CHAPTER 13

POWER SCREWS

Rudolph J. Eggert, Ph.D., P.E.
Associate Professor of Mechanical Engineering
University of Idaho
Boise, Idaho

LIST OF SYMBOLS

A	Area
$A(t)$	Screw translation acceleration
C	End condition constant
d	Major diameter
d_c	Collar diameter
d_m	Mean diameter
d_r	Root or minor diameter
E	Modulus of elasticity
F	Load force
F_c	Critical load force
G	Shear modulus
h	Height of engaged threads
I	Second moment of area
J	Polar second moment of area
k	Radius of gyration
L	Thread lead
L_c	Column length
n	Angular speed, r/min
n_s	Number of thread starts

N_e	Number of engaged threads
P_i	Basic load rating
p	Thread pitch
S_y	Yield strength
T_c	Collar friction torque
T_i	Basic static thrust capacity
T_R	Raising torque
T_L	Lowering torque
t	Time
$V(t)$	Screw translation speed
w	Thread width at root
W_i	Input work
W_o	Output work
α	Flank angle
α_n	Normalized flank angle
β	Thread geometry parameter
Δx	Screw translation
$\Delta\theta$	Screw rotation
η	Efficiency
λ	Lead angle
μ_t	Coefficient of thread friction
μ_c	Coefficient of collar friction
σ	Normal stress
σ'	von Mises stress
τ	Shear stress
Ψ	Helix angle

13.1 INTRODUCTION

Power screws convert the input rotation of an applied torque to the output translation of an axial force. They find use in machines such as universal tensile testing machines, machine tools, automotive jacks, vises, aircraft flap extenders, trench braces, linear actuators, adjustable floor posts, micrometers, and C-clamps. The mechanical advantage inherent in the screw is exploited to produce large axial forces in response to small torques. Typical design considerations, discussed in the following sections, include kinematics, mechanics, buckling and deflection, and stresses.

Two principal categories of power screws are machine screws and recirculating-ball screws. An example of a machine screw is shown in Fig. 13.1. The screw threads are typically formed by thread rolling, which results in high surface hardness, high strength, and superior surface finish. Since high thread friction can cause self-locking when the applied torque is removed, protective brakes or stops to hold the load are usually not required.

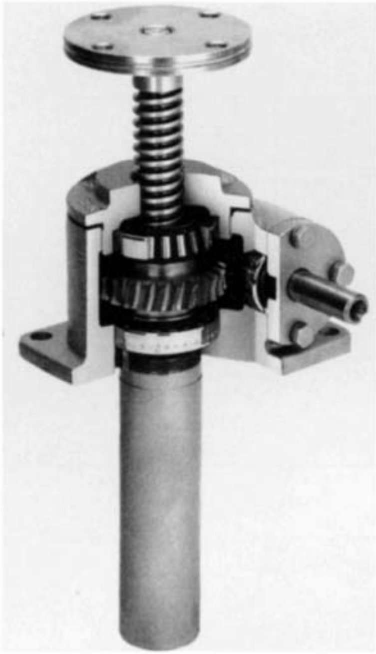


FIGURE 13.1 Power screw assembly using rolled thread load screw driven by worm shaft and gear nut. *(Simplex Uni-Lift catalog UC-101, Templeton, Kenly & Co., Inc., Broadview, Ill., with permission.)*

Three thread forms that are often used are the *Acme thread,* the *square thread,* and the *buttress thread.* As shown in Fig. 13.2, the Acme thread and the square thread exhibit symmetric leading and trailing flank angles, and consequently equal strength in raising and lowering. The Acme thread is inherently stronger than the square thread because of the larger thread width at the root or minor diameter. The general-purpose Acme thread has a 14½-degree flank angle and is manufactured in a number of standard diameter sizes and thread spacings, given in Table 13.1. The buttress thread is proportionately wider at the root than the Acme thread and is typically loaded on the 7-degree flank rather than the 45-degree flank. See Refs. [13.1], [13.2], [13.3], and [13.4] for complete details of each thread form.

