



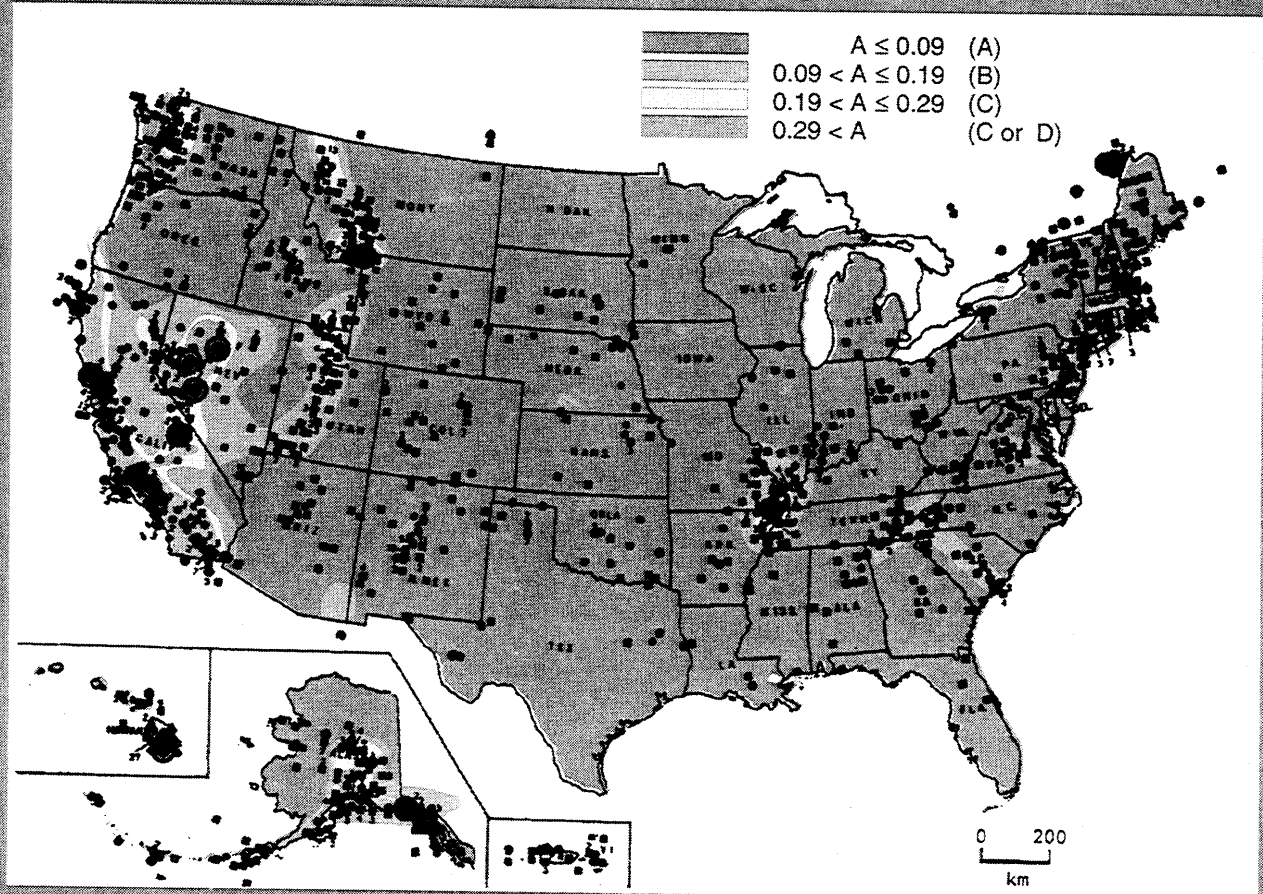
U.S. Department
of Transportation
Federal Highway
Administration

October 1996

Seismic Design of Bridges

Design Example No. 4

Three-Span Continuous CIP Concrete Bridge



Publication No. FHWA-SA-97-009



SEE NOTES ON PAGE 3 - 123

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| 16. Abstract This document describes one of seven seismic design examples that illustrate "how" to apply AASHTO's seismic analysis and design requirements on actual different bridge types across the United States. Each provides a complete set of "designer's notes" covering the seismic analysis, design, and details for that particular bridge including flow charts, references to applicable AASHTO requirements, and thorough commentary that explains each step. In addition, each example highlights separate issues (skew effects, wall piers, elastomeric bearings, pile foundations, etc.). The first example is a 242' reinforced concrete box girder two span overcrossing with spread footing foundations, SPC-C & A = 0.28g. The second example is a 400' 3-span skewed steel plate girder bridge over a river in New England with spread footing foundations, SPC-B & A = 0.15g. The third example is a skewed 70' single span prestressed concrete girder bridge with tall-closed seat-type abutments on spread footings, SPC-C & A = 0.36g. The fourth example is a 320' reinforced concrete box girder 3-span skewed bridge in the western United States with spread footing foundations, SPC-C & A = 0.30g. The fifth example is a 1488' steel plate girder bridge in the inland Pacific Northwest with pile foundations, SPC-B & A = 0.15g. It has nine spans and consists of two units: a four-span tangent (Unit 1) and a five-span with a 1300-foot radius curve (Unit 2). The sixth example is a 290' sharply curved (104 degrees) 3-span concrete box girder bridge in the Northwestern United States with pile abutment foundations and drilled shaft pier foundations, SPC-C & A = 0.20g. The seventh example is a 717' 10-span prestressed girder bridge with open pile bents, SPC-B & A = 0.10g. The superstructure consists of three continuous span units arranged in a 3-4-3 span series. | | | |
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Seismic Design Course

Design Example No. 4

Prepared for

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Federal Highway Administration
Central Federal Lands Highway Division**

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PLEASE NOTE

Data, specifications, suggested practices, and drawings presented herein are based on the best available information, are delineated in accord with recognized professional engineering principles and practices, and are provided for general information only. None of the procedures suggested or discussed should be used without first securing competent advice regarding their suitability for any given application.

This document was prepared with the help and advice of FHWA, State, academic, and private engineers. The intent of this document is to aid practicing engineers in the application of the AASHTO seismic design specification. BERGER/ABAM and the United States Government assume no liability for its contents or use thereof.

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**PURPOSE
OF DESIGN
EXAMPLE**

This is the fourth in a series of seismic design examples developed for the FHWA. A different bridge configuration is used in each example. The bridges are in either Seismic Performance Category B or C sites. Each example emphasizes different features that must be considered in the seismic analysis and design process. The matrix below is a summary of the features of the first seven examples.

| DESIGN EXAMPLE NO. | DESIGN EXAMPLE DESCRIPTION | SEISMIC CATEGORY | PLAN GEOMETRY | SUPER- STRUCTURE TYPE | PIER TYPE | ABUTMENT TYPE | FOUNDATION TYPE | CONNECTIONS AND JOINTS |
|--------------------------|--|---------------------|------------------------------|--|--|-----------------------------|---|--|
| 1 | Two-Span Continuous | SPC - C | Tangent Square | CIP Concrete Box | Three-Column Integral Bent | Seat Stub Base | Spread Footings | Monolithic Joint at Pier Expansion Bearing at Abutment |
| 2 | Three-Span Continuous | SPC - B | Tangent Skewed | Steel Girder | Wall Type Pier | Tall Seat | Spread Footings | Elastomeric Bearing Pads (Piers and Abutments) |
| 3 | Single-Span | SPC - C | Tangent Square | AASHTO Precast Concrete Girders | (N/A) | Tall Seat (Closed-In) | Spread Footings | Elastomeric Bearing Pads |
| 4 | Three-Span Continuous | SPC - C | Tangent Skewed | CIP Concrete | Two-Column Integral Bent | Seat | Spread Footings | Monolithic at Col. Tops Pinned Column at Base Expansion Bearings at Abutments |
| 5 | Nine-Span Viaduct with Four-Span and Five-Span Continuous Struts. | SPC - B | Curved Square | Steel Girder | Single-Column (Variable Heights) | Seat | Steel H-Piles | Conventional Steel Pins and PTFE Sliding Bearings |
| 6 | Three-Span Continuous | SPC - C | Sharply- Curved Square | CIP Concrete Box | Single Column | Monolithic | Drilled Shaft at Piers, Steel Piles at Abutments | Monolithic Concrete Joints |
| 7 | 12-Span Viaduct with (3) Four-Span Structures | SPC - B | Tangent Square | AASHTO Precast Concrete Girders | Pile Bents (Battered and Plumb) | Seat | Concrete Piles and Steel Piles | Pinned and Expansion Bearings |

**REFERENCE
AASHTO
SPECIFICATIONS**

The examples conform to the following specifications.

AASHTO Division I (herein referred to as “Division I”)

Standard Specifications for Highway Bridges, American Association of State Highway and Transportation Officials, Inc., 15th Edition, as amended by the Interim Specifications-Bridges-1993 through 1995.

AASHTO Division I-A (herein referred to as “Division I-A” or the “Specification”)

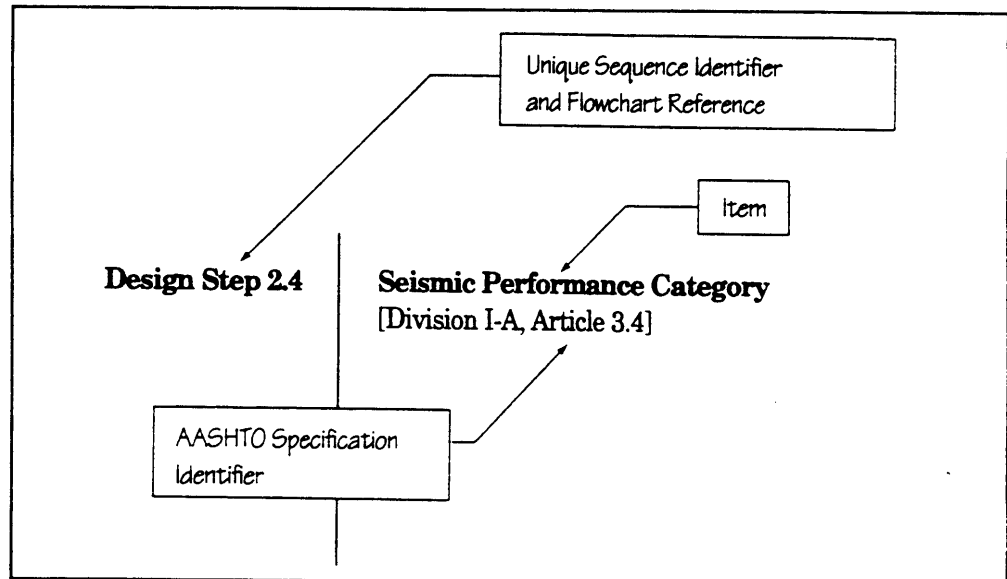
Standard Specifications for Highway Bridges, Division I-A, Seismic Design, American Association of State Highway and Transportation Officials, Inc., 15th Edition, as amended by the Interim Specifications-Bridges-1995.

**FLOWCHARTS
AND
DESIGN STEPS**

This fourth example follows the outline given in detailed flowcharts presented in Section II, Flowcharts. The flowcharts include a main chart, which generally follows the one currently used in AASHTO Division I-A, and several subcharts that detail the operations that occur for each Design Step.

The purpose of Design Steps is to present the information covered by the example in a logical and sequential manner that allows for easy referencing within the example itself. Each Design Step has a unique number in the left margin of the calculation document. The title is located to the right of the Design Step number. Where appropriate, a reference to either Division I or Division I-A of the AASHTO Specification follows the title.

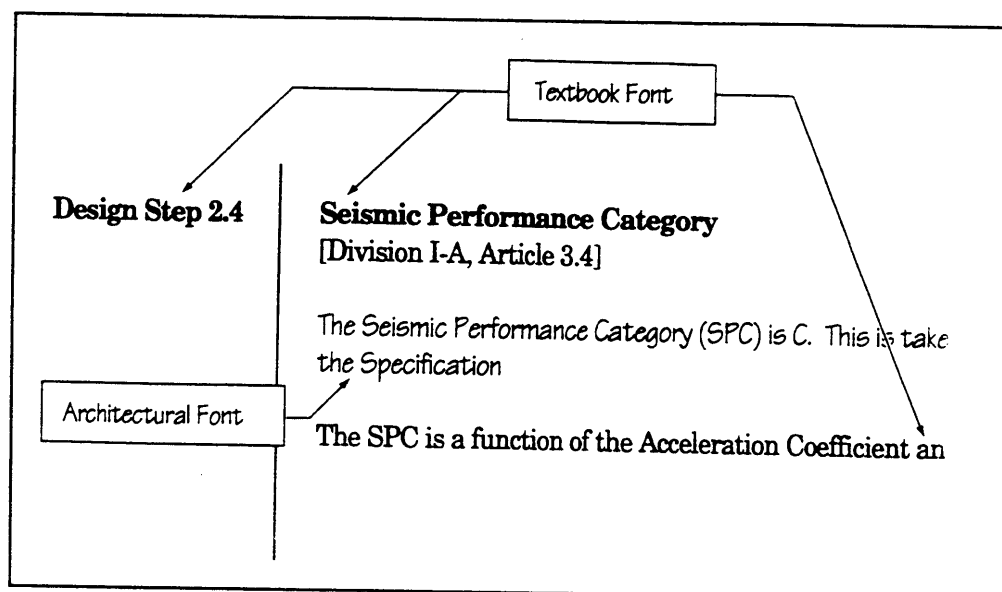
An example is shown below.



**USE OF
DIFFERENT
TYPE FONTS**

In the example, two primary type fonts have been used. One font, similar to the type used for textbooks, is used for all section headings and for commentary. The other, an architectural font that appears hand printed, is used for all primary calculations. The material in the architectural font is the essential calculation material and essential results.

An example of the use of the fonts is shown below.

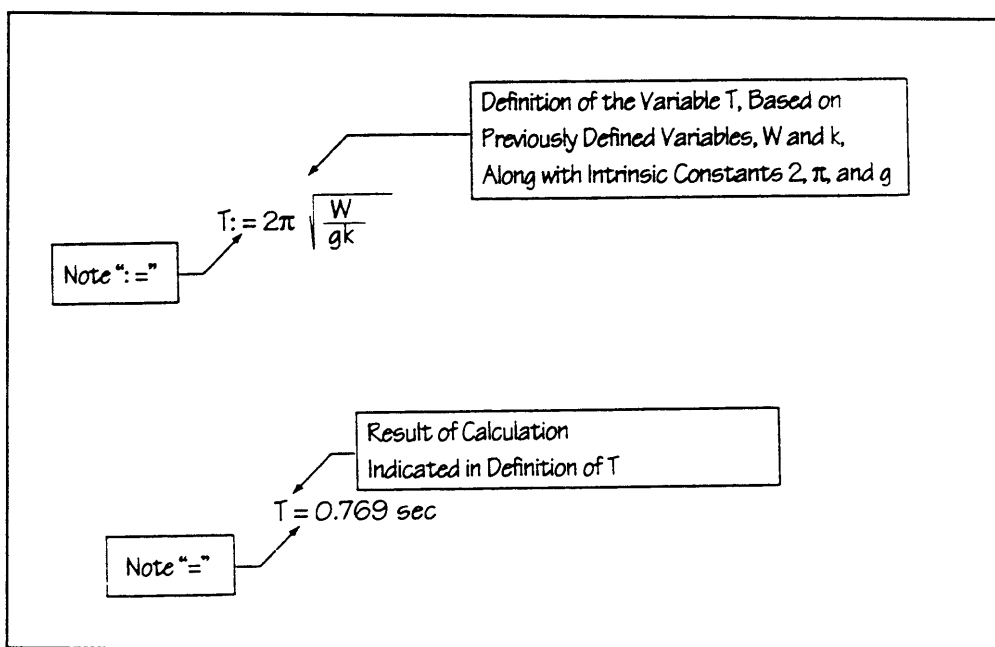


**USE OF
MATHCAD®**

To provide consistent results and quality control, all calculations have been performed using the program Mathcad®.

The variables used in equations calculated by the program are defined before the equation, and the **definition** of either a variable or an equation is distinguished by a ':= ' symbol. The **echo** of a variable or the result of a calculation is distinguished by a '=' symbol, i.e., no colon is used.

An example is shown below.



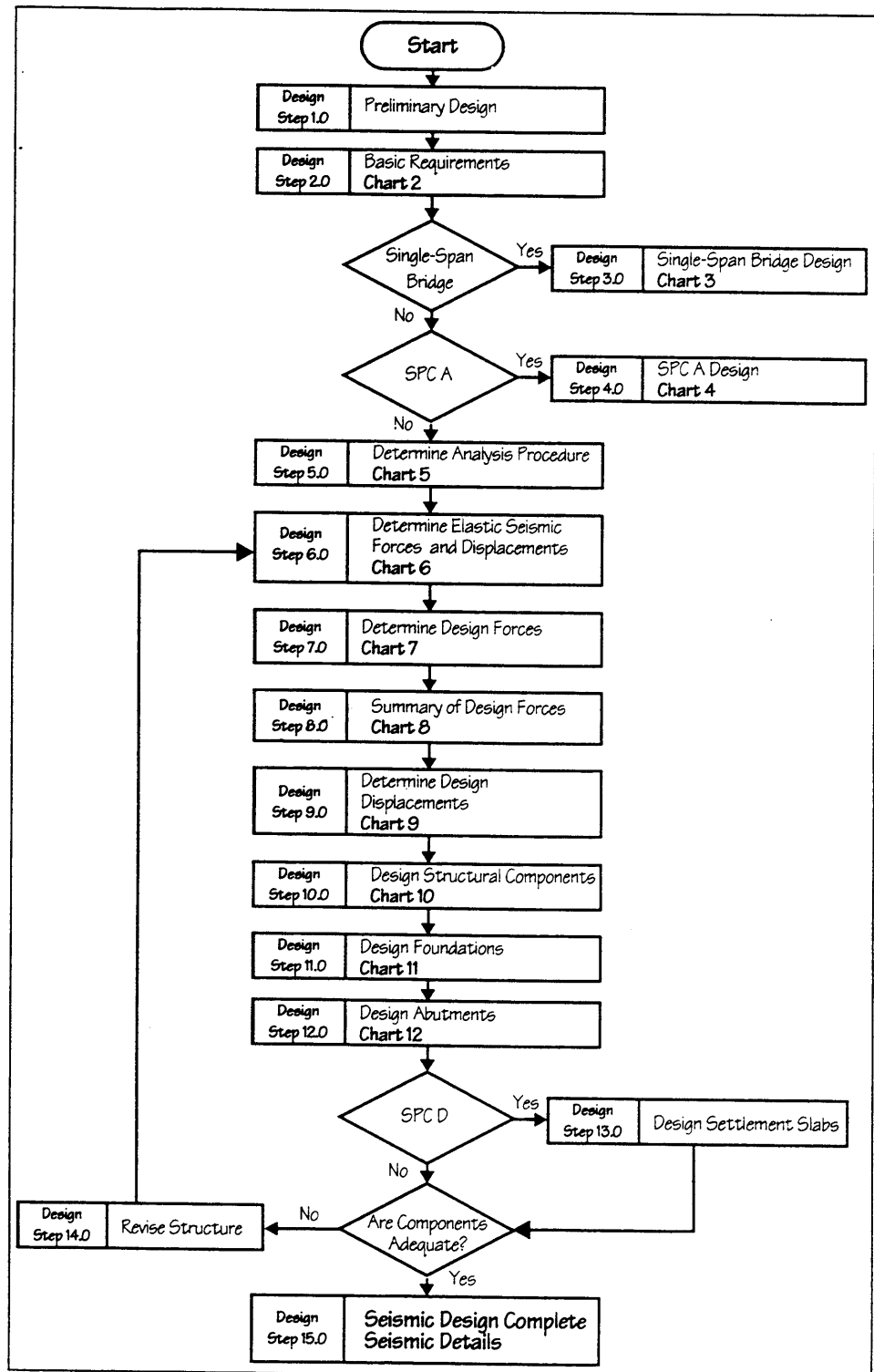
Note that Mathcad® carries the full precision of the variables throughout the calculations, even though the listed result of a calculation is rounded off. Thus, hand-calculated checks made using intermediate rounded results may not yield the same result as the number being checked.

Also, Mathcad® does not allow the superscript “ \cdot ” to be used in a variable name. Therefore, the specified compressive strength of concrete is defined as f_c in this example (not f^{\cdot}_c).

Section II

Flowcharts

FLOWCHARTS

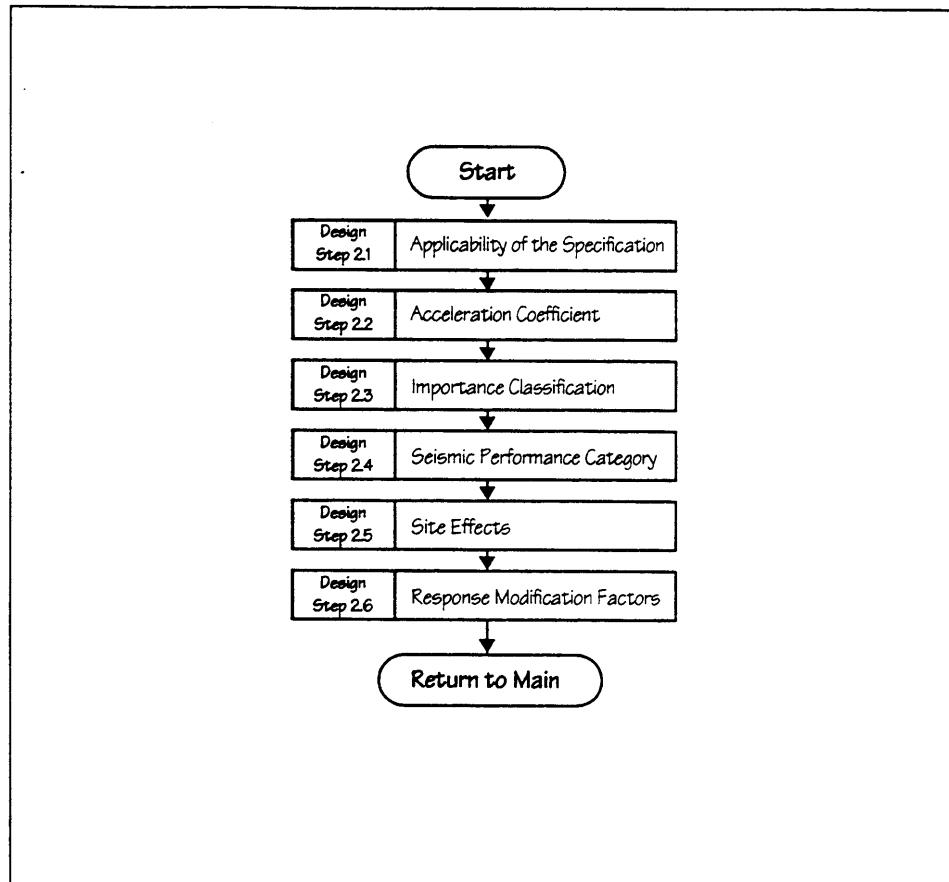


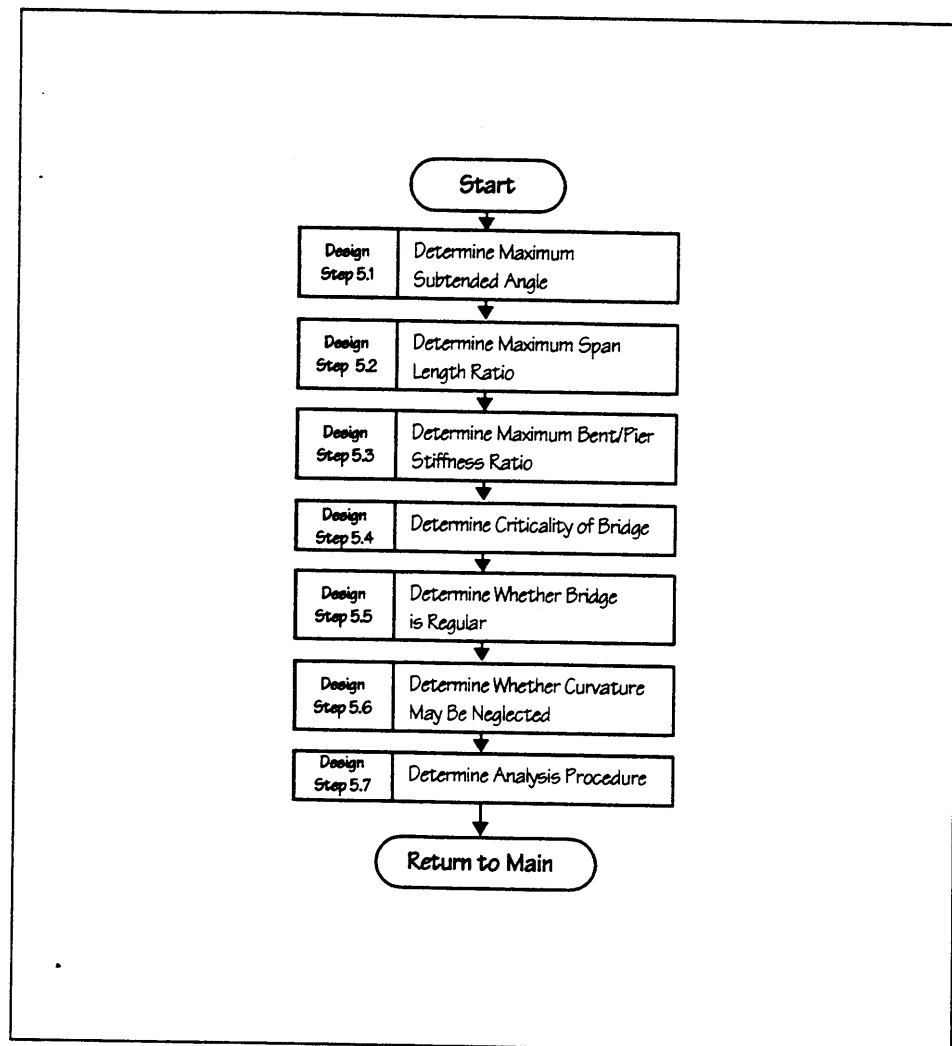
Main Flowchart — Seismic Design AASHTO Division I-A

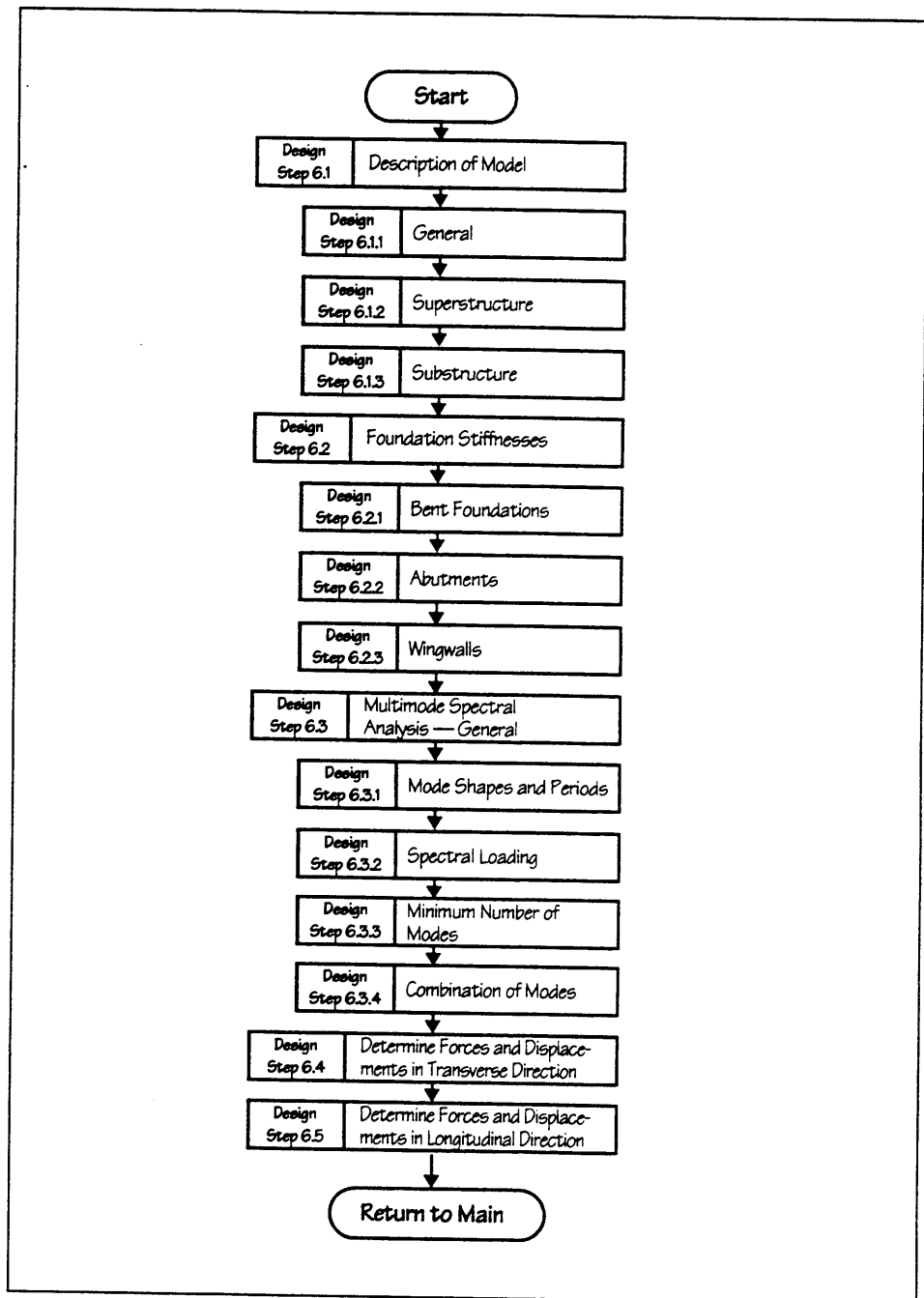
FLOWCHARTS
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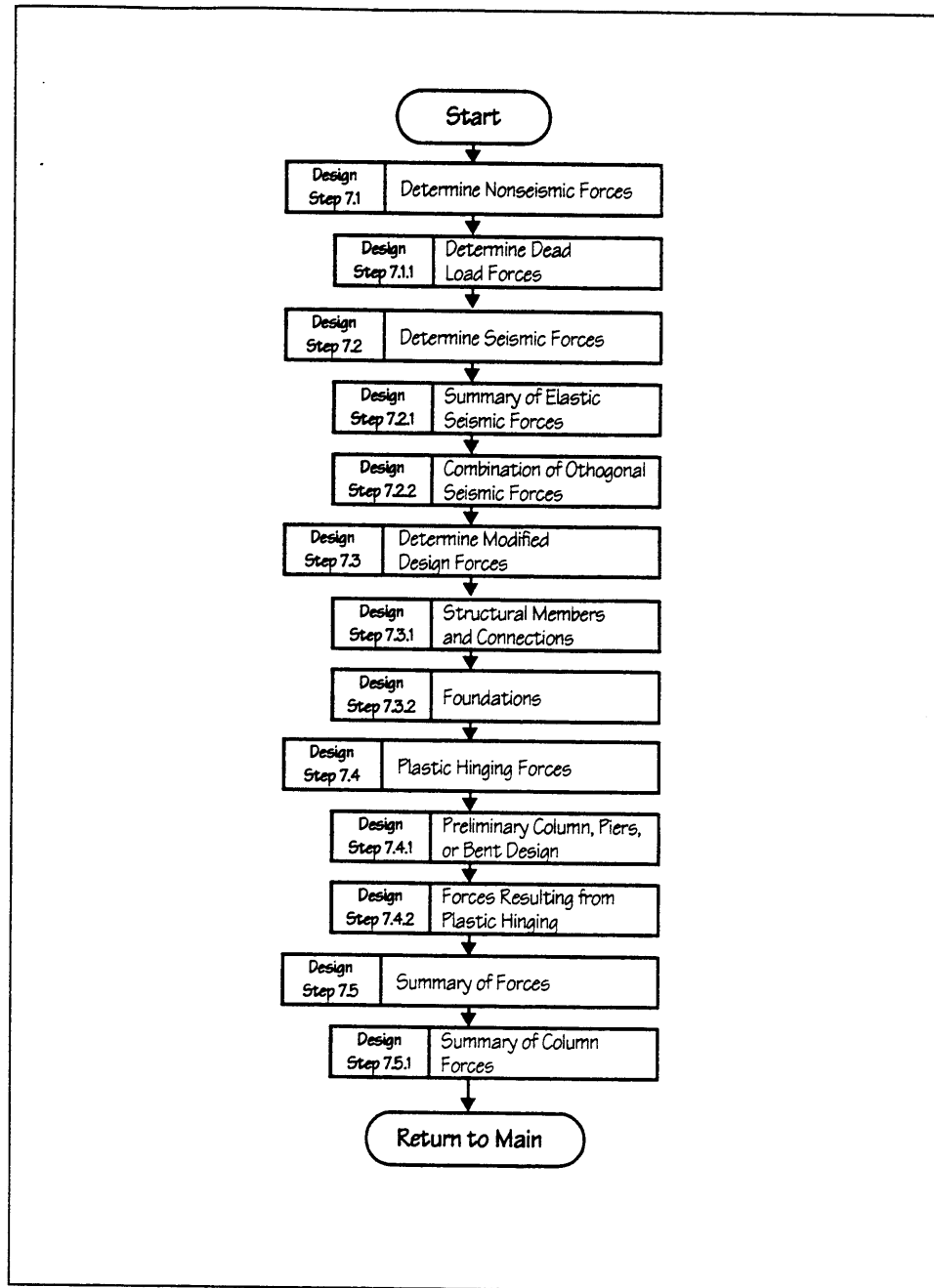
Key to Detailed Flowcharts

