Constrained Axial Buckling About Non-Principal Axes

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When certain structural members are used in axial compression, framing considerations dictate the orientation of the cross section about the member's longitudinal axis. When a second member is framed to the first, a constraint is imposed which forces the member to buckle about a non-principal axis. The author investigates axial buckling for this case, and formulates equations for the compound buckling of axial compression members. Formulas are derived using the differential equations of classical structural stability mechanics.

DISCUSSION

For certain structural members, such as Z-shapes and single angles, practical framing considerations dictate member orientation when used as columns or bracing. Consider the case shown in Figure 1a where a member is in axial compression, is supported continuously about one of its non-principal axes, and is simply supported at its ends about its other non-principal axis as shown in Figure 1b.

In Figure 1c, X' and Y' are principal axes and X and Y are non-principal axes. Normally engineers calculate the Euler buckling load about the major and minor principal axes of a cross section. In this present case, the cross section is forced to buckle about the Y-Y axis, since it is constrained by the continuous bracing. The question arises as to whether it is correct to calculate the radius of gyration r_y and to use this value in the normal AISC design equations for compression members since the Y axis is a non-principal axis.

To address the behavior, one must start with the fundamental equilibrium equations of stability. The pertinent equations for moments are taken from *Theory of Elastic Stability* (Timoshenko and Gere, 1961, p. 242).

$$M_x = EI_x \frac{d^2v}{dz^2} + EI_{xy} \frac{d^2u}{dz^2} \tag{1}$$

$$M_y = EI_y \frac{d^2u}{dz^2} + EI_{xy} \frac{d^2v}{dz^2} \tag{2}$$

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where the sign convention for the moments are shown in Figure 2. When one considers equilibrium for an arbitrary length of the member shown in Figure 3, the following equations can be written in each respective orthogonal plane:

$$Pu + M_y = 0 \tag{3}$$

$$Pv + M_x = 0 \tag{4}$$

Substituting the expressions from Equations 1 and 2, yields the following second order differential equations:

$$Pu + EI_y \frac{d^2u}{dz^2} + EI_{xy} \frac{d^2v}{dz^2} = 0 \tag{5}$$

$$Pv + EI_x \frac{d^2v}{dz^2} + EI_{xy} \frac{d^2u}{dz^2} = 0 \tag{6}$$

The equations of equilibrium for second order analysis are coupled due to the product of inertia. Since the member is continuously braced in the y direction, for every value of z:

$$v(z) = 0 \qquad v'(z) = 0 \qquad v''(z) = 0$$

Equation 5 reduces to the classical, uncoupled differential equation:

$$Pu + EI_y \frac{d^2u}{dz^2} = 0 \tag{7}$$

Assuming a sinusoidal solution for $u(z)$:

$$u(z) = K_1 \sin(az) + K_2 \cos(az) \tag{8}$$

and applying the following boundary conditions:

$$u(0) = 0 \qquad u(L) = 0 \qquad u''(0) = 0 \qquad u''(L) = 0$$

yields the classical Euler solution:

$$a = \sqrt{\frac{P}{EI_y}} = \frac{\pi}{L} \rightarrow P_{cr} = \frac{\pi^2 EI_y}{L^2} \rightarrow P_{cr} = \frac{\pi^2 EA}{(L/r_y)^2}$$

Note however that r_y is about the non-principal Y-Y axis. Note also that, in the buckled configuration, a moment is induced about the x axis given by the expression:

$$M_x = EI_{xy} \frac{d^2 u}{dz^2} \tag{9}$$

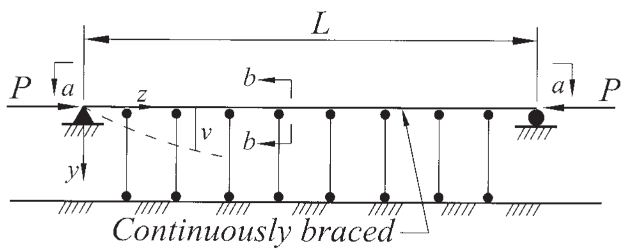


Fig. 1a. Member in axial compression with continuous bracing about one non-principal axis.

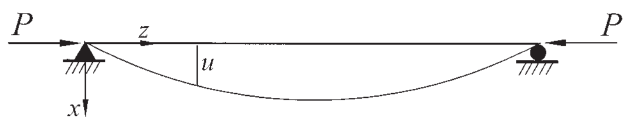


Fig. 1b. View a-a.