Ball screws recirculate ball bearings between the screw rod and the nut, as shown in Fig. 13.3. The resulting rolling friction is significantly less than the sliding friction of the machine screw type. Therefore less input torque and power are needed. However, motor brakes or screw stops are usually required to prevent ball screws from *self-lowering* or *overhauling.*

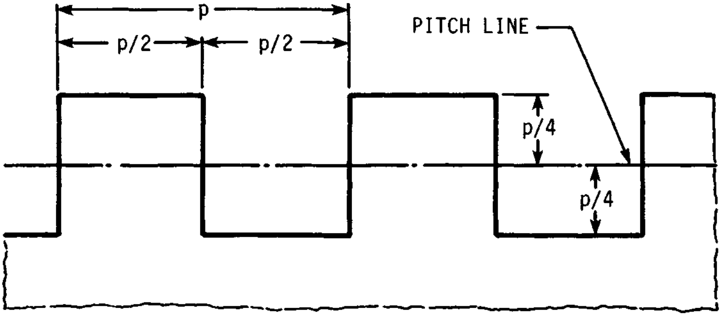
13.2 KINEMATICS

The primary function or design requirement of a power screw is to move an axial load F through a *specified* linear distance, called the *travel.* As a single-degree-of-freedom mechanism, screw travel is constrained between the fully *extended* position x_{max} and the closed or *retracted* position x_{min}. The output *range of motion,* therefore, is $x_{max} - x_{min}$. As the input torque T is applied through an angle of rotation $\Delta\theta$, the screw travels Δx in proportion to the screw lead L or total number of screw turns N_t as follows:

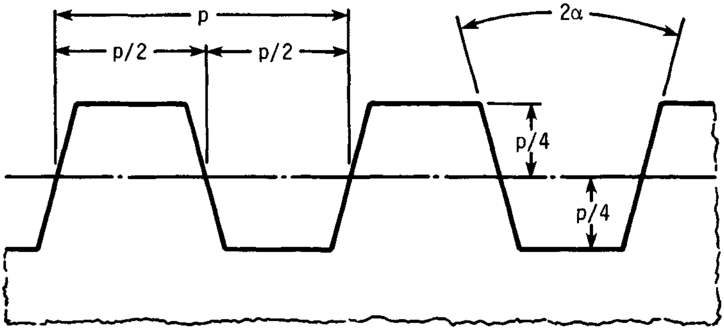
$$\Delta x = L \frac{\Delta\theta}{2\pi} = LN_t \tag{13.1}$$

In addition to range of motion specifications, other kinematic requirements may be prescribed, such as velocity or acceleration. The linear screw speed V, in/min, is obtained for a constant angular speed of n, r/min, as

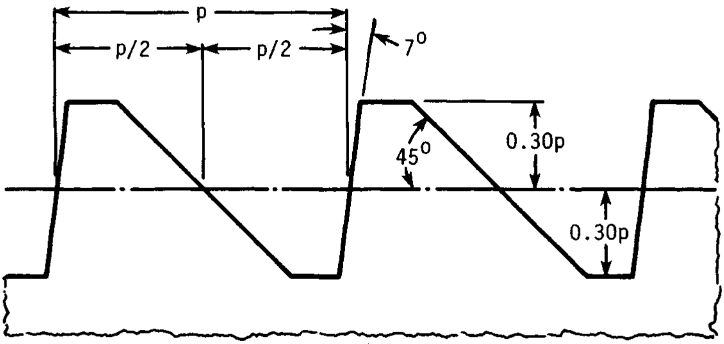
$$V = nL \tag{13.2}$$



(a)



(b)



(c)

FIGURE 13.2 Basic thread forms. (*a*) Square; (*b*) general-purpose Acme; (*c*) buttress. The stub Acme thread height is $0.3p$.