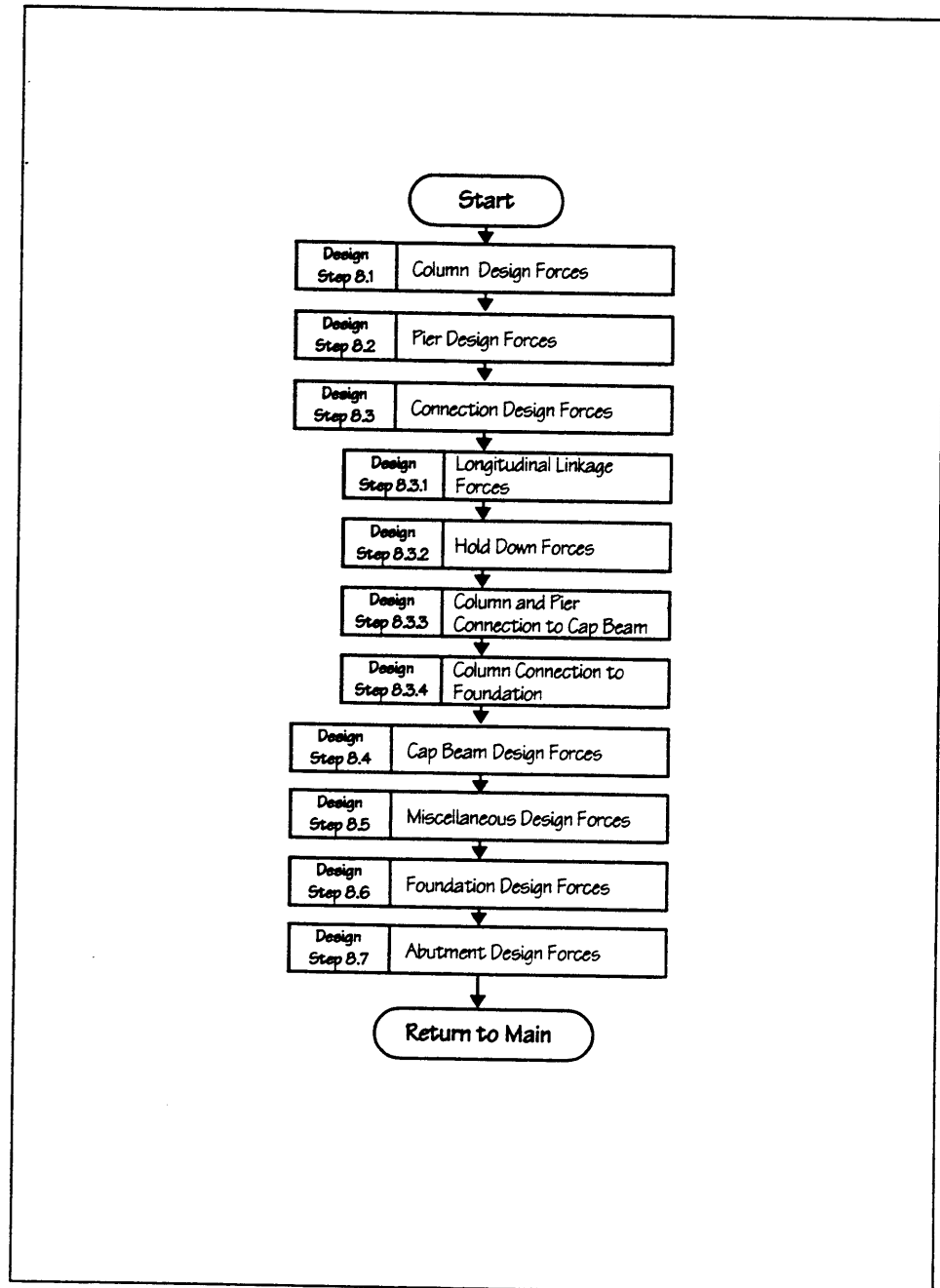
- Design Step 1.0 — Not Focused on in Example No. 4/Not Included
- Design Step 2.0 — Page 2-3
- Design Step 3.0 — Not Applicable for Example No. 4
- Design Step 4.0 — Not Applicable for Example No. 4
- Design Step 5.0 — Page 2-4
- Design Step 6.0 — Page 2-5
- Design Step 7.0 — Page 2-6
- Design Step 8.0 — Page 2-7
- Design Step 9.0 — Page 2-8
- Design Step 10.0 — Page 2-9
- Design Step 11.0 — Page 2-10
- Design Step 12.0 — Page 2-11
- Design Step 13.0 — Not Required for Example No. 4

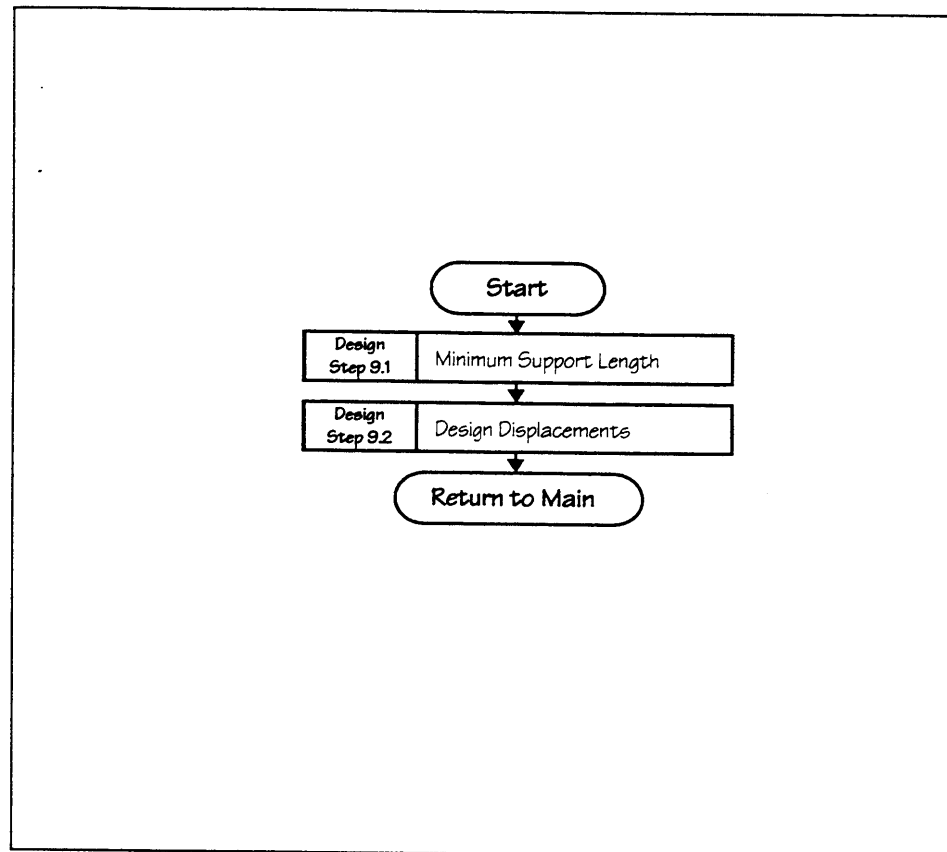
FLOWCHARTS
(continued)**Chart 2 — Basic Requirements**

FLOWCHARTS
(continued)**Chart 5 — Determine Analysis Procedure**

FLOWCHARTS
(continued)**Chart 6 — Determine Elastic Seismic Forces and Displacements**

FLOWCHARTS
(continued)**Chart 7 — Determine Design Forces**

FLOWCHARTS
(continued)**Chart 8 — Summary of Design Forces**

FLOWCHARTS
(continued)**Chart 9 — Determine Design Displacements**

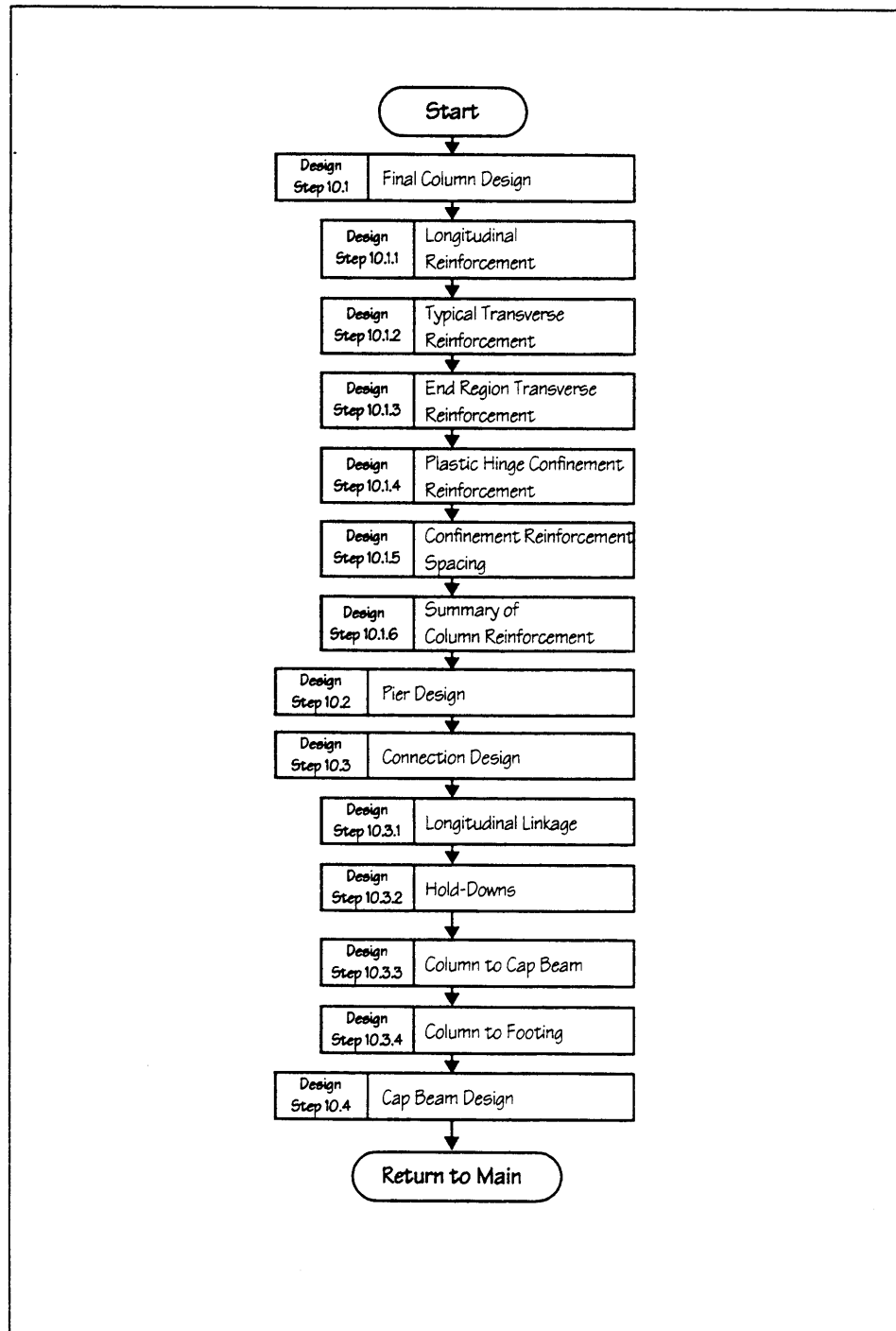
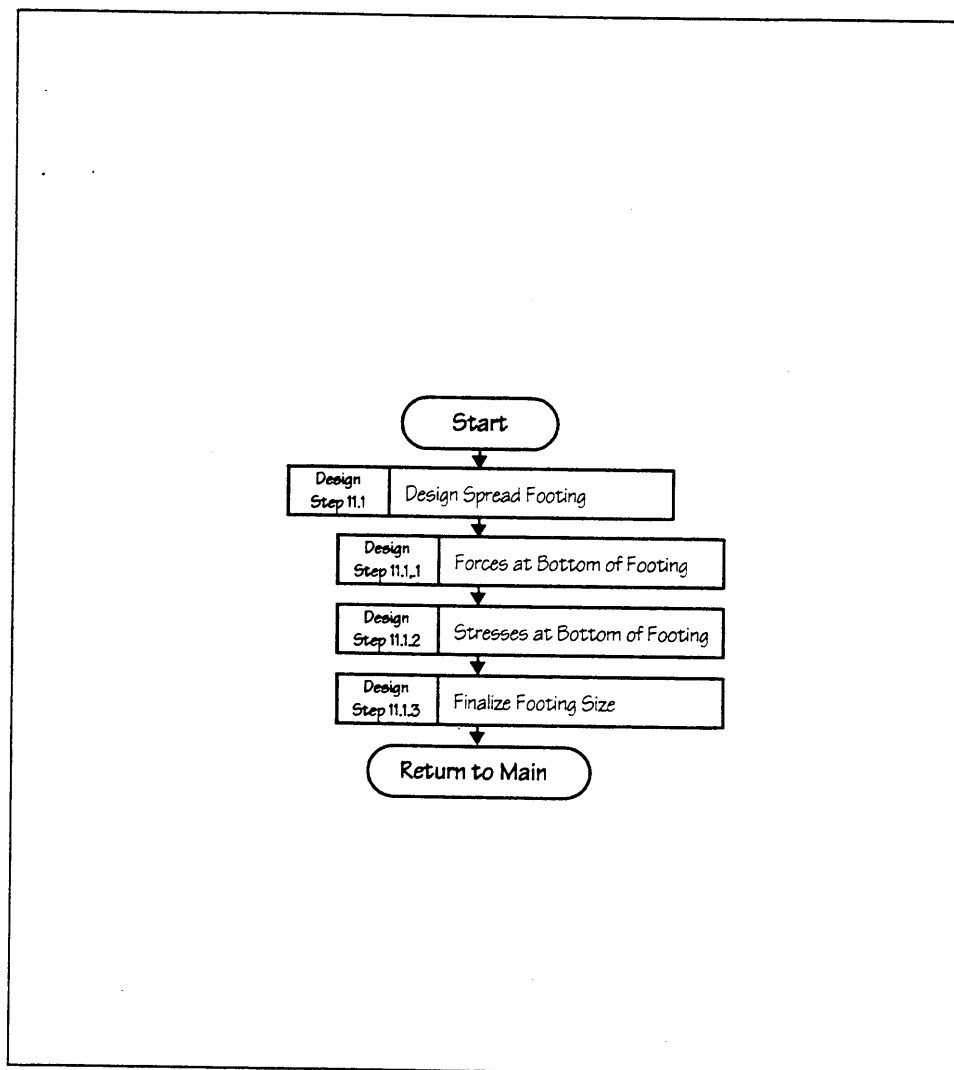
FLOWCHARTS
(continued)

Chart 10 — Design Structural Components

FLOWCHARTS
(continued)**Chart 11 — Design Foundations**

FLOWCHARTS
(continued)

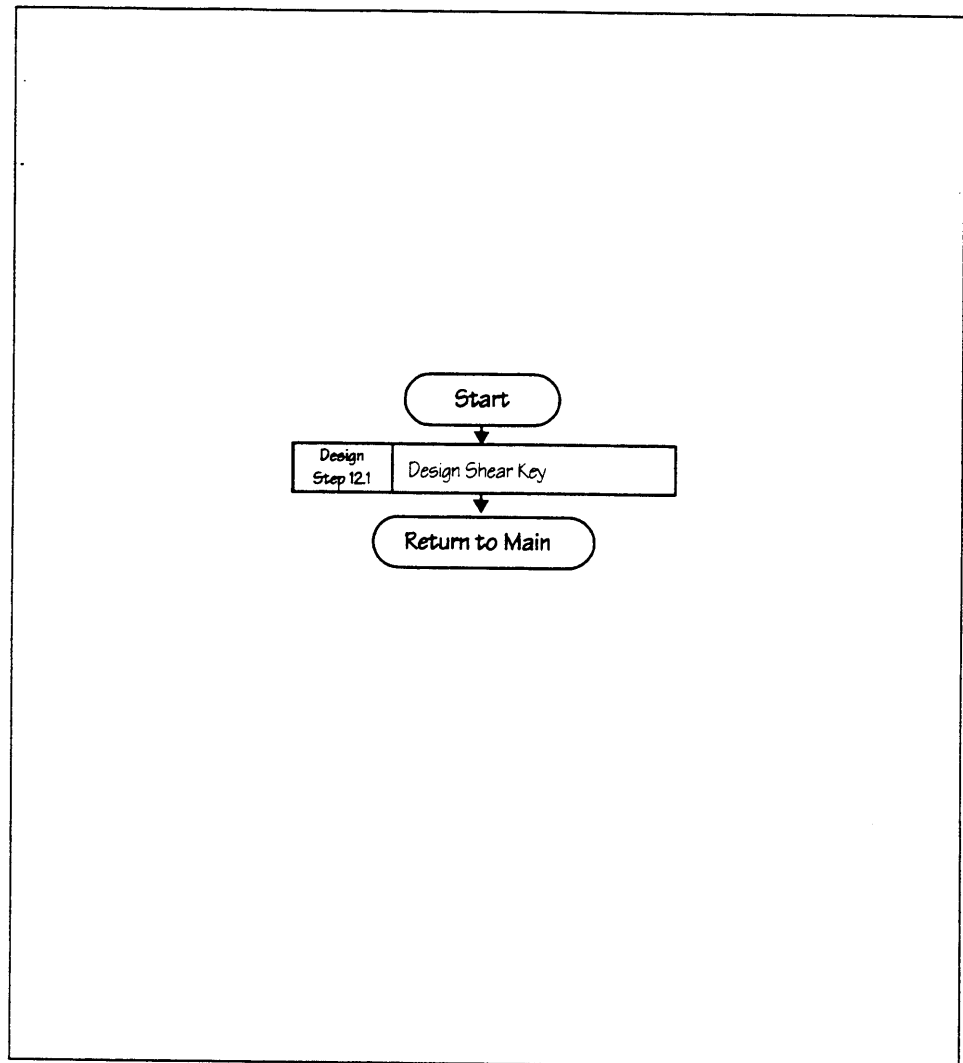


Chart 12 — Design Abutments

Section III
Analysis and Design

SECTION III

ANALYSIS AND DESIGN

DATA

The bridge is to be built in the western United States in a seismic zone with an acceleration coefficient of 0.30g. The subsurface conditions consist of a 250-foot-deep glacial deposit of dense sand and gravel overlying rock. Geotechnical information for the site is provided in Appendix A.

The three-span bridge is 320 feet long with spans of 100, 120, and 100 feet. All substructure elements are oriented at a 30-degree skew from a line perpendicular to a straight bridge centerline alignment. Figure 1a shows a plan and elevation of the bridge. The superstructure is a cast-in-place concrete box girder with two interior webs. The intermediate bents have a cross beam integral with the box girder and two round columns that are pinned at the top of spread footing foundations. Figure 1b shows a cross section through the bridge with an elevation of an intermediate bent. The seat-type abutments are on spread footings, as shown in Figure 1c, and the intermediate bents are all cast-in-place concrete. Framing of the box girder superstructure is shown in Figure 1d.

REQUIRED

Design the bridge for seismic loading, using the *Standard Specifications for Highway Bridges, Division I-A, Seismic Design*, American Association of State Highway and Transportation Officials, Inc., 15th Edition, as amended by the Interim Specification-Bridges-1995.

FEATURES

ISSUES EMPHASIZED FOR THIS EXAMPLE

- Skew Effects
- Foundation Springs for Spread Footings
- Two-Column Bent Behavior
- Pinned Base Column Design

BRIDGE DATA
(continued)

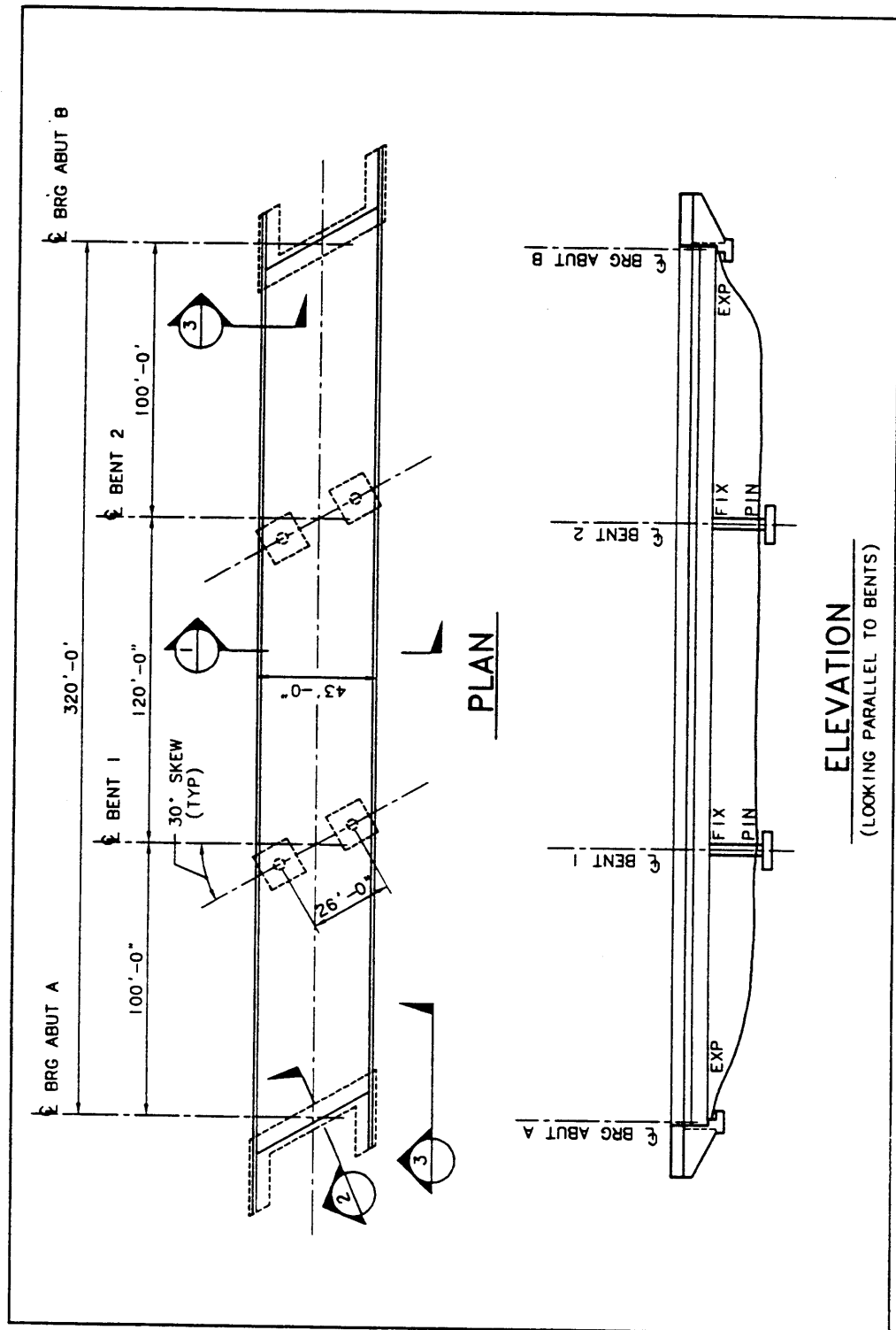


Figure 1a — Bridge No. 4 - Plan and Elevation

BRIDGE DATA
(continued)

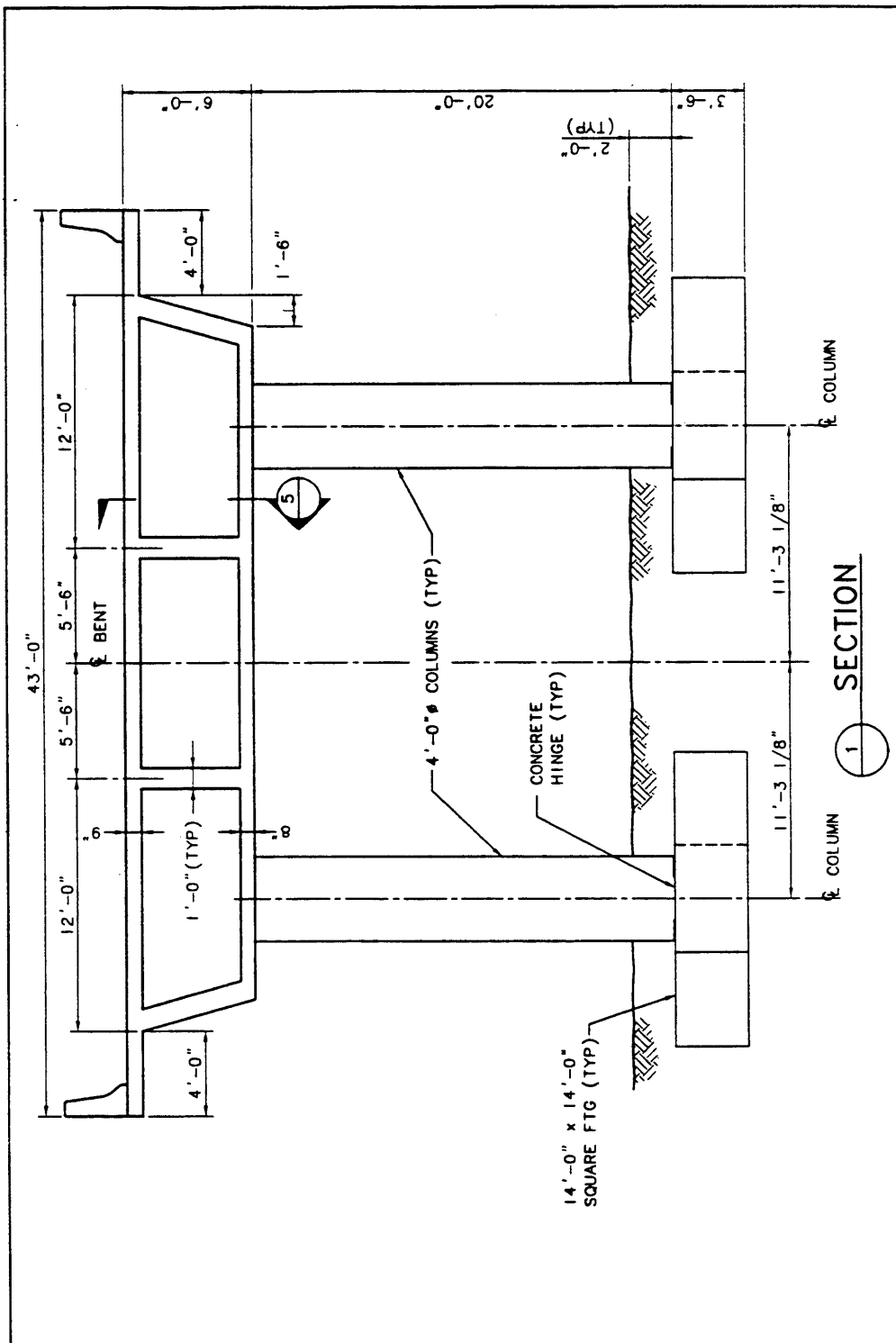


Figure 1b — Bridge No. 4 - Typical Cross Section

BRIDGE DATA
(continued)

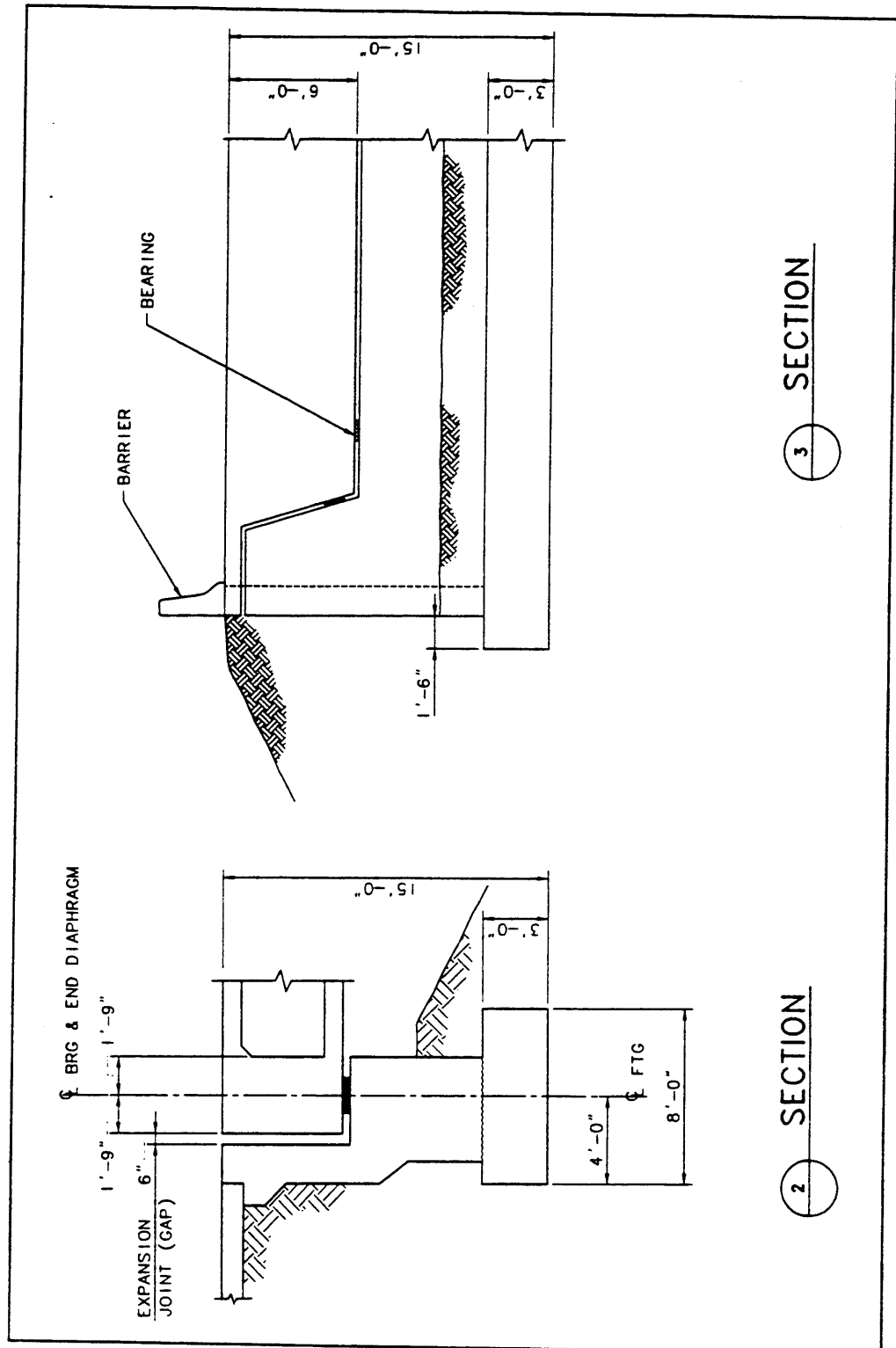


Figure 1c — Bridge No. 4 - Seat-Type Abutment

BRIDGE DATA
(continued)

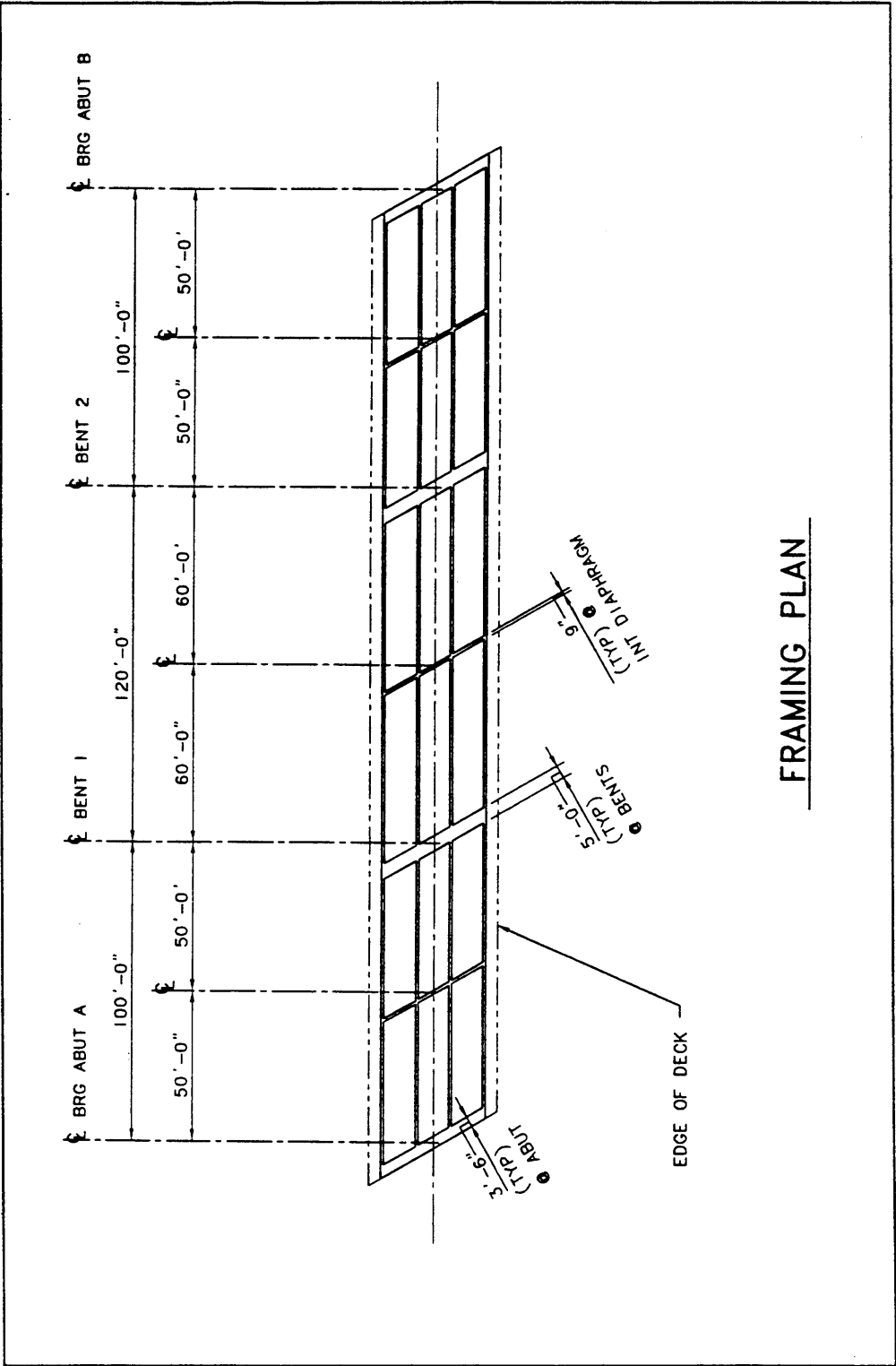


Figure 1d — Box Girder Framing Plan

SOLUTION**DESIGN STEP 1****PRELIMINARY DESIGN**

A static load design (live and dead loads) and a preliminary seismic design of the bridge have been completed. The initial configuration of the superstructure and preliminary sizes of substructure elements are as shown in Figure 1 (a to d).