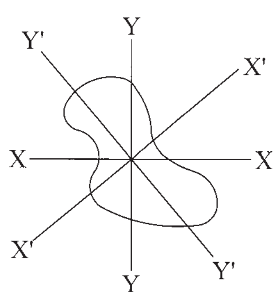
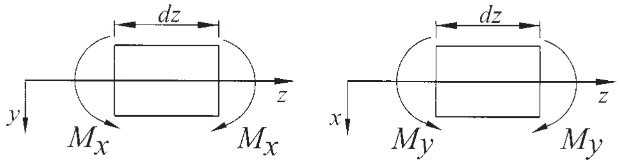


Fig. 1c. Section b-b: general cross section.



a. M_x b. M_y

Fig. 2. Sign convention for moments, M_x *and* M_y.

The conclusion is that the Euler buckling solution holds for buckling about a non-principal axis when a member is constrained to buckle in this fashion as shown in Figure 1a and Figure 1b.

COMPOUND BUCKLING

Consider a second case where a member is braced discretely at its midpoint, and admit the possibility of the simplest *compound* buckled configuration shown in Figure 4a and Figure 4b.

The equations of equilibrium are the same ones shown in the first case, but a fourth order approach is used in order to be consistent with Timoshenko and Gere's formulation (Timoshenko and Gere, 1960, p. 51). The equilibrium Equations 5 and 6 become:

$$P \frac{d^2 u}{dz^2} + EI_y \frac{d^4 u}{dz^4} + EI_{xy} \frac{d^4 v}{dz^4} = 0 \tag{10}$$

$$P \frac{d^2 v}{dz^2} + EI_x \frac{d^4 v}{dz^4} + EI_{xy} \frac{d^4 u}{dz^4} = 0 \tag{11}$$

Assume the solutions are sinusoidal of the following form:

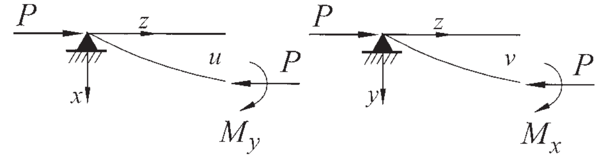
$$u(z) = K_1 \sin(az) + K_2 \cos(az) + K_3 z + K_4 \tag{12}$$

$$v(z) = K_5 \sin(bz) + K_6 \cos(bz) + K_7 z + K_8 \tag{13}$$

where the K_i terms are constants. Differentiating $u(z)$ and $v(z)$ for the second and fourth derivatives:

$$\frac{d^2 u}{dz^2} = -K_1 a^2 \sin(az) - K_2 a^2 \cos(az) \tag{14}$$

$$\frac{d^4 u}{dz^4} = K_1 a^4 \sin(az) + K_2 a^4 \cos(az) \tag{15}$$



a. x-z plane b. y-z plane

Fig. 3. Free body diagrams.

$$\frac{d^2v}{dz^2} = -K_5 b^2 \sin(bz) - K_6 b^2 \cos(bz) \tag{16}$$

$$\frac{d^4v}{dz^4} = K_5 b^4 \sin(bz) + K_6 b^4 \cos(bz) \tag{17}$$

When these terms are substituted into Equations 10 and 11, the following two equations result:

$$\begin{aligned} &P(-K_1 a^2 \sin az - K_2 a^2 \cos az) + \\ &EI_y (K_1 a^4 \sin az + K_2 a^4 \cos az) + \\ &EI_{xy} (K_5 b^4 \sin bz + K_6 b^4 \cos bz) = 0 \end{aligned} \tag{10a}$$

$$\begin{aligned} &P(-K_5 b^2 \sin bz - K_6 b^2 \cos bz) + \\ &EI_x (K_5 b^4 \sin bz + K_6 b^4 \cos bz) + \\ &EI_{xy} (K_1 a^4 \sin az + K_2 a^4 \cos az) = 0 \end{aligned} \tag{11a}$$

Through algebraic manipulation of Equations 10a and 11a, the following equation, quadratic in P, can be derived:

$$a^2 b^2 \begin{bmatrix} P^2 - P(EI_x b^2 + EI_y a^2) + \\ E^2 a^2 b^2 (I_x I_y - I_{xy}^2) \end{bmatrix} = 0 \tag{18}$$