TABLE 13.1 Standard Thread Sizes for Acme Thread Form†

Size D, in	Threads per inch n
1/4	**16**
5/16	16, **14**
3/8	16, 14, **12**, **10**
7/16	16, 14, **12**, **10**
1/2	16, 14, 12, **10**, 8
5/8	16, 14, 12, 10, **8**
3/4	16, 14, 12, 10, 8, **6**
7/8	14, 12, 10, 8, **6**, 5
1	14, 12, 10, 8, 6, **5**
1 1/8	12, 10, 8, 6, **5**, 4
1 1/4	12, 10, 8, 6, **5**, 4
1 3/8	10, 8, 6, 5, **4**
1 1/2	10, 8, 6, 5, **4**, 3
1 3/4	10, 8, 6, 4, **4**, 3, 2 1/2
2	8, 6, 5, **4**, 3, 2 1/2, 2
2 1/4	6, 5, 4, **3**, 2 1/2, 2
2 1/2	5, 4, **3**, 2 1/2, 2
2 3/4	4, **3**, 2 1/2, 2
3	4, 3, 2 1/2, **2**, 1 1/2, 1 1/3
3 1/2	4, 3, 2 1/2, **2**, 1 1/2, 1 1/3, 1
4	4, 3, 2 1/2, **2**, 1 1/2, 1 1/3, 1
4 1/2	3, 2 1/2, **2**, 1 1/2, 1 1/3, 1
5	3, 2 1/2, **2**, 1 1/2, 1 1/3, 1

† The preferred size is shown in boldface.

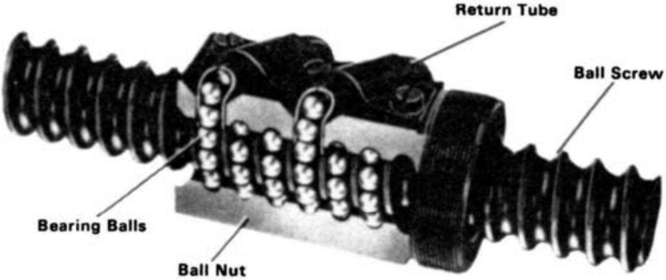


FIGURE 13.3 Ball screw assembly. *(Saginaw Steering Gear Division, General Motors Corporation.)*

The input speed may vary with respect to time t, resulting in a proportional change in output speed according to

$$V(t) = \dot{x}(t) = \frac{L}{2\pi}\dot{\theta}(t) \tag{13.3}$$

Similarly, the linear and angular accelerations of the load screw are related as follows:

$$A(t) = \ddot{x}(t) = \frac{L}{2\pi}\ddot{\theta}(t) \tag{13.4}$$

Inertia forces and *torques* are often neglected for screw systems which have small accelerations or masses. If the screw accelerates a large mass, however, or if a nominal mass is accelerated quickly, then inertia forces and torques should be analyzed. The total required input torque is obtained by *superposing* the static equilibrium torque, the torque required to accelerate the load, and the inertia torque of the screw rod itself. The inertia torque of the screw is sometimes significant for high-speed linear actuators. And lastly, impacts resulting from jerks can be analyzed using strain-energy methods or finite-element methods.

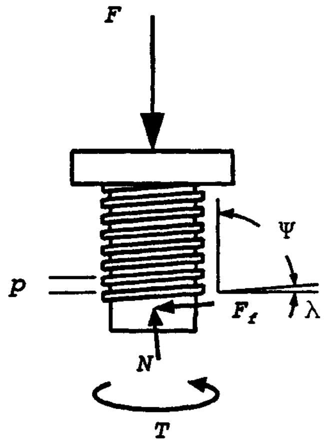


FIGURE 13.4 Free-body diagram of load screw.

13.3 MECHANICS

Under static equilibrium conditions, the screw rotates at a constant speed in response to the input torque T shown in the free-body diagram of Fig. 13.4. In addition, the load force F, normal force N, and sliding friction force F_f act on the screw. The friction force opposes relative motion. Therefore, the direction of the friction force F_f will reverse when the screw translates in the direction of the load rather than against it. The torques required to raise the load T_R (i.e., move the screw in the direction opposing the load) and to lower the load T_L are

$$T_R = \frac{Fd_m}{2}\left(\frac{\pi\mu_t d_m + L\beta}{\pi d_m\beta - \mu_t L}\right) \tag{13.5}$$

$$T_L = \frac{Fd_m}{2}\left(\frac{\pi\mu_t d_m - L\beta}{\pi d_m\beta + \mu_t L}\right) \tag{13.6}$$

where

$$d_m = d - p/2$$
$$L = pn_s$$
$$\tan\lambda = \frac{L}{\pi d_m}$$
$$\tan\alpha_n = \tan\alpha\cos\lambda$$
$$\beta = \cos\alpha_n \qquad (\beta = 1 \text{ for square threads})$$