The initial iterative process, which resulted in the sizes of the bent columns and footings, is not shown in this example. However, the assumed seismic behavior of the structure used for preliminary design is described below.

Pinned base column design is a feature of this example and is addressed in Design Step 10.3.4. For preliminary design, the bases of the bent columns are considered pinned in both the transverse and longitudinal directions. The moments of inertia of the structural elements are assumed to be identical to that of the full uncracked cross section.

In the longitudinal direction, the intermediate bent columns are assumed to resist the entire longitudinal seismic force. The seat-type abutments will allow free longitudinal movement of the superstructure and will not provide longitudinal restraint. This behavior is illustrated in Figure 2.

In the transverse direction, the superstructure is assumed to act as a simply supported beam spanning laterally between the abutments with the maximum transverse displacement at the center of the middle span. This behavior is illustrated in Figure 3. The intermediate bents and the abutments are assumed to participate in resisting the transverse seismic force along with the superstructure. Effects of the structure's skew are ignored for preliminary design.

At the abutments, several types of bearings may be used to accommodate the expected longitudinal displacements from expansion and contraction. Elastomeric bearings, such as PTFE bearings against a sliding surface, with provision for sliding between the bearing and end diaphragm under large displacements, will work. The transverse restraint will be provided by a girder stop or shear key to enable transfer of transverse seismic forces to the abutment.

DESIGN STEP 1
(continued)

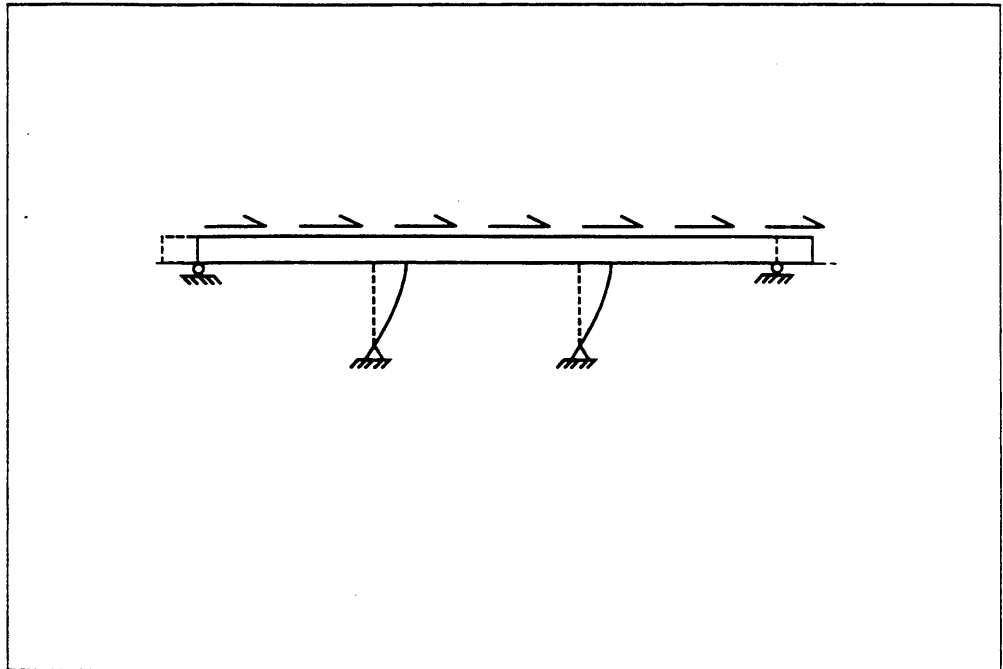


Figure 2 — Longitudinal Seismic Behavior

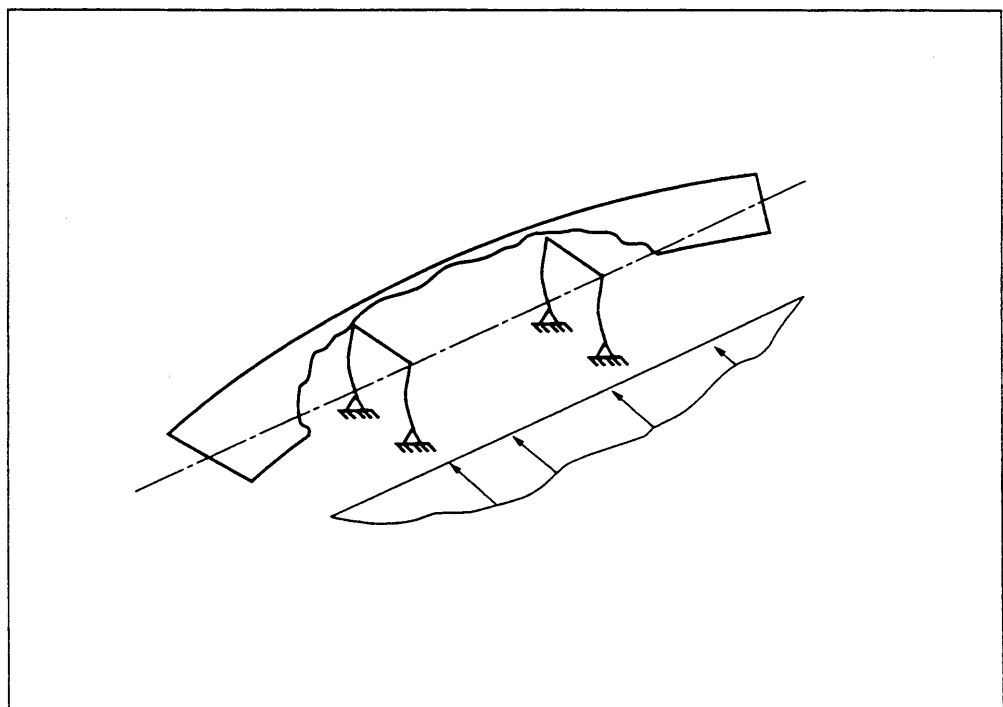


Figure 3 — Transverse Seismic Behavior

DESIGN STEP 2**BASIC REQUIREMENTS****Design Step
2.1****Applicability of Specification**

[Division I-A, Article 3.1]

The bridge has three spans that total 320 feet and is a cast-in-place concrete box girder with a reinforced concrete substructure. Thus, the Specification applies.

**Design Step
2.2****Acceleration Coefficient**

[Division I-A, Article 3.2]

For this example, the Acceleration Coefficient (A) is 0.30 (normally taken from Figure 3 of the Specification).

A site investigation by a qualified geotechnical engineer or seismic hazard assessment specialist may be used to develop more accurate acceleration data. Such an investigation is required if the structure is near an active fault, if long-duration earthquakes are expected, or if design for a long return period is required due to great importance of the structure. In addition, some agencies may require acceleration coefficients different from those given in the AASHTO Specification.

**Design Step
2.3****Importance Classification**

[Division I-A, Article 3.3]

The Importance Classification (IC) of this bridge is taken to be II. It is assumed not to be essential for use following an earthquake.

**Design Step
2.4****Seismic Performance Category**

[Division I-A, Article 3.4]

The Seismic Performance Category (SPC) is C, as taken from Table 1 of the Specification.

The SPC is a function of both the Acceleration Coefficient and the Importance Classification.

**Design Step
2.5****Site Effects**

[Division I-A, Article 3.5]

The site conditions affect the design through a coefficient based on the soil profile. In this case, SOIL PROFILE TYPE II is used, since it corresponds to stable deposits of sands and gravels over 200 feet deep.

**Design Step
2.5**
(continued)

The Site Coefficient (S) for this type of soil is 1.2, per Table 2 of the Specification.

A geotechnical investigation may be made by qualified professionals to establish site-specific seismic response information (e.g., site-specific response spectra). This investigation is typically done on a site-by-site basis. In some cases, State Departments of Transportation (DOTs) have developed representative spectra for soil types and seismic hazards in their jurisdictions. These spectra are then used in lieu of the information in Article 3.5. Lacking such specific information, the structural engineer should decide whether to have site-specific information generated or use the approach described in this section.

**Design Step
2.6**

Response Modification Factors
[Division I-A, Article 3.7]

Since this bridge is classified as SPC C, appropriate Response Modification Factors (R Factors) must be selected for later use in establishing appropriate design force levels.

In this case, Table 3 of the Specification gives the following R Factors.

- | | |
|-----------|---|
| $R = 5$ | For the substructure, since multiple-column bents are used |
| $R = 0.8$ | For the superstructure to abutment connection (bearings and girder stops) |
| $R = 1.0$ | For the connection of the bent columns to the cap beam or superstructure, and the connection of the bent columns to the foundations |

These factors will be used to ensure that inelastic effects are restricted to elements that can be designed to provide reliable, ductile response that can be inspected after an earthquake to assess damage, and that can be repaired relatively easily. The foundations do not fit this constraint, and thus will be designed not to experience inelastic effects. For bridges classified as SPC C or D, it is recommended that the connections of the bent columns to the superstructure and foundation be designed for the maximum forces capable of being developed by plastic hinging of the bent column. These forces will often be significantly less than those obtained using an R Factor of 1. In this example, the pinned column base has a relatively small plastic moment capacity that limits the forces capable of being transmitted to the foundation, allowing the footing size to be considerably smaller than if a fixed column base were used.

DESIGN STEP 3

SINGLE-SPAN BRIDGE DESIGN

Not applicable.

DESIGN STEP 4

SEISMIC PERFORMANCE CATEGORY A DESIGN

Not applicable.

DESIGN STEP 5**DETERMINE ANALYSIS PROCEDURE****Design Step
5.1****Determine Maximum Subtended Angle**
[Division I-A, Article 4.2]

The bridge is not curved in the horizontal plane.

**Design Step
5.2****Determine Maximum Span Length Ratio**
[Division I-A, Article 4.2]

The maximum span length ratio is $1.20 = 120 \text{ ft}/100 \text{ ft}$.

**Design Step
5.3****Determine Maximum Bent/Pier Stiffness Ratio**
[Division I-A, Article 4.2]

There are two identical intermediate bents; thus, the maximum bent/pier stiffness ratio is 1.0.

**Design Step
5.4****Critical Bridge**
[Division I-A, Article 4.2.3]

Assume that the bridge is not critical.

If the bridge is large, expensive, required to be functional immediately following the design earthquake, or complex geometrically, then the Specification recommends that the Time-History Method (Division I-A Procedure 4) be used to analyze the structure.

**Design Step
5.5****Regular Bridge**
[Division I-A, Article 4.2]

Table 5 of the Specification gives the requirements for determining whether a bridge is regular. The requirements are based on limiting values of the parameters determined in the steps above.

Because there is no curve, the span length ratio is less than 2, and the maximum bent/pier stiffness ratio is less than 4. The bridge is regular.

**Design Step
5.6****Curved Bridge**
[Division I-A, Article 4.2.2]

Not applicable; there is no curvature.

**Design Step
5.7****Analysis Procedure**

[Division I-A, Article 4.2]

Because this is not a single-span bridge or an SPC A bridge, the analysis requirements of Article 4 must be satisfied. Table 4 of the Specification is used to select the minimum analysis requirements.

From Table 4 of the Specification, *Minimum Analysis Requirements*, either the Uniform Load Method (Procedure 1) or the Single-Mode Spectral Method (Procedure 2) may be used to analyze this structure since it has less than six spans.

These methods represent the minimum analysis requirements that must be satisfied; the Multimode Spectral Method (Procedure 3) or the Time-History Method (Procedure 4) may be used in lieu of Procedures 1 and 2.

For this example, Procedure 3 is used for the analysis because performing this analysis by computer is just as easy as by other means, yet is more rigorous and can more closely model the stiffness and response of the structure.

DESIGN STEP 6

Design Step 6.1

Design Step 6.1.1

DETERMINE ELASTIC SEISMIC FORCES AND DISPLACEMENTS

Description of Model

[Division I-A, Article 4.5.2]

General

The structural analysis program SAP90 Version BETA 6.00 (CSI, 1994) was used for the analyses. The model used is shown in Figure 4, and includes a single line of frame elements for the superstructure and individual elements for the cap beam and columns of the intermediate bents. A copy of the SAP90 input file for the analyses is provided in Appendix B.

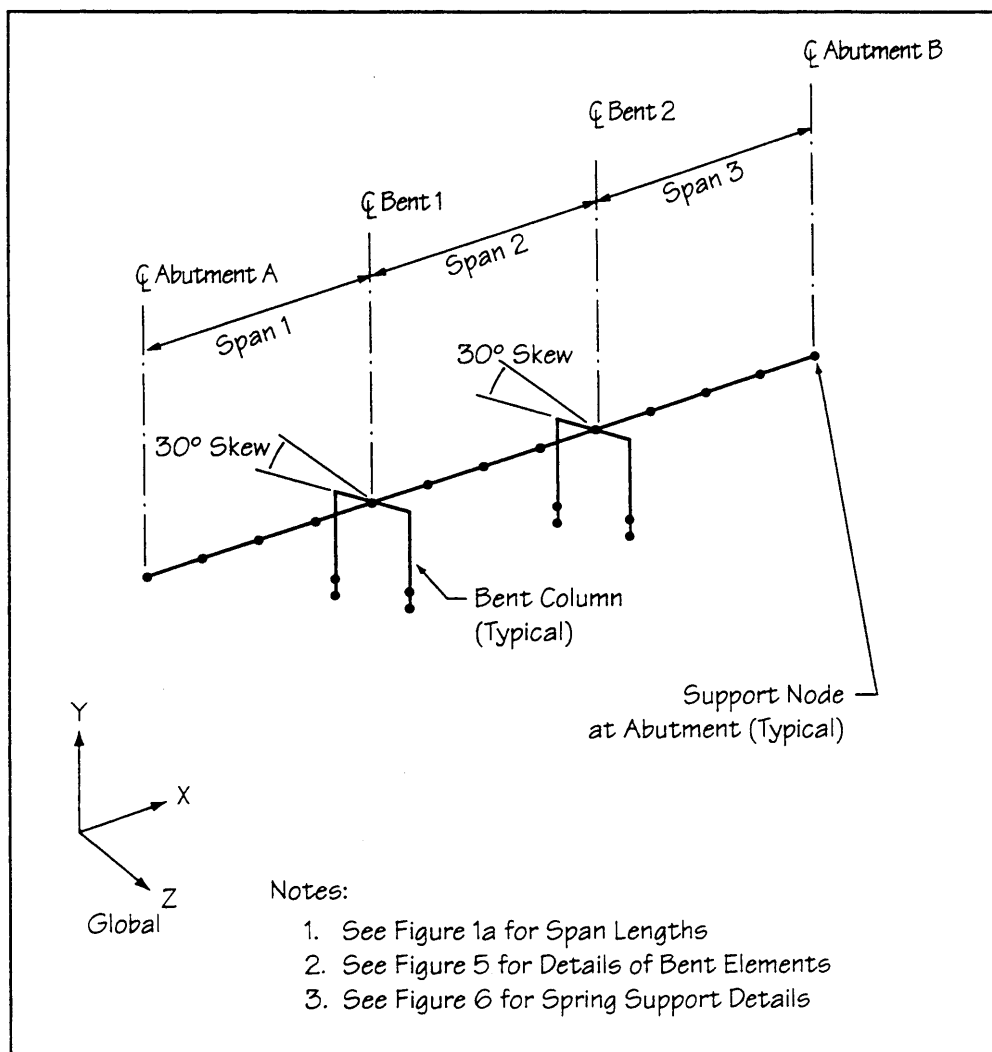


Figure 4 — Structural Model of Bridge

**Design Step
6.1.2**

Superstructure

The superstructure has been modeled with four elements per span and the work lines of the elements are located along the centroid of the superstructure.

The properties of the elements used for the model are for the structure configuration shown in Design Step 1, Preliminary Design. The superstructure density used for the modal analysis has been adjusted to include additional dead loads from traffic barriers and wearing surface overlay. The total weight of these additional dead loads is 2.35 kips per lineal foot of superstructure. The properties of the structure used in the seismic model (both superstructure and substructure) are shown in Table 1.

As shown in Figure 4, the superstructure has been collapsed into a single line of 3-D frame elements which follow the horizontal geometry of the bridge centerline. This “stick” model is used solely for the determination of seismic forces for this example. Such a model does not give exactly the same forces for other loadings (for instance, dead loads) since the weight of the superstructure is not distributed uniformly across the cap beam. However, because weight or mass is an important parameter in dynamic analysis, the total weight of the structure should be close to that obtained from an accurate dead load analysis or check.

Enough nodes must be used along the length of the superstructure to accurately characterize the response and forces bearing in mind that SAP90 and most other programs lump mass at the nodes. For a bridge, such as this one, with uniform cross section and a straight alignment, nodes at the quarter points are sufficient. Determination of moments of inertia and torsional stiffness of the superstructure is based on uncracked cross-sectional properties.

The presence of the skew is accounted for only in the orientation of the substructure elements, and is not considered in determination of the superstructure properties.

Design Step
6.1.2
(continued)

**Table 1
Section Properties for Model**

| | Model Element | | |
|--|---------------------------|------------------|------------------------|
| | CIP Box Superstructure | Bent Cap Beam | Bent Columns (Each) |
| Area (ft ²) | 72.74 | 27.00 | 12.57 |
| I _x - Torsion (ft ⁴) | 1,177 | 100,000 (1) | 25.13 |
| I _y (ft ⁴) | 9,697 | 100,000 (2) | 12.57 |
| I _z (ft ⁴) | 401 | 100,000 (3) | 12.57 |
| Density (lb / ft ³) | 182 | 150 | 150 |

(1) This value has been increased for force distribution to bent columns.

Actual value is I_x = 139 ft⁴.

(2) This value has been increased for force distribution to bent columns.

Actual value is I_y = 90 ft⁴.

(3) This value has been increased for force distribution to bent columns.

Actual value is I_z = 63 ft⁴.

Design Step
6.1.3

Substructure

The bents and abutments are skewed 30 degrees; therefore, the properties of the bent elements are rotated in the model to properly account for the skew. (There are no elements to model the abutments, only support nodes as shown in Figure 4). The bents are modeled with 3-D frame elements that represent the cap beam and individual columns. Figure 5 shows the relationship between the actual bent and the "stick" model of 3-D frame elements. A single element was used for each column between the top of footing and the soffit of the box girder superstructure. The connection of the column top at the soffit of the box girder to the center of gravity of the cap (at the superstructure centroid) beam is made with rigid link elements. The node at the top of the footing (4xx) is released for rotation in both plan directions to model the pinned column base. Foundation springs are connected to the node (3xx) at

Design Step
6.1.3
(continued)

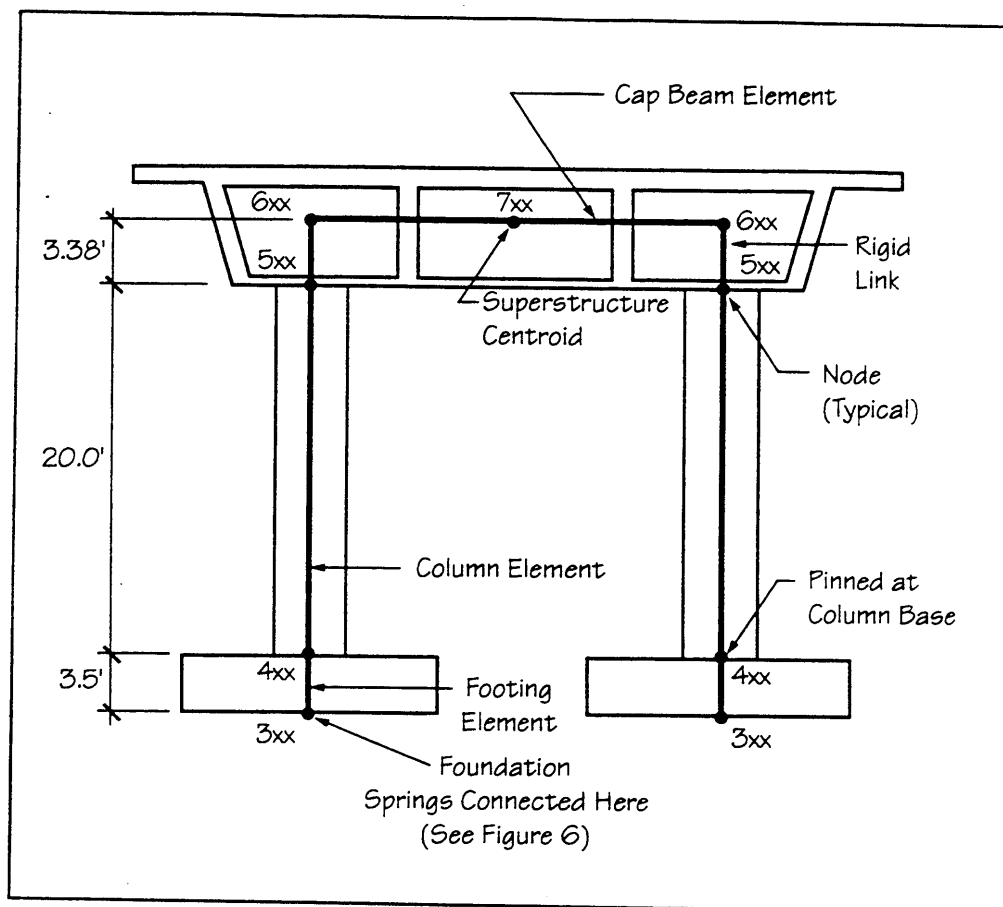


Figure 5 — Details of Bent Elements

the base of the footing. For this model, the moments of inertia and torsional properties of the columns are based on uncracked sections.

In the actual structure, internal forces are transferred between the superstructure and the bent almost uniformly along the cap beam. In the seismic model, the superstructure forces are transferred at the single point where the superstructure and bent intersect. Therefore, in the seismic model the forces in the cap beam are not representative of actual forces, and the distribution of forces to the columns may not be accurate. For this example, the torsional stiffness and moments of inertia of the model's cap beams were increased in order to provide a more representative distribution of forces to the columns. These adjusted properties are shown in Table 1, along with the actual calculated properties.

In some cases, modeling could be simplified by eliminating the footing elements and placing the foundation springs at the bottom of the

**Design Step
6.1.3
(continued)**

column. However, in this example since the column base is modeled as a pinned connection, rotational releases for the pin would be incompatible with application of the foundation springs for the spread footing. The determination of foundation spring stiffnesses to model the foundations is discussed in Design Step 6.2.

The use of uncracked versus cracked section properties for column elements is a subject of ongoing debate within the bridge design community. The use of gross or uncracked moments of inertia for reinforced concrete elements with small axial loads results in a stiffer model of the structure, which would have smaller displacements. However, during seismic events, cracking along the height of the column would occur and would reduce the stiffness from the gross value to some effective stiffness value, resulting in larger displacements of the structure. Values for effective bridge column moments of inertia related to axial load and reinforcing percentages have been developed by Priestley, Seible, and Chai (1992); and FHWA, *Seismic Retrofitting Manual* (1995) recommends their use in evaluating structure displacements.

Some groups advocate using the uncracked moments of inertia to determine internal forces and reactions, then using the effective or reduced stiffness to determine maximum displacements. For this example, the uncracked stiffnesses of the columns are used, and the footing elements are also modeled with uncracked section properties.

**Design Step
6.2**

Foundation Stiffnesses

**Design Step
6.2.1**

Bent Foundations

The intermediate bent foundations were modeled with equivalent spring stiffnesses for the spread footing. Figure 6 shows details of the spring supports. For this example, all of the intermediate bent footings use the same foundation springs. The spring stiffnesses are developed for the local bent support coordinate geometry, and are input into the SAP90 model with the same orientation as the local bent columns.

For a program that can only accommodate global directions for spring releases, the stiffnesses computed here would require some transformation from local to global coordinate geometry for input into the model.

Establishing meaningful soil stiffnesses for bridge foundations is a complex problem which is often simplified to linear springs for static or modal analyses. There are several methods available for establishing

Design Step
6.2.1
(continued)

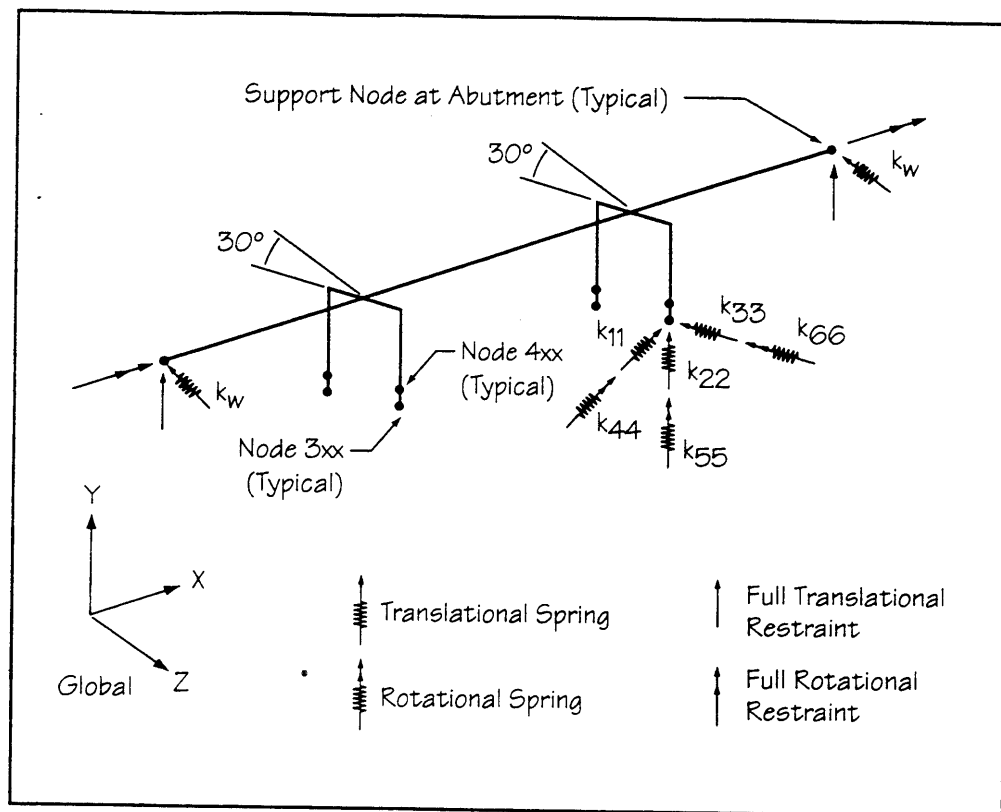


Figure 6 — Details of Spring Supports

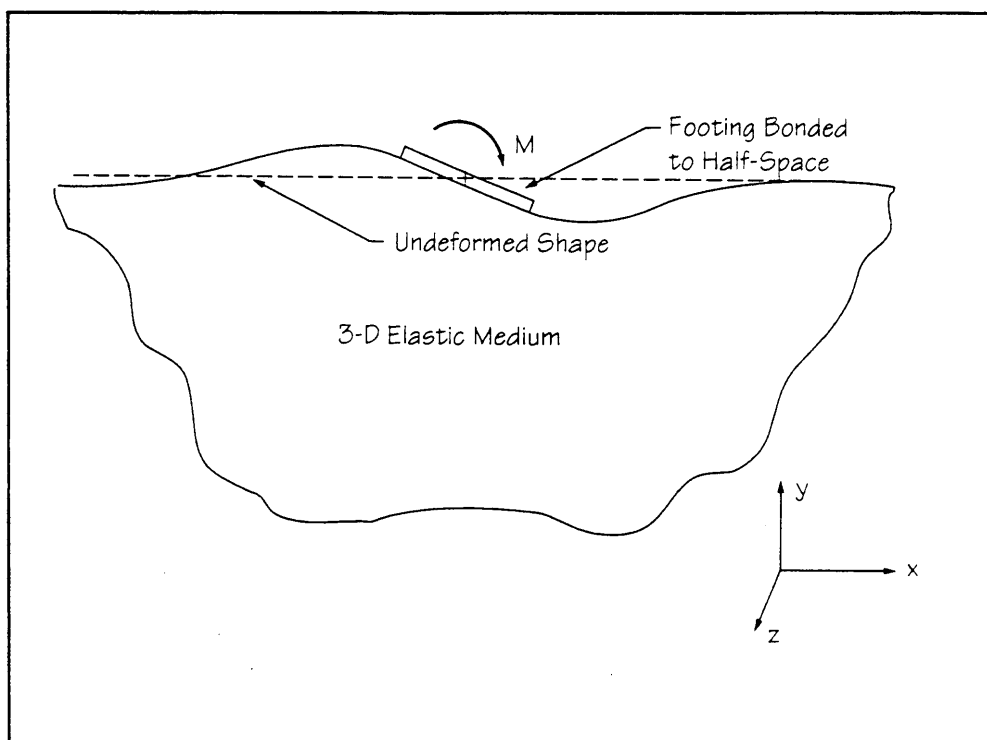
spread footing spring constants for use in a seismic analysis. The complexity of the methods varies widely, as does the input information required. Generally, any reasonable estimate of foundation stiffness will produce satisfactory results for dynamic analysis. The use of springs computed by some rational method, or by modifying substructure stiffness with an equivalent length to fixity, will provide better results than no foundation stiffness considerations at all.

For this example, the spring stiffnesses of the spread footings at Bents 1 and 2 will be calculated using an elastic half-space approach. The method used here is from FHWA, *Seismic Design and Retrofit Manual for Highway Bridges* (1987). Development of the elastic half-space formulae for equivalent radii and stiffness coefficients is beyond the scope of this example. Formulae and constants used here are from the FHWA manual referenced above.

**Design Step
6.2.1
(continued)**

The Elastic Half-Space Method essentially characterizes the spread footing as a rigid body bonded to an elastic medium which extends to infinity below the foundation, as shown in Figure 7. Steps needed to compute spring constants are

1. Calculate the equivalent radii for the footing using the formulae from Figure 8. (Note that the axes are identified differently than those from the support node local coordinate system.)
2. Determine the equation for the stiffness coefficients (K_0) from Table 2.
3. Using values for the shape modification factor and the embedment factor (from Figures 9 and 10), calculate the spring constants, using the previously determined equation for the stiffness coefficient.



**Figure 7 — Elastic Half-Space Deformation
in X-Y Plane Due to an Applied Moment**

Design Step
6.2.1
(continued)

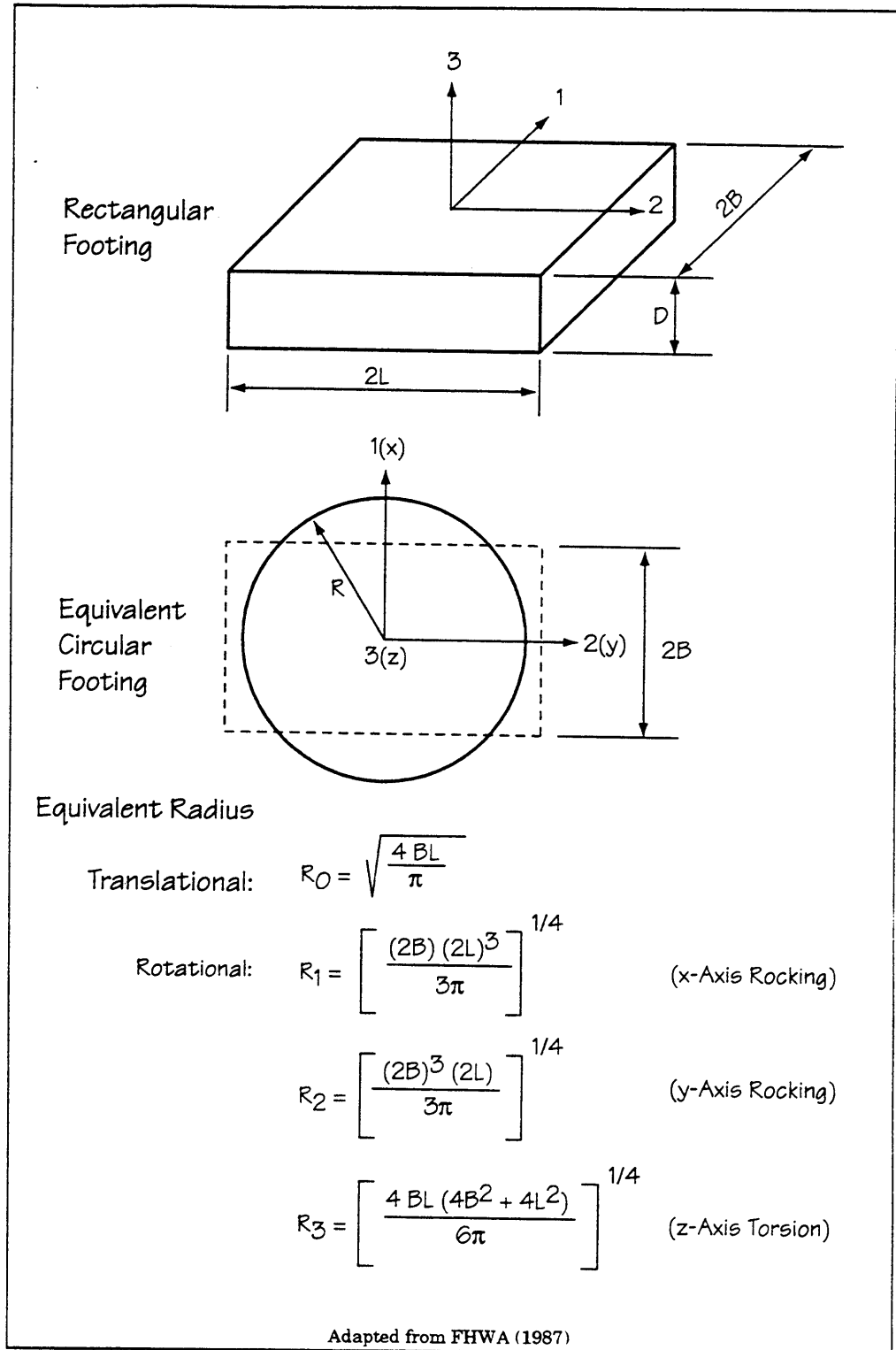


Figure 8 — Equivalent Radii for Rectangular Footings

Design Step
6.2.1
(continued)

Table 2
Stiffness Coefficients for a Circular Surface Footing

| Displacement Degree-of-Freedom | K_0 |
|--------------------------------|-------------------|
| Vertical Translation | $4GR/(1-\nu)$ |
| Horizontal Translation | $8GR/(2-\nu)$ |
| Torsional Rotation | $16GR^3/3$ |
| Rocking Rotation | $8GR^3/3 (1-\nu)$ |

Note: G and ν Are the Shear Modulus and Poisson Ratio for Elastic Half-Space Material; R is the Equivalent Radius of the Footing.