This characteristic equation must be satisfied for a non-trivial solution to the problem. Applying the quadratic formula to the term in brackets, the critical buckling load P_{cr} is found by calculating the smaller, non-negative root of Equation 19:

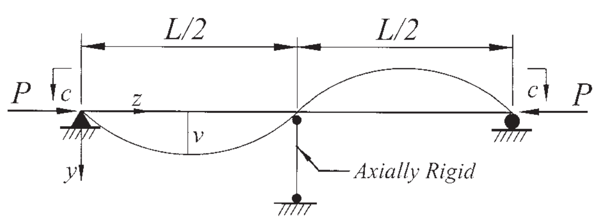


Fig. 4a. Second buckling mode of a simply supported column.

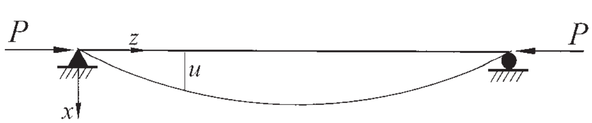


Fig. 4b. First buckling mode of a simply supported column.

$$\begin{aligned} P_{cr} = &\frac{EI_x b^2 + EI_y a^2}{2} \pm \\ &\frac{E}{2}\sqrt{b^4 I_x^2 - 2a^2 b^2 I_x I_y + a^4 I_y^2 + 4a^2 b^2 I_{xy}^2} \end{aligned} \tag{19}$$

For the example shown in Figure 4a and Figure 4b, the ends are pinned in each orthogonal plane, which yield the following boundary conditions:

$$u(0) = 0 \qquad u''(0) = 0 \qquad u(L) = 0 \qquad u''(L) = 0$$

$$v(0) = 0 \qquad v''(0) = 0 \qquad v(L) = 0 \qquad v''(L) = 0$$

One should note at this point that the boundary conditions are applied to Equations 12 and 13 in each orthogonal plane independently. The respective values of a and b which satisfy the boundary conditions can be found in Timoshenko and Gere (1960, pp. 46-56) where the deflected shape shown in Figure 4a corresponds to the second buckling mode of a simply supported column, and the deflected shape shown in Figure 4b is the first buckling mode of a simply supported column. This implies:

$$a = \frac{\pi}{L} \qquad\qquad b = \frac{2\pi}{L}$$

By substituting the values of a and b into Equation 19, and simplifying, the critical buckling load is found by taking the smaller, non-negative root.

$$P_{cr} = \frac{\pi^2 E}{L^2}\left[(2I_x + \frac{1}{2}I_y) - \sqrt{4I_x^2 - 2I_x I_y + \frac{1}{4}I_y^2 + 4I_{xy}^2}\right] \tag{19a}$$

The moment of inertia and the product of inertia terms within the brackets can be considered as an effective or quasi moment of inertia for compound buckling:

$$P_{cr} = \frac{\pi^2 EI_{eff}}{L^2} \tag{20}$$

where I_{eff} is defined as:

$$I_{eff} = \left[(2I_x + \frac{1}{2}I_y) - \sqrt{4I_x^2 - 2I_x I_y + \frac{1}{4}I_y^2 + 4I_{xy}^2}\right] \tag{21}$$

The effective radius of gyration for compound buckling is simply:

$$r_{eff} = \sqrt{\frac{I_{eff}}{A}} \tag{22}$$

Equation 21 isolates the effect of the product of inertia on the critical buckling load when a member is constrained to buckle as shown in Figure 4a and Figure 4b. One interesting point to note is that the classical buckling solution about each respective, orthogonal axis is contained in Equation 19a. This is evident for instance, if one considers only the I_x terms and ignores the other terms. (One must use the larger root in this case since the smaller root is zero.)

EXAMPLE

Consider a Z section, shown in Figure 5 in place of the generalized cross section shown in Figure 1c, and consider the member is supported as shown in Figure 4a and Figure 4b.