The thread geometry parameter β includes the effect of the flank angle α as it is projected normal to the thread and as a function of the lead angle. For general-purpose single-start Acme threads, α is 14.5 degrees and β is approximately 0.968, varying less than 1 percent for diameters ranging from ¼ in to 5 in and thread spacing ranging from 2 to 16 threads per inch. For square threads, $\beta = 1$.

In many applications, the load slides relative to a collar, thereby requiring an additional input torque T_c:

$$T_c = \frac{F\mu_c d_c}{2} \tag{13.7}$$

Ball and tapered-roller thrust bearings can be used to reduce the collar torque.

The *starting torque* is obtained by substituting the *static* coefficients of friction into the above equations. Since the sliding coefficient of friction is roughly 25 percent less than the static coefficient, the *running torque* is somewhat less than the starting torque. For precise values of friction coefficients, specific data should be obtained from the published technical literature and verified by experiment.

Power screws can be *self-locking* when the coefficient of friction is high or the lead is small, so that $\pi \mu_t d_m > L$ or, equivalently, $\mu_t > \tan \lambda$. When this condition is not met, the screw will *self-lower* or *overhaul* unless an opposing torque is applied.

A measure of screw efficiency η can be formulated to compare the work output W_o with the work input W_i:

$$\eta = \frac{W_o}{W_i} = \frac{F\,\Delta x}{T\,\Delta\theta} \tag{13.8}$$

where T is the total screw and collar torque. Similarly, for one revolution or 2π radians and screw translation L,

$$\eta = \frac{FL}{2\pi T} \tag{13.9}$$

Screw manufacturers often list output travel speed V, in in/min, as a function of required motor torque T in lbf·in, operating at n r/min, to lift the rated capacity F, in lbf. The actual efficiency for these data is therefore

$$\eta = \frac{FV}{2\pi n T} \tag{13.10}$$

Efficiency of a square-threaded power screw with respect to lead angle λ, as shown in Fig. 13.5, is obtained from

$$\eta = \frac{1 - \mu \tan \lambda}{1 + \mu \cot \lambda} \tag{13.11}$$

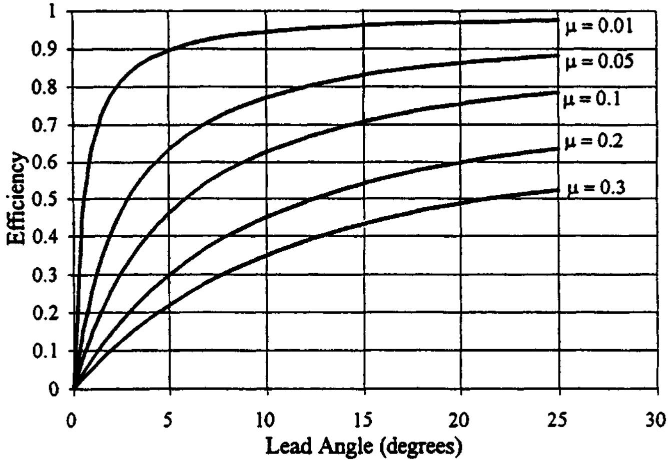


FIGURE 13.5 Screw efficiency η versus thread lead angle λ.

Note the importance of proper lubrication. For example, for $\lambda = 10$ degrees and $\mu = 0.05$, η is over 75 percent. However, as the lubricant becomes contaminated with dirt and dust or chemically breaks down over time, the friction coefficient can increase to $\mu = 0.30$, resulting in an efficiency $\eta = 35$ percent, thereby doubling the torque, horsepower, and electricity requirements.

