general expression for internal work stored by the yield line pattern is

$$W_i = \sum_{n=1}^{N} \left(m_p \theta_{nx} L_{nx} + m_p \theta_{ny} L_{ny} \right) \tag{2.4}$$

where θ_{nx} and θ_{ny} are the x- and y-components of the relative rotation of the rigid plate segments along the yield line, L_{nx} and L_{ny} are the *x*- and *y*- components of the yield line length, and m_p is the plastic moment strength of the end plate per unit length,

$$m_p = F_{yp} Z_p = F_{yp} \left(\frac{(1)\ t_p^2}{4} \right) \tag{2.5}$$

The internal work, W_i, includes the distance from the inner bolts to the edge of the yield line pattern, for example, the distance *s* in Figure 2.3. Minimization of W_i with respect to the *s*-distance results in the least internal energy for the yield line pattern.

The external work due to the unit virtual rotation is given by

$$W_e = M_{pl}\ \theta = M_{pl} \left(\frac{1}{h} \right) \tag{2.6}$$

where M_{pl} is the end plate flexural strength and θ is the applied virtual displacement. The applied virtual displacement is equal to $1/h$, where h is the distance from the centerline of the compression flange to the tension side edge of the end plate.

The flexural strength of the end plate is found by setting W_i equal to W_e and solving for M_{pl}. Or, by rearranging the expression, the required end plate thickness can be determined.

To reduce the complexity of the yield line equations, the following simplifications have been incorporated into their development. No adjustment in end plate or column flange strength is made to account for the plate material removed by bolt holes. The width of the beam or column web is considered to be zero in the yield line equations. The width of fillet welds along the flange or stiffeners and web is not considered in the yield line equations. Finally, the very small strength contribution from yield lines in the compression region of the connections is neglected.

There have been relatively few studies conducted to determine the column flange strength in beam-to-column end-plate moment connections. In a beam-to-column end-plate moment connection, the beam flange tension forces are transmitted directly to the column flange by the connection bolts. The column flange must provide adequate strength to resist the applied bolt tensile forces. The column flanges can be configured as stiffened or unstiffened. A stiffened column flange has flange stiffener plates, often called continuity plates, installed perpendicular to the column web and in-line with the connecting beam flanges. An unstiffened column flange does not have stiffener or continuity plates.

Yield line analysis has been used to develop solutions for the stiffened and unstiffened column flange configurations

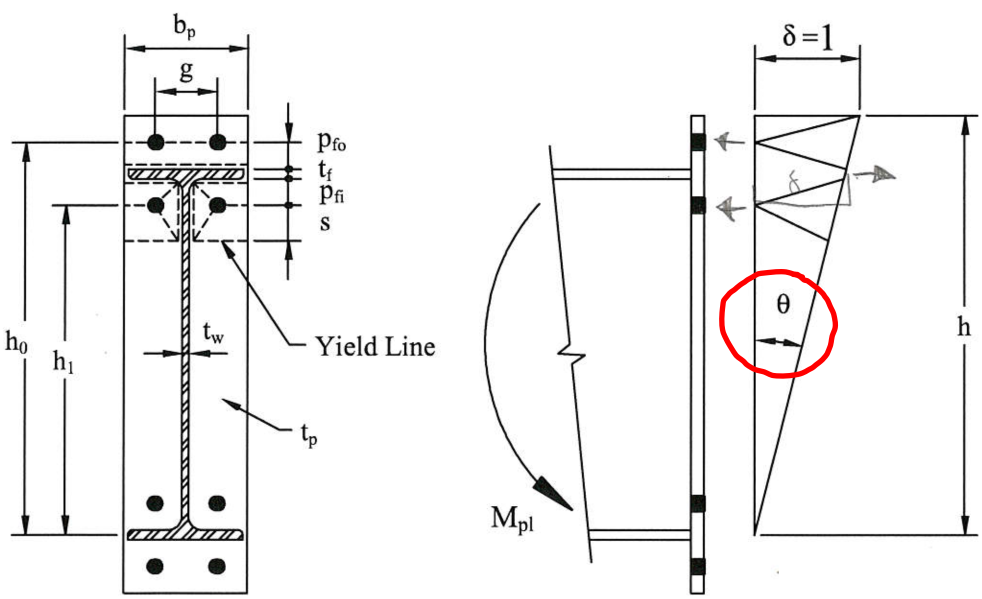
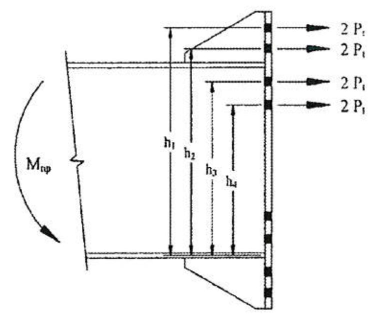


Fig. 2.3. Yield line pattern and virtual displacement of a four-bolt extended unstiffened connection.

Table 3.3. Summary of Eight-Bolt Extended Stiffened End Plate Design Strength

<table>
<tr><td colspan="2">End Plate Geometry and Yield Line Pattern</td><td>Bolt Force Model</td></tr>
<tr><td>Case 1 ($d_e \le s$)</td><td>Case 2 ($d_e > s$)</td><td rowspan="2">
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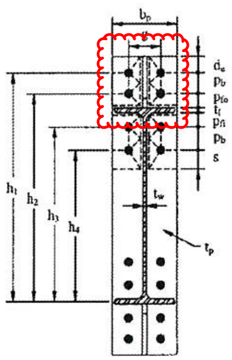
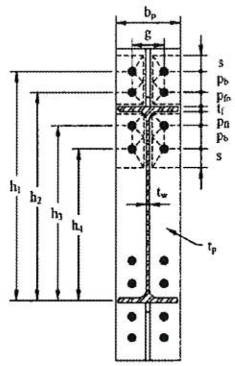
<tr><td>
</td><td>
</td></tr>
</table>

End Plate	$\phi M_{pl} = \phi_b F_{yp} t_p^2 Y_p$ **Case 1 ($d_e < s$)** $Y_p = \frac{b_p}{2}\left[h_1\left(\frac{1}{2d_e}\right) + h_2\left(\frac{1}{p_{fo}}\right) + h_3\left(\frac{1}{p_{fi}}\right) + h_4\left(\frac{1}{s}\right)\right] + \frac{2}{g}\left[h_1\left(d_e + \frac{p_b}{4}\right) + h_2\left(p_{fo} + \frac{3p_b}{4}\right) + h_3\left(p_{fi} + \frac{p_b}{4}\right) + h_4\left(s + \frac{3p_b}{4}\right) + p_b^2\right] + g$ **Case 2 ($d_e > s$)** $Y_p = \frac{b_p}{2}\left[h_1\left(\frac{1}{s}\right) + h_2\left(\frac{1}{p_{fo}}\right) + h_3\left(\frac{1}{p_{fi}}\right) + h_4\left(\frac{1}{s}\right)\right] + \frac{2}{g}\left[h_1\left(s + \frac{p_b}{4}\right) + h_2\left(p_{fo} + \frac{3p_b}{4}\right) + h_3\left(p_{fi} + \frac{p_b}{4}\right) + h_4\left(s + \frac{3p_b}{4}\right) + p_b^2\right] + g$ $s = \frac{1}{2}\sqrt{b_p\, g}$ $\phi_b = 0.90$ Note: If $p_{fi} > s$, use $p_{fi} = s$
Bolt Rupture	$\phi M_{np} = \phi\, 2\, P_t\, (h_1 + h_2 + h_3 + h_4)$ $\phi = 0.75$