Adapted from FHWA (1987)

Design Step
6.2.1
(continued)

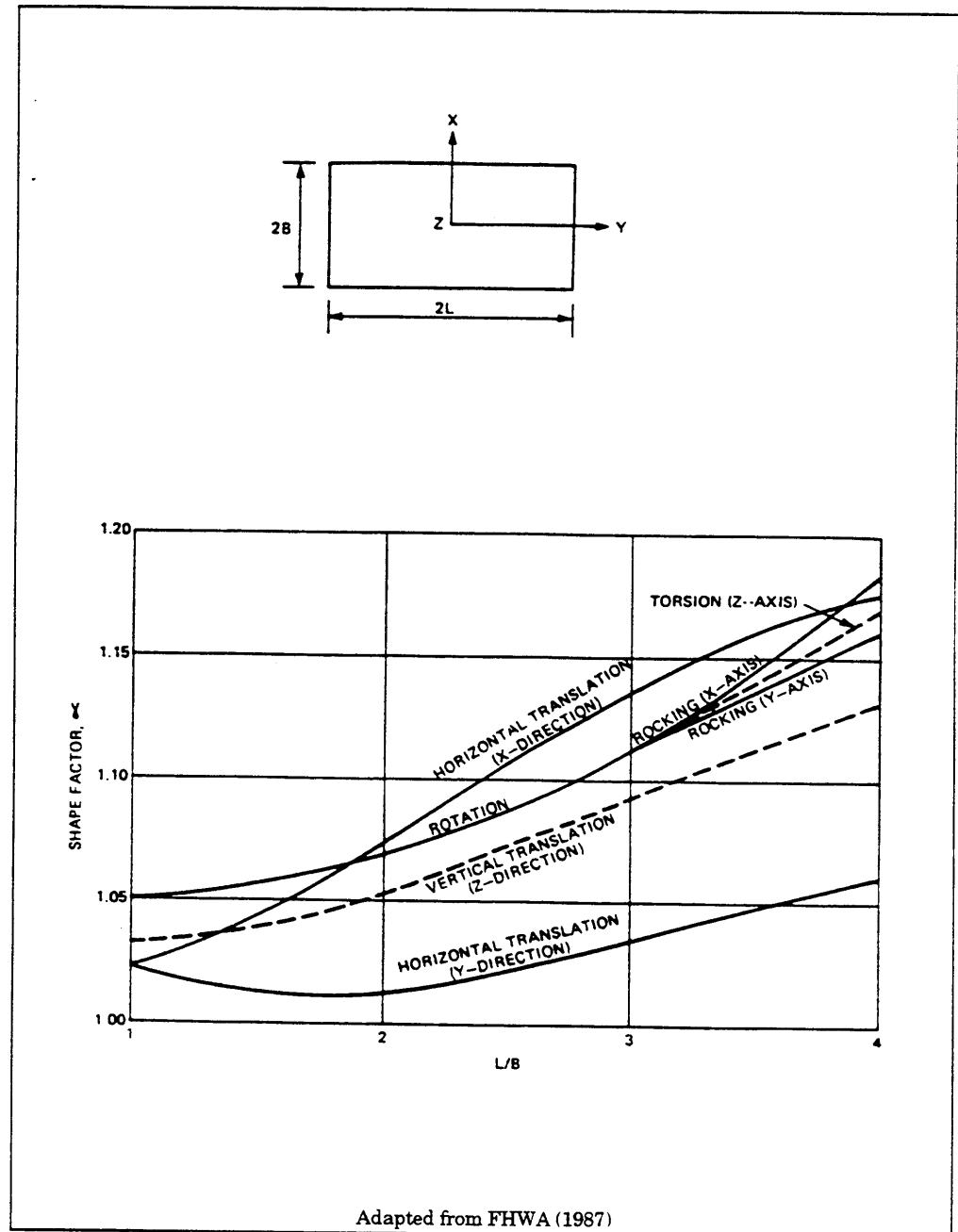


Figure 9 — Shape Factors for Rectangular Footings

Design Step
6.2.1
(continued)

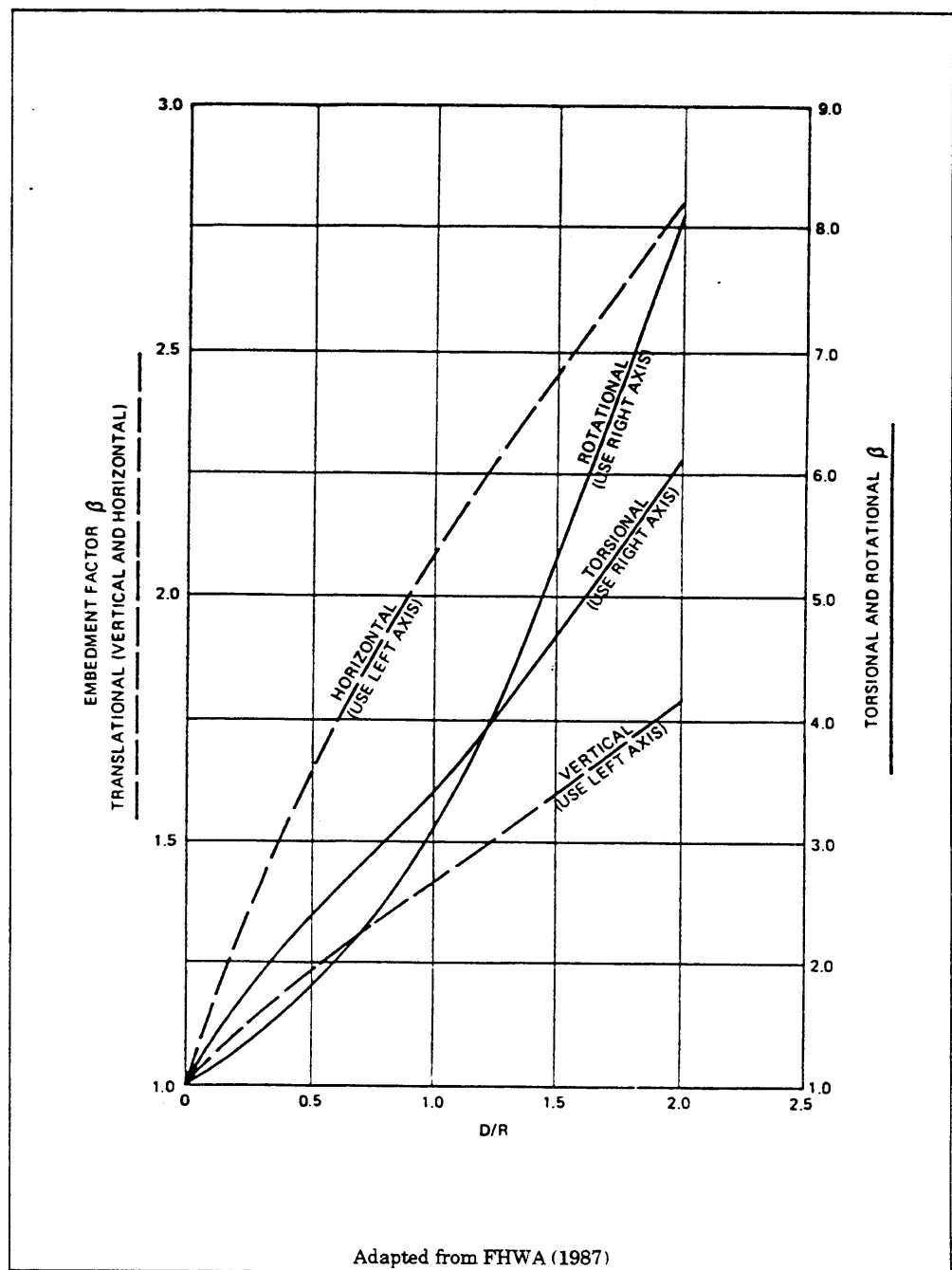


Figure 10 — Embedment Factors for Footings

**Design Step
6.2.1
(continued)**

a) Calculate the Equivalent Radii for the Footing

Assume the following dimensions.

$L := 7\text{-ft}$ Half of footing plan dimension parallel to bent plane

$B := 7\text{-ft}$ Half of footing plan dimension perpendicular to bent plane

$D := 5.5\text{-ft}$ Depth to bottom of footing from soil surface
(2-ft cover + 3.5-ft footing depth)

Calculate the equivalent radii from the Figure 8 equations. (Use the coordinate axes convention shown in the figure.)

$$R_o := \sqrt{\frac{4 \cdot B \cdot L}{\pi}}$$

Translational (all directions)

$$R_o = 7.9\text{-ft}$$

$$R_x := \left[\frac{(2 \cdot B) \cdot (2 \cdot L)^3}{3 \cdot \pi} \right]^{\frac{1}{4}}$$

For x-axis rotation
(rocking)

$$R_x = 7.99\text{-ft}$$

$$R_y := \left[\frac{(2 \cdot B)^3 \cdot (2 \cdot L)}{3 \cdot \pi} \right]^{\frac{1}{4}}$$

For y-axis rotation
(rocking)

$$R_y = 7.99\text{-ft}$$

Design Step
6.2.1
(continued)

$$R_z := \left[\frac{4 \cdot B \cdot L \cdot (4 \cdot B^2 + 4 \cdot L^2)}{6 \cdot \pi} \right]^{\frac{1}{4}} \quad \text{For z-axis rotation (torsion)}$$

$$R_z = 7.99 \cdot \text{ft}$$

b) Determine the Stiffness Coefficients

Spring constants (K) for shallow rectangular footings are obtained by modifying the solution for a circular footing bonded to the surface of an elastic half-space from the following.

$$K = \alpha \beta K_0 \quad \text{Equation (28) from FHWA (1987)}$$

Table 2 gives stiffness coefficient (K_0) equations for different displacement degrees of freedom to be used in computing the spring constants.

G and v are the shear modulus and Poisson's ratio, respectively, for the elastic half-space material. The geotechnical engineer may supply these values for the actual soil conditions, or representative values for the soil type may be used. R is the equivalent radius of the footing previously calculated.

$$G := 10 \cdot \text{ksi} \quad \text{Shear modulus for dense sand}$$

$$G = 1440 \cdot \text{ksf} \quad \text{gravel (from Bowles)}$$

$$v := 0.35 \quad \text{Poisson's ratio for dense sand (from Bowles)}$$

The terms α and β are the shape modification factor and the embedment factor, respectively. α may be determined from Figure 9, and β from Figure 10.

c) Calculate the Spring Constants

Compute the L/B ratio for the shape factor, and use Figure 9 to determine values of α .

$$\text{Shape} := \frac{L}{B} \quad \text{Shape} = 1$$

Design Step
6.2.1
(continued)

Shape factors from Figure 9.

$\alpha_x := 1.02$ For x translation (lateral)

$\alpha_y := 1.02$ For y translation (lateral)

$\alpha_z := 1.03$ For z translation (vertical)

$\alpha_{R_x} := 1.05$ For x-axis rotation (rocking)

$\alpha_{R_y} := 1.05$ For y-axis rotation (rocking)

$\alpha_{R_z} := 1.05$ For z-axis rotation (torsion)

Compute the D/R ratio for the shape factor, and use Figure 10 to determine values of β .

$$\text{Embedment} := \frac{D}{R}$$

$$E_o := \frac{D}{R_o} \quad E_o = 0.7 \quad \text{For translations}$$

$$E_x := \frac{D}{R_x} \quad E_x = 0.69 \quad \text{For x-axis rotation (rocking)}$$

$$E_y := \frac{D}{R_y} \quad E_y = 0.69 \quad \text{For y-axis rotation (rocking)}$$

$$E_z := \frac{D}{R_z} \quad E_z = 0.69 \quad \text{For z-axis rotation (torsion)}$$

**Design Step
6.2.1**

(continued)

Embedment factors from Figure 10.

$\beta_x := 1.83$ For x translation (lateral)

$\beta_y := 1.83$ For y translation (lateral)

$\beta_z := 1.31$ For z translation (vertical)

$\beta_{Rx} := 2.25$ For x-axis rotation (rocking)

$\beta_{Ry} := 2.25$ For y-axis rotation (rocking)

$\beta_{Rz} := 2.80$ For z-axis rotation (torsion)

Compute the elastic half-space spring constants for the footing. Substitute appropriate R (equivalent radii) values into the K_0 coefficient expressions from Table 2, and factor by appropriate α and β values to obtain the final springs.

Vertical stiffness (z translation)

$$k_{22} := \left[\frac{4 \cdot G \cdot R_0}{(1 - \nu)} \right] \cdot \alpha_z \cdot \beta_z$$

$$k_{22} = 9.44 \cdot 10^4 \cdot \frac{\text{kip}}{\text{ft}}$$

Lateral stiffness (x translation)

$$k_{11} := \left[\frac{8 \cdot G \cdot R_0}{(2 - \nu)} \right] \cdot \alpha_x \cdot \beta_x$$

$$k_{11} = 1.03 \cdot 10^5 \cdot \frac{\text{kip}}{\text{ft}}$$

Design Step
6.2.1
(continued)

Lateral stiffness (y translation)

$$k_{33} := \left[\frac{8 \cdot G \cdot R_o}{(2 - \nu)} \right] \cdot \alpha_y \cdot \beta_y$$

$$k_{33} = 1.03 \cdot 10^5 \cdot \frac{\text{kip}}{\text{ft}}$$

Torsional stiffness (z rotation)

$$k_{55} := \left(\frac{16 \cdot G \cdot R_z^3}{3} \right) \cdot \alpha_{Rz} \cdot \beta_{Rz} \quad \text{z-axis rotation (torsion)}$$

$$k_{55} = 1.15 \cdot 10^7 \cdot \text{kip} \cdot \frac{\text{ft}}{\text{rad}}$$

Rocking rotational stiffness (x and y axes)

$$k_{44} := \left[\frac{8 \cdot G \cdot R_x^3}{3 \cdot (1 - \nu)} \right] \cdot \alpha_{Rx} \cdot \beta_{Rx} \quad \text{x-axis rotation (rocking)}$$

$$k_{44} = 7.12 \cdot 10^6 \cdot \text{kip} \cdot \frac{\text{ft}}{\text{rad}}$$

$$k_{66} := \left[\frac{8 \cdot G \cdot R_y^3}{3 \cdot (1 - \nu)} \right] \cdot \alpha_{Ry} \cdot \beta_{Ry} \quad \text{y-axis rotation (rocking)}$$

$$k_{66} = 7.12 \cdot 10^6 \cdot \text{kip} \cdot \frac{\text{ft}}{\text{rad}}$$

The following is a summary of the spread footing springs calculated.

$$k_{11} = 1.03 \cdot 10^5 \cdot \frac{\text{kip}}{\text{ft}} \quad \text{Translation, x axis}$$

**Design Step
6.2.1
(continued)**

$$k_{22} = 9.44 \cdot 10^4 \cdot \frac{\text{kip}}{\text{ft}}$$

Translation, z (vertical) axis

$$k_{33} = 1.03 \cdot 10^5 \cdot \frac{\text{kip}}{\text{ft}}$$

Translation, y axis

$$k_{44} = 7.12 \cdot 10^6 \cdot \text{kip} \cdot \frac{\text{ft}}{\text{rad}}$$

Rotation, x axis (rocking)

$$k_{55} = 1.15 \cdot 10^7 \cdot \text{kip} \cdot \frac{\text{ft}}{\text{rad}}$$

Rotation, z axis (torsion)

$$k_{66} = 7.12 \cdot 10^6 \cdot \text{kip} \cdot \frac{\text{ft}}{\text{rad}}$$

Rotation, y axis (rocking)

Use these springs to model the foundation stiffnesses at each of the spread footings for the intermediate bents in the Multimode Spectral Analysis. These details are input into the SAP90 model in the local bent support node coordinate system, as shown in Figure 6.

Care should be taken to obtain the correct orientation for input of the springs into the model. This precision is especially important for a rectangular footing that would have significantly different stiffnesses for each of its orthogonal directions.

**Design Step
6.2.2**

Abutments

The abutments were modeled with a combination of full restraints (vertical translation and superstructure torsional rotation) and an equivalent spring stiffness (transverse translation), as shown in Figure 6. Other degrees of freedom are released.

The model allows longitudinal translational response that is unrestrained at the abutment. A gap between the end of the superstructure and the abutment backwall that is larger than the expected seismic displacement must be included if no longitudinal force is to be developed. (See Figure 11.) Depending on the site acceleration coefficient, soil conditions, and bridge configuration, the superstructure may come into contact with the abutment backwall. In that case, a longitudinal force will develop. This effect can be modeled, and is described in the *Seismic Design and Retrofit Manual for Highway Bridges*, FHWA (1987).

Design Step
6.2.2
(continued)

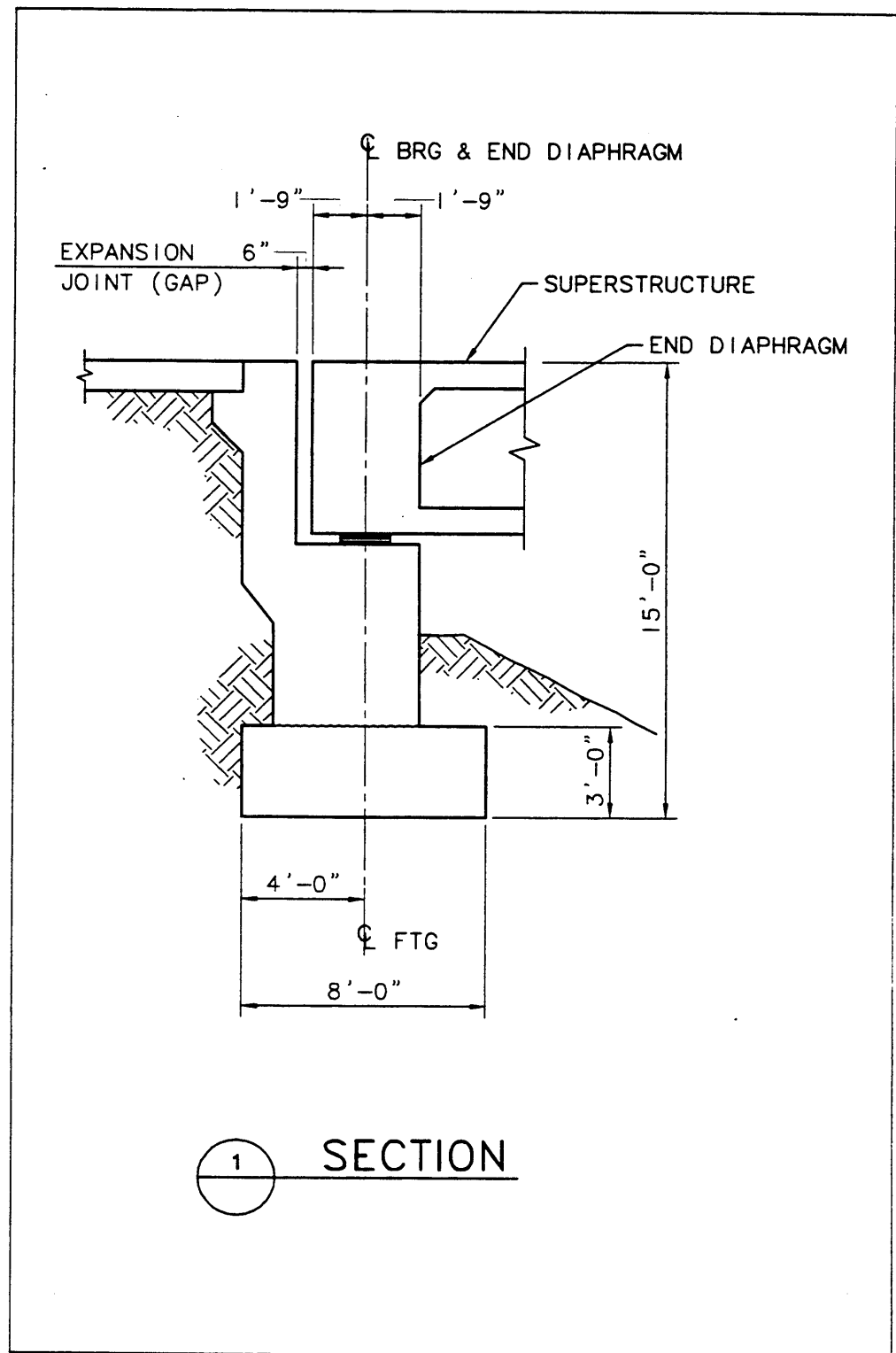


Figure 11 — Section at Abutment

**Design Step
6.2.2
(continued)**

An unrestrained longitudinal response also assumes that the bearings are free to translate in the longitudinal direction, which may not be exactly the case. The actual stiffness and movement characteristics of the bearings must be assessed. However, since the intent of this example is to provide all of the longitudinal resistance at the bents, the assumption of “free bearings” in the longitudinal direction is conservative and desirable for design of the bents.

In the model, torsional response of the superstructure is considered fully restrained at the abutments. Such fixity is assumed to occur as a result of the gravity contact forces existing between the superstructure and the bearings, as well as the significant resisting moment arm provided between the bearing reactions.

The ends of the superstructure are restrained against translation in the transverse direction at the abutments by girder stops at each side of the bridge. The transverse force resulting from this restraint is taken through the girder stops into the abutment, and is then resisted by the soil acting against the abutment and its wingwalls. The transverse stop and wingwall elevation are shown in Figure 12. A simple method used by Caltrans (1989) will be used to determine a translational spring stiffness for the transverse direction, based upon the stiffness of soil against the wingwalls, which resist translation in the transverse direction.

The support location at the abutment node, where the full restraints and the transverse translational spring act, is at the intersection of the superstructure work line (cgc of box girder) and the centerline of the bearings. The actual plan location of the transverse springs is near the centroid of the wingwall. With a skew of 30 degrees and a superstructure width of 43 feet, this would place the transverse springs about 10 feet 9 inches from the support node (measured parallel to the superstructure work line). For this example, it was assumed that the restraint and spring location at the abutment node located at the intersection of the superstructure work line and centerline of the bearings was still a reasonable approach. However, for severely skewed, wide bridges the designer may wish to directly model the geometry at the abutments with rigid link elements to distribute forces to the actual transverse spring locations at each corner of the abutment, where the wingwalls are located.

The transverse translational spring at the abutments should also represent the resistance to displacement of the abutment, which is significantly greater than the contribution from the wingwalls, when using the Caltrans approach. This stiffness can be evaluated by calculating the

Design Step
6.2.2
(continued)

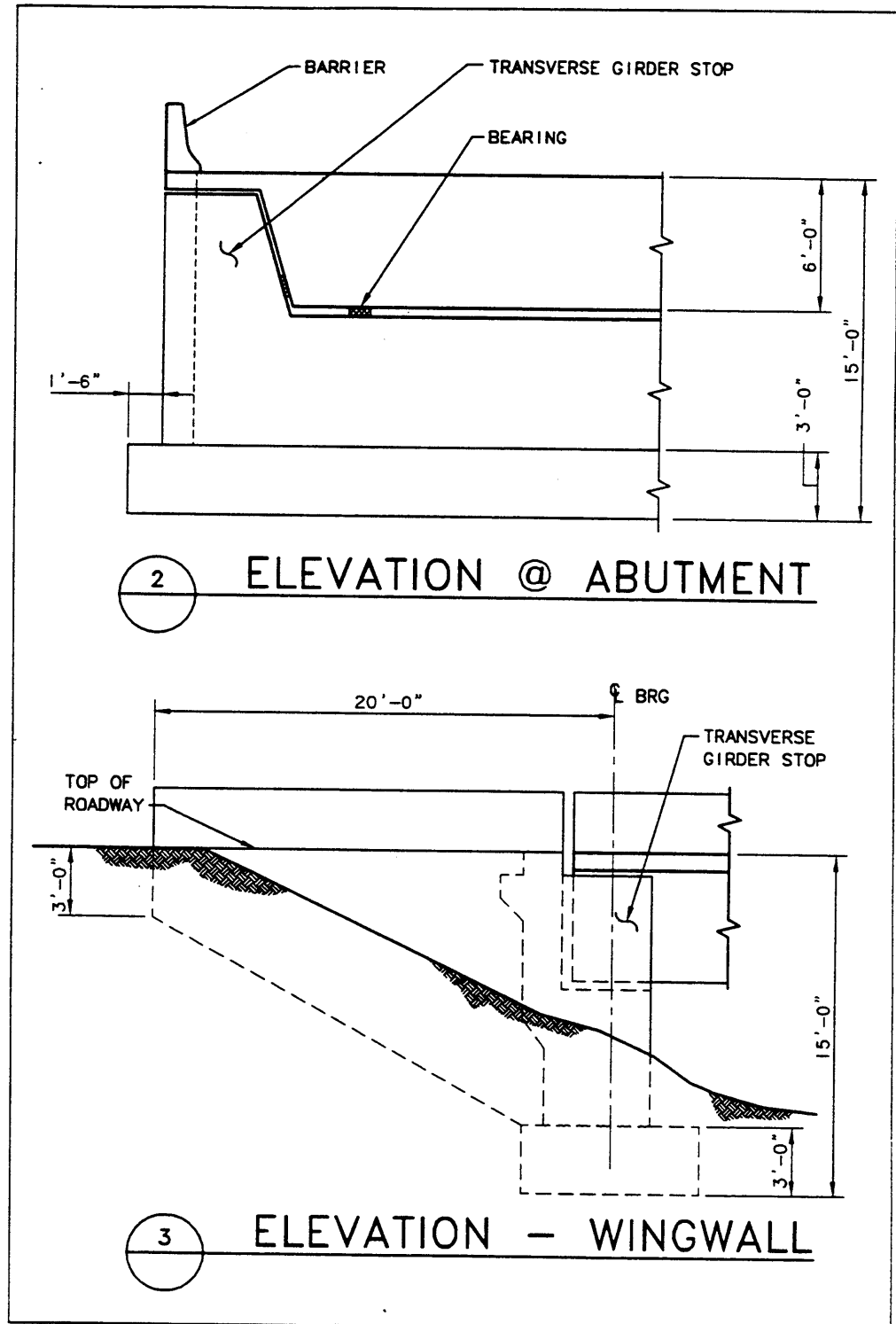


Figure 12 — Abutment and Wingwall Elevations

**Design Step
6.2.2
(continued)**

force required to slide the abutment, including the weight of soil trapped by the wingwalls, and using an iterative process to evaluate the spring constant. For severely skewed, wide bridges the effects of the skew must be included.

In this example, in order to bound the response of the structure to variation in transverse translational stiffness at the abutments, three SAP90 runs were made. Each run had a different spring stiffness or restraint condition at the abutment node for translation in the transverse direction. The lowest stiffness was obtained by using the simplified Caltrans method to compute a spring value for resistance by the wingwalls. Next a spring stiffness five times greater was calculated, to include the sliding resistance of the abutment. And finally, a full transverse translational restraint was used. These analyses showed that the structure's response to the transverse seismic loading resulted in a 5 percent difference in the transverse shear force at the abutments and a 10 percent difference in transverse shears at the intermediate bents. The highest abutment shear value was associated with the fully restrained condition, and the highest bent shear forces were associated with the lowest spring stiffness at the abutment. When making iterative analyses to provide bounding for resulting forces in a structure, the designer may wish to use forces from more than one analysis to be conservative. Because the focus of this example is the intermediate bent design, the analysis results used are those from the SAP90 run with the lowest abutment transverse translational spring stiffness, as calculated by the Caltrans method.

**Design Step
6.2.3**

Wingwalls

An appropriate transverse spring value to model the resistance provided by the abutment wingwalls may be obtained by the following approach. The behavior of soils resisting seismic loads provided by the wingwalls is nonlinear, because the soil exerts different pressures for the structure moving into or away from the soil. Caltrans simplifies this problem by considering only the compressive soil stress — that is, the stiffness of the structure moving into the soil. A uniform compressive stress of 200 kips per inch per foot of width is used as an appropriate stiffness for an 8-foot-high wall. Only the soil captured between the wingwalls (under the roadway) is assumed to participate. Accordingly, only one wingwall at a time is resisting the transverse force from the abutment. Resistance from soil on the outside face of the opposite wingwall is neglected.

**Design Step
6.2.3
(continued)**

Note that the transverse springs at the abutments act in a direction perpendicular to the superstructure work line since the wingwalls are oriented parallel to the bridge centerline.

The simplified calculations of the transverse spring constant for the abutment follow.

Establish a transverse linear spring stiffness for the abutments using the Caltrans method.

$$k_p := 200 \cdot \frac{\text{kip}}{\text{in} \cdot \text{ft}}$$

Basic stiffness per foot of width based
on an 8-ft-high wall and well-compacted
backfill material

From Figure 12, the tallest height of the wingwall is 12 feet. Then the wall height tapers to 3 feet. To account for the wall taper, a stiffness reduction is made. For this example, an effective stiffness of 2/3 (or 0.67) is used. This reduction is applied to the basic stiffness value of 200 kips per inch per foot of wall, which was not adjusted for wall height, because the average height of the wingwall is about 8 feet.

The effective stiffness per foot of wall for the wingwall is

$$k_{\text{eff}} := 0.67 \cdot k_p \qquad k_{\text{eff}} = 134 \cdot \frac{\text{kip}}{\text{in} \cdot \text{ft}}$$

Therefore, the total spring stiffness for the 20-foot wingwall is

$$L_w := 20 \cdot \text{ft}$$

$$k_w := L_w \cdot k_{\text{eff}}$$

$$k_w = 32160 \cdot \frac{\text{kip}}{\text{ft}}$$

Use this spring in the model for the
transverse translational stiffness
at the abutments

The spring k_w is oriented in the global Z direction at the abutment support nodes, as shown in Figure 6.

**Design Step
6.3**

Multimode Spectral Analysis - General
[Division I-A, Article 4.5 and 3.6.2]

**Design Step
6.3.1**

Mode Shapes and Periods
[Division I-A, Article 4.5.3]

The structure has been discretized using four elements per span and elements at each bent cap and column, as discussed previously. Twelve vibration modes have been used in the multimodal spectral analysis, which involves the superposition of individual modal responses to estimate the overall structural seismic response.

The SAP90 program (or any other dynamic spectral analysis program) lumps the tributary mass of each element at the adjacent nodes. Spring elements, which provide foundation flexibility, are massless. SAP90 determines the vibration periods and shapes for each of the vibration modes of the structure. The number of modes is dependent on the number of masses, the number of constrained degrees of freedom, and the number of foundation restraints for the system. Enough modes have to be specified so that the modal superposition to determine forces and displacements is sufficiently accurate. Typically, the modes are numbered sequentially from the longest period to the shortest.

The natural periods of vibration for the bridge for the first 12 modes are shown in Table 3. Figures 13 and 14 show two selected modes for the structure. Figure 13 shows the first mode, which is associated with the fundamental period in the longitudinal direction. The longitudinal period for this mode is 0.832 seconds. Figure 14 shows the second mode, which is the mode associated with the fundamental period in the transverse direction. The period for the second mode is 0.493 second.

