued)

Step 1. Check for sliding due to the resultant lateral shear force. Include passive resistance in front of the footing.

Total vertical force on bottom of footing is

$$P_v = 56.5 \cdot \frac{\text{kip}}{\text{ft}} \quad \text{Total vertical load on footing, per unit length}$$

$$L := 49.8 \cdot \text{ft} \quad \text{Length of abutment}$$

Total vertical load on footing is

$$P := P_v \cdot L \quad P = 2814 \cdot \text{kip}$$

Lateral force in local transverse direction of abutment is

$$F_{h_{AE}} = 29.1 \cdot \frac{\text{kip}}{\text{ft}} \quad \text{Total lateral load on footing, per unit length}$$

$$L = 49.8 \cdot \text{ft} \quad \text{Length of abutment}$$

Total transverse lateral load on footing is

$$V_T := F_{h_{AE}} \cdot L \quad V_T = 1450 \cdot \text{kip}$$

Lateral force in local longitudinal direction of abutment is

From Design Step 7.2.2, for LC2, the longitudinal shear is

$$V_L := 168.5 \cdot \text{kip} \quad \text{Total longitudinal load on footing}$$

Resultant of shear due to orthogonal forces is

$$V_{AE} := \sqrt{V_T^2 + V_L^2} \quad V_{AE} = 1460 \cdot \text{kip}$$

Design Step  
12.3.2  
(continued)

Calculate the lateral passive resistance force in front of the abutment footing.

$$E_{PE} = 8.6 \cdot \frac{\text{kip}}{\text{ft}} \quad \text{Passive resistance in front of footing}$$

Total transverse lateral load on footing is

$$V_{PE} := E_{PE} \cdot L \quad V_{PE} = 430 \cdot \text{kip}$$

Find the factor of safety against sliding, where

$$\mu_{eq} = 0.55 \quad \text{Sliding friction coefficient under seismic loads}$$

$$P = 2814 \cdot \text{kip} \quad \text{Total vertical load on footing}$$

$$V_{PE} = 430 \cdot \text{kip} \quad \text{Passive resistance force}$$

$$V_{AE} = 1460 \cdot \text{kip} \quad \text{Total longitudinal load on footing}$$

$$FS_{\text{sliding}} := \frac{\mu_{eq} \cdot P + V_{PE}}{V_{AE}} \quad FS_{\text{sliding}} = 1.35 > 1.0 \text{ okay}$$

Step 2. Check for additional soil stress due to the longitudinal shear at the top of the abutment. See Figure 9.

$$V_L = 168.5 \cdot \text{kip} \quad \text{Longitudinal shear force at top of abutment wall}$$

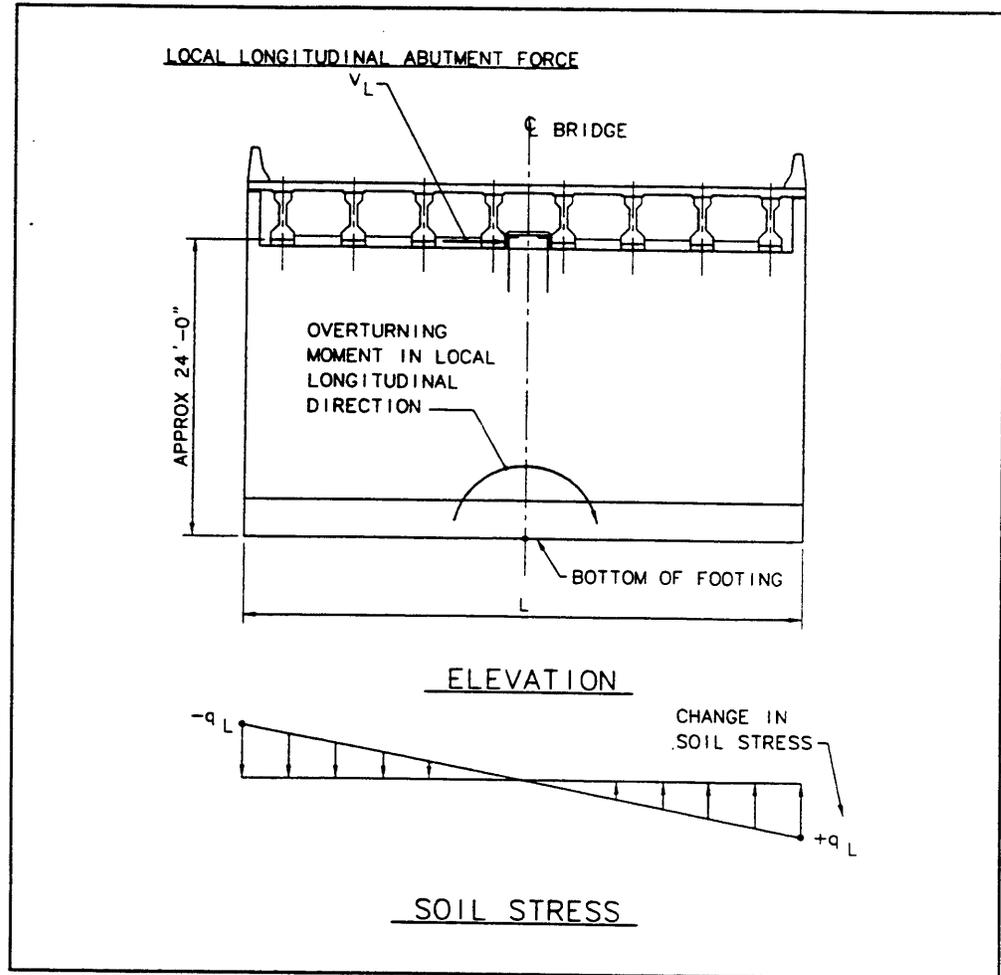
Calculate the approximate location of the shear force above the bottom of the footing.

$$y := 24.0 \cdot \text{ft}$$

$$M_L := V_L \cdot y \quad M_L = 4044.0 \cdot \text{kip} \cdot \text{ft}$$

$$L = 49.8 \cdot \text{ft} \quad \text{Length of abutment}$$

Design Step  
12.3.2  
(continued)



**Figure 9 — Effects of Local Longitudinal Force on Abutment**

Calculate the change in the soil stress, due to this longitudinal shear.

$B = 18.0 \cdot \text{ft}$                       Width of footing

$S_L := \frac{B \cdot L^2}{6}$                       Section modulus of footing

$S_L = 7440.1 \cdot \text{ft}^3$

Design Step  
12.3.2  
(continued)

Now the change in soil stress due to the longitudinal shear force can be calculated. Note that this stress adds to or subtracts from the final stress.

$$q_L := \frac{M_L}{S_L} \qquad q_L = 0.54 \text{ ksf}$$

The additional soil stress due to longitudinal forces is 0.5 ksf, which is small compared to the soil stress of 8.8 ksf due to vertical and transverse forces. For this case, it is insignificant and is not considered in the design.

**Conclusion:** The effect of adding the loads in the longitudinal direction is too small to be significant for the abutment in this example. However, consideration of these effects may be important in the case of taller, narrower abutments.

Design Step  
12.3.3

## Finalize the Footing Size

The maximum soil pressure under the seismic load combination is  $q = 8.8$  ksf, which is less than the allowable limit of  $q = 24$  ksf. Also, the half-uplift criterion is not exceeded, although this criterion is more critical than the soil stress limit.