For member properties:

I_x = 27.96 in.^{4} I_y = 5.27 in.^{4} I_{xy} = -8.59 in.^{4}

I_x' = 30.85 in.^{4} I_y' = 2.39 in.^{4} I_{eff} = 2.57 in.^{4}

For additional parameters, assume:

L = 20 ft = 240 in. and E = 29,000 ksi

The member can buckle in one of three configurations: 1) Solely in the Y-Z plane shown in Figure 4a, 2) Solely in the X-Z plane shown in Figure 4b or 3) *Compoundly*, meaning in a configuration which is a combination of Figure 4a and Figure 4b *simultaneously*. The engineer cannot know a *priori* which will control the design. The buckling load for each case must be calculated in order to find which is the most critical.

In the Y-Z plane:

$$P_{cr} = \frac{\pi^2 EI_x}{(L/2)^2} = \frac{\pi^2 29000(27.96)}{(240/2)^2} \approx 556\,\text{kips} \tag{23}$$

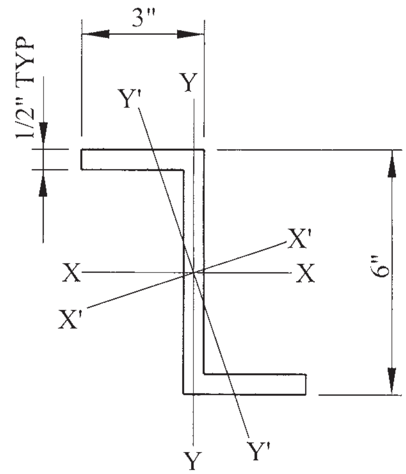


Fig. 5. Z section.

In the X-Z plane:

$$P_{cr} = \frac{\pi^2 EI_y}{L^2} = \frac{\pi^2 29000(5.27)}{240^2} \approx 26\,\text{kips} \tag{24}$$

For compound buckling:

$$P_{cr} = \frac{\pi^2 EI_{eff}}{L^2} = \frac{\pi^2 29000(2.54)}{240^2} \approx 13\,\text{kips} \tag{25}$$

The compound buckling case gives the lowest theoretical buckling load, and therefore controls the design of the member. Note that in the first two calculations, buckling is checked in each orthogonal plane independently, and the respective moment of inertias are about non-principal axes. Although a member is braced about one axis discretely at its midpoint rather than continuously, it is admissible that the member may buckle solely within each respective orthogonal plane before buckling in a compound configuration.

If the member were used as a simple compression member without any secondary brace at its midpoint, the normal Euler buckling equation applies about the weaker principal Y'-Y' axis. In this case, I_y' = 2.39 in.^{4} and P_{cr} is found:

$$P_{cr} = \frac{\pi^2 EI_y'}{L^2} = \frac{\pi^2 29000(2.39)}{240^2} \approx 12\,\text{kips} \tag{26}$$

Note the small difference between P_{cr} for the compound configuration versus P_{cr} for the simple brace ~ 1 kip. This makes perfect sense since the Y-Y axis and the Y'-Y' are closely aligned in orientation for this specific example. One sees little increase in axial capacity by adding a secondary brace in this case.

If the member is rotated 90 degrees in the X and Y plane, one can check compound buckling in this orientation by simply interchanging values of I_x and I_y in Equation 21 to find the respective I_{eff}. In this case I_{eff} = 7.00 in.^{4} and P_{cr} is found to be:

$$P_{cr} = \frac{\pi^2 EI_{eff}}{L^2} = \frac{\pi^2 29000(7.00)}{240^2} \approx 35\,\text{kips} \tag{27}$$

From this example, one can see the effect on the buckling load of a compression member when a secondary brace is provided which braces the primary member about a non-principal axis, and serves to illustrate the importance of checking the buckling load for *compound buckling*.

One point the reader should note is that, for the case of a simple compression member with no secondary brace at its midpoint, the boundary conditions are applied in the same orientation as that of the non-principal axes. For example, consider how the Z-shape would connect to a column using a gusset plate. Lutz (1992) formulated a method to account for this fact. It is possible to calculate an "r_{eff}", the effective minimum radius of gyration, for the case when effective length factors are estimated for non-principal axes. The "r_{eff}" in Lutz's case is unrelated to the r_{eff} in this paper shown in Equation 22. Depending on the choice of equivalent length factors (K factors) for each respective axis, one may gain additional axial capacity due to end connection conditions as outlined by Lutz.

