4. **Compute ϕM_n.**

$$\phi M_n = \phi\left[A_s f_y\left(d - \frac{g}{3}\right)\right]$$
$$= \frac{0.9[2.58 \times 60{,}000(21.5 - 6.46/3)]}{12{,}000} \qquad (5\text{-}18)$$
$$= 225 \text{ ft-kips}$$

Thus the design moment capacity of the section shown in Fig. 5-23 is 225 ft-kips. Note that the moment calculation is based on the lever arm measured *vertically* (parallel to the plane of loading). ■

5-5 ANALYSIS OF MOMENT CAPACITY BASED ON STRAIN COMPATIBILITY

The analysis procedures presented so far in this chapter and Chapter 4 have been restricted to problems involving

1. Elastic–plastic reinforcement with a constant yield strength;
2. Tension reinforcement and compression reinforcement in two groups of bars that can be represented by compact layers at the centroids of the respective groups;
3. All concrete of the same strength;
4. A rectangular, T, or other easily definable cross-sectional shape.

If any of these restrictions does not apply, a trial-and-error solution based on strain compatibility can be used. In the strain-compatibility calculations, *tensile* stresses, strains, and forces are taken to be *negative*, as subsequently assumed in Chapter 11. As noted in step 7 of the calculation, moments will be summed about the centroidal axis of the gross cross section. The following steps are required in such a solution:

1. Assume the strain distributions is defined by a strain ϵ_{cu} of 0.003 in the extreme compressive fiber and an assumed value of the depth c to the neutral axis.
2. Compute the depth of the rectangular stress block, $a = \beta_1 c$.
3. Compute the strains in each layer of reinforcement from the assumed strain distribution.
4. From the stress–strain curve for the reinforcement and the strains from step 3, determine the stress in each layer of reinforcement.
5. Compute the force in the compression zone and in each layer of reinforcement.
6. Compute $P = C - T$. For a beam without axial force, P equals zero. If the calculated value of P is not equal to zero, adjust the strain distribution and repeat steps 1 to 6 until P is as close to zero as desired. The imbalance should not exceed 0.1 to 0.5 percent of C.
7. Sum the moments of the internal forces. If $P = 0$, this can be about any convenient axis. We shall sum the moments about the centroid of the cross section. This axis is normally used in columns where P is not zero, as explained in Chapter 11.

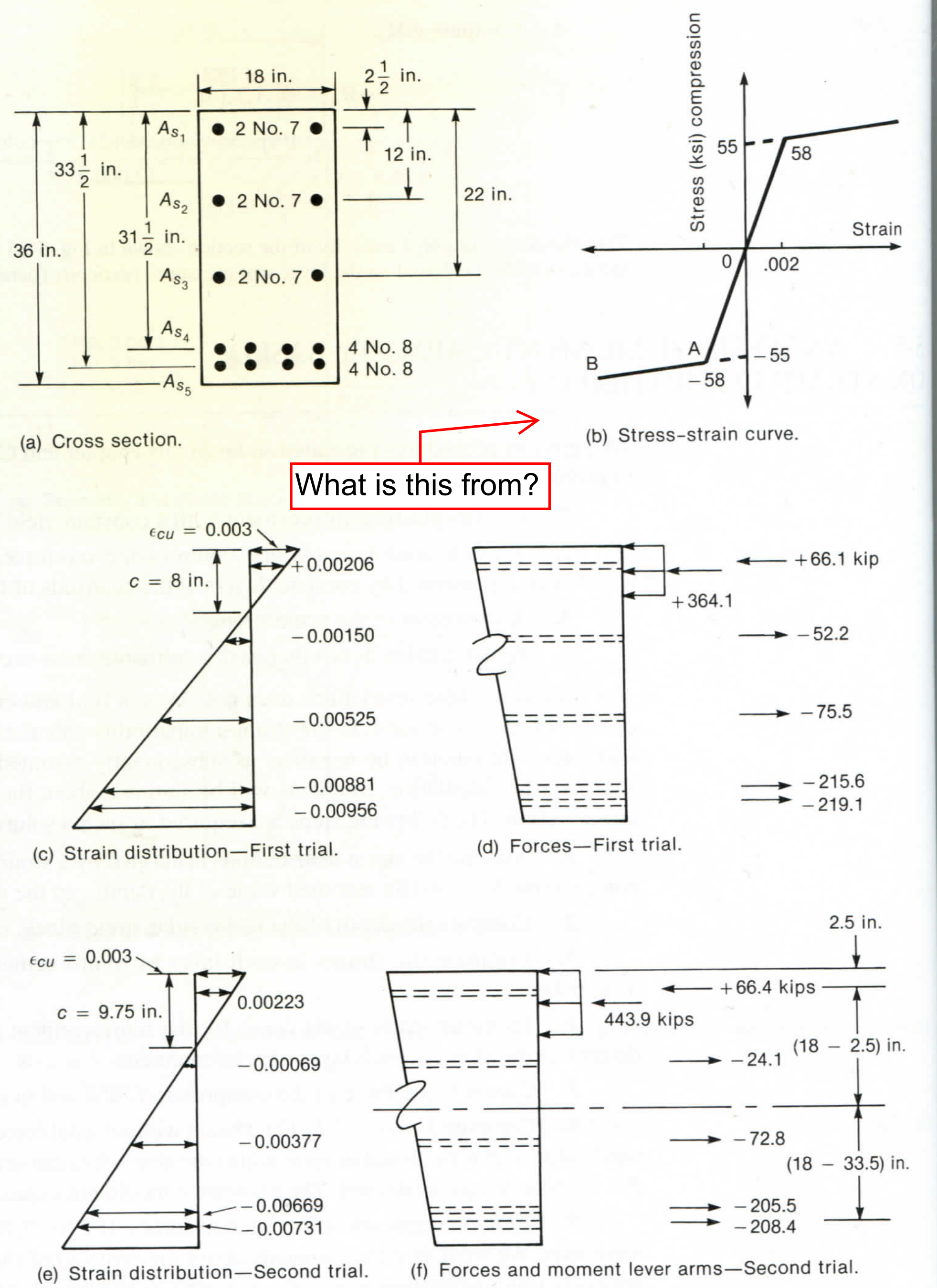
Example 5-8 Strain-Compatibility Analysis of Moment Capacity

