# Calculating alpha

Bob

November 20, 2012

## 1 calculating angle of attack

During the meeting the idea to write a script for the calculation of the blade angle came to mind. While looking at the case information another way was discovered. What we want is to optimize the efficiency, for this we know the formula:

$$\eta = \frac{v_{kruis} * T}{\omega * Q} = \frac{N * 0.5 * v_{kruis} * V_R^2 * \rho * c * [C_L * \cos(\phi) - C_D * \sin(\phi)]}{\omega * r * N * 0.5 * V_R^2 * \rho * c * [C_L * \sin(\phi) + C_D * \cos(\phi)]} \tag{1}$$

While looking for a way simplify this we're going to make a few assumptions, at every part of the propeller we're taking the average, thus we assume the values are more or less equal to the average of that part and not radius dependent; furthermore we assume that RE is more or less the same and that $\alpha(angleofattack)$ ranges between small values.

$$\eta = \frac{v_{kruis} * T}{\omega * Q} = \frac{v_{kruis} * [C_L * \cos(\phi) - C_D * \sin(\phi)]}{\omega * r * [C_L * \sin(\phi) + C_D * \cos(\phi)]} \tag{2}$$

From several references the following relations can be found $C_L \propto \alpha$ otherwise said $C_L = K_0 * \alpha$ and for $C_D = K_1 + K_2 * \alpha^2$. These $K$ values are a combination of constants and these $K$ values could be determined by using propdesign. Setting propdesign to 2D and plotting $dT/dr$ against $x$, $dQ/dr$ against $x$ and $phi(target)$ against $x$. Then using different $\alpha's$ we could determine we could solve this as a set of equations since we know:

$$\eta = \frac{v_{kruis} * T}{\omega * Q} \tag{3}$$

Now if we differentiate our equation (2) and set this equal to zero we could determine our optimal angle of attack at certain values of $\phi$.

$$\frac{d\eta}{d\alpha} = \frac{(derivativeofnumerator) * denominator - (derivativeofdenominator) * numerator}{denominator^2} = 0 \tag{4}$$

$$0 = (derivativeofnumerator) * denominator - (derivativeofdenominator) * numerator \tag{5}$$

$$\begin{split}0 = v_{kruis} * [K_0 * \cos(phi) - (2 * K_2 * \alpha) * \sin(\phi)] * \omega * r * [K_0 * \alpha * \sin(\phi) + (K_1 + K_2 * \alpha^2) * \cos(\phi)] - \\ v_{kruis} * [K_0 * \alpha * \cos(phi) - (K_1 + K_2 * \alpha^2) * \sin(\phi)] * \omega * r * [K_0 * \sin(\phi) + (2 * K_2 * \alpha) * \cos(\phi)]\end{split} \tag{6}$$

$$\begin{split}0 = [K_0 * \cos(phi) - (2 * K_2 * \alpha) * \sin(\phi)] * [K_0 * \alpha * \sin(\phi) + (K_1 + K_2 * \alpha^2) * \cos(\phi)] - \\ [K_0 * \alpha * \cos(\phi) - (K_1 + K_2 * \alpha^2) * \sin(\phi)] * [K_0 * \sin(\phi) + (2 * K_2 * \alpha) * \cos(\phi)]\end{split} \tag{7}$$

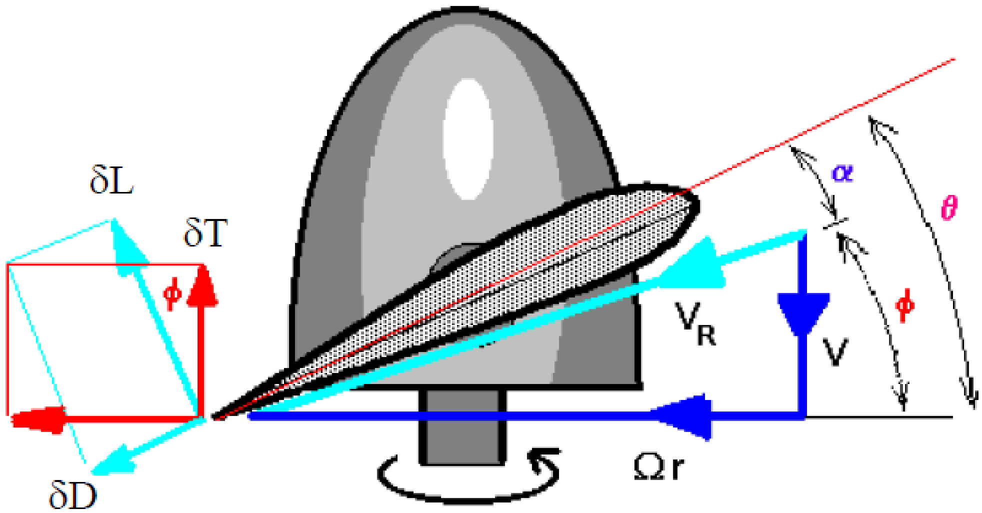


Figure 1: Propeller Sketch

$$0 = K_0^2*\alpha*\cos(\phi)*\sin(\phi)+K_0(K_1+K_2*\alpha^2)*\cos^2(\phi)-(2*K_2*K_0*\alpha^2)*\sin^2(\phi)-2*K_2*\alpha*(K_1+K_2*\alpha^2)*\cos(\phi)*\sin(\phi)-[K_0^2*\alpha*\cos(\phi)*\sin(\phi)+(2*K_2*K_0*\alpha^2)*\cos^2(\phi)-K_0*(K_1+K_2*\alpha^2)*\sin^2(\phi)-(2*K_2*\alpha)*(K_1+K_2*\alpha^2)*\cos(\phi)*\sin(\phi)] \quad (8)$$