13.4 BUCKLING AND DEFLECTION

Power screws subjected to compressive loads may buckle. The Euler formula can be used to estimate the critical load F_c at which buckling will occur for relatively long screws of column length L_c and second moment of area $I = \pi d_r^4/64$ as

$$F_c = \frac{C\pi^2 E}{L_c^2}\left(\frac{\pi d_r^4}{64}\right) \tag{13.12}$$

where C is the theoretical end-condition constant for various cases given in Table 13.2. Note that the critical buckling load F_c should be reduced by an appropriate load factor of safety as conditions warrant. See Chap. 30 for an illustration of various end conditions and effective length factor K, which is directly related to the end-condition constant by $C = 1/K^2$.

A column of length L_c and radius of gyration k is considered long when its slenderness ratio L_c/k is larger than the critical slenderness ratio:

$$\frac{L_c}{k} > \left(\frac{L_c}{k}\right)_{\text{critical}} \tag{13.13}$$

$$\frac{L_c}{k} > \left(\frac{2\pi^2 CE}{S_y}\right)^{1/2} \tag{13.14}$$

The radius of gyration k, cross-sectional area A, and second moment of area I are related by $I = Ak^2$, simplifying the above expression to

$$\frac{L_c}{d_r} > \frac{1}{4}\left(\frac{2\pi^2 CE}{S_y}\right)^{1/2} \tag{13.15}$$

For a steel screw whose yield strength is 60 000 psi and whose end-condition constant is 1.0, the critical slenderness ratio is about 100, and L_c/d_r is about 25. For steels whose slenderness ratio is less than critical, the Johnson parabolic relation can be used:

$$\frac{F_c}{A} = S_y - \frac{1}{CE}\left(\frac{S_y L_c}{2\pi k}\right)^2 \tag{13.16}$$

TABLE 13.2 Buckling End-Condition Constants

End condition	C
Fixed-free	¼
Rounded-rounded	1
Fixed-rounded	2
Fixed-fixed	4

which can be solved for a circular cross section of minor diameter d_r as

$$d_r = \sqrt{\frac{F_c}{\pi S_y} + \frac{S_y L_c^2}{\pi^2 CE}} \tag{13.17}$$

The load should be externally guided for long travels to prevent eccentric loading.

Axial compression or extension δ can be approximated by

$$\delta = \frac{FL_c}{AE} = \frac{4FL_c}{\pi d_r^2 E} \tag{13.18}$$

And similarly, angle of twist ϕ, in radians, can be approximated by

$$\phi = \frac{TL_c}{JG} = \frac{32TL_c}{\pi d_r^4 G} \tag{13.19}$$

13.5 STRESSES

Using St. Venants' principle, the nominal *shear* and *normal stresses* for cross sections of the screw rod away from the immediate vicinity of the load application may be approximated by

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d_r^4} \tag{13.20}$$

$$\sigma_x = \frac{F}{A} = \frac{4F}{\pi d_r^2} \tag{13.21}$$

Failure due to yielding can be estimated by the ratio of S_y to an equivalent, von Mises stress σ' obtained from

$$\sigma' = \sqrt{\left(\frac{4F}{\pi d_r^2}\right)^2 + 3\left(\frac{16T}{\pi d_r^3}\right)^2} = \frac{4}{\pi}\sqrt{\left(\frac{F}{d_r^2}\right)^2 + 48\left(\frac{T}{d_r^3}\right)^2} \tag{13.22}$$

The nominal *bearing* stress σ_b on a nut or screw depends on the number of engaged threads $N_e = h/p$ of pitch p and engaged thickness h and is obtained from

$$\sigma_b = \frac{F}{A_{\text{projected}}} = \frac{4F}{\pi\,(d^2 - d_r^2)}\left(\frac{p}{h}\right) \tag{13.23}$$

Threads may also shear or strip off the screw or nut because of the load force, which is approximately parabolically distributed over the cylindrical surface area A_{cyl}. The area depends on the width w of the thread at the root and the number of engaged threads N_e according to $A_{\text{cyl}} = \pi dwN_e$. The maximum shear stress is estimated by

$$\tau = \frac{3}{2}\frac{F}{A_{\text{cyl}}} \tag{13.24}$$

For square threads such that $w = p/2$, the maximum shear stress for the nut thread is

$$\tau = \frac{3F}{\pi dh} \tag{13.25}$$

To obtain the shear stress for the screw thread, substitute d_r for d. Since d_r is slightly less than d, the stripping shear stress for the screw is somewhat larger.