Compute the design moment capacity, ϕM_n, of the cross section shown in Fig. 5-24a. The concrete strength is 3500 psi. The reinforcement has the stress–strain curve shown in Fig. 5-24b.

In this solution, *compressive* strains and stresses are taken as *positive*, tensile strains and stresses as negative. As a result, the stress–strain curve for the reinforcement in tension has the following equations:

Part O–A, $\epsilon \geq -0.002$:

$$f_s = (29 \times 10^3 \epsilon) \text{ ksi} \qquad (5\text{-}19a)$$



What is this from?

Fig. 5-24
Strain compatibility solution—Example 5-8.

Part A–B, $\epsilon < -0.002$:

$$f_s = (-55 + 1.5 \times 10^3 \epsilon) \text{ ksi} \tag{5-19b}$$

Similar equations can be derived for the compressive branch of the stress–strain curve of the steel:

1. **Assume a strain distribution.** The first trial strain distribution in Fig. 5-24c is defined by $\epsilon_{cu} = 0.003$ and $c = 8$ in.

2. **Compute the depth of the equivalent rectangular stress block.**

$$a = \beta_1 c$$
$$= 0.85 \times 8 = 6.8 \text{ inches}$$

TABLE 5-1

Layer
Compression zone
A_{s1}
A_{s2}
A_{s3}
A_{s4}
A_{s5}
Compression zone
A_{s1}
A_{s2}
A_{s3}
A_{s4}
A_{s5}

[a]ϵ, f_s, and F_s a

Therefore, $a = 6.8$ in. for first trial.

3. **Compute the strains in each layer of reinforcement.**
4. **Compute stress in each layer of reinforcement.**
5. **Compute forces in the compression zone and in each layer of reinforcement.**
6. **Compute $P = C - T$.**

Steps 3 to 6 are carried out in Table 5-1. For a bar located at distance y below the top of the beam, the strain is

$$\epsilon = 0.003 - \left(0.003\frac{y}{c}\right) \tag{5-20}$$

For A_{s1}, y is less than a. As a result, this layer of steel displaces concrete assumed to be stressed in compression. If $y < a$,

$$f_s = (f_s \text{ from (5-19)}) - 0.85f_c'$$

The strains and forces in the various layers are illustrated in Fig. 5-24c and d. The sum of the forces in the bars and the concrete is 130.2 kips tension. Since there is no axial force in this member, the sum should be zero. Thus the assumed compressed zone is too small.

As a second trial, try $c = 9.75$ in., which results in $a = 8.29$ in. Steps 3 to 6 of the second trial are also given in Table 5-1. At the end of the second trial, the forces have converged to within 1 kip. Since this is less than 0.1percent of the force in the compression zone, it will be assumed to

TABLE 5-1 Calculation of Internal Forces—Example 5-8[a]

