### 1.21 Julian to Calendar Date Conversion

## Introduction

This document will provide the correct conversion from the Julian Date (positive, only) to the Julian or Gregorian calendar date and fraction of a day. The day of the week corresponding to a given calendar date will also be calculated.

Astronomical days beginning at Greenwich mean noon (12 ${ }^{\text {h }}$ UT) are numbered consecutively from an epoch far in the past. The ordinal number assigned to these days is the Julian Day Number which is defined to be 0 for the day starting at Greenwich mean noon on 1 January 4713 B.C.. Astronomers and historians disagree on counting the years preceding year 1 (A.D. 1). In astronomy the year preceding year +1 is 0 , in history 1 B.C.. Thus for our purposes 4713 B.C. corresponds to the year -4712.

The Julian Date (JD) corresponding to any instant is the Julian Day Number followed by the decimal fraction of the day elapsed since noon. The unit of day is the mean solar day.
The Julian Date can be expressed in Universal Time (UT) or dynamical time. The term Julian Ephemeris Date (JED) was used prior to 1984 when the Julian Date was based upon Ephemeris Time (ET). Since then ET has been replaced with Terrestrial Dynamical Time (TDT). The Julian Ephemeris Date is the same as the Julian Date expressed in TDT. We will use the notation JED when either ET or TDT is used.

For many astronomical applications, in order to keep the numbers small, a Modified Julian Date (MJD) is used. MJ(E)D $=\mathrm{J}(E) \mathrm{D}-2400000.5$. Note that a day MJD begins at midnight of the civil day. Confusing, eh?

The Julian calendar (not to be confused with the Julian Date, which is named after Julius Scaliger, the father of the sixteenth century chronologist Joseph Justus Scaliger) was introduced by Julius Caesar in -45 (46 B.C.). This year is known as the "year of confusion". This calendar served as a standard, for better or worse, for European civilization until Pope Gregory's reform of + 1582. Beginning 15 October 1582 (JD 2299160.5) the Gregorian calendar ("our" calendar) was used.

## Method

Following Meeus (op. cit.), the conversion of the Julian Date (JD) to the calendar date proceeds from the method below.

Let

$$
\begin{aligned}
& Z=\text { floor(JD }+0.5) \\
& z=\operatorname{FRAC}(J D+0.5)
\end{aligned}
$$

where JD is the Julian Date, floor( $x$ ) is the integer part of the number $x$, and $\operatorname{FRAC}(x)$ is the fractional (decimal) part of the number x .

Then

$$
\begin{aligned}
& A=Z, \quad \text { if } Z<2299161 \\
& A=Z+1+a-\operatorname{loor}(\mathrm{a} / 4), \quad \text { if } Z^{3} 2299161
\end{aligned}
$$

where

$$
\text { - } a=\operatorname{floor}((Z-1867216.25 / 36524.25))
$$

If we write the calendar date as YYYY.MMDDdddddd, where

- YYYY = the year
- $M M=$ the month (1 for January, 2 for February, etc.)
- $D D=$ the day of the month
- dddddd = the decimal fraction of the day,
and calculate the following intermediate values

```
B=A+1524
C= floor((B-122.1)/365.25)
D = floor(365.25C)
E = floor((B-D)/30.6001), then
DD = floor(B - D - floor(30.6001E) + z)
dddddd = FRAC(B - D - floor(30.6001E) + z)
MM = E-1, if E < 14
MM = E-13, if E=14 or 15
YYYY = C - 4716, if MM > 2
YYYY = C - 4715, if MM = 1 or 2
```

The day of the week corresponding to the Julian Date is given by

$$
w=\bmod (\mathrm{JD}-\operatorname{ddddd}+1.5,7)
$$

where $\bmod (x, y)$ is the remainder on dividing $x$ by $y$.
If $\mathbf{w}=0$, it is Sunday, 1 Monday, 2 Tuesday, etc.