Design Step
6.3.1
(continued)

Table 3
Modal Periods and Vibrations

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PROGRAM SAP90, VERSION BETA6.00
FHWA BRIDGE NO 4 PRELIMINARY DESIGN CALCS

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FILE:exam4.OUT

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E I G E N V A L U E S   A N D   F R E Q U E N C I E S

```

| MODE | PERIOD (TIME) | FREQUENCY (CYC/TIME) | FREQUENCY (RAD/TIME) | EIGENVALUE (RAD/TIME)**2 |
|------|------------------|-------------------------|-------------------------|-----------------------------|
| 1 | 0.831827 | 1.202172 | 7.553472 | 57.054941 |
| 2 | 0.492683 | 2.029703 | 12.753000 | 162.639019 |
| 3 | 0.297626 | 3.359927 | 21.111042 | 445.676084 |
| 4 | 0.229760 | 4.352366 | 27.346720 | 747.843098 |
| 5 | 0.208742 | 4.790611 | 30.100294 | 906.027677 |
| 6 | 0.204992 | 4.878243 | 30.650903 | 939.477861 |
| 7 | 0.125186 | 7.988121 | 50.190842 | 2519.121 |
| 8 | 0.101884 | 9.815115 | 61.670188 | 3803.212 |
| 9 | 0.082165 | 12.170569 | 76.469940 | 5847.652 |
| 10 | 0.081362 | 12.290709 | 77.224804 | 5963.670 |
| 11 | 0.076875 | 13.008090 | 81.732243 | 6680.160 |
| 12 | 0.068666 | 14.563152 | 91.502984 | 8372.796 |

Design Step
6.3.1
(continued)

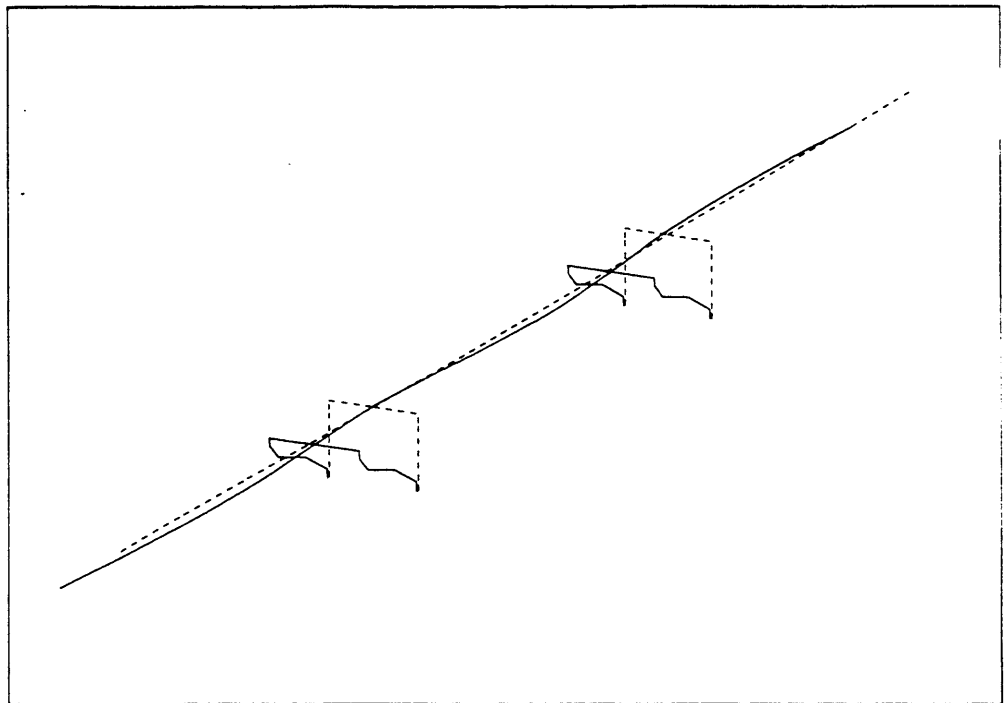


Figure 13 — Deformed Shape for Mode 1

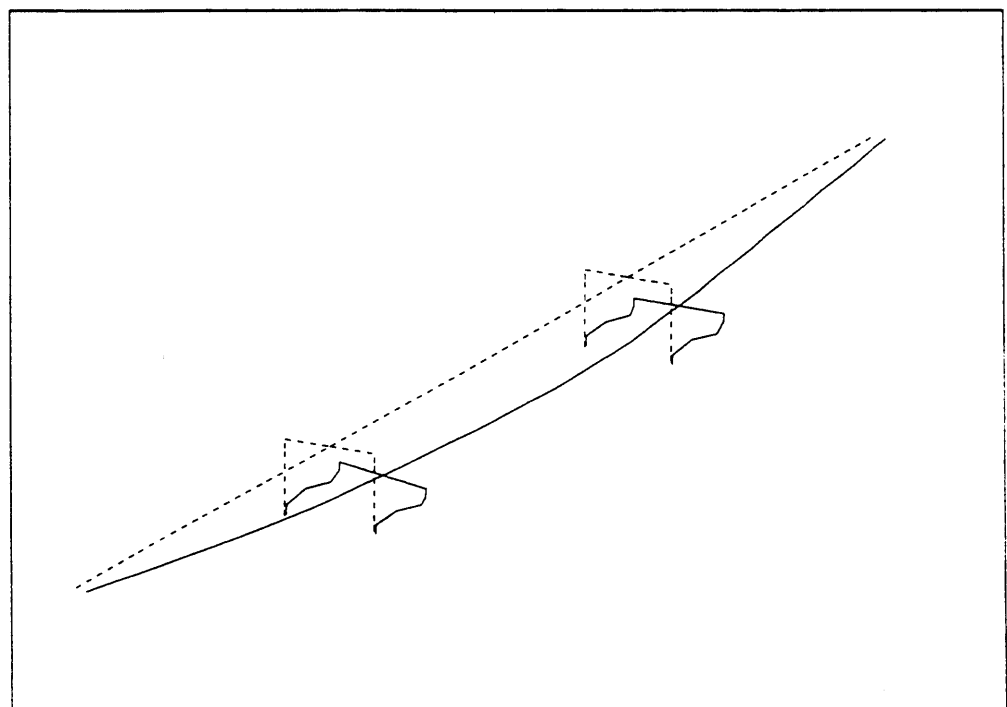


Figure 14 — Deformed Shape for Mode 2

**Design Step
6.3.1
(continued)**

Hand Check ✓ Check Fundamental Period in the Longitudinal Direction

As a check, compare the longitudinal period from the Multimode Analysis with a simple hand calculation. For this example, a simple Single Degree of Freedom is used with a single mass (W) and spring (K = stiffness of participating bent columns).

The mass of the superstructure is the primary mass moving in this case. The total superstructure weight is made up of the following.

$$w_{\text{box}} := 11.65 \cdot \frac{\text{kip}}{\text{ft}} \quad \text{Weight of cast-in-place concrete box girder (including diaphragms and bent cap beam)}$$

$$w_{\text{misc}} := 2.35 \cdot \frac{\text{kip}}{\text{ft}} \quad \text{Weight of barriers and overlay}$$

$$w_{\text{super}} := w_{\text{box}} + w_{\text{misc}}$$

$$w_{\text{super}} = 14 \cdot \frac{\text{kip}}{\text{ft}} \quad \text{Weight per foot of superstructure}$$

Compute the total superstructure weight.

$$L := 320 \cdot \text{ft} \quad \text{Overall length of bridge}$$

$$W_{\text{super}} := w_{\text{super}} \cdot L$$

$$W_{\text{super}} = 4480 \cdot \text{kip} \quad \text{Superstructure weight}$$

As discussed in Design Step 1, Preliminary Design, for the longitudinal direction, the pinned base intermediate bent columns are assumed to resist the entire longitudinal seismic force as shown in Figure 2. All columns are 4-foot diameter, with a total of four columns participating.

The column stiffness is calculated as a cantilever using $k = 3EI / H^3$, assuming the superstructure is very rigid compared to the columns, and the columns are pinned at the base.

**Design Step
6.3.1
(continued)**

$$E_c := 3600 \cdot \text{ksi}$$

Young's Modulus of concrete

$$d := 48 \cdot \text{in}$$

Outside diameter of column

The effective height (H) used to compute the column stiffness is the clear height from the top of the footing to the soffit of the cap beam, because the column is pinned at the base and assumed to be very flexible compared to the stiffness of the superstructure.

$$H := 20.0 \cdot \text{ft}$$

Clear height of column

Compute the stiffness for a single column.

$$I_{col} := \frac{\pi \cdot d^4}{64}$$

$$I_{col} = 12.57 \cdot \text{ft}^4$$

Moment of inertia of a single column

$$K_{col} := \frac{3 \cdot E_c \cdot I_{col}}{H^3}$$

$$K_{col} = 2443 \cdot \frac{\text{kip}}{\text{ft}}$$

Stiffness for a single column as
a cantilever

For all four of the columns, the stiffness for the total resisting system is

$$K_{total} := 4 \cdot K_{col}$$

$$K_{total} = 9772 \cdot \frac{\text{kip}}{\text{ft}}$$

Include the weight of one-half of the column height in the total weight used to calculate the period.

$$\gamma_{conc} := .150 \cdot \frac{\text{kip}}{\text{ft}^3}$$

Unit weight of concrete

Design Step
6.3.1
(continued)

$$A_{col} := \frac{\pi \cdot d^2}{4}$$

Cross-sectional area of column

$$A_{col} = 12.57 \cdot \text{ft}^2$$

The weight of one-half of a single column is

$$W_{col} := \left(\frac{H}{2} \right) \cdot A_{col} \cdot \gamma_{conc}$$

$$W_{col} = 18.85 \cdot \text{kip}$$

For a single column

Therefore, the total weight used for the calculation of the period.

$$W_{total} := W_{super} + 4 \cdot W_{col}$$

$$W_{total} = 4555 \cdot \text{kip}$$

Calculate the longitudinal period.

$$T := 2 \cdot \pi \cdot \sqrt{\frac{W_{total}}{K_{total} \cdot g}}$$

Division I-A
Eqn (4-3)

$$T = 0.756 \cdot \text{sec}$$

Compare this period with the period obtained from SAP90 for vibration in the longitudinal direction from the first mode.

$$T_{SAP} := 0.832 \cdot \text{sec}$$

$$\frac{T}{T_{SAP}} = 0.909$$

Close, say okay

Design Step
6.3.1
(continued)

The period obtained from the Multimode Analysis is slightly longer than that obtained from the hand calculation. The discrepancy is expected, because the SAP90 analysis includes spring supports to model the foundation, and the hand calculation assumes a stiffer condition with cantilever columns. The model's stiffness is, therefore, lower than the hand-calculated stiffness, and so the period is longer, as shown in the relationship between period and stiffness in Division I-A, Equation 4-3.

Hand Check ✓ Check Fundamental Period in the Transverse Direction

As a check, compare the transverse period from the Multimode Analysis with a simple hand calculation. For this example, the Uniform Load Method described in FHWA (1981) is used.

Calculate the transverse fundamental period by the Uniform Load Method, considering the stiffness of the superstructure as a simple beam spanning the abutments, superimposed with the cantilever stiffness of the four bent columns, as used in the calculation of the longitudinal period. Assume a single, simple-span uniform load deflection, as shown in Figure 3.

The previous longitudinal period check yields the following.

$$W_{\text{total}} = 4555 \cdot \text{kip}$$

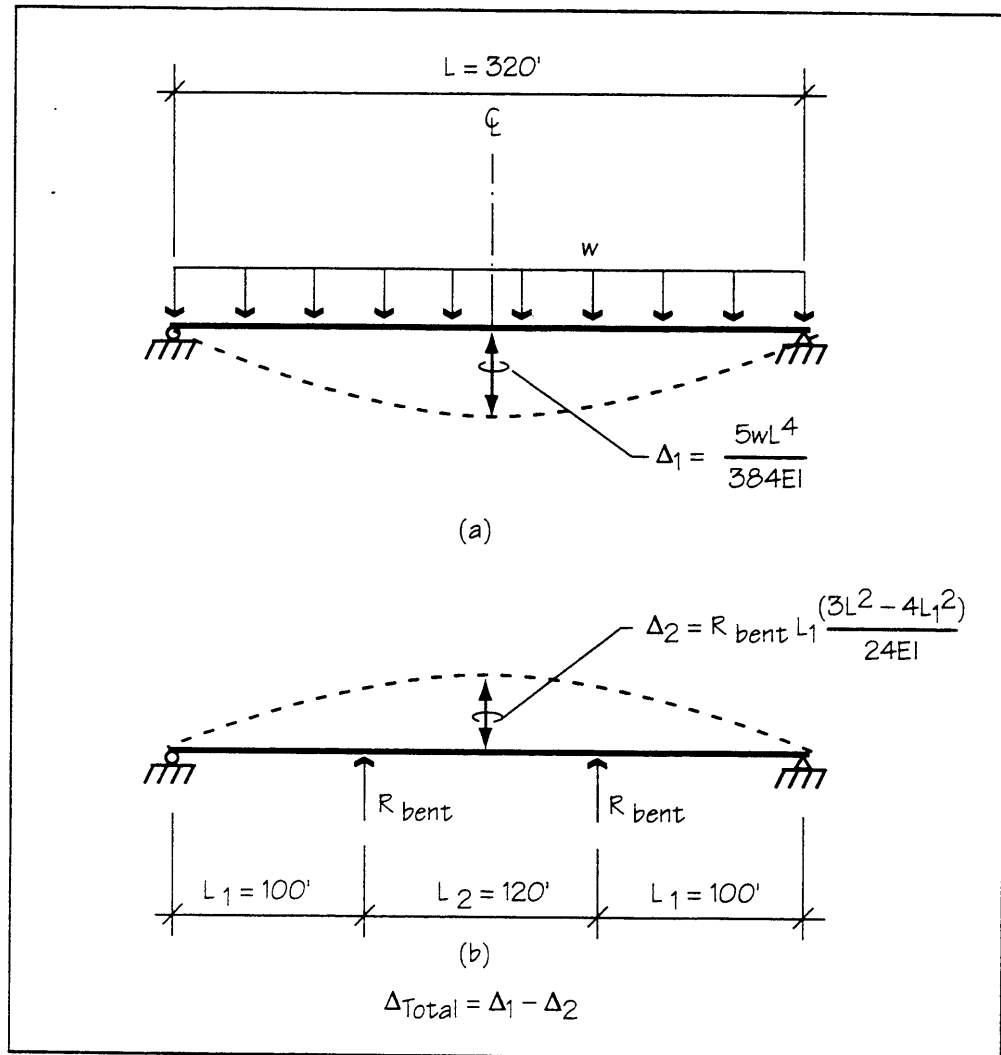
$$K_{\text{col}} = 2443 \cdot \frac{\text{kip}}{\text{ft}} \quad \text{Stiffness for a single column as a cantilever}$$

The maximum total transverse deflection, Δ_{total} , is at the midspan of Span 2. The stiffness contribution of the superstructure is from the deflection relationship of a simply supported beam with a uniform load, as shown in Figure 15(a), with a maximum deflection Δ_1 . This simple span deflection is resisted by the stiffness of the bent columns acting as two concentrated loads on the simple beam, as shown in Figure 15(b), where Δ_2 is the deflection at midspan of Span 2. A superposition of these deflections yields a total deflection for the system with $\Delta_{\text{total}} = \Delta_1 + \Delta_2$.

$$E_c = 3600 \cdot \text{ksi} \quad \text{Young's modulus of concrete}$$

$$I = 9697 \cdot \text{ft}^4 \quad \text{Moment of inertia of box girder about a vertical axis (from Table 1)}$$

Design Step
6.3.1
(continued)



**Figure 15 — Transverse Seismic Deflections
for Hand Check**

$$L = 320 \cdot \text{ft}$$

Overall length of bridge

$$L_1 = 100 \cdot \text{ft}$$

Length of bridge end spans

Taking stiffness of a bent as equal to two times the stiffness of a single column.

$$K_{bent} := 2 \cdot K_{col}$$

$$K_{bent} = 4886 \cdot \frac{\text{kip}}{\text{ft}}$$

**Design Step
6.3.1
(continued)**

Compute the following coefficients for ease of calculation (from Figure 15).

$$A := \frac{5 \cdot L^3}{384} \qquad A = 426667 \cdot \text{ft}^3$$

$$B := L_1 \cdot \frac{(3 \cdot L^2 - 4 \cdot L_1^2)}{24} \qquad B = 1113333 \cdot \text{ft}^3$$

Deflections may be expressed in terms of the above-computed coefficients as

$$\Delta_1 := w \cdot L \cdot \frac{A}{E_c \cdot I}$$

$$\Delta_2 := R_{\text{bent}} \cdot \frac{B}{E_c \cdot I}$$

where

$$R_{\text{bent}} := K_{\text{bent}} \cdot \Delta_{\text{bent}}$$

The total final deflection at the midspan of Span 2 is

$$\Delta_{\text{total}} := \Delta_1 - \Delta_2$$

Assume that the final deflection at the bent is 85 percent of the total deflection at the midspan of Span 2.

$$\Delta_{\text{bent}} := 0.85 \cdot \Delta_{\text{total}}$$

Substituting expressions for Δ_1 and Δ_2 into the relationship for the total system deflection, and solving for K_{total} ($= wL/\Delta_{\text{total}}$).

The total stiffness of the system is

$$K_{\text{total}} := \left(\frac{E_c \cdot I}{A} + K_{\text{bent}} \cdot 0.85 \cdot \frac{B}{A} \right)$$

Design Step
6.3.1
(continued)

$$K_{total} = 22618 \cdot \frac{\text{kip}}{\text{ft}}$$

Calculate the transverse period.

$$T := 2 \cdot \pi \cdot \sqrt{\frac{W_{total}}{K_{total} \cdot g}} \quad \begin{array}{l} \text{Division I-A} \\ \text{Eqn (4-3)} \end{array}$$

$$T = 0.497 \cdot \text{sec}$$

Compare this period with the period obtained from SAP90 for vibration in the transverse direction from the second mode.

$$T_{SAP} := 0.493 \cdot \text{sec}$$

$$\frac{T}{T_{SAP}} = 1.008 \quad \text{Very close, say okay}$$

Design Step
6.3.2

Spectral Loading
[Division I-A, Article 3.6.2]

The input response spectra for this bridge is shown in Figure 16 below. The curve shown in the figure is given by the equation for C_{sm} , the elastic seismic response coefficient.

$$C_{sm}(T_m) := \frac{1.2 \cdot A \cdot S}{\frac{2}{T_m^{\frac{2}{3}}}} < 2.5 \cdot A \quad \begin{array}{l} \text{Division I-A} \\ \text{Eqn (3-2)} \end{array}$$

where

A is the acceleration coefficient

S is the site coefficient

T_m is the period of the m th mode of vibration

A design response spectrum must be input to provide loading for the model. This spectrum is specified in Article 3.6.2 of the Specification, and it applies in both the transverse and longitudinal directions.

Design Step
6.3.2
(continued)

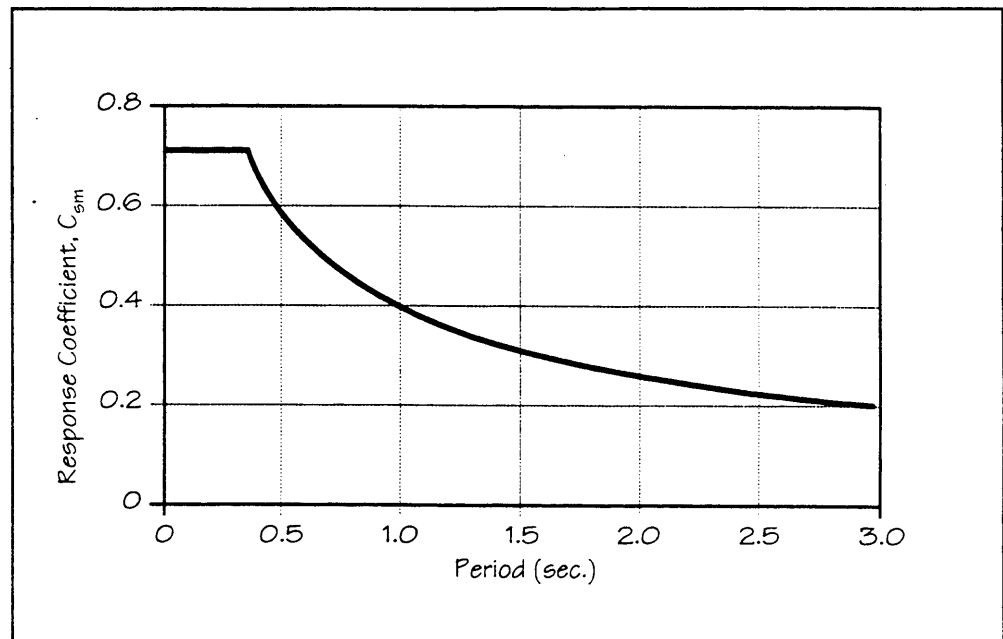


Figure 16 — Relationship Between Elastic Seismic Response Coefficient and Period

For this example, the longitudinal direction is a straight line parallel to the centerline of the bridge. The transverse direction is applied at 90 degrees to the longitudinal direction.

The spectrum is defined as a function of period T by Equation 3-2 of the Specification and the corresponding upper limit of two and a half times A . Most programs will require period-spectrum data pairs to be input. The user must, therefore, calculate the C_{sm} values that will define a smooth function within the analysis software. (C_{sm} is the modal analysis version of C_s .) The range must cover the entire range of expected periods for the structure.

Some programs allow the user to control interpolation between the input points. The typical choices are linear interpolation on an arithmetic scale or linear interpolation on a log-log scale. Logarithmic interpolation results in a curvilinear representation between data points on an arithmetic scale. Thus, the user should be aware of how the program will interpolate, so that a smooth representation of the input spectrum can be obtained, and the spectrum will be what the user expects. Use of small increments between input points will ensure that the proper spectrum is obtained, regardless of how the interpolation is performed.

Figure 16 and Equation 3-2 are based on 5 percent damping.

**Design Step
6.3.3**

**Minimum Number of Modes
[Division I-A, Article 4.5]**

Twelve modes have been included to provide an accurate estimate of the response and internal forces.

As discussed previously, a sufficient number of modes has to be specified so that the modal superposition determines forces and displacements with sufficient accuracy.

One way of assessing whether the number of modes is sufficient to accurately characterize response is to check whether the percentage of cumulative mass that participates for the specified number of modes is at least 90 percent of the total for each of the directions of the applied loading. In this example, no loading is applied in the vertical (y) direction, and it is not critical that mass participation in that direction be at least 90 percent. However, the designer should not rely only on mass participation. Mode shapes should be inspected to determine that important masses, such as all of the substructure elements, are excited by the selected modes.

Results for the multimode analysis are given in Table 4. The three columns under Individual Mode (percent) show the participating mass in each direction for each mode. The next three columns under Cumulative Sum (percent) show the cumulative participating mass in each direction. Seven modes are required in order to obtain over 90 percent of the cumulative mass participating in each of the two global plan directions (X and Z), which are the directions of the applied lateral loading.

Design Step
6.3.3
(continued)

In Table 4

UX = Longitudinal direction
UY = Vertical direction
UZ = Transverse direction

Table 4
Modal Participating Mass

| PROGRAM SAP90, VERSION BETA6.00 | | | | | | FILE:exam4.OUT | | |
|---|------------|---------------------------|----------|-----------|--------------------------|----------------|---------|--|
| FHWA BRIDGE NO 4 PRELIMINARY DESIGN CALCS | | | | | | | | |
| M O D A L P A R T I C I P A T I N G M A S S | | | | | | | | |
| MODE | PERIOD | INDIVIDUAL MODE (PERCENT) | | | CUMULATIVE SUM (PERCENT) | | | |
| | | UX | UY | UZ | UX | UY | UZ | |
| 1 | 0.831827 | 90.2050 | 0.0000 | 0.1295 | 90.2050 | 0.0000 | 0.1295 | |
| 2 | 0.492683 | 0.1709 | 0.0000 | 79.2038 | 90.3759 | 0.0000 | 79.3333 | |
| 3 | 0.297626 | 0.0000 | 0.2074 | 0.0000 | 90.3759 | 0.2074 | 79.3333 | |
| 4 | 0.229760 | 0.3867 | 0.0000 | 0.1247 | 90.7626 | 0.2074 | 79.4580 | |
| 5 | 0.208742 | 0.0000 | 0.2449 | 0.0000 | 90.7626 | 0.4523 | 79.4580 | |
| 6 | 0.204992 | 0.0000 | 79.3444 | 0.0000 | 90.7626 | 79.7967 | 79.4580 | |
| 7 | 0.125186 | 0.0000 | 0.0000 | 11.0219 | 90.7626 | 79.7967 | 90.4799 | |
| 8 | 0.101884 | 0.0328 | 0.0000 | 0.0057 | 90.7954 | 79.7967 | 90.4856 | |
| 9 | 0.082165 | 0.0000 | 12.3200 | 0.0000 | 90.7954 | 92.1168 | 90.4856 | |
| 10 | 0.081362 | 0.0000 | 0.1055 | 0.0000 | 90.7954 | 92.2222 | 90.4856 | |
| 11 | 0.076875 | 0.0354 | 0.0000 | 0.0061 | 90.8308 | 92.2222 | 90.4917 | |
| 12 | 0.068666 | 0.0000 | 0.0047 | 0.0000 | 90.8308 | 92.2269 | 90.4917 | |
| T O T A L U N R E S T R A I N E D M A S S A N D L O C A T I O N | | | | | | | | |
| DIRECTION | MASS | X | Y | Z | | | | |
| UX | 157.015449 | 160.000000 | 0.935100 | -8.82E-18 | | | | |
| UY | 146.736971 | 160.000000 | 0.763842 | 1.48E-17 | | | | |
| UZ | 157.015449 | 160.000000 | 0.935100 | -8.82E-18 | | | | |

**Design Step
6.3.4**

**Combination of Modes
[Division I-A, Article 4.5]**

The response of the model in each of the calculated modes must be superimposed to establish the overall response. Because the modal maximum responses do not all occur simultaneously, a simple summation of the modal absolute values is not appropriate. Most programs use either the Square Root of the Sum of the Squares (SRSS) Method or the Complete Quadratic Combination (CQC) method. The simplest is the SRSS method, and it is adequate when the modal periods are well spaced. When the periods are quite close, coupling between modal responses can occur, and the CQC method should be used. This method accounts for coupling between modes, preserves the sign of the cross-modal terms, and is based on random vibration fundamentals. Most programs now include the CQC method as an option. It requires very little additional run time for most models, and should be used exclusively to eliminate the judgment of what constitutes closely spaced periods. The default combination method for SAP90 is CQC.

**Design Step
6.4**
(continued)

Table 5
Response for Transverse Direction (E_{Qtrans})

| Support/Location | | Forces and Moments - EQtrans | | | | Axial (kips) |
|------------------------|--------|------------------------------|--------------------|-----------------|--------------------|-----------------|
| | | Longitudinal (1) | | Transverse (2) | | |
| | | Shear (kips) | Moment (kip-ft) | Shear (kips) | Moment (kip-ft) | |
| Abutment A | | 0 | 0 | 781 | 0 | 38 |
| Bent 1 Left Column | Top | 99 | 1,981 | 274 | 5,481 | 492 |
| | Bottom | 99 | 0 | 274 | 0 | 492 |
| Bent 1 Right Column | Top | 148 | 2,951 | 274 | 5,481 | 548 |
| | Bottom | 148 | 0 | 274 | 0 | 548 |
| Bent 2 Left Column | Top | 148 | 2,951 | 274 | 5,481 | 548 |
| | Bottom | 148 | 0 | 274 | 0 | 548 |
| Bent 2 Right Column | Top | 99 | 1,981 | 274 | 5,481 | 492 |
| | Bottom | 99 | 0 | 274 | 0 | 492 |
| Abutment B | | 0 | 0 | 781 | 0 | 38 |

(1) For bent columns, the longitudinal direction is 90 degrees to the plane of the bent frame.

(2) For bent columns, the transverse direction is parallel to the plane of the bent frame.

**Design Step
6.4**
(continued)

**Table 6
Displacements**

| Location | Displacements at CGC of Superstructure | | | |
|------------------|--|------------------|------------------|------------------|
| | EQtrans | | EQlong | |
| | Global X (ft) | Global Z (ft) | Global X (ft) | Global Z (ft) |
| Abutment A | 0.0117 | 0.0254 | 0.2752 | 0.0022 |
| Center of Span 1 | 0.0116 | 0.0947 | 0.2750 | 0.0092 |
| Bent 1 | 0.0116 | 0.1476 | 0.2743 | 0.0144 |
| Center of Span 2 | 0.0116 | 0.1732 | 0.2746 | 0.0169 |
| Bent 2 | 0.0116 | 0.1476 | 0.2743 | 0.0144 |
| Center of Span 3 | 0.0116 | 0.0947 | 0.2750 | 0.0092 |
| Abutment B | 0.0117 | 0.0254 | 0.2752 | 0.0022 |

**Design Step
6.4**

Determine Forces and Displacements in Transverse Direction
[Division I-A, Article 4.5]

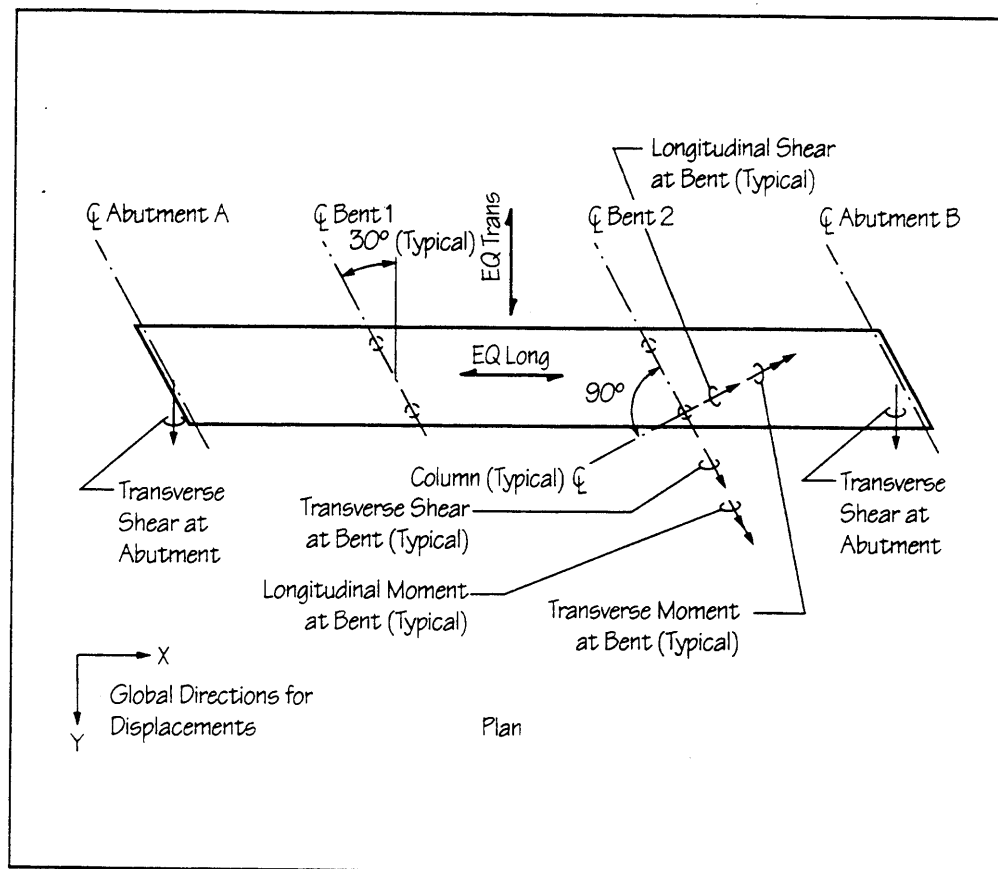
Using the Multimode Spectral Method, perform a transverse analysis. Transverse analysis means that the input response spectrum was assigned to the transverse direction, and in this case no longitudinal or vertical spectra were used. The longitudinal direction is along a straight line parallel to the centerline of the bridge (global X). The transverse direction is applied at 90 degrees to the longitudinal direction (global Z). These directions are shown in Figure 17. In most cases, when the same model is used for both directions of loading, both the transverse and longitudinal analyses are performed in the same computer run, as is the case for this example.

The analysis program handles all the calculations, including the modal combinations. In this case, 12 modes were used to characterize the response. This number was kept constant for all the analyses.

The results are given in Table 5. The SAP90 input file for this analysis is EXAM4. Shown in the table are forces and moments. Directions for forces and moments at the bents are shown in Figure 17, and are oriented along the local coordinate system for the bent elements. For bent columns, the transverse direction is parallel to the plane of the bent frame, and the longitudinal direction is 90 degrees to the plane of the bent frame. Abutment transverse forces are oriented in the global coordinate system (global Z direction) as shown in Figure 17.

Displacements for both transverse and longitudinal analyses are given in Table 6. Directions for the displacements are in the global coordinate directions which are shown in Figure 17.

**Design Step
6.4**
(continued)



**Figure 17 — Key to Force, Moment,
and Displacements Directions**

**Design Step
6.5**

Determine Forces and Displacements in Longitudinal Direction
[Division I-A, Article 4.5]

Perform the analysis for loading in the longitudinal direction.

The resulting forces and moments at the intermediate piers for the spectral analysis in the longitudinal direction are given in Table 7. The SAP90 input file for this analysis is EXAM4. Displacements for both transverse and longitudinal analyses are given in Table 6. Directions for displacements are in the global coordinate system, which is shown in Figure 17.

Table 7
Response for Longitudinal Direction (EQ_{long})

| Support/Location | | Forces and Moments - EQlong | | | | Axial (kips) |
|------------------------|--------|-----------------------------|--------------------|-----------------|--------------------|-----------------|
| | | Longitudinal (1) | | Transverse (2) | | |
| | | Shear (kips) | Moment (kip-ft) | Shear (kips) | Moment (kip-ft) | |
| Abutment A | | 0 | 0 | 71 | 0 | 115 |
| Bent 1 Left Column | Top | 474 | 9,477 | 288 | 5,754 | 209 |
| | Bottom | 474 | 0 | 288 | 0 | 209 |
| Bent 1 Right Column | Top | 478 | 9,562 | 288 | 5,754 | 37 |
| | Bottom | 478 | 0 | 288 | 0 | 37 |