Therefore, use the 18-foot-wide footing.

Using the triangular-shaped soil pressure computed above, the designer can now design the footing for flexure and shear using Division I of AASHTO. Note that top reinforcement should be included in the footing in order to support the weight of the soil above the footing due to the uplift condition.

**Design Step  
12.4**

**Design Abutment Stemwall and Footing**

Keeping in mind the final forces on the abutment calculated in Design Step 12.2, and the abutment stability and soil stresses checked in Design Step 12.3, design both the abutment footing and stemwall. The backwall is not included in this design example.

**Design Step  
12.4.1**

**Abutment Stemwall Design**

Table 5 summarizes the shears and moments due to the individual loads contributing to the forces on the abutment stemwall. See Figure 10 for stress and moment shapes.

**Table 5  
Factored Shear and Moment  
in Abutment Stemwall**

Description	Force	Load Case (DL+E+EQ) Forces		
		$V_{wall_u} =$ $Wt * k_r$ (kip)	$y$ (ft)	$M_{wall_u} =$ $F_r * y$ (k-ft)
Shear on Bearings	$V_{brg}$	2.10	20.0	42.0
Stemwall Inertia	$V_w$	2.16	10.0	21.6
Backwall Inertia	$V_b$	0.17	22.5	3.8
Inertia of Soil Above Heel	$V_h$	4.68	12.5	58.5
Active Soil Pressure	$E_A$	12.74	6.3	80.7
EQ Overpressure	$\Delta E_{AE}$	5.81	13.8	80.2
Summation	$\Sigma$	27.7		286.8

Design Step  
12.4.1  
(continued)

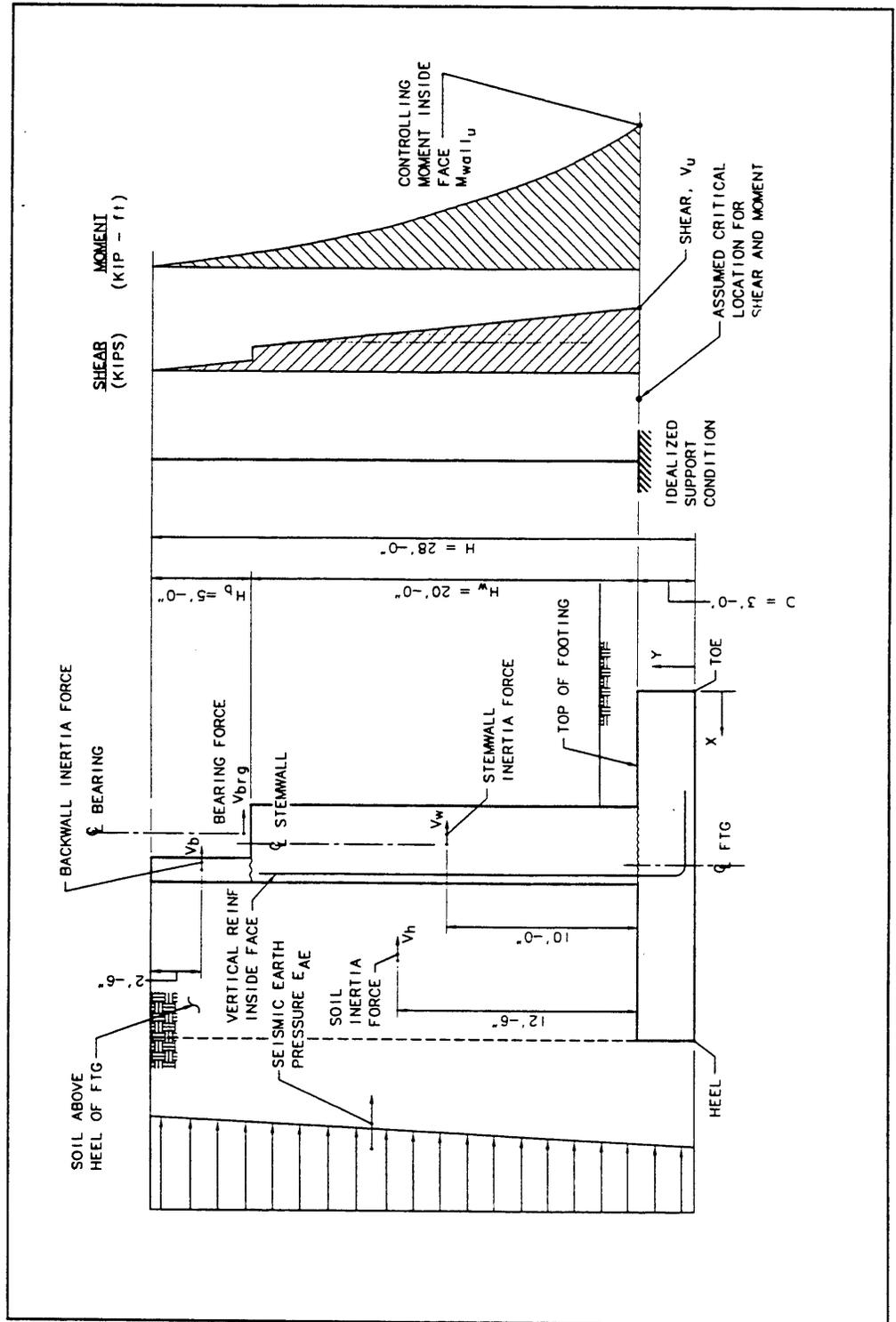


Figure 10 — Transverse Forces on Abutment Stemwall

Design Step  
12.4.1  
(continued)

From the results in Table 5, several observations can be made about forces on the abutment. These observations may not apply to other abutments.

- The shear due to the additional seismic earth pressure is approximately half the magnitude due to the normal active earth pressure (5.81 verses 12.74 kip).
- The resulting overturning moment (OTM) due to the additional seismic earth pressure is similar in magnitude to the OTM due to the normal active earth pressure (80.2 kip-ft verses 80.7 kip-ft) because the c.g. of the force is higher (0.6H verses H/3).
- The inertia of the soil above the heel of the footing, and the inertia of the abutment itself, are significant. The code assumes this mass is accelerated laterally in combination with the lateral earth pressure on the abutment. Some state DOTs consider this effect when checking for sliding and overturning of the abutment, but do not consider it when calculating forces in the wall and footing.

*a) Check Shear in Stemwall of Abutment, Determine if Shear Reinforcement Is Required*

$f_c := 4000 \cdot \text{psi}$	Concrete strength
$f_y := 60 \cdot \text{ksi}$	Yield stress of reinforcement
$\phi_m := 0.9$	Phi factor for flexure
$\phi_v := 0.85$	Phi factor for shear
$V_u := 27.7 \cdot \text{kip}$	Factored shear
$d := 44 \cdot \text{in}$	Depth of steel centroid
$b := 12 \cdot \text{in}$	Unit length of wall

Calculate the concrete shear capacity of the abutment wall.

$$V_c := 2 \cdot \sqrt{f_c} \cdot b \cdot d \qquad V_c := 66.8 \cdot \text{kip}$$

Design Step  
12.4.1  
(continued)

The concrete shear capacity is greater than the factored shear. Per Division 1, 8.19.1.1, no shear reinforcement is required where the following formula is met.

$$V_u < \frac{1}{2} \cdot \phi_v \cdot V_c \quad V_u = 27.7 \cdot \text{kip}$$

$$\phi_v := 0.85 \quad \text{Phi factor for shear}$$

$$\frac{1}{2} \cdot \phi_v \cdot V_c = 28.4 \cdot \text{kip}$$

Because the factored shear is less than the above requirement, (27.7 < 28.4 kip), no shear reinforcement is required.