$$0 = K_0(K_1 + K_2 * \alpha^2) * \cos^2(\phi) - (2 * K_2 * K_0 * \alpha^2) * \sin^2(\phi) - (2 * K_2 * K_0 * \alpha^2) * \cos^2(\phi) + K_0 * (K_1 + K_2 * \alpha^2) * \sin^2(\phi) \quad (9)$$

$$0 = K_0(K_1 + K_2 * \alpha^2) * [\cos^2(\phi) + \sin^2(\phi)] - (2 * K_2 * K_0 * \alpha^2) * [\cos^2(\phi) + \sin^2(\phi)] \quad (10)$$

$$0 = (K_1 + K_2 * \alpha^2) - (2 * K_2 * \alpha^2) \quad (11)$$

$$\alpha^2 = \frac{K_1}{K_2}, \alpha = \sqrt{\frac{K_1}{K_2}} \quad (12)$$

All in all keep in mind that the $\alpha$ and $\phi$ we have to use and can calculate are effective values and not the targeted values. For $\phi$ there's a simple formula so you can incorporate the rotation speed, but this is for the targeted value and thus useless. This means that these values are only true at a certain rotation speed and for a different rotation speed you have to do the whole process over again. Also to calculate the K values we need to use 3 different angles of attack, create a matrix and swipe it to calculate the answer. Furthermore $T$ and $Q$ have to be determined by using the graphs we earlier got from Propdesign. As you can see in the following image this is quite an exhausting process.

A last note the angles used, $\alpha$ and $\phi$ are used with their effective values and not their target values.

## References


[1] Wiki about aircraft engines `http://en.wikipedia.org/wiki/Model_airplane`

[2] Nasa about the lift coefficient of wings `http://www.grc.nasa.gov/WWW/k-12/airplane/incline.html`

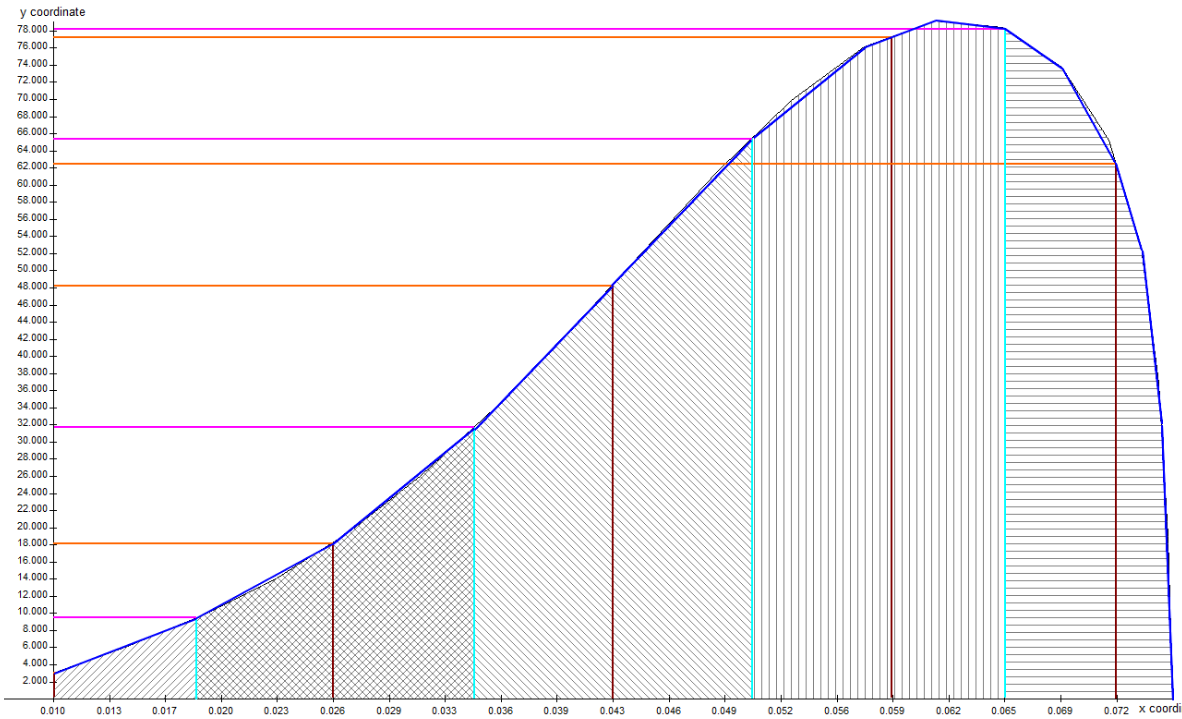


Figure 2: dT/dr tegen x

[3] Nasa about the drag coefficient of wings `http://www.grc.nasa.gov/WWW/k-12/airplane/induced.html`

[4] Journal of Aircraft `http://mysite.verizon.net/res77pzy/sitebuildercontent/sitebuilderfiles/smac_ja_10.pdf`