ADDITIONAL SOLUTIONS

When different permutations of boundary conditions are considered, one can derive the corresponding expression for I_{eff} in each case. Consider a third case shown in Figure 6a and Figure 6b: The boundary conditions in this case are as follows:

$$u(0)=0 \quad u''(0)=0 \quad u(L)=0 \quad u''(L)=0$$

$$v(0)=0 \quad v'(0)=0 \quad v(L)=0 \quad v'(L)=0$$

which give the values for a and b as derived in Timoshenko and Gere's *Theory of Elastic Stability*.

$$a=\frac{\pi}{L} \qquad b=\frac{2.86\pi}{L}$$

By substituting values of a and b into Equation 19, and taking the smaller, non-negative root, the critical buckling load P_{cr} is found:

$$P_{cr}=\frac{\pi^2 E}{L^2}\left[(4.09\,I_x+\frac{1}{2}I_y)-\sqrt{16.73\,I_x^2-4.09\,I_x I_y+\frac{1}{4}I_y^2+8.18\,I_{xy}^2}\right] \tag{19b}$$

Define I_{eff} as:

$$I_{eff}=\left[(4.09\,I_x+\frac{1}{2}I_y)-\sqrt{16.73\,I_x^2-4.09\,I_x I_y+\frac{1}{4}I_y^2+8.18\,I_{xy}^2}\right] \tag{28}$$

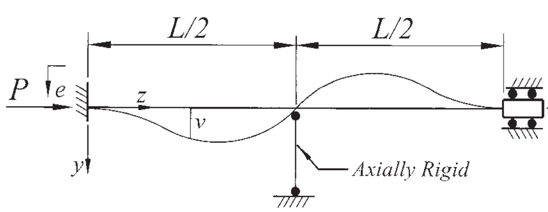


Fig. 6a. Member in axial compression with varying boundary conditions in each plane.

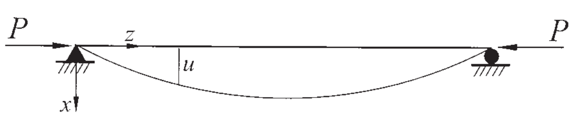


Fig. 6b. View e-e.

As a fourth case, consider Figure 7a and Figure 7b. The boundary conditions in this case are as follows:

$$u(0)=0 \quad u'(0)=0 \quad u(L)=0 \quad u'(L)=0$$

$$v(0)=0 \quad v''(0)=0 \quad v(L)=0 \quad v''(L)=0$$

which give the values for a and b:

$$a=\frac{2\pi}{L} \qquad b=\frac{2\pi}{L}$$

By substituting values of a and b into Equation 19, and taking the smaller, non-negative root, the critical buckling load P_{cr} is found:

$$P_{cr}=\frac{\pi^2 E}{L^2}\left[(2I_x+2I_y)-\sqrt{4I_x^2-8I_x I_y+4I_y^2+16I_{xy}^2}\right] \tag{19c}$$

Again, define I_{eff} as:

$$I_{eff}=\left[(2I_x+2I_y)-\sqrt{4I_x^2-8I_x I_y+4I_y^2+16I_{xy}^2}\right] \tag{29}$$

For a fifth case, consider Figure 8a and Figure 8b. The boundary conditions for this case are as follows:

$$u(0)=0 \quad u'(0)=0 \quad u(L)=0 \quad u'(L)=0$$

$$v(0)=0 \quad v'(0)=0 \quad v(L)=0 \quad v'(L)=0$$

which give the values for a and b:

$$a=\frac{2\pi}{L} \qquad b=\frac{2.86\pi}{L}$$

By substituting values of a and b into Equation 19, and taking the smaller non-negative root, the critical buckling load P_{cr} is found:

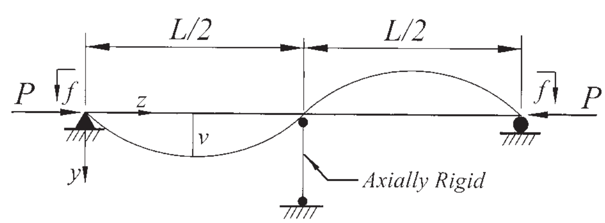


Fig. 7a. Member in axial compression with bracing at mid-point.