*b) Design Flexural Reinforcement for Inside Face of Stemwall*

$$M_{\text{wall}_u} := 286.8 \cdot \text{kip} \cdot \text{ft} \quad \text{Factored moment at base of wall}$$

$$\phi_m := 0.9 \quad \text{Phi factor for flexure}$$

Compute the percentage of moment reinforcement required.

$$f_c = 4000 \cdot \text{psi}$$

$$b = 12 \cdot \text{in} \quad \text{Unit length of wall}$$

$$d = 44 \cdot \text{in} \quad \text{Depth of steel centroid}$$

$$\text{Ratio} := \frac{\frac{M_{\text{wall}_u}}{\phi_m}}{f_c \cdot b \cdot d^2} \quad \text{Ratio} = 0.0412$$

$$\omega := .0412 \quad \text{Reinforcement index}$$

Design Step  
12.4.1  
(continued)

The reinforcement ratio required in the wall, assuming that 1.2 x M crack is met, is

$$\rho := \frac{\omega \cdot f_c}{f_y} \quad \rho = 0.0027$$

Calculate the area of reinforcement on the backface of the abutment wall, per foot length of wall.

$b = 12 \cdot \text{in}$                       Unit width

$d = 44 \cdot \text{in}$                       Depth of steel centroid

$A_s := \rho \cdot b \cdot d$                        $A_s = 1.45 \cdot \text{in}^2$

Use #11 at 12 inches (1.56 in<sup>2</sup>/ft provided) in the backwall. See Figure 10.

*c) Design Flexural Reinforcement for Outside Face of Stemwall*

Design reinforcement on “outside face” of the abutment wall. As a worse-case scenario, assume that the superstructure acts as a pinned support at the top of the abutment wall when the wall tries to move away from the soil. The pressure behind the wall will be based on  $k_h = 0.5 \times A$ , rather than a higher force for a truly restrained wall. The wall is assumed to act as a propped-cantilever, with the maximum moment on the outside face near the middle height of the wall. See Figure 11.

Calculate the “equivalent” uniform lateral load on the abutment wall.

$E_{AE} := 18.55 \cdot \text{kip}$                       Total active earth pressure force

$V_h := 4.68 \cdot \text{kip}$                       Shear due to inertia of soil on heel

$V_w := 2.16 \cdot \text{kip}$                       Shear due to inertia of stemwall

Design Step  
12.4.1  
(continued)

Approximate the equivalent uniform load on the stemwall. See Figure 11.

$$w := \frac{E_{AE}}{28 \cdot \text{ft}} + \frac{V_h}{25 \cdot \text{ft}} + \frac{V_w}{20 \cdot \text{ft}} \qquad w = 0.958 \cdot \frac{\text{kip}}{\text{ft}}$$

The wall is assumed to span between the footing and the top of the stemwall.

$$H_w := 20 \cdot \text{ft}$$

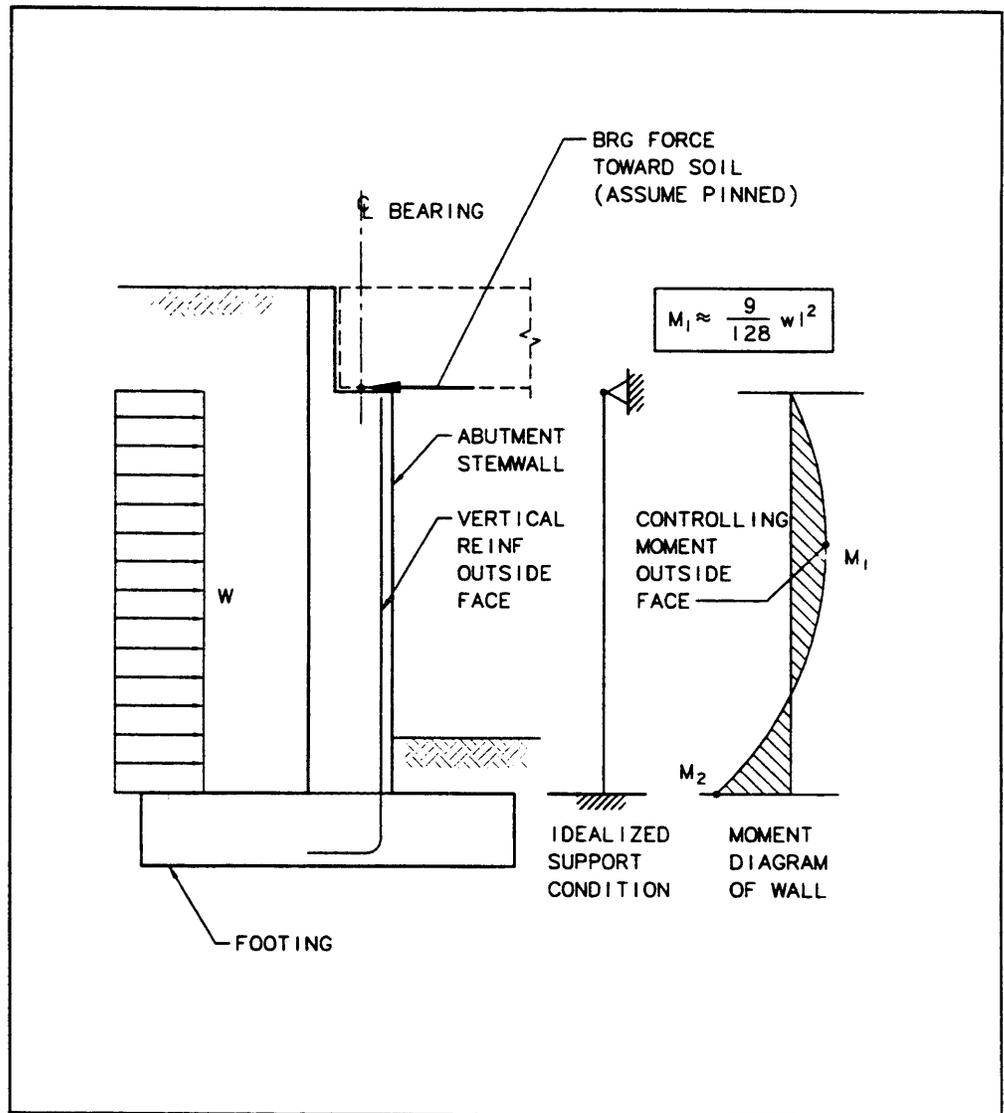


Figure 11 — Moment on Outside Face of Abutment Stemwall

Design Step  
12.4.1  
(continued)

The maximum moment causing tension on the exterior face of the stemwall is

$$M_1 := \frac{9}{128} \cdot w \cdot H_w^2 \qquad M_1 = 27 \cdot \text{kip} \cdot \text{ft}$$

This bending moment is small. Provide minimum reinforcement per AASHTO Division I, Article 8.17.1 or per local agency guidelines.