Enter Date

Julian Date (JD or JED; non-negative): JD := 2446113.75

Calculations
Intermediate Variables

$$
\begin{aligned}
& \mathbf{Z}:=\text { floor }(\mathbf{J D}+0.5) \quad \text { a }:=\text { floor }\left(\frac{\mathbf{Z}-1867216.25}{36524.25}\right) \\
& \mathbf{Z}:=\text { JD }+0.5-\mathbf{Z} \\
& \mathbf{A}:=\text { if }\left(\mathbf{Z}<2299161, \mathbf{Z}, \mathbf{Z}+1+\mathbf{a}-\text { floor }\left(\frac{\mathbf{a}}{4}\right)\right) \\
& \mathbf{B}:=\mathbf{A}+1524 \quad \underset{\text { Cu }}{\mathbf{C}}:=\text { floor }\left(\frac{\mathbf{B}-122.1}{365.25}\right) \\
& \mathbf{D}:=\text { floor }(365.25 \cdot \mathbf{C}) \quad \text { E }:=\text { floor }\left(\frac{\mathbf{B}-\mathbf{D}}{30.6001}\right) \\
& \text { DDdddddd }:=\text { B - D - floor }(30.6001 \cdot E)+\mathbf{z}
\end{aligned}
$$

## General Formulas to Calculate the Date

The day of the month:
DD := floor(DDdddddd)

The decimal fraction of the day.
dddddd := DDdddddd - DD

The month:

$$
\mathbf{M M}:=\mathbf{i f}(\mathbf{E}<14, \mathbf{E}-1, \mathbf{E}-13)
$$

The year:

$$
\text { YYYY := if }(\mathbf{M M}>2, \mathbf{C}-4716, \mathbf{C}-4715)
$$

## Results

## In Sexagesimal Notation

The functions necessary to convert between sexagesimal and decimal notations are defined past the right margin of the page.

$$
\text { TOD := dec2sexa(dddddd• } 24 \text { ) }
$$

The calendar date is then
Year: $\quad$ YYYY $=1985$
Month: $\quad \mathbf{M M}=2.0000$
Day of Month: $\quad$ DD $=17.0000$
Time of day: $\quad$ TOD $=\left(\begin{array}{lll}6 & 0 & 0\end{array}\right)$
UT; ET or TDT, if JED used.

## The Day of the Week

$$
\mathbf{w}:=\bmod (\mathrm{JD}-\text { dddddd }+1.5,7)
$$

| w | day of week |  |
| :---: | :---: | :---: |
| $\mathbf{w}=0.0000$ | 1 | Monday |
| 2 | Tuesday |  |
| 3 | Wednesday |  |
| 4 | Thursday |  |
| 5 | Friday |  |
| 6 | Saturday |  |
| 0 | Sunday |  |

Sexagesimal (angles or time) to decimal

The conversion function is given by

$$
\operatorname{sexa2dec}(\mathbf{a}, \mathbf{b}, \mathbf{c}):=\mathbf{a}+\frac{\mathbf{b}}{60}+\frac{\mathbf{c}}{3600}
$$

where
$a=$ integral number of degrees or hours
$b=$ integral number of arc minutes or minutes of time
$c=$ integral number of arc seconds or seconds of time

Decimal (angles or time) to sexagesimal
We must first define some intermediate functions:

$$
\begin{aligned}
& \text { hr_deg }\left(d_{-} h\right):=\text { floor }\left(\left|d_{-} h\right|\right) \\
& \left.\min _{\text {min }}(\mathbf{d} \mathbf{h}):=\text { floor[(|d_h| } \mathbf{h r} \text { _deg }\left(\mathbf{d} \_\mathbf{h}\right)\right) \cdot 60 \text { ] } \\
& \underset{\sec }{\sec }(\mathbf{d} \mathbf{h}):=\left(|\mathbf{d} \mathbf{-}|-\mathbf{h r} \mathbf{d e g}(\mathbf{d} \mathbf{h})-\frac{\boldsymbol{\operatorname { m i n }}(\mathbf{d} \mathbf{\prime})}{60}\right) \cdot 3600
\end{aligned}
$$

where $d \_h$ is the angle or time in decimal notation.
The conversion function is then given by

$$
\left.\operatorname{dec} 2 \operatorname{sexa}\left(d_{-} h\right):=\left(\operatorname{hr}_{-} \operatorname{deg}^{\left(d \_h\right.}\right) \quad \min \left(d_{-} h\right) \quad \sec \left(d_{-} h\right)\right) \cdot \frac{d_{-} h}{\left|d_{-} h\right|}
$$

## Unit Definitions:

```
hours \equiv1
degrees =1
seconds \equiv1
arc_seconds \equiv1
centuries =1
```