| Bent 2 Left Column | Top | 478 | 9,562 | 288 | 5,754 | 37 |
| | Bottom | 478 | 0 | 288 | 0 | 37 |
| Bent 2 Right Column | Top | 474 | 9,477 | 288 | 5,754 | 209 |
| | Bottom | 474 | 0 | 288 | 0 | 209 |
| Abutment B | | 0 | 0 | 71 | 0 | 115 |

(1) For bent columns, the longitudinal direction is 90 degrees to the plane of the bent frame.

(2) For bent columns, the transverse direction is parallel to the plane of the bent frame.

DESIGN STEP 7**DETERMINE DESIGN FORCES****INTRODUCTION**

Under seismic loading, the bridge behaves much differently in the longitudinal direction than it does in the transverse direction. Under longitudinal seismic loads, the bridge is free to slide at the abutments. All of the longitudinal seismic load is, therefore, assumed to be taken by the intermediate bent columns.

In the transverse direction, the bridge is wide relative to its length and the superstructure acts as a horizontal diaphragm spanning the abutments under seismic loads. The bent columns participate with the abutments in resisting the transverse seismic load. In the simple hand calculation from Design Step 6.1.3(b) for the fundamental period in the transverse direction, the stiffness of the bent columns is about half of the total transverse stiffness used to compute the period.

For this bridge, inelastic response is expected in both the longitudinal and transverse directions. This response means that the full elastic seismic moments in both directions are larger than the plastic hinging moment capacity of the columns. Primary reinforcement for the column, which along with the column cross section dimensions determines the plastic hinging moment capacity, is designed using the modified design forces for structural members and connections that are determined in Design Step 7.3.1.

Plastic hinging forces will be calculated for the transverse direction, that is, in the plane of the bent frame, and for the longitudinal direction which is at 90 degrees to the plane of the bent frame. The resulting plastic hinging forces will be used throughout the remainder of the example. Because this example contains symmetrical, identical bents with circular columns, plastic hinging shears and moments for the longitudinal direction can be expected to be within the range of plastic shears and moments associated with hinging in the transverse direction. For a column section with different properties in each of its orthogonal directions, that may not be the case.

**Design Step
7.1****Determine Nonseismic Forces****Design Step
7.1.1****Determine Dead Load Forces**

The dead load forces obtained from a previously performed static analysis are summarized in Table 8. Because this is a continuous post-tensioned bridge,

Design Step
7.1.1
(continued)

the secondary effects due to post-tensioning should also be included, though they do not appear in this example.

Table 8
Dead Load Forces

| Support/Location | | Forces and Moments - Dead Load | | | | |
|------------------------|--------|--------------------------------|-----------------|----------------|-----------------|--------------|
| | | Longitudinal (1) | | Transverse (2) | | Axial (kips) |
| | | Shear (kips) | Moment (kip-ft) | Shear (kips) | Moment (kip-ft) | |
| Abutment A | | 0 | 0 | 0 | 0 | 504 |
| Bent 1 Left Column | Top | 3 | 70 | 35 | 810 | 862 |
| | Bottom | 3 | 0 | 0 | 0 | 900 |
| Bent 1 Right Column | Top | 3 | 70 | 35 | 810 | 859 |
| | Bottom | 3 | 0 | 35 | 0 | 897 |
| Bent 2 Left Column | Top | 3 | 70 | 35 | 810 | 859 |
| | Bottom | 3 | 0 | 35 | 0 | 897 |
| Bent 2 Right Column | Top | 3 | 70 | 35 | 810 | 862 |
| | Bottom | 3 | 0 | 35 | 0 | 900 |
| Abutment B | | 0 | 0 | 0 | 0 | 504 |

(1) For bent columns, the longitudinal direction is 90 degrees to the plane of the bent frame.

(2) For bent columns, the transverse direction is parallel to the plane of the bent frame.

Design Step
7.2**Determine Seismic Forces**Design Step
7.2.1**Summary of Elastic Seismic Forces**

The Multimode Spectral Method results are used to determine the modified design forces.

A summary of the full elastic seismic forces for an earthquake along each of the principal axes (both longitudinal and transverse) is shown in Table 9, which contains results from Tables 5 and 7. Because forces at both abutments are identical, and forces at both bents are identical (but opposite hand)

Design Step
7.2.1
(continued)

in Tables 5 and 7, results from one abutment and one bent (with left and right columns) are reported here.

Table 9
Full Elastic Seismic Forces

| Seismic Direction | Location | Full Elastic Seismic Forces and Moments | | | | |
|-------------------|---------------|---|-----------------|----------------|-----------------|--------------|
| | | Longitudinal (1) | | Transverse (2) | | Axial (kips) |
| | | Shear (kips) | Moment (kip-ft) | Shear (kips) | Moment (kip-ft) | |
| EQlong | Abutment | 0 | 0 | 71 | 0 | 115 |
| | Left Col Top | 474 | 9,477 | 288 | 5,754 | 209 |
| | Left Col Bot | 474 | 0 | 288 | 0 | 209 |
| | Right Col Top | 478 | 9,562 | 288 | 5,754 | 37 |
| | Right Col Bot | 478 | 0 | 288 | 0 | 37 |
| EQtrans | Abutment | 0 | 0 | 781 | 0 | 38 |
| | Left Col Top | 99 | 1,981 | 274 | 5,481 | 492 |
| | Left Col Bot | 99 | 0 | 274 | 0 | 492 |
| | Right Col Top | 148 | 2,951 | 274 | 5,481 | 548 |
| | Right Col Bot | 148 | 0 | 274 | 0 | 548 |

(1) For bent columns, the longitudinal direction is 90 degrees to the plane of the bent frame.

(2) For bent columns, the transverse direction is parallel to the plane of the bent frame.

Design Step
7.2.2

Combination of Orthogonal Seismic Forces
[Division I-A, Article 3.9]

Before the seismic forces are combined with the dead load to create the modified design forces, the seismic forces along the two principal axes must be combined in load combinations LC1 and LC2 (without dead load). See Table 10 for a summary of these forces.

The definition of LC1 and LC2 is as follows, from Division I-A, Article 3.9.

LC1 = 100 percent of the Longitudinal Analysis Results + 30 percent of the Transverse Analysis Results

Design Step
7.2.2

LC2 = 30 percent of the Longitudinal Analysis Results + 100 percent of the Transverse Analysis Results

Note that all of the forces in LC1 and LC2 are the full elastic seismic forces.

Table 10
Orthogonal Seismic Force Combinations
LC1 and LC2

$$LC1 = 1.0 \cdot EQ_{long} + 0.3 \cdot EQ_{trans}$$

$$LC2 = 0.3 \cdot EQ_{long} + 1.0 \cdot EQ_{trans}$$

| Location | Load Case | Forces and Moments | | | | |
|---------------------|-----------|--------------------|-----------------|----------------|-----------------|--------------|
| | | Longitudinal (1) | | Transverse (2) | | Axial (kips) |
| | | Shear (kips) | Moment (kip-ft) | Shear (kips) | Moment (kip-ft) | |
| Abutment | LC1 | 0 | 0 | 305 | 0 | 126 |
| | LC2 | 0 | 0 | 802 | 0 | 73 |
| Left Column Top | LC1 | 504 | 10,071 | 370 | 7,398 | 357 |
| | LC2 | 241 | 4,824 | 360 | 7,207 | 555 |
| Left Column Bottom | LC1 | 504 | 0 | 370 | 0 | 357 |
| | LC2 | 241 | 0 | 360 | 0 | 555 |
| Right Column Top | LC1 | 522 | 10,447 | 370 | 7,398 | 201 |
| | LC2 | 291 | 5,820 | 360 | 7,207 | 559 |
| Right Column Bottom | LC1 | 522 | 0 | 370 | 0 | 201 |
| | LC2 | 291 | 0 | 360 | 0 | 559 |

(1) For bent columns, the longitudinal direction is 90 degrees to the plane of the bent frame.

(2) For bent columns, the transverse direction is parallel to the plane of the bent frame.

These forces are combinations using the full elastic seismic results, and have not yet been modified by the R Factor. (The R Factor is discussed in Design Step 2.6.) At this stage, the designer could elect to compare these forces (as Group VII when combined with dead load) with other load cases for the substructure design, to see if they control. If other load cases, such as stream flow or temperature, control the seismic design forces given in Table 10 may be used without further modification.

Design Step
7.2.2

A sample calculation derives the longitudinal right column top moment for LC1 as follows.

$$M = (1.0 * M_{EQlong}) + (0.3 * M_{EQtrans})$$

$$M = (1.0 * 9,561) + (0.3 * 2,951) = 10,447 \text{ k-ft}$$

All other forces in Table 10 are calculated similarly.

Design Step
7.3**Determine Modified Design Forces**
[Division I-A, Article 7.2.1(A)]

For design of members and foundations, the modified design forces in Table 10 replace the Group VII load combination found in Table 3.22.1A of Division I. These modified forces, along with the forces associated with plastic hinging in the columns, are used in the seismic design of the various components of the bridge.

The modified design forces use the R Factor in modifying the elastic seismic forces. Viewing the entire bridge as a system, the intent of the Specification is to force the plastic hinging to occur in the columns. Therefore, inelastic action is prevented from occurring in the cap beam or foundation, where damage may not be detectable by visual inspection and may be very difficult or costly to repair.

Design Step
7.3.1**Modified Design Forces for Structural Members and Connections**

The Specification makes a distinction between the modified forces for members and connections verses the modified forces for foundations calculated in Design Step 7.3.2. Use Equation 7-1 in Division I-A to calculate the maximum forces in each member.

$$\text{Group Load} = 1.0 (D + B + SF + E + EQM) \quad \begin{array}{l} \text{Division I-A} \\ \text{Eqn (7-1)} \end{array}$$

For this example, forces B, SF, and E are assumed zero, and only D and EQM forces are combined. The equation reduces to

$$\text{Group Load} = 1.0 (D + EQM)$$

where

$$EQM = (\text{LC1 or LC2 forces}) \text{ divided by } R$$

Design Step
7.3.1
(continued)*a) Response Modification Reduction Factor, R*
[Division I-A, Article 3.7, Table 3]

The R Factor is used to modify EQM, and applies to specific forces for specific members. The question of which R value to apply to each member is a critical one. For bridges in SPC C and D, R is never applied directly to the axial load, nor to the shear force in a column; although when the plastic hinging forces in the column are calculated in Design Step 7.4, it will be shown how both axial and shear forces are affected indirectly.

In this example, R reduces the seismic column moments, but increases the seismic lateral shear force on the connection of the superstructure to the abutment. Recall that R was determined in Design Step 2.6. A summary of the R values used to modify EQM is presented below.

R = 5.0 For moments in multiple-column bents
R = 0.8 For shear and axial connection force of superstructure to abutment
R = 1.0 For connection of column to superstructure or foundation

b) Calculate the Modified Design Forces with EQM

Once the R values have been established, the value of EQM can be calculated.

Table 11 summarizes the modified design forces. The R values used for specific forces are shown.

For example, the longitudinal right column top moment using LC1 is derived as follows.

$$M = (D + EQ/R)$$
$$M = (70 + 10,447/5) = 2159 \text{ k-ft}$$

All other forces in Table 11 are calculated similarly.

Design Step
7.3.1
(continued)

Table 11
Modified Design Forces
for Structural Members and Connections

Group LC1 = 1.0*Dead Load + 1.0*LC1/R

Group LC2 = 1.0*Dead Load + 1.0*LC2/R

| | | |
|-----|-----|-------------------------------|
| R = | 5.0 | Column Moments |
| R = | 0.8 | Abutment Shear Forces |
| R = | 1.0 | Axial and Column Shear Forces |

| | | Modified Design Forces | | | | | |
|------------------|-----------|------------------------|-----------------|----------------|-----------------|-------------------|-------------------|
| Location | Load Case | Longitudinal (1) | | Transverse (2) | | Axial Pmin (kips) | Axial Pmax (kips) |
| | | Shear (kips) | Moment (kip-ft) | Shear (kips) | Moment (kip-ft) | | |
| Abutment | LC1 | 0 | 0 | 382 | 0 | 378 | 630 |
| | LC2 | 0 | 0 | 1,003 | 0 | 432 | 577 |
| Left Column Top | LC1 | 507 | 2,084 | 405 | 2,290 | 505 | 1,219 |
| | LC2 | 244 | 1,035 | 395 | 2,251 | 307 | 1,417 |
| Right Column Top | LC1 | 525 | 2,159 | 405 | 2,290 | 658 | 1,060 |
| | LC2 | 294 | 1,234 | 395 | 2,251 | 300 | 1,418 |

(1) For bent columns, the longitudinal direction is 90 degrees to the plane of the bent frame.

(2) For bent columns, the transverse direction is parallel to the plane of the bent frame.

Design Step
7.3.2

Modified Design Forces for Foundations
[Division I-A, Article 7.2.1(B)]

Use Equation 7-2 in Division I-A to calculate the maximum forces in the bent column foundations.

$$\text{Group Load} = 1.0 (D + B + SF + E + EQF) \quad \text{Division I-A Eqn (7-2)}$$

For this example, forces B, SF, and E are assumed zero, and only D and EQF forces are combined. The equation reduces to

$$\text{Group Load} = 1.0 (D + EQF)$$

where

$$EQF = (\text{LC1 or LC2 forces at the bottom of the bent columns}) \text{ divided by } R$$

Design Step
7.3.2
(continued)

a) Recall the Response Modification Reduction Factor, R
[Division I-A, Article 7.2.1(B) and Article 3.7, Table 3]

$R = 1$ For calculating the modified design forces in the foundation

b) Calculate the Modified Design Forces with EQF

Table 12 summarizes the values of EQF modified design forces for the bent columns. (The abutments will not be designed for this example.)

For example, the longitudinal right column bottom shear using LC1 is derived as follows.

$$V = (D + EQ/R)$$

$$V = (3 + 522/1.0) = 525 \text{ kips}$$

All other forces in Table 12 are calculated similarly.

Table 12
Modified Design Forces for Foundations

Group LC1 = 1.0*Dead Load + 1.0*LC1/R
Group LC2 = 1.0*Dead Load + 1.0*LC2/R

$R = 1.0$ Foundation Moments

| | | Modified Design Forces for Foundations | | | | | |
|---------------------|-----------|--|-----------------|----------------|-----------------|------------|------------|
| Location | Load Case | Longitudinal (1) | | Transverse (2) | | Axial Pmin | Axial Pmax |
| | | Shear (kips) | Moment (kip-ft) | Shear (kips) | Moment (kip-ft) | (kips) | (kips) |
| Left Column Bottom | LC1 | 507 | 0 | 370 | 0 | 543 | 1,257 |
| | LC2 | 244 | 0 | 360 | 0 | 345 | 1,455 |
| Right Column Bottom | LC1 | 525 | 0 | 405 | 0 | 696 | 1,098 |
| | LC2 | 294 | 0 | 395 | 0 | 338 | 1,456 |

(1) For bent columns, the longitudinal direction is 90 degrees to the plane of the bent frame.

(2) For bent columns, the transverse direction is parallel to the plane of the bent frame.

**Design Step
7.4****Plastic Hinging Forces**

Before the forces due to plastic hinging can be calculated, the longitudinal column reinforcement must be determined.

**Design Step
7.4.1****Preliminary Column, Pier, or Bent Design**
[Division I-A, Article 7.6.2 (A and B)]

The previous design step derived the Seismic Group Loads to be used in the seismic design of the bridge. This design step focuses on the preliminary design of the bent columns. Both the longitudinal and transverse reinforcement in the columns will be designed for the seismic load case.

Depending on the Seismic Performance Category, the column may be controlled by dead load combined with seismic loads, or by other loads, such as live loads or stream flow loads. This example deals only with the seismic load combinations.

Division I-A, Article 7.6.2(B) mentions moment magnification in the columns. Currently, the magnitude and method of computing magnified moments for seismic loadings are under review by AASHTO. Some engineers refer to the Division I, Article 8.16.5; others feel that moment magnification during seismic loadings should not be included for concrete columns.

For concrete columns, one approximation to account for a magnified moment is to multiply the maximum axial load in the column by the full elastic deflection at the column top. This additional moment is then added to the primary seismic moment before designing the reinforcement. In *Bridge Design Specifications* (1995), Caltrans neglects slenderness effects in reinforced concrete compression members when proportioning them for Group VII load combination. In this example, magnification has been neglected.

Basic substructure data for column design is

$f_c := 4000 \cdot \text{psi}$ Concrete compressive strength

Circular column section properties are

$b_w := 4.0 \cdot \text{ft}$ Diameter of column

Design Step
7.4.1
(continued)

$$A_g := \frac{\pi \cdot b_w^2}{4}$$

$$A_g = 12.57 \cdot \text{ft}^2$$

Cross-sectional area of column

a) Summary of Modified Design Forces for the Preliminary Column Design
[Division I-A, Article 7.2.3]

Below is a summary of the controlling Modified Design Forces for the preliminary column design, taken from Table 11 for the Right Column Top. On inspection, it can be seen that Group LC1 controls. Note that the transverse direction is in the plane of the bent frame, and the longitudinal direction is 90 degrees to the plane of the bent frame.

$$P_{\max_U} := 1060 \cdot \text{kip}$$

Maximum axial load

$$P_{\min_U} := 658 \cdot \text{kip}$$

Minimum axial load

$$M_L := 2159 \cdot \text{kip} \cdot \text{ft}$$

Longitudinal moment

$$M_T := 2290 \cdot \text{kip} \cdot \text{ft}$$

Transverse moment

For a circular column, the modified biaxial bending moment can be converted to a moment about a single axis by calculating that

$$M_U := \sqrt{M_L^2 + M_T^2}$$

$$M_U = 3147 \cdot \text{kip} \cdot \text{ft}$$

The above forces will be used in the design of the longitudinal reinforcement in the column.

b) Calculate the ϕ Factor for Both the Maximum and Minimum Axial Load for Use with M_U

Division I-A, 7.6.2(B) specifies a ϕ factor that varies from 0.9 to 0.5 depending on the axial load. Because the lower bound of ϕ equals 0.5

Design Step
7.4.1
(continued)

when the axial stress exceeds $0.2 \cdot f'_c$, column charts that use a lower bound of ϕ equal to 0.7 are not applicable.

For this sample problem, the nominal capacity of the column is plotted as P_n , M_n without a ϕ factor. Therefore, the factored loads must be divided by ϕ before being plotted on the capacity chart. See Figure 18 for the general curves for design capacity, nominal capacity, and plastic capacity.

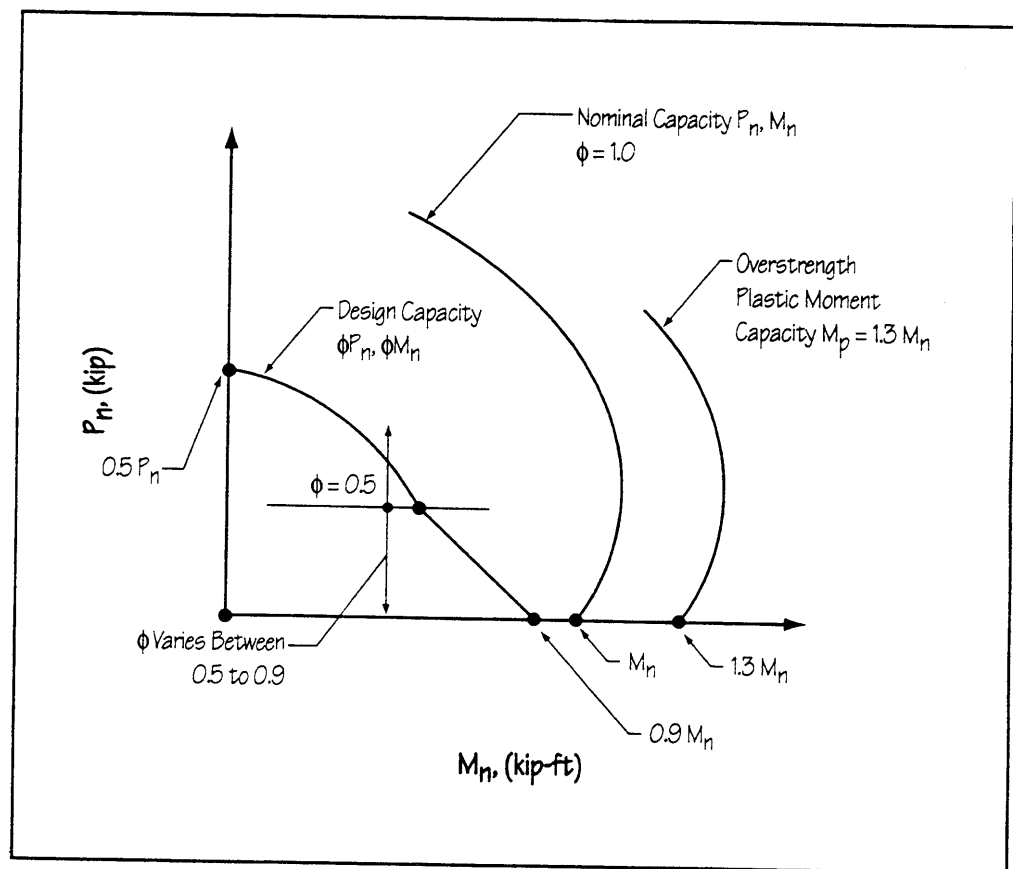


Figure 18 — Column Interaction Curves, General

Compute the maximum and minimum column axial stresses, σ_{\max} and σ_{\min} .

$$P_{\max_u} = 1060 \cdot \text{kip}$$

$$P_{\min_u} = 658 \cdot \text{kip}$$

$$A_g = 12.57 \cdot \text{ft}^2$$

Gross area of the column
cross section

Design Step
7.4.1
(continued)

$$\sigma_{\max} := \frac{P_{\max_u}}{A_g}$$

$$\sigma_{\max} = 586 \cdot \text{psi}$$

$$\sigma_{\min} := \frac{P_{\min_u}}{A_g}$$

$$\sigma_{\min} = 364 \cdot \text{psi}$$

Per Division I-A, Article 7.6.2(B), $\phi = 0.5$ when the maximum axial stress exceeds $0.2 \cdot f'_c$, and varies linearly from 0.5 to 0.9 for stresses less than $0.2 \cdot f'_c$. See Figure 19.

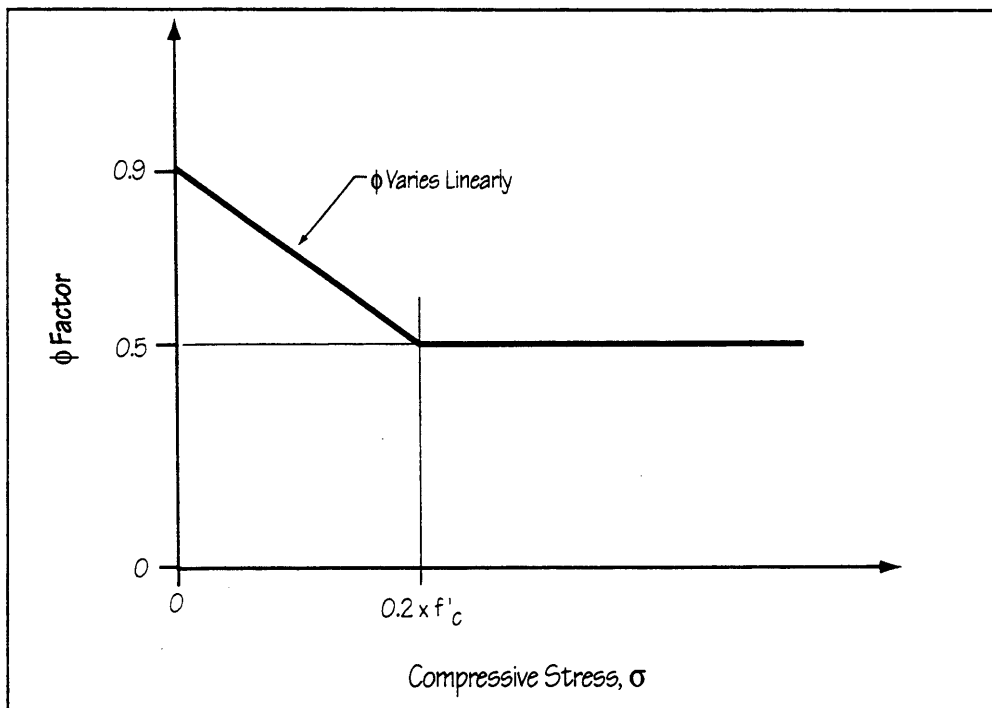


Figure 19 — ϕ Factor versus Compressive Stress

$$f'_c = 4000 \cdot \text{psi}$$

$$0.2 \cdot f'_c = 800 \cdot \text{psi}$$

Because the column axial stresses σ_{\max} and σ_{\min} are less than $0.2 \cdot f'_c$, both ϕ_{\max} and ϕ_{\min} are between 0.9 and 0.5.

Design Step
7.4.1
(continued)

ϕ_{\max} is associated with the maximum axial load.

$$\phi_{\max} := 0.9 - \frac{\sigma_{\max}}{0.2 \cdot f_c} \cdot (0.9 - 0.5) \quad \phi_{\max} = 0.61$$

ϕ_{\min} is associated with the minimum axial load.

$$\phi_{\min} := 0.9 - \frac{\sigma_{\min}}{0.2 \cdot f_c} \cdot (0.9 - 0.5) \quad \phi_{\min} = 0.72$$

c) Calculate the P_u / ϕ and M_u / ϕ forces and plot on the P_n, M_n Column Capacity Curve

The application of the understrength factor ϕ in this example is based on Article 8.16.4.2.4 of Division I. While the seismic provisions of Division I-A provide alternate values for the ϕ factor for SPC C and D as computed in Part (b) of this step, the provisions imply that the application of the factor is identical to that specified in Division I. In Division I, the nominal strength interaction diagram is reduced “radially” by the ϕ factor. This fact means that both the axial strength P_n and the flexural strength M_n are reduced by the same ϕ factor to obtain the design strengths ϕP_n and ϕM_n , as shown qualitatively in Figure 18.

Some designers prefer to apply the ϕ factor only to the nominal moment strengths when the axial loads are in the tension controlled region (below the balance point of the interaction diagram). Because the moment strength increases with axial load, not applying the ϕ factor to the axial load effectively implies a nominal flexural strength that is less than if ϕ had been used with the axial load. In design, this practice will result in more reinforcement than would result from use of the standard Division I practice. Stated differently, the practice is similar to using a slightly smaller R Factor for determining the required moment strengths.