Design Step  
12.4.2

**Abutment Footing Design**

Calculation of the soil pressure under the toe of the footing is based on the triangular soil distribution shown in Figure 8. Figure 12 shows the general shear and moment diagram for this loading.

*a) Check the Shear in the Toe of the Footing, at a Distance “d” from the Face of the Wall*

First calculate the distance d.

$$D := 3.0 \cdot \text{ft} \qquad \text{Depth of footing}$$

$$d := D - 3 \cdot \text{in} \qquad \text{Depth of steel centroid}$$

$$d = 33 \cdot \text{in}$$

Then calculate the distance x.

$$L_t := 6.0 \cdot \text{ft} \qquad \text{Length of footing toe}$$

$$x := L_t - d \qquad \text{Distance from critical section to the footing toe}$$

$$x = 3.25 \cdot \text{ft}$$

Design Step  
12.4.2  
(continued)

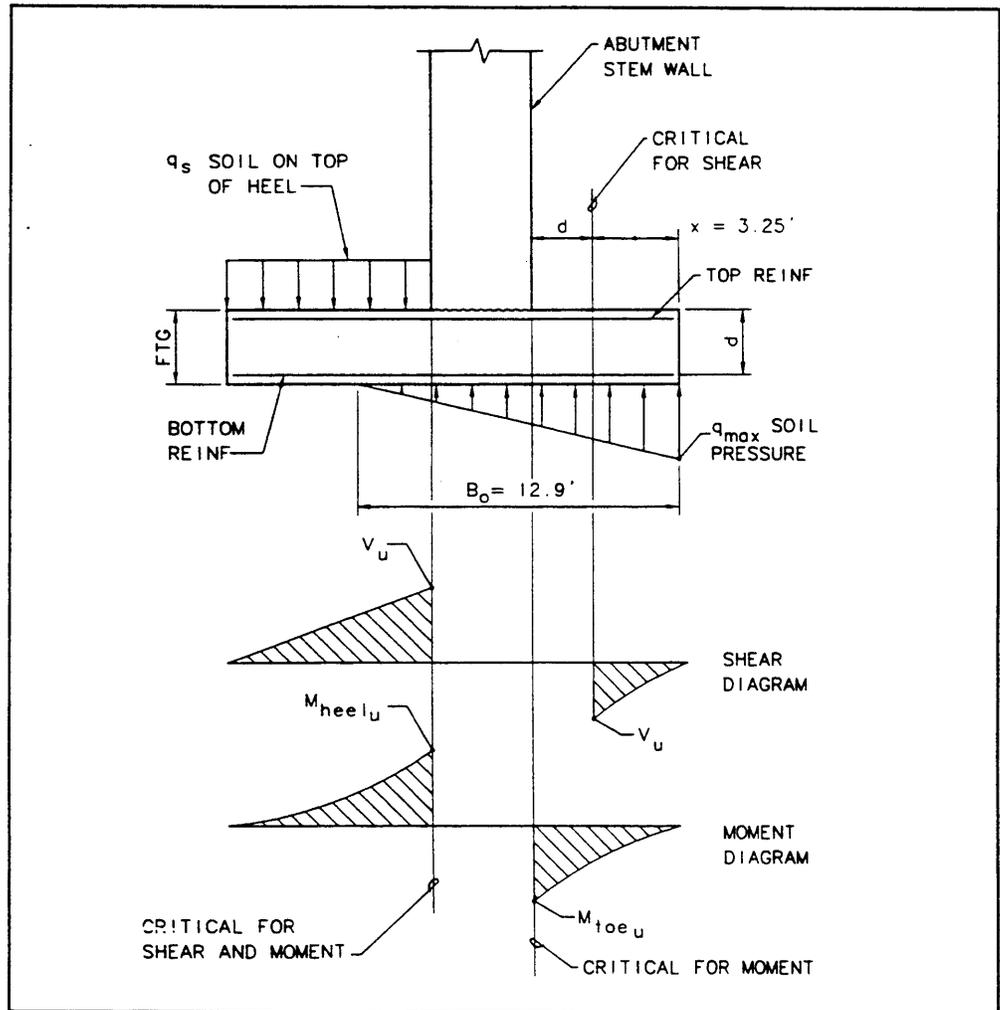


Figure 12 — Design Forces on Abutment Footing

The soil pressure is related to the contact length of the soil,  $B_o$ .

$B := 18 \cdot \text{ft}$  Width of footing

$e := 4.7 \cdot \text{ft}$  Eccentricity of load on footing

$B_o := 3 \cdot \left( \frac{B}{2} - e \right)$  Length of triangular soil stress block

$B_o = 12.9 \cdot \text{ft}$

Design Step  
12.4.2  
(continued)

The soil pressure at distance  $x$  is

$$q_{\max} := 8.8 \cdot \text{ksf} \quad \text{Max. soil pressure at } x$$

$$q_x := \frac{(B_o - x)}{B_o} \cdot q_{\max} \quad q_x = 6.6 \cdot \text{ksf}$$

Calculate the shear force at distance  $x$ .

$$b = 1 \cdot \text{ft} \quad \text{Unit width}$$

$$x = 3.25 \cdot \text{ft} \quad \text{Critical section}$$

$$V := \frac{(q_{\max} + q_x)}{2} \cdot b \cdot x \quad V = 25 \cdot \text{kip}$$

For determining the shear value at  $x$ , the dead weight of both the footing toe, and the soil above the footing toe, should be subtracted from the total shear. Therefore, the design shear force is

$$V_u := V - (2.7 \cdot \text{kip} + 1.6 \cdot \text{kip}) \quad V_u = 20.7 \cdot \text{kip}$$

For this shear, determine if shear reinforcement is required. The basic properties of the footing are

$$d := 33 \cdot \text{in} \quad \text{Depth to flexural reinforcement}$$

$$b := 12 \cdot \text{in} \quad \text{Unit length of footing}$$

$$f_c := 4000 \cdot \text{psi} \quad \text{Concrete strength}$$

$$f_y := 60 \cdot \text{ksi} \quad \text{Reinforcement yield strength}$$

The shear capacity of the concrete is

$$V_c := 2 \cdot \sqrt{f_c} \cdot b \cdot d \quad V_c := 50.1 \cdot \text{kip}$$

Design Step  
12.4.2  
(continued)

Per Division I, 8.16.6.6.3(d), no shear reinforcement is required where

$$V_u < \phi_v \cdot V_c \qquad V_u = 20.7 \cdot \text{kip}$$

$$\phi_v \cdot V_c = 42.6 \cdot \text{kip}$$

Therefore, because the concrete shear capacity is greater than the shear in the footing toe, no shear reinforcement is required in the toe.