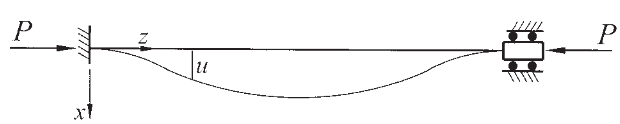


Fig. 7b. View f-f.

$$P_{cr} = \frac{\pi^2 E}{L^2}\left[(4.09 I_x + 2 I_y) - \sqrt{16.73\, I_x^2 - 16.36\, I_x I_y + 4\, I_y^2 + 32.72\, I_{xy}^2}\right] \quad (19d)$$

where I_{eff} is defined as:

$$I_{eff} = \left[(4.09 I_x + 2 I_y) - \sqrt{16.73\, I_x^2 - 16.36\, I_x I_y + 4\, I_y^2 + 32.72\, I_{xy}^2}\right] \quad (30)$$

Consider the sixth and final case to be investigated shown in Figure 9a and Figure 9b.

This final case is meant as a model for an X-braced system. K_s represents the out of the plane stiffness provided by a cross tension member. Picard and Beaulieu (1987) investigated this condition for X bracing, and derived an expression for K_s based on both members being continuous through their common intersection point. In practice, in order to design the member correctly, the engineer must assess carefully the nature of the intersection and end conditions. This is contingent on the sort of bracing members used and the connection detail at the intersection and ends. One may vary K_s to find any one of a number of solutions, but this is beyond the scope and intent of this paper. In this final case, the author assumed the stiffness K_s is infinite so that there is effectively a pinned support at this location. For additional discussion of effective length factors of X-braced systems, see the paper by El-Tayem and Goel (1986).

The boundary conditions for this final case are as follows:

$$u(0) = 0 \qquad u''(0) = 0 \qquad u(L) = 0 \quad u''(L) = 0$$

$$v(0) = 0 \qquad v''(0) = 0 \qquad v(L) = 0 \quad v''(L) = 0$$

which give the values for a and b:

$$a = \frac{2\pi}{L} \qquad\qquad b = \frac{2\pi}{L}$$

By coincidence, this yields the same solution as the fourth case above. The corresponding solution is found by substituting values of a and b into Equation (19), and by taking the smaller, non-negative root, the critical buckling load P_{cr} is found:

$$P_{cr} = \frac{\pi^2 E}{L^2}\left[(2 I_x + 2 I_y) - \sqrt{4 I_x^2 - 8 I_x I_y + 4 I_y^2 + 16 I_{xy}^2}\right] \quad (19e)$$

with I_{eff} defined as:

$$I_{eff} = \left[(2 I_x + 2 I_y) - \sqrt{4 I_x^2 - 8 I_x I_y + 4 I_y^2 + 16 I_{xy}^2}\right] \quad (31)$$

Note however that Equation (19e) can be rewritten in the following form by using the Mohr's circle transformation equations:

$$P_{cr} = \frac{\pi^2 E I_y}{(L/2)^2} \quad (19f)$$

With further reflection and considering Figure 9a and Figure 9b, it is obvious that this ought to be the case: P_{cr} is simply the Euler buckling load with ends pinned about the weak principal axis of the member, and using the half length $L/2$.

One question which may arise for all the cases analyzed: How does the theory address the behavior of a member

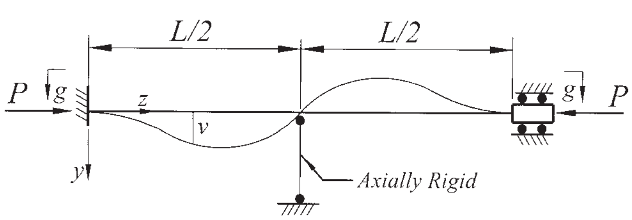


Fig. 8a. Member in axial compression with bracing at mid-point.

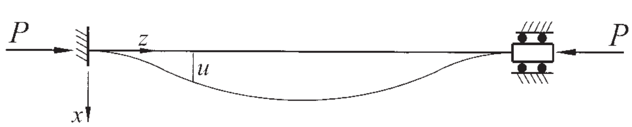


Fig. 8b. View g-g.

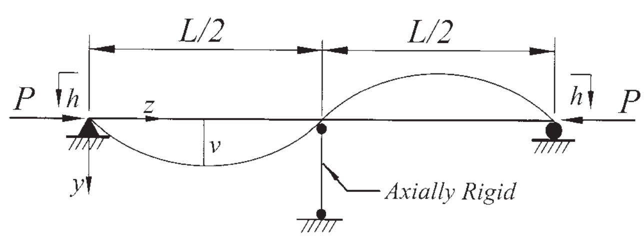


Fig. 9a. Member in axial compression with cross tension member.