The practice of using ϕ only with the moment is more conservative than the standard practice, although probably only slightly so. However, the practice is not consistent with the philosophy of using the ϕ factor, in which ϕ is intended to account for potential material, member, or system understrength. In this example, ϕ factors are applied to both moments and axial loads.

Design Step
7.4.1
(continued)

From Part (a) of this step

$$M_U = 3147 \cdot \text{kip} \cdot \text{ft}$$

Controlling forces associated with maximum axial load (plotted as Point No. 1 in Figure 20) are

$$P_{\max_U} = 1060 \cdot \text{kip}$$

$$\frac{P_{\max_U}}{\phi_{\max}} = 1746 \cdot \text{kip}$$

$$\frac{M_U}{\phi_{\max}} = 5184 \cdot \text{kip} \cdot \text{ft}$$

Controlling forces associated with minimum axial load (plotted as Point No. 2 in Figure 20) are

$$P_{\min_U} = 658 \cdot \text{kip}$$

$$\frac{P_{\min_U}}{\phi_{\min}} = 916 \cdot \text{kip}$$

$$\frac{M_U}{\phi_{\min}} = 4382 \cdot \text{kip} \cdot \text{ft}$$

Try a 48-inch-diameter column with 34 #11 bars (2.93 percent reinforcement).

The column capacity curve in Figure 20 graphs the nominal capacity of P_n versus M_n for a 48-inch-diameter column with a reinforcement ratio of 2.93 percent. Note that the ϕ factor used for this curve is 1.0 to obtain an interaction diagram for nominal capacity.

The forces for the two load cases calculated above are plotted on the curve in Figure 20.

Design Step
7.4.1
(continued)

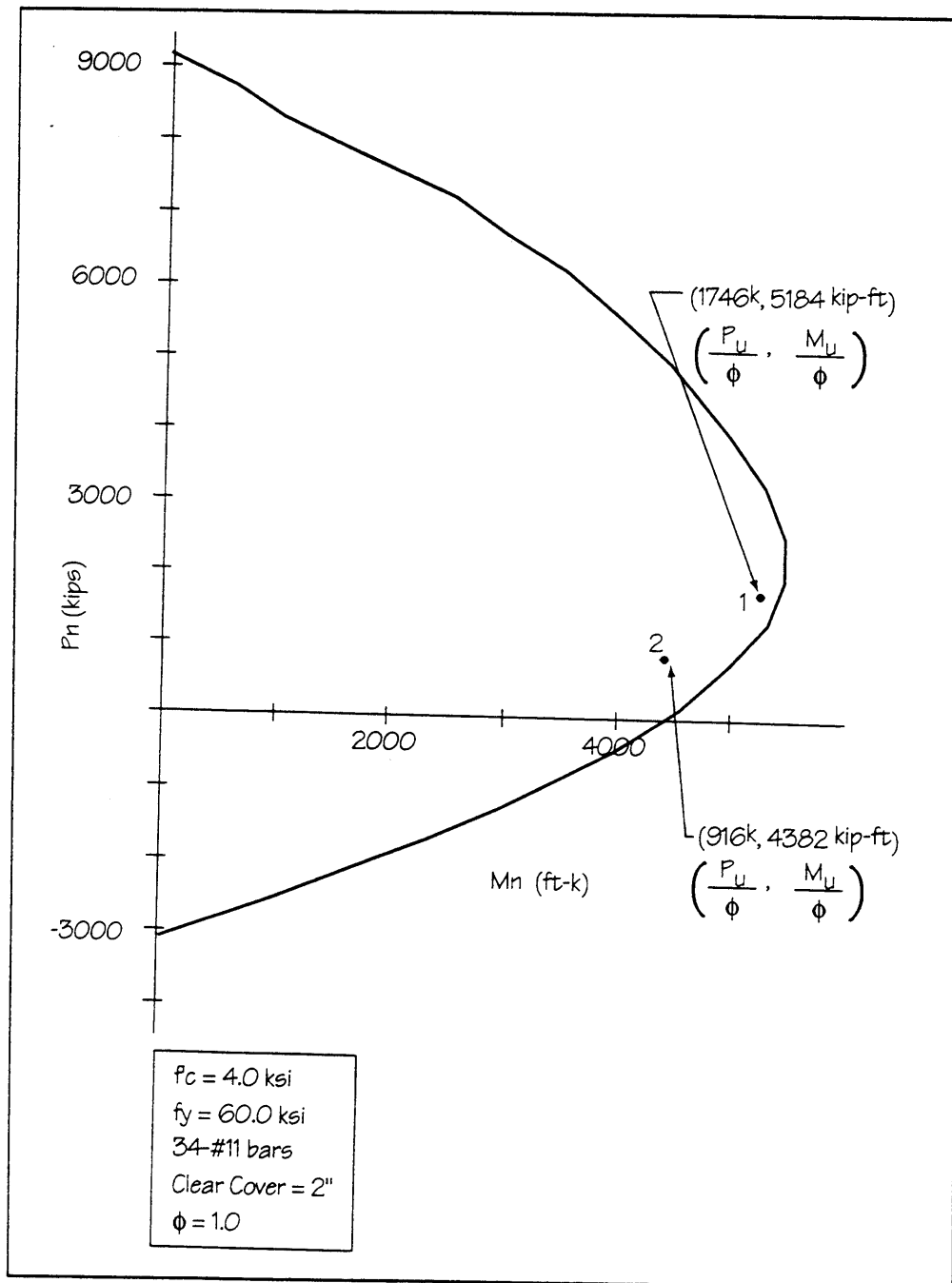


Figure 20 — Column Interaction Diagram
with Factored Loads

Design Step
7.4.1
(continued)*d) Select the Preliminary Longitudinal Reinforcement*

Because the forces for both load cases plot inside the capacity curve for a column with 34 #11 bars, this reinforcement is sufficient. The 2.93 percent reinforcement provided is between 1 and 6 percent allowed per Division I-A, Article 7.6.2(A).

Therefore, the 48-inch-diameter column with 34 #11 bars is selected.

Design Step
7.4.2Forces Resulting from Plastic Hinging
[Division I-A, Article 7.2.2(B)]

As discussed in the introduction of Design Step 7, plastic hinging forces will be computed for directions perpendicular to the plane of the bent frame (longitudinal) and parallel to the plane of the bent frame (transverse).

After the column cross-sectional dimensions and longitudinal reinforcement have been determined, the forces resulting from plastic hinging in the columns must be computed before the spiral transverse reinforcement can be established. The plastic hinging forces at the top and bottom of the column are also necessary to establish the connection forces transferred to the cap beam and foundation. See Figure 21.

Plastic hinging forces are calculated for bridges in Seismic Categories C and D. The importance of using these forces versus having to use the full elastic forces in design cannot be overemphasized. When the columns are allowed to form a plastic hinge at the top and bottom of the column during a design level earthquake, the hinge acts as a “fuse” to limit the forces transferred to the bridge components. This fuse limits the forces transferred into both the footing at the bottom of the column and the cap beam at the top of the column. It also limits the shear (and sometimes the axial force) that the column has to be designed for. The Specification does allow the above components to be designed without calculating the plastic hinging forces. Typically, plastic hinging forces are less than full elastic seismic forces in SPC C and D.

Article 7.2.2(A) of Division I-A covers plastic hinging perpendicular to the plane of the bent. The shear associated with plastic hinging is the sum of the overstrength plastic moments at the top and bottom of the column, divided by the distance between column hinging points (in this example, the clear height of the column). The bases of the columns are pinned at

Design Step
7.4.2
(continued)

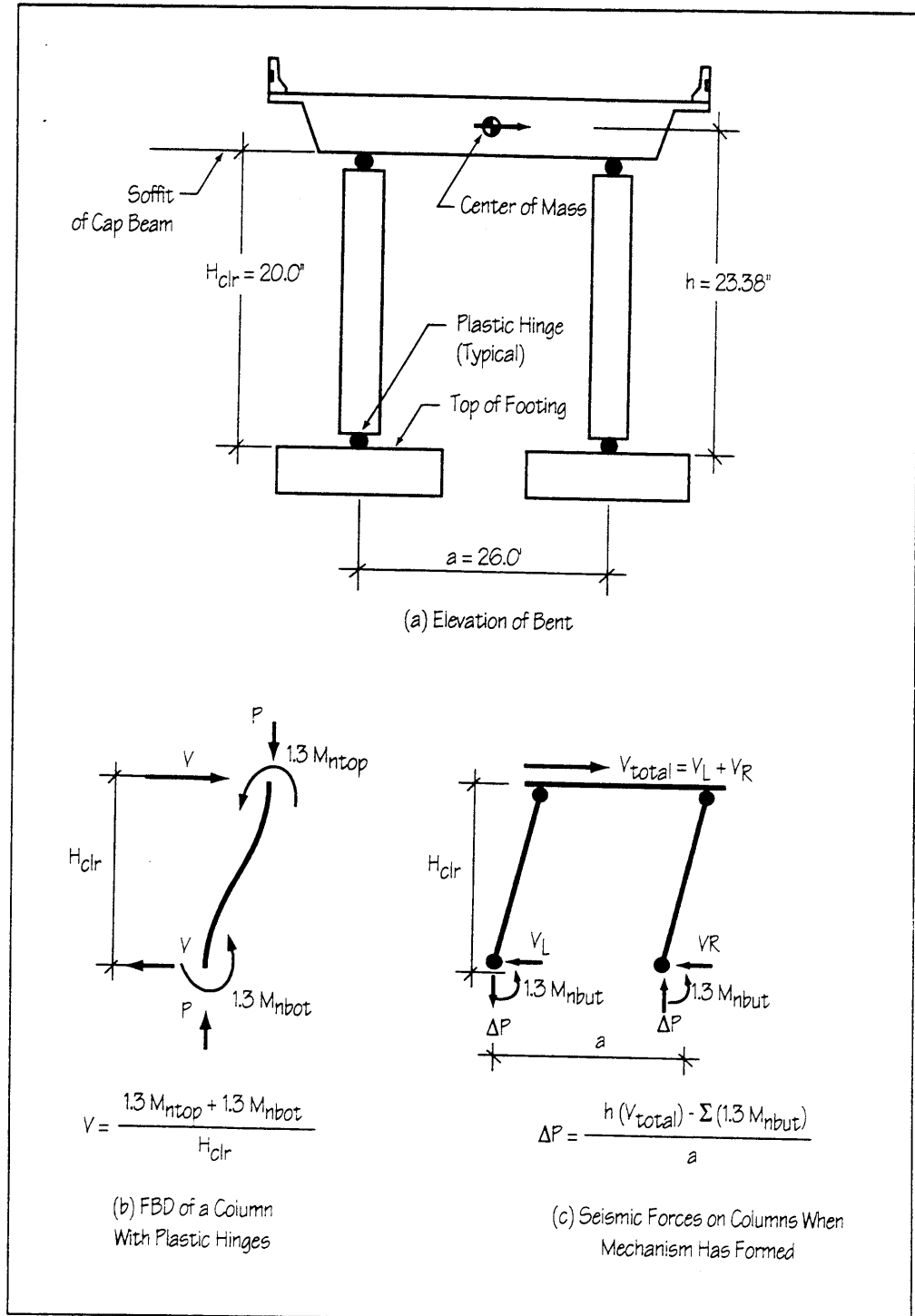


Figure 21 — Plastic Hinging Mechanism in Bent

Design Step
7.4.2
(continued)

the top of the footing. For the analysis, the node at the column base was released for rotation in both plan directions with no moment capacity at the column base. Though the design specifies a pinned connection, there will in fact be some nominal moment capacity at the base. For computing plastic hinging forces, a moment capacity equal to 10 percent of the column top moment capacity will be assumed at the column base. The shears for plastic hinging perpendicular to the plane of the bent are computed as follows.

From Figure 22, for an axial load corresponding to the dead load ($P_n = 859$ kips), the moment capacity at the top of the column is

$$M_{ntop} = 5000 \text{ k-ft} \qquad 1.3 * M_{ntop} = 6500 \text{ k-ft}$$

Using 10 percent of the column top capacity at the bottom of the column (assumed capacity), the moment capacity at the bottom of the column is

$$M_{nbot} = 500 \text{ k-ft} \qquad 1.3 * M_{nbot} = 650 \text{ k-ft}$$

Calculate the shear force associated with plastic hinging by taking the sum of the overstrength plastic moment capacities at the top and bottom of the column, divided by the column clear height.

$$V = (1.3 * M_{ntop} + 1.3 * M_{nbot}) / H = (6500 + 650) / 20 = 358 \text{ kips}$$

Article 7.2.2(B) of Division I-A gives four steps to calculate the plastic hinging forces for bents with two or more columns in the plane of the bent. In this design example, these steps are broken down into 11 intermediate steps (1a through 4f) in order to add further clarification. Table 13 is a tool to tabulate the results of each step. Each step has a brief narrative and an example force.

For this example, the nominal capacities for the column are taken from Figure 22 using a ϕ factor of 1.0, and then an overstrength plastic capacity is computed by multiplying by 1.3. Alternately, an interaction diagram plotted for a ϕ factor of 1.3 could be used directly, thus eliminating steps 1b and 4b below.

Given: The preliminary design of the column established the longitudinal reinforcement (34 #11 bars in a 48-inch-diameter column). The column capacity curve of P_n versus M_n will again be used in this section. The axial column dead load is 859 kips.

Design Step
7.4.2
(continued)

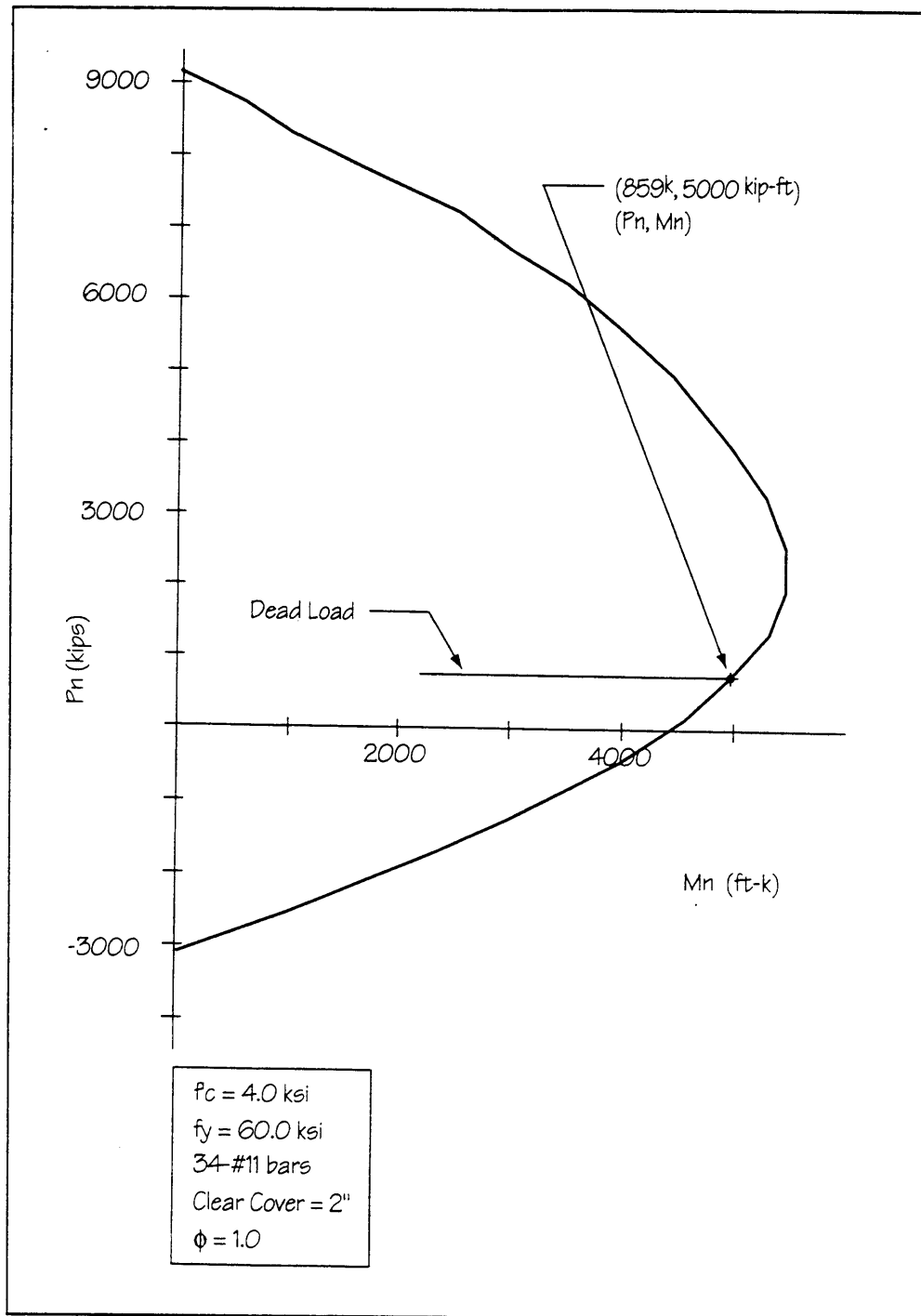


Figure 22 — Column Interaction Diagram
Step 1a Nominal Moment Capacity

Design Step 7 — Determine Design Forces

Design Example No. 4 Bridge with Two-Column Bents

Design Step
7.4.2
(continued)

Table 13
Plastic Hinging Forces in Columns

$$\begin{aligned} h &= 23.38 \text{ ft} & V &= 1.3 \cdot M_{n\text{top}} + 1.3 \cdot M_{n\text{bot}} / H & \text{Eq. (1)} \\ H &= 20.00 \text{ ft} & \Delta P &= [\Sigma(V) \cdot h - \Sigma(1.3 \cdot M_{n\text{bot}})] / a & \text{Eq. (2)} \\ a &= 26.00 \text{ ft} & \text{Adjusted } P &= P \pm \Delta P & \text{Eq. (3)} \end{aligned}$$

| Step | Column Moments (kip-ft) | | | | Column Shears (kips) | | | ΔP (kips) | Column Axial Forces | |
|-------|---|------|--------|-------|----------------------|------|-------|----------------------|---------------------|-------|
| | M | Left | | Right | | Left | Right | Total | P (kips) | |
| | | Top | Bottom | Top | Bottom | | | | Left | Right |
| Given | | | | | | | | | 859 | 859 |
| 1a | Mn | 5000 | 500 | 5000 | 500 | | | | | |
| 1b | 1.3*Mn | 6500 | 650 | 6500 | 650 | | | | | |
| 2 | | | | | | 358 | 358 | 715 | | |
| 3a | | | | | | | | | 593 | |
| 3b | | | | | | | | | 266 | 1452 |
| 4a | Mn | 4600 | 460 | 5300 | 530 | | | | | |
| 4b | 1.3*Mn | 5980 | 598 | 6890 | 689 | | | | | |
| 4c | | | | | | 329 | 379 | 708 | | |
| 4d | | | | | | | | | 587 | |
| 4e | | | | | | | | | 272 | 1446 |
| 4f | Difference in Column Plastic Shear Capacities (%) = $[(715 - 708) / 715] \cdot 100 = 0.98\% < 10\%$ Okay (from Steps 2 and 4c) | | | | | | | | | |

The forces in the individual columns in the plane of the bent associated with plastic hinging of the bent columns are

| | | | |
|----------------------------|---------------|---------------------------------------|-------------|
| <u>Minimum Axial Load:</u> | a) Axial Load | $P_{\min_p} =$ | 272 kips |
| | b) Moment | $M_{\min_p} = 1.3 \cdot M_{\min_n} =$ | 5980 kip-ft |
| | c) Shear | $V_{\min_p} =$ | 329 kips |
| <u>Maximum Axial Load:</u> | a) Axial Load | $P_{\max_p} =$ | 1446 kips |
| | b) Moment | $M_{\max_p} = 1.3 \cdot M_{\max_n} =$ | 6890 kip-ft |
| | c) Shear | $V_{\max_p} =$ | 379 kips |

Design Step
7.4.2
(continued)

Step 1. For an axial load corresponding to the dead load, determine the moment capacity, M_n , at the top of each column in the bent, using Figure 22. Assume 10 percent of the column top capacity for the hinge capacity at the bottom of the column.

$$M_{ntop} = 5000 \text{ k-ft}$$

$$M_{nbot} = 500 \text{ k-ft} \quad (\text{assumed})$$

Step 1b. Calculate the column overstrength plastic moment capacities $1.3 \cdot M_n$ for each column.

$$1.3 \cdot M_{ntop} = 6500 \text{ k-ft}$$

$$1.3 \cdot M_{nbot} = 650 \text{ k-ft}$$

Step 2. Use Equation (1) in Table 13 to calculate the corresponding column shear force, V , in each column.

$$V = (1.3 \cdot M_{ntop} + 1.3 \cdot M_{nbot}) / H = (6500 + 650) / 20 = 358 \text{ kips}$$

Step 3a. Compute the overturning axial force, ΔP , in the columns, using Equation (2) in Table 13. Note that this is a +/- load because the earthquake is bidirectional.

$$\Delta P = [\Sigma(V) \cdot h - \Sigma(1.3 \cdot M_{nbot})] / a$$

$$\Delta P = [(715 \text{ kips}) \cdot (23.38 \text{ ft}) - (650 \text{ k-ft} + 650 \text{ k-ft})] / (26.0 \text{ ft}) = 593 \text{ kips}$$

Step 3b. Calculate the adjusted axial force in the columns, using Equation (3) in Table 13, which assumes a simple rigid frame.

Example: For the right column, the adjusted P_{max_p} is computed as follows.

$$P_{max_p} = P + \Delta P = 859 + 593 = 1452 \text{ kips}$$

Step 4a. Determine the revised moment capacities, M_n , for each column, using the new axial loads in Step 3b. See Figure 23 for the plot.

Example: For the right column, $M_{ntop} = 5300 \text{ k-ft}$

Again, use 10 percent of M_{ntop} for the bottom of the column.

$$M_{nbot} = 530 \text{ k-ft}$$

Design Step
7.4.2
(continued)

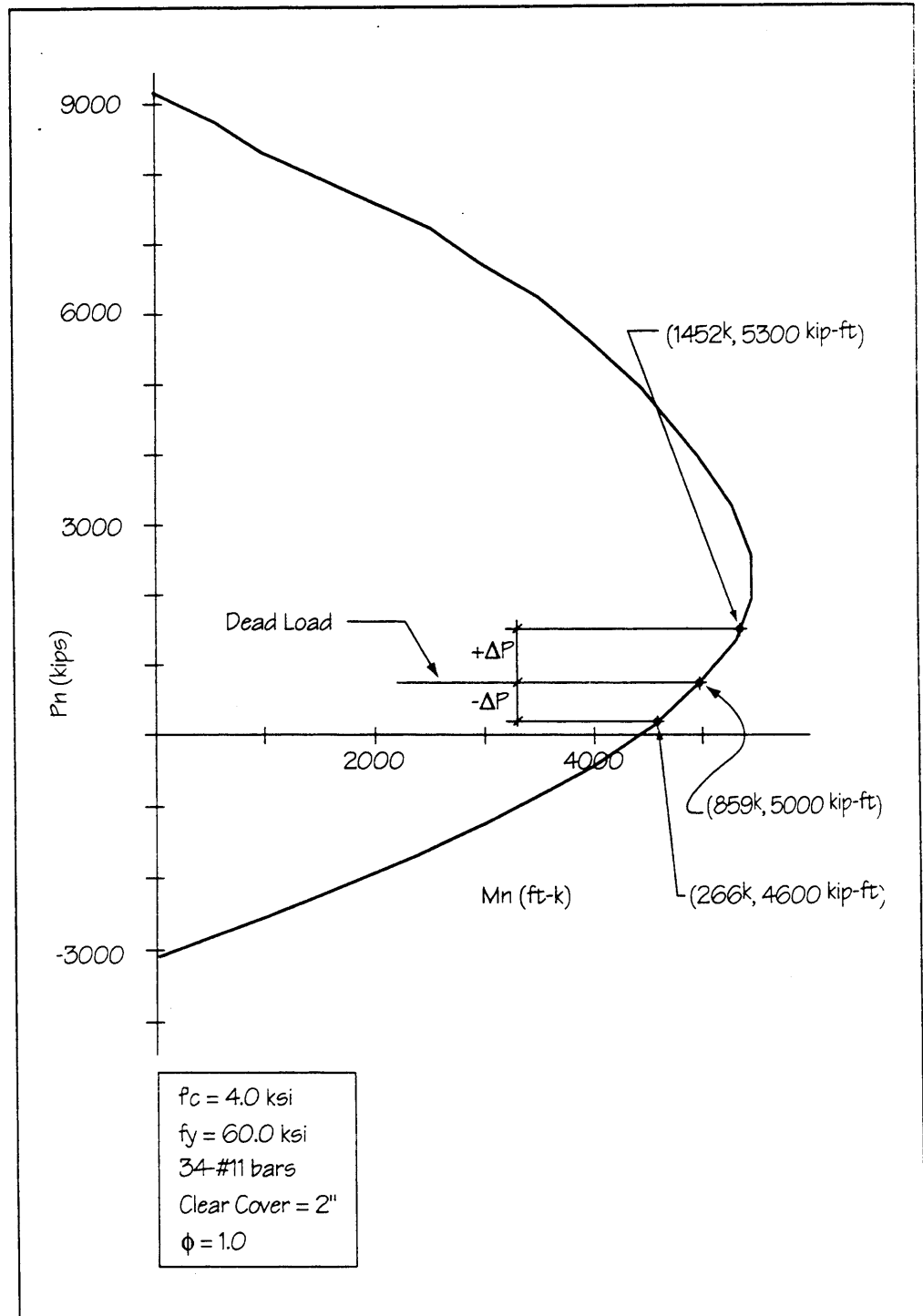


Figure 23 — Column Interaction Diagram
Step 4a Nominal Moment Capacities

Design Step
7.4.2
(continued)

Step 4b. Calculate the revised column overstrength plastic moment capacities, $1.3 \cdot M_n$ for each column top and bottom.

Example: For the right column, $1.3 \cdot M_{n_{top}} = 6890 \text{ k-ft}$

Step 4c. Find the new column shears, V , using the revised values of $1.3 \cdot M_n$ from Step 4b. (Note that these become the final shear forces associated with the overstrength plastic moment capacity of the column for this example.)

Example: For the right column, $V_{max_p} = 379 \text{ kips}$

Step 4d. Calculate the revised overturning forces in the columns, using the revised column shears in Step 4c.

Example: For the right column, $\Delta P = +587 \text{ kips}$

Step 4e. Calculate the revised axial forces in the exterior columns, using the revised overturning forces derived in Step 4d.

Example: For the right column, the adjusted $P_{max_p} = 1446 \text{ kips}$

Step 4f. Because the difference between the total shear forces in Steps 2 and 4c is no greater than 10 percent, it is not necessary to return to Step 4a for another iteration.

A summary of the final plastic hinging forces follows.

| | |
|---|--|
| Forces associated with minimum axial load | $P_{min_p} = 272 \text{ kip}$ $M_{min_p} = 5980 \text{ kip-ft}$ $V_{min_p} = 329 \text{ kip}$ |
| Forces associated with maximum axial load | $P_{max_p} = 1446 \text{ kip}$ $M_{max_p} = 6890 \text{ kip-ft}$ $V_{max_p} = 379 \text{ kip}$ |

Design Step
7.5Design Step
7.5.1

Summary of Forces

Summary of Column Forces

Table 14 summarizes the forces for the bent columns. These forces were derived in Design Steps 7.1 through 7.4. All forces, other than the Modified Foundation Forces, are for the bent column top.

Table 14
Summary of Bent Column Forces

| Force | Load Case | Longitudinal (1) | | Transverse (2) | | Axial | |
|---------------------------------|--------------|------------------|-----------------|----------------|-----------------|-------------|------------------|
| | | Shear (kips) | Moment (kip-ft) | Shear (kips) | Moment (kip-ft) | Pmin (kips) | Pmax or P (kips) |
| Dead Load | Dead Load | 3 | 70 | 35 | 810 | - | 859 |
| Full Elastic Seismic Forces | EQlong | 474 | 9,477 | 288 | 5,754 | - | 37 |
| | EQtrans | 148 | 2,951 | 274 | 5,481 | - | 548 |
| Orthogonal Seismic Combinations | LC1 | 522 | 10,447 | 370 | 7,398 | - | 201 |
| | LC2 | 291 | 5,820 | 360 | 7,207 | - | 559 |
| Modified Member Forces | LC1 | 525 | 2,159 | 405 | 2,290 | 658 | 1,060 |
| | LC2 | 294 | 1,234 | 395 | 2,251 | 300 | 1,418 |
| Modified Foundation Forces | LC1 | 525 | 0 | 405 | 0 | 696 | 1,098 |
| | LC2 | 294 | 0 | 395 | 0 | 338 | 1,456 |
| Plastic Hinging Forces | Longitudinal | 358 | 6,500 | - | - | - | 859 |
| | Trans (Pmax) | - | - | 379 | 6,890 | - | 1,446 |
| | Trans (Pmin) | - | - | 329 | 5,980 | 272 | - |

(1) The longitudinal direction is 90 degrees to the plane of the bent frame.

(2) The transverse direction is parallel to the plane of the bent frame.

DESIGN STEP 8**SUMMARY OF DESIGN FORCES**

This step provides a summary of design forces to be used for Design Steps 10, 11, and 12.

**Design Step
8.1****Column Design Forces**
[Division I-A, Article 7.2.3]

The selection of forces used to design the column reinforcement for seismic loads is outlined in Division I-A, Article 7.2.3. The shear force associated with plastic hinging in the column is generally smaller than the full elastic force in Seismic Performance Categories C and D. For this example, column forces in the transverse direction are used for column design.

a) Axial Forces per Division I-A, Article 7.2.3(a)

Use either the modified forces calculated in Design Step 7.3.1 (same as elastic forces, because $R = 1$ for axial forces) or the plastic hinging forces calculated in Design Step 7.4.2. Both modified forces and plastic hinging values are summarized in Table 14.

Modified Elastic Forces for LC1

$$P = 859 \pm 201 \text{ kips}$$

$$P_{\min U} = 658 \text{ kips} \qquad P_{\max U} = 1060 \text{ kips}$$

Hinging Forces (transverse direction)

$$P_{\min p} = 272 \text{ kips} \qquad P_{\max p} = 1446 \text{ kips}$$

In this design example, the modified axial forces are selected for column design.

b) Moments per Division I-A, Article 7.2.3(b)

Use the modified moments calculated in Design Step 7.4.1(a) and summarized in Table 14. The biaxial moment is

$$M_U = 3147 \text{ kip-ft}$$

**Design Step
8.1
(continued)***c) Shear Forces per Division I-A, Article 7.2.3(c)*

Use either the vector sum (transverse and longitudinal directions) of the modified forces calculated in Design Step 7.3.1 (same as elastic forces, because $R = 1$ for shear forces), or the plastic hinging forces calculated in Design Step 7.4.2. Both modified forces and plastic hinging values are summarized in Table 14.

Elastic (Modified) Forces

$V_L := 525 \cdot \text{kip}$ Shear for the longitudinal direction

$V_T := 405 \cdot \text{kip}$ Shear for the transverse direction

Take the vector sum as the design shear.

$$V_U := \sqrt{V_L^2 + V_T^2} \qquad V_U = 663 \cdot \text{kip}$$

Hinging Forces

$$V_{\max_p} := 379 \cdot \text{kip}$$

As is normally the case, the shear forces associated with plastic hinging are smaller than the elastic forces.

Therefore, use the shear force associated with hinging.

d) Summary of Column Forces

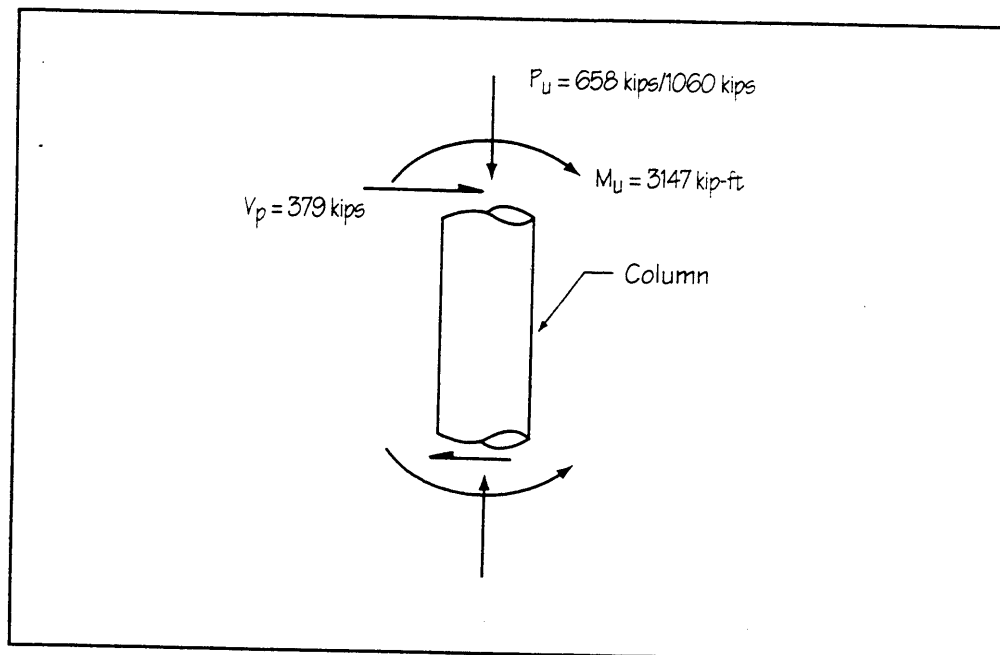
See Figure 24.

$$P_{\min_U} = 658 \text{ kips}$$

$$M_U = 3147 \text{ kip-ft}$$

$$V_{\max_p} = 379 \text{ kips}$$

$$P_{\max_U} = 1060 \text{ kips}$$

**Design Step
8.1
(continued)****Figure 24 — Summary of Column Forces****Design Step
8.2****Pier Design Forces**

Not applicable.

**Design Step
8.3****Connection Design Forces**
[Division I-A, Article 7.2.5]**Design Step
8.3.1****Longitudinal Linkage Connections**
[Division I-A, Article 7.2.5(A)]

There are no linkage connections; therefore, those provisions are not applicable.

**Design Step
8.3.2****Hold-Down Forces**
[Division I-A, Article 7.2.5(B)]

Hold-down devices must be supplied if the vertical force at the supports due to earthquake loading exceeds 50 percent of the dead load reaction. The effects due to torsion in the superstructure are negligible.

The maximum vertical seismic force at the abutment is 126 kips, as specified in Table 10. The dead load reaction from Table 8 is 504 kips.

Design Step
8.3.3

The seismic load is about 25 percent of the dead load case. Thus, no hold-down devices are required.

Column and Pier Connection to Cap Beam
[Division I-A, Article 7.2.5(C)]

For seismic loads, the recommended connection design forces between the column and superstructure cap beam are the forces developed at the top of the column due to column hinging, as determined in Design Step 7.4.2. (The plastic hinging values are summarized in Table 14.) The code allows the designer to use the modified forces calculated in Design Step 7.3.1 if they are smaller, but that choice is not recommended because the connection should be able to develop the plastic hinging capacity of the column to ensure desired ductility.

A summary of the maximum forces due to column hinging follows.

Maximum axial forces due to hinging are

$$P_{\max_p} = 1446 \text{ kips}$$

Maximum moment due to hinging is

$$M_{\max_p} = 1.3 \cdot M_{\max_n} = 6890 \text{ kip-ft}$$

Maximum shear force due to hinging is

$$V_{\max_p} = 379 \text{ kips}$$

Design Step
8.3.4Column Connection to Foundation
[Division I-A, Article 7.2.5(C)]

The recommended connection design forces between the column and foundation are the forces developed at the bottom of the column due to column hinging. These forces are the same as those specified in Design Step 8.3.3, except that the moment at the bottom of the column is taken to be the overstrength capacity of the hinged base, which is assumed to be 10 percent of the column top capacity. The actual pinned connection (see Design Step 10.3.4) will have a minimal overstrength plastic moment capacity which will be included in the foundation design. Forces for the column connection to the foundation are summarized as follows.

Design Step
8.4

$$P_{max_p} = 1446 \text{ kips (without column weight)}$$

$$M_U = 689 \text{ kip-ft (assumed)}$$

$$V_{max_p} = 379 \text{ kips}$$

Cap Beam Design Forces

The cap beam must be designed for the load combinations in Division I, Table 3.22.1A, except for Group VII seismic loads where Division I-A is to be used instead. (This design example does not present the dead or live load forces on the cap beam, only the seismic forces.)

It is recommended that the forces due to plastic hinging at the top of the column be transferred into the cap beam. These forces are summarized in Design Step 8.3.3 above. Figure 25 identifies how these forces are applied to the cap beam. In order to restrict the formation of plastic hinges to the top of column sections, cap beam sections must remain elastic at internal force levels that form plastic hinges in sections at the top of columns.

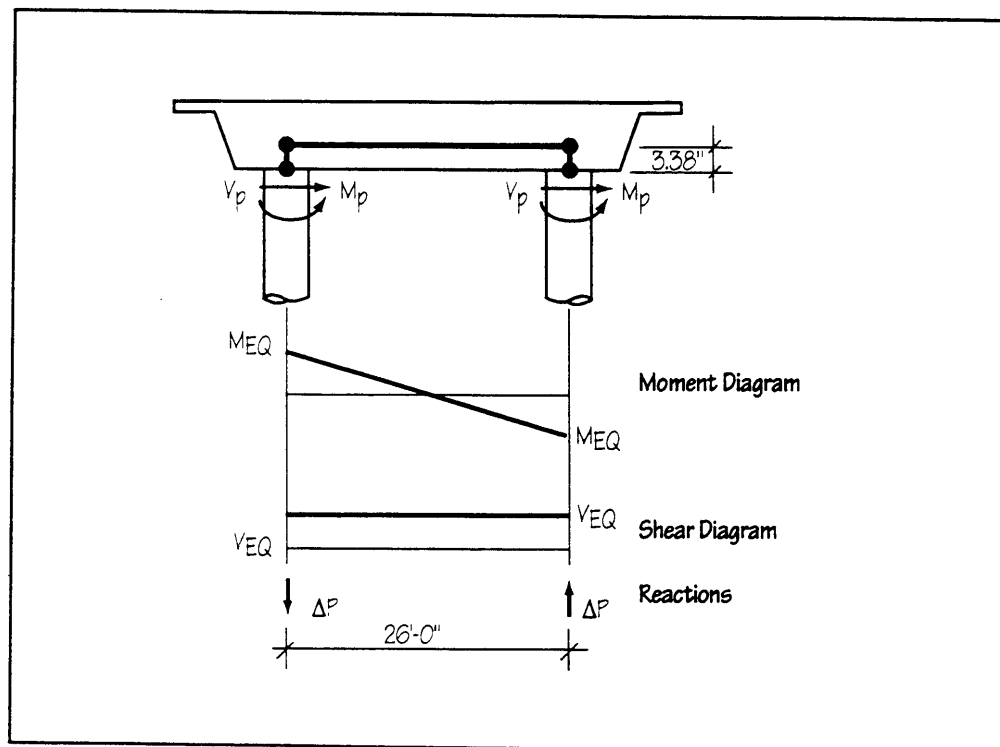


Figure 25 — Plastic Hinging Forces on Cap Beam

Design Step
8.4
(continued)

Because of symmetry, the maximum peak moment in the cap beam occurs at the columns. The plastic moment transferred from the column is M_p .

$$M_{\max_p} = 6890 \text{ kip-ft}$$

The column shear associated with the plastic moment V_p acts at a distance 3.38 feet below the c.g. of the cap beam.

$$V_{\max_p} = 379 \text{ kips}$$

Because of symmetry, the maximum seismic moment in the cap beam due to these column forces occurs over the columns.

$$\begin{aligned} M_{\max_{EQ}} &= M_{\max_p} + (3.38' \cdot V_{\max_p}) \\ &= 6890 + (3.38 \cdot 379) = 8171 \text{ kip-ft} \end{aligned}$$

Therefore, due to these column plastic hinging forces, the seismic shear in the cap beam is constant between the columns.

For a two column bent

$$V_{EQ} = 2M_{EQ} / 26.0 \text{ ft} = 629 \text{ kips}$$