*b) Design Bottom Reinforcement in Footing Toe*

The flexural moment in the toe of the abutment footing, due to the triangular soil stress, is computed at the front face of the abutment wall. The resulting moment is given below.

$$M := 134 \cdot \text{kip} \cdot \text{ft}$$

When the moment due to the weight of the footing toe and soil above the toe is subtracted, the factored moment reduces to

$$M_u := M - (4.3 \cdot \text{kip}) \cdot 3 \cdot \text{ft} \qquad M_u = 121 \cdot \text{kip} \cdot \text{ft}$$

Now calculate the reinforcement required in the bottom of the footing.

$$\phi_m = 0.9 \qquad \text{Phi factor for flexure}$$

$$\text{Ratio} := \frac{M_u}{\phi_m \cdot f_c \cdot b \cdot d^2} \qquad \text{Ratio} = 0.0309$$

$$\omega := .031 \qquad \text{Reinforcement index}$$

The reinforcement ratio required in the footing, assuming that 1.2 x M crack is met, is

$$\rho := \frac{\omega \cdot f_c}{f_y} \qquad \rho = 0.0021$$

Design Step  
12.4.2  
(continued)

Calculate the area of reinforcement in the bottom mat of the footing, due to soil pressure under the toe of the footing, per unit length.

$$b = 12 \cdot \text{in} \quad \text{Unit width}$$

$$d = 33 \cdot \text{in} \quad \text{Depth of steel centroid}$$

$$A_s := \rho \cdot b \cdot d \quad \text{Area of flexural reinforcement}$$

$$A_s = 0.82 \cdot \text{in}^2$$

Use #9 at 12 inches (1.00 in<sup>2</sup>/ft provided).

*c) Check the Shear in the Heel of the Footing, at the Face of the Wall*

The heel of the abutment footing must support both the weight of the soil above the footing and the footing weight itself. This extreme case requirement assures that the footing will not fail if the abutment tips about the toe of the abutment. See Figure 13 at the end of this design step.

The weight of the soil above the heel of the abutment is

$$P_h := 26.0 \cdot \text{kip}$$

The weight of the footing heel is

$$P_{\text{heel}} := 3.6 \cdot \text{kip}$$

The total shear force on the heel of the abutment footing is computed at the face of the wall, not at distance “d” from the face of the wall, because the supporting stemwall does not induce compression into the end region of the footing.

$$V_u := P_h + P_{\text{heel}} \quad V_u = 29.6 \cdot \text{kip}$$

$$\phi_v \cdot V_c = 42.6 \cdot \text{kip} \quad \text{Shear capacity}$$

Design Step  
12.4.2  
(continued)

Because the capacity is greater than the load, no shear reinforcement is required in the heel of the footing.

*d) Design Top Reinforcement in Heel of Footing*

First, the total vertical loading on the footing heel must be expressed as a uniform load,  $w$ .

$$V_u = 29.6 \cdot \text{kip} \quad \text{Total shear force}$$

$$L_h := 8 \cdot \text{ft} \quad \text{Length of the heel}$$

$$w := \frac{V_u}{L_h} \quad w = 3.7 \cdot \frac{\text{kip}}{\text{ft}}$$

The moment in the heel of the abutment footing, computed at the back face of the abutment wall, is

$$M_u := w \cdot \frac{L_h^2}{2} \quad M_u = 118.4 \cdot \text{kip} \cdot \text{ft}$$

Compute the area of reinforcement required in the top of the footing.

$$\phi_b := 0.9 \quad \text{Phi factor for flexure}$$

$$\text{Ratio} := \frac{\frac{M_u}{\phi_b}}{f_c \cdot b \cdot d^2} \quad \text{Ratio} = 0.0302$$

$$\omega := .030 \quad \text{Reinforcement index}$$

Design Step  
12.4.2  
(continued)

The reinforcement ratio required in the footing, assuming that 1.2 x M crack is met, is

$$\rho := \frac{\omega \cdot f_c}{f_y} \quad \rho = 0.0020$$

$b = 12 \cdot \text{in}$  Unit width

$d = 33 \cdot \text{in}$  Depth of steel centroid

$A_s := \rho b d$  Area of flexural reinforcement

$A_s = 0.80 \cdot \text{in}^2$  As per foot length of wall

Use #9 at 12 inches (1.00 in<sup>2</sup>/ft provided).

Design Step  
12.4.3

#### Summary of Abutment Design

A summary of the reinforcement specifications required in the abutment due to Group VII loading is highlighted below and in Figure 13.

##### a) Abutment Stemwall

Although no shear reinforcement is required, provide a nominal amount per local agency requirements.

Use #11 at 12 inches on inside face of abutment.

Use minimum steel on outside face per local agency requirements.

##### b) Abutment Footing

Although no shear reinforcement is required, provide a nominal amount per local agency requirements.

Bottom flexural reinforcement in toe of footing, #9 at 12 inches

Top flexural reinforcement in heel of footing, #9 at 12 inches

Design Step  
12.4.3  
(continued)

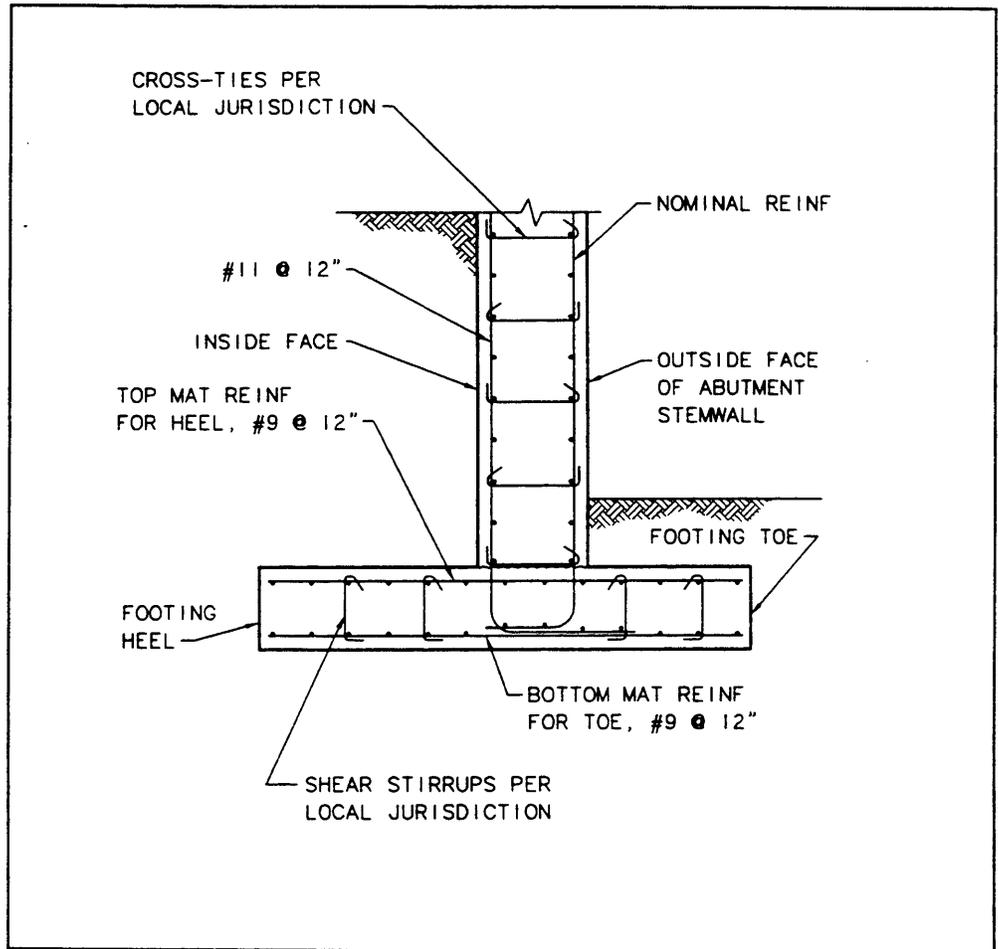


Figure 13 — Summary of Abutment Reinforcement

DESIGN STEP 13

DESIGN SETTLEMENT SLABS

Not applicable.