Layer	y (in.)	ϵ	f_s (ksi)	A_s(in.^{2})	F_s (kips)	C_c (kips)
First trial: Assume that $c = 8$ in., $a = \beta_1 c = 6.8$ in.						
Compression zone	—	—	—	—	—	$6.8 \times 18 \times 0.85 \times 3.5 = +364.1$
A_{s1}	2.5	+0.00206	$58.1 - 0.85 \times 3.5 = 55.1$	1.20	+66.1	—
A_{s2}	12	−0.00150	−43.5	1.20	−52.2	—
A_{s3}	22	−0.00525	−62.9	1.20	−75.5	—
A_{s4}	31.5	−0.00881	−68.2	3.16	−215.6	—
A_{s5}	33.5	−0.00956	−69.3	3.16	−219.1	—
					$\Sigma F_s = -496.3$ kips	$\Sigma C_c = +364.1$ kips
					$\Sigma F_s + \Sigma C_c = -132.2$ kips	
Second trial: Assume that $c = 9.75$ in., $a = 8.29$ in.						
Compression zone	—	—	—	—	—	$8.29 \times 18 \times 0.85 \times 3.5 = 443.9$
A_{s1}	2.5	+0.00223	$58.3 - 0.85 \times 3.5 = 55.4$	1.20	+66.4	—
A_{s2}	12	−0.00069	−20.1	1.20	−24.1	—
A_{s3}	22	−0.00377	−60.7	1.20	−72.8	—
A_{s4}	31.5	−0.00669	−65.0	3.16	−205.5	—
A_{s5}	33.5	−0.00731	−66.0	3.16	−208.4	
					$\Sigma F_s = -444.4$ kips	$\Sigma F_c = +443.9$ kips
					$\Sigma F_s + \Sigma C_c = -0.5$ kips	

Fs > Fy?

[a] f_s and F_s are positive in compression.

be close enough. It is sometimes useful to plot $P = C - T$ versus c to help in the choice of c for future trials.

7. Check whether $f_s = f_y$ and whether the section is tension-controlled. The stresses in each layer of steel have been computed in Table 5-1, and no further check of f_s is needed. $a/d_t = 9.75/36 = 0.271$. Since this is less than $a_{TCL}/d_t = 0.319$, the section is tension-controlled and $\phi = 0.90$.

If the design were carried out according to ACI Section 10.3.3, it would be necessary to check whether ϵ_t was larger than 0.004 tensile strain, or whether $\rho \leq 0.75\ \rho_b$. From Fig. 5-24c the strain in the extreme tensile layer of steel at ultimate is 0.00956 tensile. Therefore the section is tension-controled and $\phi = 0.9$.

8. Compute the moments about midheight. Once P has converged to zero, the moments can be computed. The forces from the second trial are shown in Fig. 5-24f. The distances from the midheight are taken positive upward, negative downward. A counterclockwise moment is taken as positive. We have

$$
\begin{aligned}
M_n = 443.9\left(\frac{36}{2} - \frac{8.29}{2}\right) &+ 66.4(18 - 2.5) \\
&+ [-24.1(18 - 12)] + [-72.8(18 - 22)] \\
&+ (-205.5(18 - 31.5)] + [-208.4(18 - 33.5)] \\
&= 13{,}330 \text{ in.-kips}
\end{aligned}
$$

Thus, the nominal moment capacity of the section shown in Fig. 5-24 is $M_n = 13{,}330$ in.-kips and the design moment capacity $\phi M_n = 12{,}000$ ~~ft-~~kips.
in-kips

This type of problem is ideally suited for solution via a spreadsheet.

5-6 DESIGN OF MEMBERS WITH FRP REINFORCEMENT-DESIGN ACCORDING TO ACI 440 REPORT

Properties of FRP Reinforcement

The three most common types of FRP are GFRP with glass fibers, AFRP with Aramid fibers, and CFRP with carbon fibers. The material properties are described briefly in Section 3-14. In tests, FRP bars have a brittle, nearly linear stress–strain relationship in compression or tension. The design procedure is controlled by the brittle nature of the FRP reinforcement.

Most types of fiber-reinforced polymer reinforcement (FRP) cannot be bent once the polymer has set. As a result, if bent or hooked bars are required, they must be bent during manufacture.

The polymer resins used to make FRP bars undergo a phase change between 150° and 250°F, causing a reduction in strength. By the time the temperature of the bar reaches 480°F, the tensile strength of a typical FRP drops to about 20 percent of the strength at room temperature.

Design of Members with FRP Reinforcement

This section is based on the recent reports by ACI Committee 440. [5-9] There is no accepted American consensus standard covering the design of members reinforced with fiber-reinforced polymer (FRP) reinforcement. Currently, design is based on ultimate-strength design using FRP material properties supplied by the manufacturer of the reinforcement. Generally the properties provided are low fractiles of the distributions of strengths and moduli. The three most important properties are the initial ultimate tensile strength, f^*_{fu}, the modulus of elasticity, E_f, and the initial rupture strain, ε^*_{fu}.

The design philosophy for FRP-reinforced structures differs significantly from that used for steel-reinforced structures. For the latter, the designer ensures that the steel reinforcement