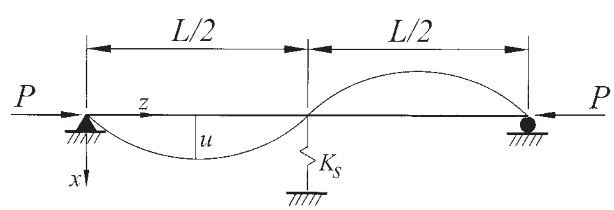


Fig. 9b. View h-h.

when the cross section shown in Figure 1c is oriented such that the principal axes are oriented vertically and horizontally respectively? This would be the case for a wide flange brace for example. In such an instance, one would consider buckling in each orthogonal plane separately as is normally done. The effective radius of gyration r_{eff} may still be calculated, but one will find that r_{eff} is not the smallest radius of gyration, and theoretically, no compound buckling will take place. Therefore, there is no inherent contradiction with normal buckling theory as commonly implemented and the theory derived in this paper. The equations derived provide the engineer with the capability to check for compound buckling for any compression member regardless of the orientation of the cross section about its longitudinal axis. The example provided illustrates the importance of checking this mode of buckling.

Another significant question for members braced about a non-principal axis is whether compound buckling is *always* more critical than in-plane buckling. The critical buckling load P_{cr} for compound buckling is a function of end support conditions and specific section properties, and therefore, an energy formulation of each case can be done to ascertain whether this is true generally, by comparing strain energies among different cases. Note that in the current approach, once all pertinent radii of gyration are calculated for a specific example, one can see readily which mode of buckling controls the design based on each respective L/r ratio. Further research is required.

TORSIONAL BUCKLING

For the case of thin-walled, open cross sections, the potential for torsional buckling or torsional-flexural buckling must be considered. This mode of failure must be checked, since it often is the controlling mode of failure for thin-walled members. The engineer must give careful consideration to the torsional boundary conditions, which are highly dependent on the actual detailing of connections. For a comprehensive theoretical treatment of torsional buckling and flexural-torsional buckling, see Timoshenko and Gere (1961), Section 5.4. Studies relating to this subject were done by Earls and Galambos (1995 and 1996) on single angles. Additionally, for guidance in designing members for torsional stability, see *Load and Resistance Factor Design Manual of Steel Construction*, Appendix E (AISC, 1998) and *Guide to Stability Design Criteria for Metal Structures*, 4th Edition, Section 13.3.5 (Galambos, 1988).

CONCLUSION

For the initial case analyzed, where a member is continuously supported about a non-principal axis, the author discovered that the classical Euler buckling equation holds, but the radius of gyration is calculated about the respective non-principal axis. Due to coupling from the product of inertia, a moment is induced about the orthogonal non-principal axis. See Equation 9.

For the additional cases of various boundary conditions investigated, where a member is constrained at its midpoint by a support or secondary member, and which is therefore constrained to buckle about a non-principal axis, the author has shown through stability analysis, that the axial buckling solution can be formulated analogously to the classical Euler buckling solution by defining an effective or quasi moment of inertia, I_{eff}, which depends on both end support conditions, non-principal moments of inertia and the corresponding product of inertia. In turn, an effective or quasi radius of gyration can be calculated. The buckled shape is *compound*, meaning simultaneous displacement in two orthogonal planes. The expressions derived are unique for compound buckling of axial members and may be applied for any orientation of the cross section of a member about its longitudinal axis. Actual physical tests will be required to corroborate the validity of the derived equations.

NOMENCLATURE

A	=	cross-sectional area
a, b	=	buckling parameters
E	=	modulus of elasticity
I_{eff}	=	effective moment of inertia for compound buckling
I_x, I_y	=	moments of inertia about non-principal axes
I_{xy}	=	product of inertia
I_x', I_y'	=	moments of inertia about the principal axes
K_i	=	constant coefficients
K_s	=	spring support term
L	=	length of a member
M_x	=	bending moment about x axis
M_y	=	bending moment about y axis
P	=	axial compressive load
P_{cr}	=	critical buckling load P
r_{eff}	=	effective radius of gyration for compound buckling
r_y	=	radius of gyration about the Y axis
T	=	axial tensile load
u, v	=	displacements

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