DESIGN STEP 14

REVISE STRUCTURE

Not required.

DESIGN STEP 15

SEISMIC DETAILS

Not applicable.



**Section IV**  
**Closing Statements**

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**SECTION IV**

**CLOSING STATEMENTS**

The specifications prescribe the force level for the connection between the superstructure and the abutment for single-span bridges. This connection force must be accounted for in the design of the abutment. The phrase “no detailed analysis” does not imply “no detailed design” for single-span bridges.

The Mononobe-Okabe (M-O) equations, presented in the Commentary of AASHTO Division I-A, were used to calculate the lateral earth pressure on the abutment during a seismic event. It was demonstrated in the example that these complicated equations are simply modified Coulomb earth-pressure theory equations. By calculating the Coulomb value, one can check the order of magnitude of the M-O results. The equation for “active” pressure was verified in a lab for unsaturated soils. The equation for passive pressure should be used cautiously.

The magnitude of the lateral forces behind the abutment wall depends on whether the abutment can displace laterally during an earthquake. If the abutment is restrained from moving laterally, the forces behind the wall during an earthquake will be much larger than if the system is detailed to allow a small lateral displacement. For the sake of stability, the wall should be proportioned to slide before it overturns.

For the calculation of lateral forces on the abutment, the inertia effect of both the soil sitting on the heel of the footing and the concrete abutment itself was included. These forces were included in stability checks against sliding and overturning, and for the design of the reinforcement in the abutment. Some DOTs include these effects in the stability calculations, but not in the design of the reinforcement. Others include only one-half the full value in the both calculations. Therefore, it is important to determine how the local jurisdiction includes these effects. In this example, the full values were used, both in the stability checks and for the design of the reinforcement.



**Section V**  
**References**

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SECTION V

REFERENCES

- AAHSTO (1993). *Standard Specifications for Highway Bridges*, American Association of State Highway and Transportation Officials, Inc., 15th Edition, as amended by the Interim Specifications-Bridges-1993 through 1995.
- AAHSTO (1995). *Standard Specifications for Highway Bridges, Division I-A, Seismic Design*, American Association of State Highway and Transportation Officials, Inc., 15th Edition, as amended by the Interim Specifications-Bridges-1995.
- Bowles, J.E. (1982). *Foundation Analysis and Design*, McGraw-Hill, Inc., Third Edition, New York, NY.
- Goh, A.T.C, and Donald, I.B. (1984). *Investigation of Soil-Concrete Interface Behavior by Simple Shear Apparatus*, Fourth Edition, Australia-New Zealand, Vol. 1, pp. 101-106.
- Potyondy, J.G. (1961). *Slain Friction Between Various Soils and Construction Materials*, Geotechnique, Vol. XI, No. 4, pp. 339-353.



**Appendix A**  
**Geotechnical Data**

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<b>APPENDIX A</b>	<b>GEOTECHNICAL DATA</b>
<b>SUBSURFACE CONDITIONS</b>	Subsurface conditions were derived from two borings drilled at the site, as indicated in Figure A1. The borings encountered medium dense sand and gravel and moderately hard, fine-grained moderately jointed, fresh to slightly weathered limestone. The water table is located approximately 35 feet below the ground surface, perched above the rock. New fill will be required to construct the approach embankments. The fill is anticipated to have properties similar to those of the underlying native sand and gravel.
<b>SOIL PROPERTIES</b>	<p><b>Sand and Gravel, Fill</b></p> <p><math>\phi = 35^\circ</math>  <math>c = 0</math>  <math>\gamma = 130</math> pcf</p> <p>Allowable bearing pressure = 6 ksf  Ultimate bearing pressure = 24 ksf  Ultimate coefficient of friction along footing base = 0.55  Allowable coefficient of friction along footing base = 0.37 (FS = 1.5)</p>
<b>SOIL PROFILE TYPE</b>	Type I — Stable deposits of sand and gravel overlying rock within 200 feet of the ground surface.
<b>SITE ACCELERATION</b>	0.36g — Taken from AASHTO seismicity map.
<b>LATERAL EARTH PRESSURES</b>	<p>The abutments will be cantilever-type and as such will be free to deflect at the top. The following properties are recommended for computing lateral earth pressures.</p> <p><math>\phi = 35^\circ</math>  <math>c = 0</math>  <math>\gamma = 130</math> pcf</p> <p><math>\delta = \text{coefficient of wall friction} = \phi/2 = 17.5^\circ</math></p> <p>(Note that the use of values of <math>\delta</math> greater than <math>15^\circ</math> can result in unconservative values of passive earth pressure. A geotechnical engineer should be consulted to confirm values greater than <math>15^\circ</math>.)</p>

**LATERAL  
EARTH  
PRESSURES**  
(continued)

$k_h = A/2 = 0.18g$  (horizontal acceleration coefficient)

$k_v = 0$  (vertical acceleration coefficient)

The location of the resultant of the static and dynamic increment of the lateral earth pressure is worth noting. Typically, the static active and passive earth pressures are assumed to act at  $H/3$  from the bottom of the wall. However, Barker et. al. (1991) noted that several researchers have found experimentally that the force acts at 0.4 to 0.454. In accordance with typical design standards, a triangular earth pressure distribution is recommended (equivalent fluid density), with the horizontal resultant acting at  $H/3$ . Regarding the incremental-dynamic earth pressure (excluding the static part of the pressure), researchers have suggested that the horizontal resultant acts in the range of  $0.5H$  to  $0.6H$  from the bottom of the wall (Sherif, 1981; Seed and Whitman, 1970). For most problems, it is sufficient to assume that the dynamic increment acts as a uniformly distributed pressure with the resultant at  $H/2$ .

Passive pressure may be ignored in the upper 2 feet.

**OTHER ISSUES**

Because of the depth of the water table, the medium-dense relative density of the sand and gravel, and the presence of rock, liquefaction will not occur.

**REFERENCES**

- Barker, R.M., Duncan, J.M., Rojiani, K.B., Booi, P.S.K., Tan, C.K., and Kim, S.G. (1991). *Manuals for the Design of Bridge Foundations*, National Cooperative Research Program Report 343, pp 308, Transportation Research Board, Washington, DC.
- Seed, H.B., and Whitman, R.V. (1970). "Design of Earth Retaining Structures for Dynamic Loads," ASCE Specialty Conference — Lateral Stresses in the Ground and Design of Earth Retaining Structures, American Society of Civil Engineers.
- Sherif, M.A. (1981). *Earth Pressures on Retaining Walls*, pp 46, Crystal Press, Seattle, WA.

**ADDITIONAL  
CONSIDERATIONS**

The following are supplementary geotechnical considerations for Design Example No. 3.