Note that the load flows from the point of load application through the thread geometry to the screw rod. Because of the nonlinear strains induced in the threads at the point of load application, each thread carries a *disproportionate* share of the load. A detailed analytical approach such as finite-element methods, backed up by experiments, is recommended for more accurate estimates of the above stresses and of other stresses, such as a thread bending stress and hoop stress induced in the nut.

13.6 BALL SCREWS

The design of ball screw assemblies is similar to that of machine screw systems. Kinematic considerations such as screw or nut travel, velocity, and acceleration can be estimated following Sec. 13.2. Similarly input torque, power, and efficiency can be approximated using formulas from Sec. 13.3. Critical buckling loads can be estimated using Eq. (13.12) or (13.16). Also, nominal shear and normal stresses of the ball screw shaft (or rod) can be estimated using Eqs. (13.20) and (13.21).

Design for strength, however, is typically completed using a catalog selection procedure rather than analytical stress-versus-strength analysis. Ball screw manufacturers usually list static and dynamic load capacities for a variety of screw shaft (rod) diameters, ball diameters, and screw leads; an example is shown in Table 13.3. The static capacity for *basic static thrust capacity* T_i, lbf, is the load which will produce a ball track deformation of 0.0001 times the ball diameter. The dynamic capacity or *basic load rating* P_i, lbf, is the constant axial load that a group of ball screw assemblies can endure for a rated life of 1 million inches of screw travel. The *rated life* is the length of travel that 90 percent of a group of assemblies will complete or exceed before any signs of fatigue failure appear. The catalog ratings, developed from laboratory test results, therefore involve the effects of hertzian contact stresses, manufacturing processes, and surface fatigue failure.

The catalog selection process requires choosing the appropriate combination of screw diameter, ball diameter, and lead, so that the axial load F will be sufficiently less than the basic static thrust capacity or the basic load rating for the rated axial travel life. For a different operating travel life of X inches, the modified basic load rating P_{iX}, lbf, is obtained from

$$P_{iX} = P_i\left(\frac{10^6}{X}\right)^{1/3} \tag{13.26}$$

An equivalent load rating P can be obtained for applications involving loads $P_1, P_2, P_3, \ldots, P_n$ that occur for $C_1, C_2, C_3, \ldots, C_n$ percent of the life, respectively:

$$P = \sqrt{\frac{C_1P_1^3 + C_2P_2^3 + \cdots + C_nP_n^3}{100}} \tag{13.27}$$

For the custom design of a ball screw assembly, see Ref. [13.5], which provides a number of useful relations.

TABLE 13.3 Sizes and Capacities of Ball Screws†

Major diameter, in	Lead, in, **mm**	Ball diameter, in	Dynamic capacity, lb	Static capacity, lb
0.750	0.200	0.125	1 242	4 595
	0.250	0.125	1 242	4 495
0.875	0.200	0.125	1 336	5 234
	0.250	0.125	1 336	5 234
1.000	0.200	0.125	1 418	5 973
	0.200†	0.156	1 909	7 469
	0.250	0.125	1 418	5 973
	0.250	0.156	1 909	7 469
	0.250	0.187	—	—
	0.400	0.125	1 418	5 973
	0.400	0.187	—	—
1.250	0.200	0.125	1 904	9 936
	0.200†	0.156	2 583	12 420
	0.250	0.125	1 904	9 936
	0.250	0.156	2 583	12 420
	0.250	0.187	3 304	15 886
1.500	0.200	0.125	2 046	11 908
	0.200†	0.156	2 786	14 881
	0.250	0.156	2 786	14 881
	0.250	0.187	3 583	18 748
	0.500	0.156	2 786	14 881
	0.500	0.250	5 290	24 762
1.500	**5**†	0.125	2 046	11 908
	5	0.156	2 787	14 881
	10	0.156	2 786	14 881
	10	0.250	5 290	24 762
	10	0.312	7 050	29 324
1.750	0.200	0.125	2 179	13 879
	0.200†	0.156	2 968	17 341
	0.250	0.156	2 968	17 341
	0.250	0.187	3 829	20 822
	0.500	0.187	3 829	20 882
	0.500	0.250	5 664	27 917
	0.500	0.312	7 633	33 232
2.000	0.200	0.125	2 311	15 851
	0.200†	0.156	3 169	19 801
	0.250	0.156	3 169	19 801
	0.250	0.187	4 033	23 172
	0.400	0.250	6 043	31 850
	0.500	0.312	8 135	39 854
	5	0.125	2 311	15 851
	5†	0.156	3 169	19 801
	6	0.156	3 169	19 801
	6	0.187	4 033	23 172
	10	0.250	6 043	31 850
	10	0.312	8 135	39 854