**Seismicity Map**

The seismicity map currently contained in the Association of American State Highway Transportation Officials (AASHTO) Specification for Bridges is based largely on work conducted by the United States Geological Survey (USGS) in the early 1980s, with a subsequent revision in 1988. These maps are periodically updated as more information becomes known about faulting and tectonic structure in various regions in the country. Consequently, information contained in the existing maps may be inaccurate based on current research. While the AASHTO map does provide the basis for the seismic design of bridges on a national basis, design engineers should be aware that more recent maps have been developed by the USGS regarding earthquake hazards in the United States. Specifically, in 1990 the USGS published probabilistic earthquake acceleration and velocity charts for the United States and Puerto Rico as Map MF-2120; and currently, the USGS is updating the seismic hazard maps for the United States for the Building Seismic Safety Council (BSSC). In developing these maps, the USGS conducted several workshops in various sections of the country to obtain input from local experts on seismic source zones and earthquake activity rates. Based on this information, the national seismic hazard maps for various areas of the country have changed considerably. Specifically, the seismicity picture for the Pacific Northwest was revised based on subduction zone earthquake models that were not considered in prior analyses. These sources effectively increased the ground acceleration projections in Western Washington and Oregon by as much as 100 percent. Thus, use of the draft maps from the USGS may provide additional insights on seismic hazards that may be considered as a sound basis for alternate designs for critical structures.

**Soil Profile Type**

The current version of the AASHTO specifications includes provisions for three soil profile types that are used to characterize the site amplification effects of the underlying soils. Under this scheme, Type I is rock and Type III is considered to be a deposit that contains at least 30 feet of soft to medium stiff clay. While the verbal descriptions corresponding to these soil types may be adequate in most instances, there may be a number of occasions where these descriptions may be confusing and may result in multiple classification of the underlying soils.

**ADDITIONAL  
CONSIDERATIONS**  
(continued)

Specifically, the soils found in the Boston area underlying the Central Artery/Tunnel Project (CA/T) suggest conditions where rock is within 100 feet of the ground surface and may be underlain by Boston blue clay and estuarine silts. The different consultants working on this project have classified the soil in this area anywhere between and including Types I and III. Clearly, this illustrates a difficulty with the current scheme.

Because of the ambiguity in the classifications under the current scheme, studies were undertaken, primarily in the building sector, to provide a more rational basis for categorizing underlying subsurface soil conditions. The resulting revised classification scheme includes descriptors of soil depth, material type, Standard Penetration Test results, and shear wave velocity. The implications of this scheme are that estimation of the engineering properties of the soils underlying the site may be used to provide a better assessment of the stiffness of the soil column and the appropriate site categorization for seismic analyses. This classification scheme is briefly summarized in a paper that was presented at the Fifth U.S. National Conference on Earthquake Engineering held in Chicago in 1994 (E.E. Rinne). Similar classification systems will likely be used for the AASHTO code in the near future.

Another difficulty experienced in estimating a soil profile type for bridge design is the potential influence of liquefaction. Specifically, the soil profile type changes if liquefaction develops. Many engineers classify soil conditions as a Type III deposit if liquefaction seems likely to develop, whereas if liquefaction were not considered likely to occur, the site could be classified as either a Type I or Type II profile. The rationale for this classification is the belief that a Type III profile might provide a more conservative design for the bridge. However, the occurrence of liquefaction should not fundamentally change the classification of the underlying soils. Specifically, if liquefaction were not to occur, the underlying soil profile would not change; whereas, if liquefaction were to occur, the net effect would not be amplification of ground motions, but more probably an attenuation of ground motions or reduction of force levels to the structure. In this circumstance, it would not be necessary to change the profile type, because the more conservative design would have already been accomplished using the site acceleration and appropriate soil modification factor prior to the development of liquefaction. Therefore, the occurrence of liquefaction should not change the soil profile type classification selected for the design of the bridge.

**ADDITIONAL  
CONSIDERATIONS**  
(continued)**Uniform Hazard Response Spectra**

Currently, the AASHTO design provisions contain a normalized response spectrum shape that is anchored to the site peak acceleration to create the site design response spectrum. This procedure is based on the assumption that response spectra in the eastern and western United States are the same, and that the spectra are not affected by varying levels of ground motion. Recognizing the limitations of this practice, the building sector has approached the issue by developing seismic hazard maps corresponding to spectral ordinates at two different periods. From these maps, appropriate design response spectra may be created for different locations throughout the United States taking into account the differences between earthquakes in the eastern and western United States and the different characteristics of the earthquakes that could affect the region. These design response spectra are essentially uniform hazard response spectra. This research should be reflected in code provisions for the Building Seismic Safety Council in the near future. Similar provisions may be reflected in subsequent editions of the AASHTO specifications.

**Ultimate Bearing Capacity**

For design purposes, bearing capacities have traditionally been determined based on an allowable settlement of the foundation. Hence, the term “allowable bearing capacity” has been used quite extensively for both buildings and bridges. In checking the seismic performance of spread footing foundations, the structural engineer needs to assess whether the total foundation loads (dead load, live load, and earthquake load) exceed the ultimate bearing capacity of the underlying soil. An assessment that the loads exceed the ultimate bearing capacity of the underlying soil would necessarily prompt the structural engineer to increase the size of the footing to enable the loads to stay below the ultimate bearing capacity.

Because most footing design is based on settlement considerations, it is quite reasonable to expect a factor of safety of at least 2 and perhaps as large as 5 in the footing design. Therefore, on this basis, allowable bearing pressure values may be increased by a minimum of 2 and perhaps as much as 5 to obtain an ultimate capacity value. This ultimate capacity value reflects transitory loadings during which the loads are applied for only a fraction of a second before being released.

Consequently, because of the transitory nature of the earthquake loading, it is reasonable to expect relatively high values for the ultimate bearing capacities of spread footing foundations. Conditions that would

**ADDITIONAL  
CONSIDERATIONS**  
(continued)

suggest that footing size might need to be increased to avoid exceeding the ultimate bearing capacity of the underlying soils may need to be reviewed by the geotechnical engineer to confirm that excessive factors of safety were not used in determining the ultimate bearing capacity for the underlying soils.

**Lateral Earth Pressures**

Currently, the Mononobe-Okabe equation is pervasively used throughout the engineering profession to compute lateral earth pressures on abutments and retaining walls. This procedure is based on a Coulomb approach to pressure determination. One of the greatest uncertainties in the application of this formula is in selection of an appropriate value for use as the seismic coefficient. Specifically, the AASHTO manual suggests that a seismic coefficient equal to one-half the acceleration level may be appropriate provided that the abutment and/or retaining walls can move a distance equal to  $10A$  (where  $A$  represents the peak ground acceleration for the site). This particular assumption would appear to be somewhat flawed in that the ability to move this distance implies that the retaining wall or abutment would have a sliding factor of safety of 1.0 or less. Clearly, this is not the case because abutments and other retaining walls are typically designed for a sliding factor of safety of 1.5 or greater. This is not to suggest that all walls need to be designed for at-rest pressures, but rather to acknowledge that there is an inconsistency in the existing code, and that use of active earth pressure coefficient values may still be valid for the design of walls. Consequently, many walls or abutments may be adequately designed using a seismic coefficient equal to one-half the value of peak ground acceleration, provided that the soils underneath the abutment or foundation walls are stable and not subject to liquefaction, lateral spreading, or landsliding.

Another factor affecting calculation of lateral earth pressures is determination of the appropriate shape of the dynamic lateral earth pressure increment. While AASHTO suggests that a uniform pressure distribution be presumed for the lateral pressure evaluation, it is not clear from AASHTO if the uniform pressure represents both active and dynamic loading conditions, or strictly the dynamic increment. Nevertheless, it is recommended that the dynamic pressure increment be represented as a horizontal uniform earth pressure on the wall. This recommendation is consistent with research conducted by the late Professor M. Sherif at the University of Washington.

**ADDITIONAL  
CONSIDERATIONS**  
(continued)

A final consideration in the lateral earth pressure evaluation are the pressures contained in the Caltrans' bridge design aids for abutment walls (7.7 kips per linear foot). In relating this pressure to other structures, it must be remembered that the Caltrans' example is based on an abutment having a height of about 8 feet. Therefore, this value may not apply to retaining walls or abutments with different heights. Similarly, the subgrade modulus contained in the Caltrans design aid would be applicable only for approximately 8-foot-high abutments that have been backfilled with clean, granular material. Calculations based on use of different backfill material, or shorter abutment heights, would generate different values for the lateral earth pressure coefficient or the subgrade modulus value.

**Liquefaction**

Liquefaction potential needs to be evaluated at most bridge locations that are underlain by Holocene alluvial soils, because these soils are typically loose and may be susceptible to the development of liquefaction during a strong earthquake. Liquefaction evaluations are typically performed using empirical procedures developed by the late Professor H.B. Seed. Using the Seed procedures to calculate liquefaction can quite typically lead to a conclusion that liquefaction could occur to depths in excess of 100 feet, although there is no detailed compilation of earthquake-induced liquefaction effects to suggest that liquefaction could occur to depths in excess of 100 feet. On the contrary, most of the existing information suggests that liquefaction primarily occurs within about 50 feet of the ground surface. Therefore, many engineers have assumed that liquefaction would be limited to a depth of 50 feet. While this depth may appear to be arbitrary, it has typically been supported by the records of maximum depth to which liquefaction has occurred during recent earthquakes. Therefore, it is considered that a maximum depth of liquefaction of about 50 feet is reasonable.

A more important consideration than liquefaction is the development of lateral spreads, which can occur on slopes adjacent to shoreline areas. Lateral spreads are particularly hazardous, since they may move piers and abutments toward the water, causing bridge decks to fall off their supports. Therefore, it is essential to make an assessment of the potential occurrence of lateral spreading and to take positive steps to provide sufficient lateral support for the bridge abutments and piers.

**ADDITIONAL  
CONSIDERATIONS**  
(continued)

Potential consequences of liquefaction also need to be addressed in the foundation design in areas that are not susceptible to lateral spreading. Typically, this adjustment involves neglecting the vertical support of the soils in the area where liquefaction occurs and also in the soils above this zone. Analyses also should include a reduced value of the subgrade modulus for computing the lateral resistance of the piles in areas where liquefaction may develop. Thus, the primary focus of foundation design in relation to the occurrence of liquefaction is on providing adequate lateral and vertical support for the structure. This analysis may be further complicated by considerations of loading in that earthquake loads are considered simultaneously with reduced support from liquefaction. Many highway departments have handled this issue by treating the loads separately. While the treatment of the loads in combination most certainly results in conservative design, this approach may be rather expensive considering that liquefaction typically does not develop to the end of the earthquake ground shaking and that the development of liquefaction typically results in a lower level of ground shaking at the surface. Therefore, it would appear to be more reasonable to consider these hazards separately than in combination.

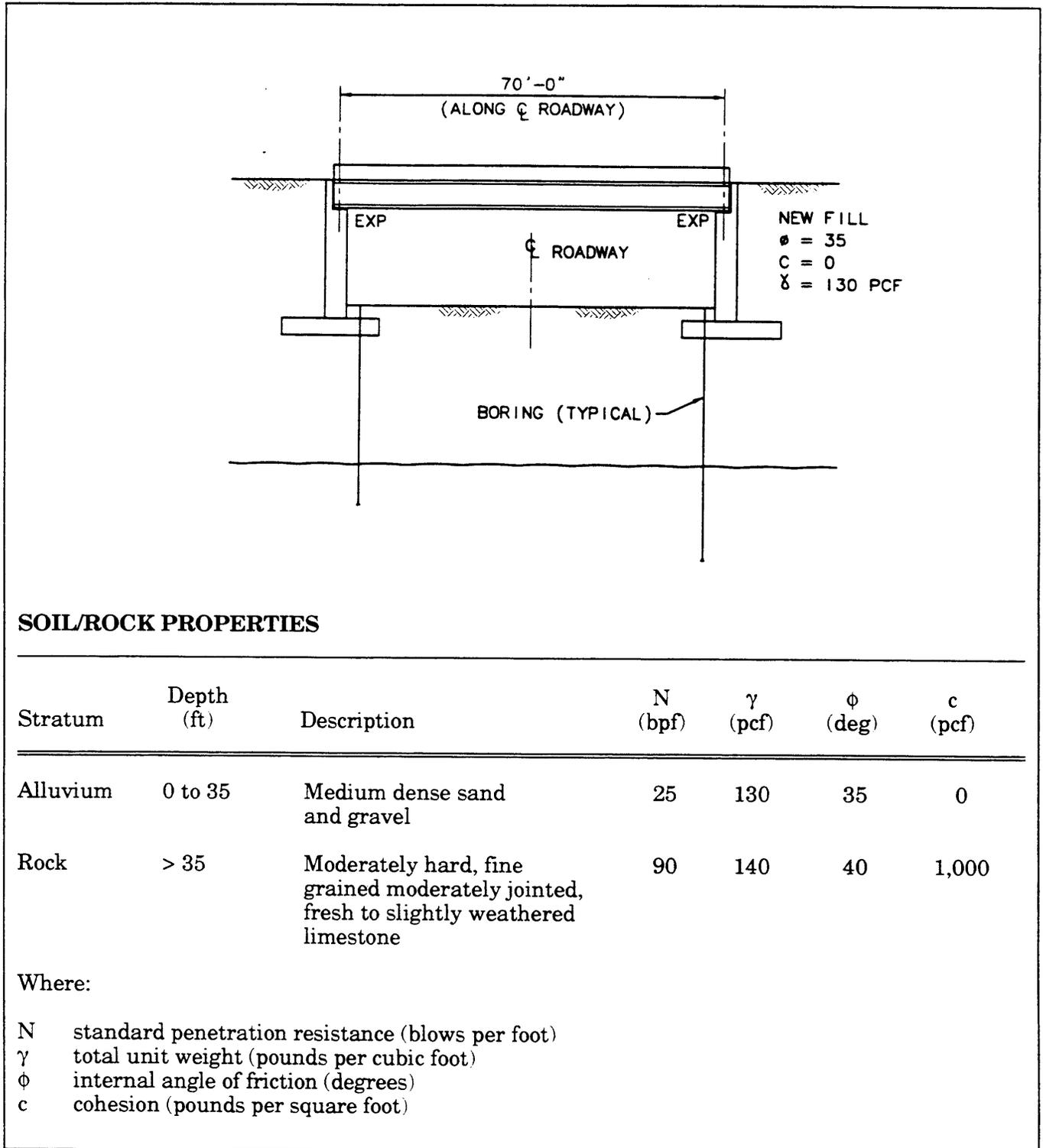


Figure A1 – Subsurface Conditions