TABLE 13.3 Sizes and Capacities of Ball Screws† *(Continued)*

Major diameter, in	Lead, in, **mm**	Ball diameter, in	Dynamic capacity, lb	Static capacity, lb
2.250	0.250	0.156	3 306	22 262
	0.250	0.187	4 266	26 684
	0.500	0.312	8 593	44 780
	0.500	0.375	10 862	53 660
2.500	0.200	0.125	2 511	19 794
	0.200	0.156	3 134	24 436
	0.250	0.187	4 410	29 671
	0.400	0.250	6 633	39 746
	0.500	0.312	9 015	49 701
	0.500	0.375	10 367	59 308
	5	0.125	2 511	19 794
	5†	0.156	3 134	24 436
	10	0.250	6 633	39 746
	10	0.312	9 015	49 701
3.000	0.250	0.187	4 810	35 570
	0.400	0.250	7 125	47 632
	0.500	0.375	12 560	71 685
	0.660	0.375	12 560	71 685
	10	0.250	7 125	47 632
	10	0.312	9 744	58 648
3.500	0.500	0.312	10 360	69 287
	0.500	0.375	13 377	83 514
	1.000	0.500	19 812	111 510
	1.000	0.625	26 752	139 585
4.000	0.500	0.375	14 088	95 343
	1.000	0.500	21 066	127 282

† These values are not recommended; consult manufacturer.
Source: 20th Century Machine Company, Sterling Heights, Mich., by permission.

13.7 OTHER CONSIDERATIONS

A number of other important design factors should also be considered. Principal among these is lubrication. Greases using lithium thickeners with antioxidants and EP additives are effective in providing acceptable coefficients of sliding friction and corrosion protection. For operating environments which expose the screw threads to dust, dirt, or water, a protective boot, made of a compatible material, is recommended. Maintenance procedures should ensure that the screw threads are free of contaminants and have a protective film of grease. Operation at ambient temperatures in excess of 200°F requires special lubricants and boot materials as recommended by the manufacturer.

Screw and nut threads will wear with use, especially in heavy-duty-cycle applications, increasing the backlash from the as-manufactured allowance. Use of adjustable split nuts and routine inspection of thread thickness is recommended.

Power screws employing electric motors are often supplied with integral limit switches to control extension and retraction. To prevent ejection of the screw in case of a limit switch failure, a stop nut can be added. In addition, a torque-limiting clutch can be integrated at the motor to prevent equipment damage.

REFERENCES

13.1 ANSI B1.7M-1984 (R2001), "Screw Threads, Nomenclature, Definitions, and Letter Symbols," American Society of Mechanical Engineers, New York, 1992.

13.2 ANSI B1.5-1999, "Acme Screw Threads," American Society of Mechanical Engineers, New York, 1977.

13.3 ANSI B1.8-1977, "Stub Acme Screw Threads," American Society of Mechanical Engineers, New York, 1988.

13.4 ANSI B1.9-1973 (R1979), "Buttress Screw Threads," American Society of Mechanical Engineers, New York, 1973.

13.5 ANSI B5.48-1977 (R1988), "Ball Screws," American Society of Mechanical Engineers, New York, 1977.