Regarding the plastic hinging forces on the cap beam which occur simultaneously (that is, using the maximum and minimum plastic hinging forces together), the actual shear in the cross beam, V_{EQ} , is equal to ΔP (= 587 kips) for a two-column bent. Using the maximum plastic hinging forces for both sides of the bent is necessary for determining the seismic design moments, because the forces are cycled from one column to the other. However, for determining the cap beam seismic shear, using only the maximum plastic hinging forces, as shown above, is conservative.

Design Step
8.5**Miscellaneous Design Forces**

This design step is not applicable.

**Design Step
8.6****Foundation Design Forces**
[Division I-A, Article 7.2.6]

The design forces for the spread footing under the bent columns may be either a) or b), as follows. The forces given below are those at the bottom of the column. Therefore, the axial force is adjusted in Design Step 11 (Foundation Design) to account for the additional weight of the footing and overlying soil. There is also an overturning moment at the base of the footing from shear applied at the column base, which is computed in Design Step 11.

a) Modified Design Forces for Foundations
[Division I-A, Article 7.2.1(B)]

These forces were calculated in Design Step 7.3.2 and are summarized in Table 14, LC1 being the controlling load case. Note that $R = 1.0$ represents the full elastic seismic forces. The column weight is included in these axial forces.

$$\begin{array}{ll} P_{min_U} = 696 \text{ kip} & P_{max_U} = 1098 \text{ kips} \\ M_{Ulong} = 0 \text{ kip-ft} & M_{Utrans} = 0 \text{ kip-ft} \\ V_{Ulong} = 525 \text{ kips} & V_{Utrans} = 405 \text{ kips} \end{array}$$

b) Forces from Column Plastic Hinging
[Division I-A, Article 7.2.2]

Because of the one-half uplift criteria, the forces associated with both the maximum and minimum axial load need to be considered. The plastic hinging forces were calculated in Design Step 7.4. For the column base, 10 percent of the column top capacity is assumed as a preliminary value. The actual overstrength plastic moment capacity of the pinned column base is determined in Design Step 10.3.4. These axial forces do not include the column weight.

Plastic hinging forces in longitudinal direction are

$$\begin{array}{ll} P_p = 859 \text{ kips} \\ M_p = 650 \text{ kip-ft} & (\text{assumed}) \\ V_p = 358 \text{ kips} \end{array}$$

Plastic hinging forces in transverse direction (minimum axial load) are

$$\begin{array}{ll} P_{min_p} = 272 \text{ kips} \\ M_{min_p} = 598 \text{ kip-ft} & (\text{assumed}) \\ V_{min_p} = 329 \text{ kips} \end{array}$$

Design Step
8.6
(continued)

Plastic hinging forces in transverse direction (maximum axial load) are

$$\begin{aligned} P_{\max p} &= 1446 \text{ kips} \\ M_{\max p} &= 689 \text{ kip-ft} \quad (\text{assumed}) \\ V_{\max p} &= 379 \text{ kips} \end{aligned}$$

Conclusion: Use the forces associated with plastic hinging in both directions. The plastic hinging shears are smaller than the modified shears. See Figure 26.

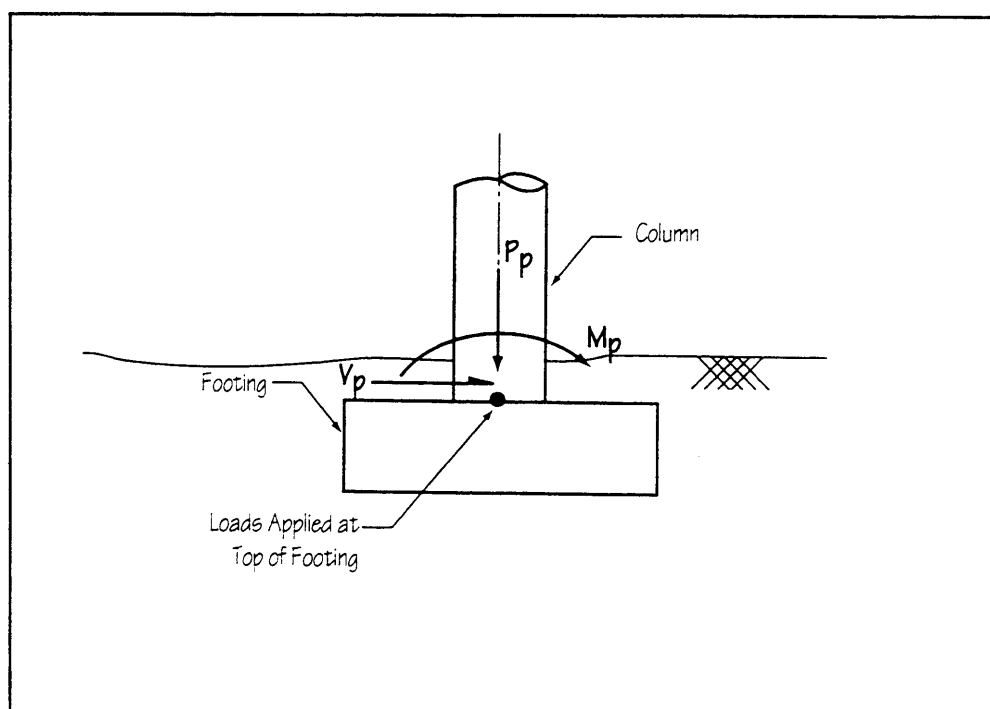


Figure 26 — Summary of Forces on Footing

Design Step
8.7**Abutment Design Forces**

[Division I-A, Articles 7.2.7 and 7.4.2(B)]

Only transverse forces transferred from the superstructure to the abutment are considered in this design example. The superstructure transfers the seismic force to the abutment through the shear key.

**Design Step
8.7
(continued)**

For design forces for the abutment, Division I-A, Article 7.4.2(B) refers back to Division I-A, Article 7.2.6. Because there are no hinging effects at the abutment, the controlling forces are the Modified Design Forces for Connections calculated in Design Step 7.3.1. Table 11 gives the seismic forces on the abutment at the level of the bearings. Note that $R = 0.8$ for connections. In summary, the forces at either abutment are

Transverse Shear (from LC2) $V_{trans} = 1003$ kip with $R = 0.8$

The transverse shear is transferred from the superstructure through the shear key to the abutment.

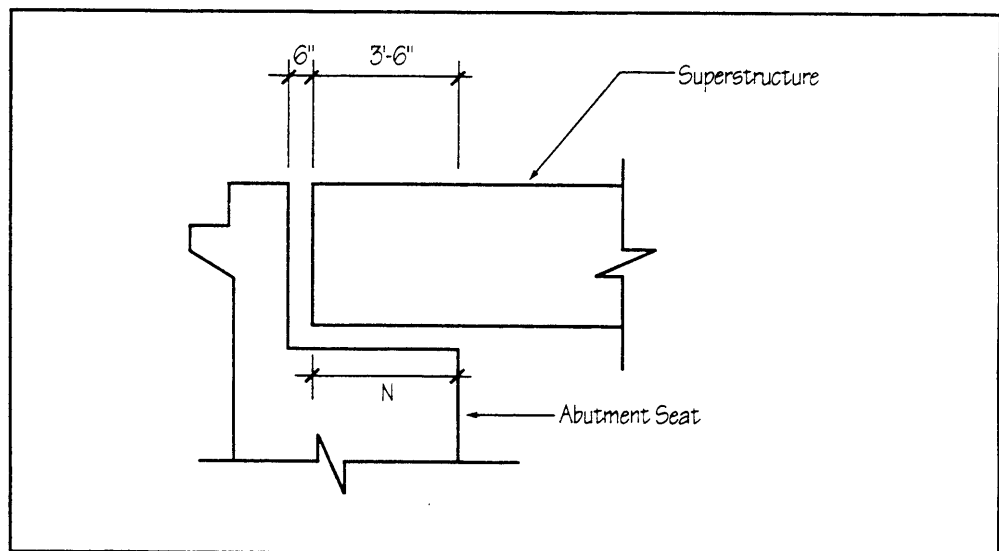
DESIGN STEP 9**DETERMINE DESIGN DISPLACEMENTS**

[Division I-A, Article 7.3]

**Design Step
9.1****Minimum Support Length**

[Division I-A, Article 7.3.1]

The bearing seats supporting the expansion ends of the bridge must provide a minimum support length at least N inches. See Figure 27.

**Figure 27 — Minimum Support Length at Abutment**

$L := 320 \cdot \text{ft}$ Length between abutments

$H := 20.0 \cdot \text{ft}$ Average height of columns
between expansion joints

$S := 30 \cdot \text{deg}$ Skew

From Division I-A, Equation 7-3A

$$N := \left(12 \cdot \text{in} + 0.03 \cdot L \cdot \frac{\text{in}}{\text{ft}} + 0.12 \cdot H \cdot \frac{\text{in}}{\text{ft}} \right) \cdot \left(1 + 0.000125 \cdot S^2 \right)$$

$N = 2 \cdot \text{ft}$ Minimum support width required

**Design Step
9.1
(continued)**

The width of the abutment seat provided (along the longitudinal axis of the bridge) is calculated taking the skew into account.

The provided width normal to the abutment backwall is 3 feet 6 inches.

$$W := 3.50 \cdot \text{ft}$$

The seat width along the longitudinal bridge axis is

$$N := \frac{W}{\cos(S)}$$

$$N = 4.04 \cdot \text{ft} \quad \text{Greater than required, okay}$$

**Design Step
9.2**

Design Displacements

The displacement from the Multimode analysis in the longitudinal direction is 0.275 foot at the abutment (from Table 6).

Design Step
10.1.2
(continued)

The required shear strength of the section, V_n , must be at least

$$V_n := \frac{V_{max_p}}{\phi} \quad \begin{array}{l} \text{Division I} \\ \text{Eqn (8-46)} \end{array}$$

$$V_n = 446 \cdot \text{kip}$$

The concrete shear strength of the column, calculated per Division I, Article 8.16.6.2, applies only to the mid-section between the “end regions” of the column. Refer to Design Steps 10.1.3 and 10.1.4 for transverse reinforcement requirements in the end regions.

For computing shear strength of circular sections, d need not be less than the distance from extreme compression fiber to centroid of the tension reinforcement in opposite half of member. Refer to Figure 28 for the variables.

$$d_{core} = 44 \cdot \text{in} \quad b_w = 48 \cdot \text{in}$$

The location of the #11 longitudinal bars from the center of the column is

$$r_b := \frac{d_{core}}{2} - 0.70 \cdot \text{in} - \frac{1.55}{2} \cdot \text{in}$$

$$r_b = 20.5 \cdot \text{in}$$

$$z_{bar} := \frac{2}{\pi} \cdot r_b \quad \begin{array}{l} \text{Centroid of tension side} \\ \text{reinforcement (Popov, 1976)} \end{array}$$

$$z_{bar} = 13.07 \cdot \text{in}$$

$$d := \frac{b_w}{2} + z_{bar} \quad d = 37.1 \cdot \text{in}$$

$$V_c := 2 \cdot \sqrt{f_c} \cdot b_w \cdot d \quad \begin{array}{l} \text{Division I} \\ \text{Eqn (8-51)} \end{array}$$

Design Step
10.1.2
(continued)

$$V_c := 225 \text{ kip}$$

The required shear strength of the reinforcement calculated per Division I, Article 8.16.6.3, is

$$V_s := V_n - V_c \quad \begin{array}{l} \text{Division I} \\ \text{Eqn (8-47)} \end{array}$$

$$V_s = 221 \text{ kip}$$

Check a spiral spacing based on a #5 spiral, as this is probably the minimum size that will be required in the column end regions for confinement at plastic hinges per Division I-A, Article 7.6.2(D). Try a spacing of 6 inches.

$$s := 6 \text{ in}$$

The resulting shear reinforcement is then

$$A_v := \frac{V_s \cdot s}{f_{yh} \cdot d} \quad \begin{array}{l} \text{Division I} \\ \text{Eqn (8-53)} \end{array}$$

$$A_v = 0.6 \text{ in}^2 \quad \begin{array}{l} \text{For two legs of the spiral} \\ \text{reinforcement} \end{array}$$

Because the cross-sectional area of #5 spiral reinforcing is 0.31 in² (per leg) or 2*0.31 = 0.62 in² (per 2 legs), the provided A_v is greater than the required A_v (0.62 > 0.60). Therefore, #5 spiral at 6-inch spacing is sufficient for typical transverse reinforcing. Check the spacing required for #5 spiral in the column end regions.

Design Step
10.1.3**Determine Minimum End Region Transverse Reinforcement**
[Division I-A, Article 7.6.2(C)]

The end regions of the column must meet the following two criteria. As the column is pinned at the base, the connection there will be specially designed as a hinge in Design Step 10.3.4. Therefore, only the top of the column is considered an end region for the transverse reinforcement requirements for confinement at plastic hinges.

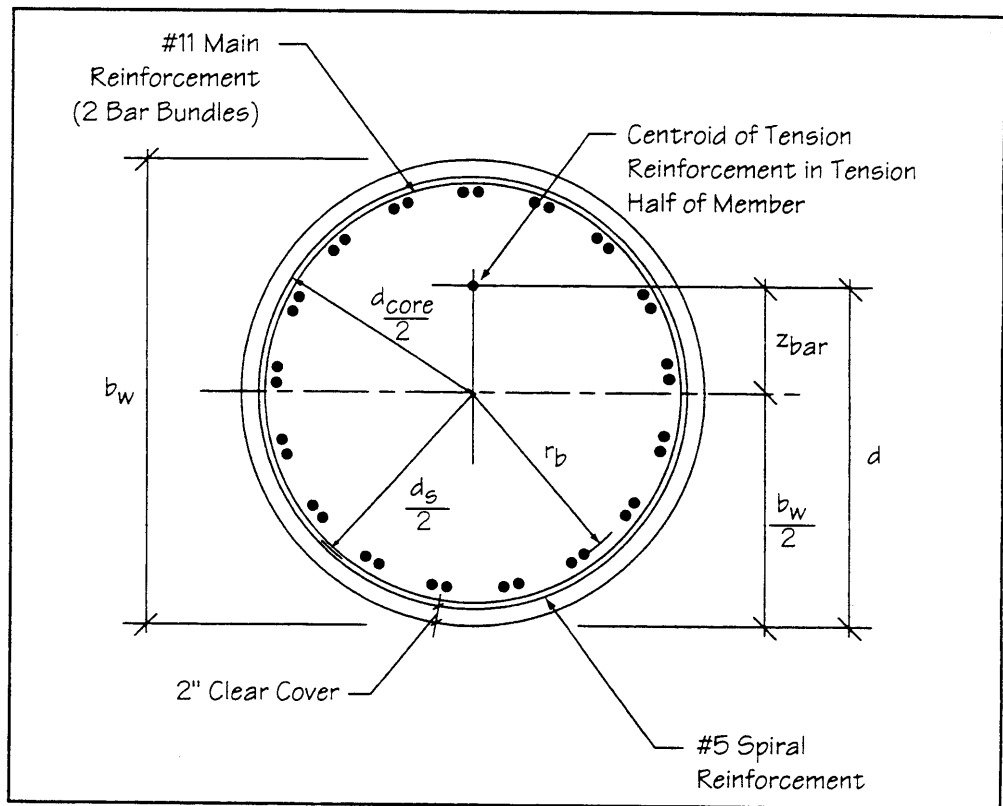
DESIGN STEP 10**DESIGN STRUCTURAL COMPONENTS**

[Division I-A, Article 7.6.2(C)]

This section concentrates on the critical components that resist the seismic forces.

**Design Step
10.1****Column Design**

For basic column data, see Figure 28 details.

**Figure 28 — Cross Section of Column**

$$f_c := 4000 \cdot \text{psi}$$

Concrete strength

$$f_{yh} := 60 \cdot \text{ksi}$$

Yield strength of spiral reinforcing

$$H_{clr} := 20.0 \cdot \text{ft}$$

Column clear height

$$b_w := 48 \cdot \text{in}$$

Outside diameter of column

**Design Step
10.1
(continued)**

Calculate the diameter of the concrete core, measured to the outside of the transverse spiral reinforcement. Assume a 2-inch clear cover.

$$d_{core} := b_w - 2 \cdot (2 \cdot \text{in}) \qquad d_{core} = 44 \cdot \text{in}$$

Summary of Controlling Column Design Forces from Design Step 8.1
[Division I-A, Article 7.2.3]

$$P_{min_U} := 658 \cdot \text{kip} \qquad P_{max_U} := 1060 \cdot \text{kip}$$

$$M_U := 3147 \cdot \text{kip} \cdot \text{ft}$$

$$V_{max_p} := 379 \cdot \text{kip}$$

**Design Step
10.1.1****Longitudinal Reinforcement**

Because there has been no change in the modified design forces for the column, the preliminary longitudinal column reinforcement computed in Design Step 7.4.1 is used. The longitudinal reinforcing is 34 #11 bars (2.93 percent reinforcing).

**Design Step
10.1.2****Determine Typical Transverse Reinforcement**
[Division I-A, Article 7.6.2(C)]

The required shear strength of the column section must be at least that calculated per Division I, Article 8.16.6.1, using the ϕ value from Division I, Article 8.16.1.2 for shear.

$$\phi := 0.85 \qquad \text{Strength reduction factor for shear}$$

$$f_c = 4000 \cdot \text{psi} \qquad \text{Concrete compressive strength}$$

$$b_w := 48 \cdot \text{in} \qquad \text{Diameter of circular section}$$

$$V_{max_p} = 379 \cdot \text{kip} \qquad \text{Factored shear force on section}$$

Design Step
10.1.3
(continued)

Criteria 1. The shear strength of the concrete, V_c , shall be in accordance with Division I, Article 8.16.6.2, when the axial load associated with the shear produces an average compression stress in excess of $0.1f'_c$ over the core concrete area of the support members. As the average compression stress increases from 0 to $0.1f'_c$, the strength V_c increases linearly from 0 to the value given in Division I, Article 8.16.6.2.

Compute the column core area, A_{core} .

$$d_{core} = 44 \cdot \text{in}$$

$$A_{core} := \frac{\pi \cdot d_{core}^2}{4} \qquad A_{core} = 1521 \cdot \text{in}^2$$

Check for the minimum column core axial stress, σ_a , even though the shear force is associated with the maximum axial force. This is conservative.

$$P_{min_U} = 658 \cdot \text{kip}$$

$$\sigma_a := \frac{P_{min_U}}{A_{core}} \qquad \sigma_a = 0.43 \cdot \text{ksi}$$

The minimum axial stress in the column, in order to use the full concrete shear capacity, is

$$0.1 \cdot f_c = 0.4 \cdot \text{ksi}$$

Because the minimum axial stress in the column is more than $0.1 \cdot f'_c$ ($0.43 \text{ ksi} > 0.40 \text{ ksi}$), the full value of V_c can be used. In some columns, the minimum axial load may be less than $0.1 \cdot f'_c$, and would, therefore, require a reduction in V_c .

$$V_c = 225 \cdot \text{kip}$$

Therefore, because the concrete shear capacity is the same as in Step 10.1.2, the minimum transverse reinforcement in the end regions, per Division I-A, Article 7.6.2(C), is the same as in the typical region.

Design Step
10.1.3
(continued)

The following criteria for extent of the end region shall be assumed.

Criteria 2. The extent of the column “End Region” below the top of the column is the maximum of the following three values.

- a. Diameter of column = 48 inches
 - b. $H_{cl}/6 = 20.0 \text{ feet}/6 = 3 \text{ feet } 4 \text{ inches} = 40 \text{ inches}$
 - c. 18 inches minimum
- <-- Controls

Design Step
10.1.4

Determine the Transverse Reinforcement for Confinement at Plastic Hinges
[Division I-A, Article 7.6.2(D)]

The core of the column must be confined by spiral in plastic hinge regions, as governed by the largest of

- The numerical criteria in this section
- The numerical criteria in Division I-A, Article 7.6.2(C) in Design Step 10.1.3

The volumetric ratio of spiral reinforcement is the greater of that required by Equation 7-4 or 7-5 in Division I-A for spiral reinforcement.

$$b_w = 48 \cdot \text{in}$$

Outside diameter of column

$$A_g := \frac{\pi \cdot b_w^2}{4}$$

$$A_g = 1810 \cdot \text{in}^2$$

$$A_{core} = 1521 \cdot \text{in}^2$$

Column core area

$$f_c = 4 \cdot \text{ksi}$$

Concrete compressive strength

$$f_{yh} := 60 \cdot \text{ksi}$$

Yield strength of spiral reinforcement

Design Step
10.1.4
(continued)

The volumetric ratio per Equation 7-4 is

$$\rho_s := 0.45 \cdot \left(\frac{A_g}{A_{core}} - 1 \right) \cdot \frac{f_c}{f_{yh}} \quad \begin{array}{l} \text{Division I-A} \\ \text{Eqn (7-4)} \end{array}$$

$$\rho_s = 0.0057$$

The volumetric ratio per Equation 7-5 is

$$\rho_s := 0.12 \cdot \frac{f_c}{f_{yh}} \quad \begin{array}{l} \text{Division I-A} \\ \text{Eqn (7-5)} \end{array}$$

$$\rho_s = 0.008 \quad \leftarrow \text{Controls}$$

Check for a reduced spiral pitch of 3.5 inches for the end regions.

$$s := 3.5 \cdot \text{in}$$

The diameter of the spiral, measured to the middle of the spiral, d_s , is

$$d_{core} = 44 \cdot \text{in}$$

$$d_s := d_{core} - 0.70 \cdot \text{in} \quad d_s = 43.3 \cdot \text{in}$$

Therefore, the area of one leg of the spiral is A_{sp} .

$$A_{sp} := \frac{\rho_s \cdot s \cdot d_{core}^2}{4 \cdot d_s} \quad (\text{Wang and Salmon, 1992})$$

$$A_{sp} = 0.31 \cdot \text{in}^2$$

This is the required area of one leg of the spiral. Because a #5 bar has a cross-sectional area of 0.31 in^2 , #5 spiral at 3-1/2 inches o.c. is sufficient in the end regions.

Design Step
10.1.5

Calculate the Spacing of Transverse Reinforcement for Confinement
[Division I-A, Article 7.6.2(E)]

Criteria 1. Transverse reinforcement for confinement must be provided over a column length as determined in Design Step 10.1.3, or per Division I-A, Article 7.6.2(E)1.

Minimum length = 48 inches at the column top

The spiral must be extended into the cap to the distance determined in Division I-A, Article 7.6.4.

Minimum length = 15 inches

or

Minimum length = one-half the column diameter = 24 inches (controls)

Criteria 2. Not applicable.

Criteria 3. Maximum spacing.

The spacing of 3.5 inches is less than the maximum spacing allowed, which is 4 inches in this case. Say okay.

Criteria 4. The spiral may not be lapped in the plastic hinge zones without full strength lap welds. This criterion should be included on the construction drawings.

The specifications of lap splices in the center region of the column outside of the plastic hinge zones must follow the criteria of Division I-A, Article 7.6.2(F).

Design Step
10.1.6

Summary of Column Reinforcement

Use 34 #11 bars longitudinally in 2-bar bundles spaced equally around the perimeter. Use #5 spiral at 3.5-inch pitch throughout the entire length. See Figure 29 for column details. Additional column reinforcement at the pinned column base is shown in Design Step 10.3.4.

The designer may wish to change the spiral pitch outside the plastic hinge zones to the 6-inch spacing computed in Design Step 10.1.2, though that was not done in this example.

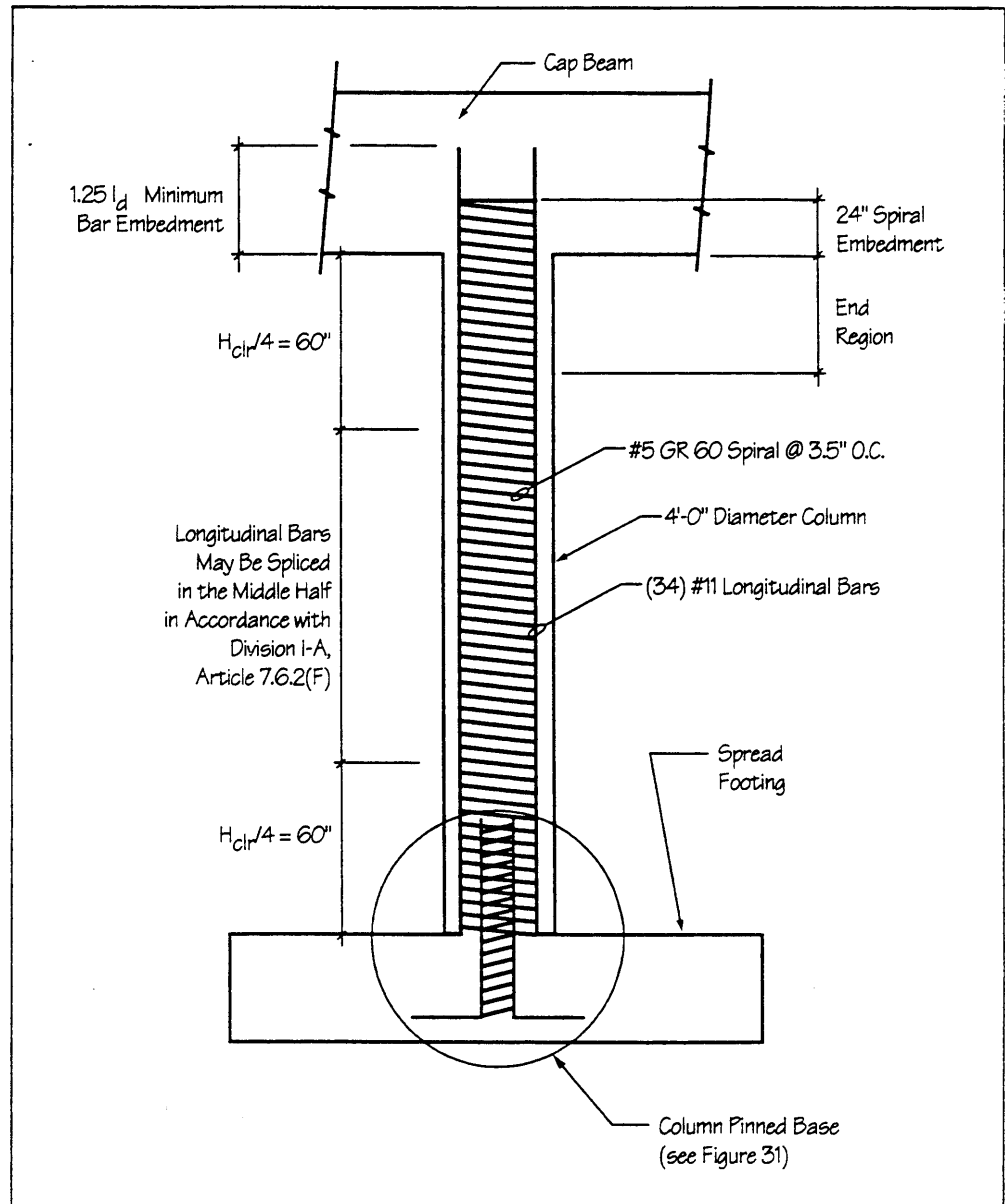
Design Step
10.1.6
(continued)

Figure 29 — Column Reinforcement Details

Design Step
10.2

Pier Design

Not applicable.

**Design Step
10.3****Connection Design****Design Step
10.3.1****Longitudinal Linkage**

Not applicable.

**Design Step
10.3.2****Hold Downs**

Not applicable.

**Design Step
10.3.3****Connection of Column to Cap Beam**

The connections of the column to the cap beam must be designed. This design should include the development of the longitudinal column steel into the cap beam and the assurance that the joint shear stresses are lower than the limits set by the Specification. Both of these conditions require the design forces to be those corresponding to plastic hinging.

a) Development Length

[Division I, Article 8.25 and Division I-A, Article 7.6.4]

The straight development length of a #11 bar, per Division I, Article 8.25, is as follows.

Basic data

$$f_c := 4000 \cdot \text{psi}$$

Concrete compressive strength

$$A_b := 1.56 \cdot \text{in}^2$$

Area of #11 bar

$$d_b := 1.410 \cdot \text{in}$$

Diameter of #11 bar

$$f_y := 60000$$

Yield strength of reinforcing
(psi)

Design Step
10.3.3
(continued)

The basic development length equation is

$$L_{db} := 0.04 \cdot A_b \cdot \frac{f_y}{\sqrt{f_c}} \quad L_{db} := 59.2 \cdot \text{in}$$

But not less than

$$0.0004 \cdot d_b \cdot f_y = 33.8 \cdot \text{in} \quad \text{Does not control}$$

Modify the basic length for bar spacing greater than 6 inches as allowed by Division I, Article 8.25.3.1. The actual spacing for 17 (2) #11 bar bundles, as shown in Figure 28, is 7.65 inches. There is no increase in development length for two-bar bundles, per Division I, Article 8.28.

$$L_d := 0.8 \cdot L_{db}$$

Modify length for possible steel overstrength capacity, 1.25 fy.
[Division I-A, Article 7.6.4]

$$L_d := 1.25 \cdot L_{db}$$

Therefore, the final development length of a straight bar must take into account both of the above effects.

$$L_d := (0.8) \cdot (1.25) \cdot L_{db}$$

$$L_d = 59 \cdot \text{in}$$

The length available for developing a straight bar in the cap beam is 72 inches less cover, say 68 inches. Therefore, there is sufficient length for development of #11 straight bars.

Design Step
10.3.3
(continued)*b) Joint Shear Stress*

[Division I-A, Article 7.6.4]

The average maximum joint shear stress in the joints of the bent is limited by the code. This limitation is an average stress, because it is calculated based on the cross-sectional area of the joint. The loading condition which produces the largest stress (typically the column plastic hinging case) should be used. The shear stress limit applies to both the horizontal and vertical directions.

In this example, both the horizontal and vertical shear stresses in the joint will be checked. These stresses are those required to transfer the forces passed into the cap beam by the column reinforcement. The maximum moment that can be applied to the joint from the column is limited to the overstrength plastic hinging capacity of the column. See Figure 30 for a free body diagram of the joint forces.

$$M_{\max_p} := 6890 \cdot \text{kip} \cdot \text{ft} \quad \text{From Design Step 8.3.3}$$

$$M_c := M_{\max_p} \quad \text{Applied moment from column in Figure 30}$$

$$M_c = 6890 \cdot \text{kip} \cdot \text{ft}$$

The moments on the left and right sides of the joint are equal to the applied moment.

$$M_c := M_L + M_R \quad \text{Where the moments on either side of the joint may be expressed as force couples with the tension (T) and compressive (C) forces, as shown in Figure 30}$$

Horizontal Joint Shear Stress

Taking the internal lever arm for the force couples from the beam as 0.75 times the beam depth.

$$h_b := 6.0 \cdot \text{ft} \quad \text{Depth of beam}$$

$$jd_b := 0.75 \cdot h_b$$

Design Step
10.3.3
(continued)

The horizontal joint shear force V_{jh} is equal to $C_L + T_R$, or approximately.

$$V_{jh} := \frac{M_L + M_R}{jd_b} \quad \text{which is equal to} \quad V_{jh} := \frac{M_c}{jd_b}$$

$$V_{jh} = 1531 \cdot \text{kip}$$

Horizontal joint shear force

The average horizontal shear stress in the joint may be calculated using the diameter of the column in the direction perpendicular to the cap beam, and an effective width along the cap equal to twice the column diameter for circular columns, unless limited by the cap beam width (from Priestley, Seible, and Chai, 1992).

$$b_w := 48 \cdot \text{in}$$

Outside diameter of the column

In this case, because twice the column diameter exceeds the cap beam width, use the cap beam width as the effective width.

$$b_{eff} := 5.0 \cdot \text{ft}$$

The average horizontal joint shear stress is

$$v_{jh} := \frac{V_{jh}}{b_w \cdot b_{eff}}$$

$$v_{jh} = 0.532 \cdot \text{ksi}$$

The allowable joint shear stress per Division I-A, Article 7.6.4 is

$$v_{allow} := 12 \cdot \sqrt{f_c}$$

$$v_{allow} := .759 \cdot \text{ksi}$$

Greater than 0.532 ksi calculated,
say okay

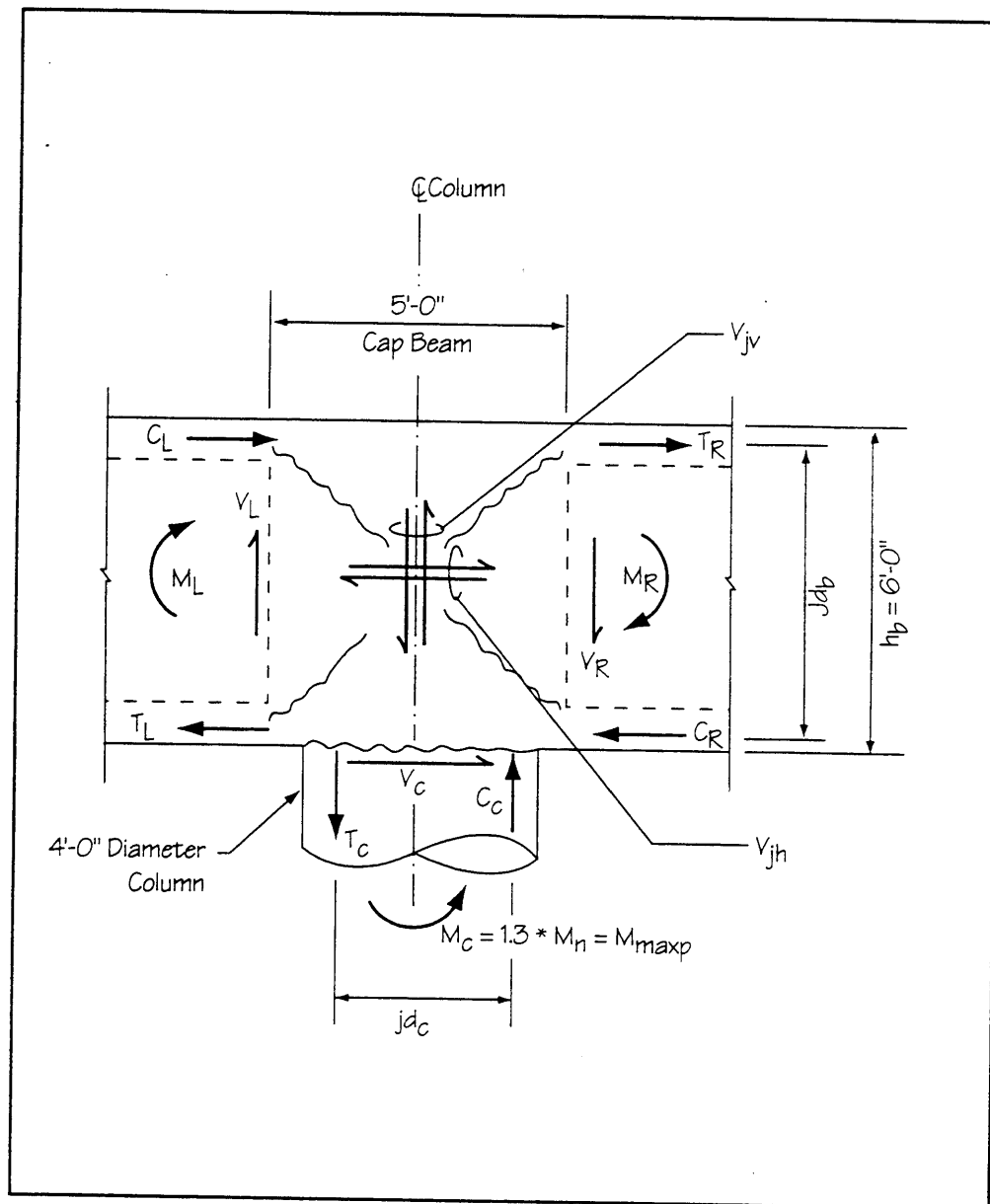
Design Step
10.3.3
(continued)

Figure 30 — Joint Shear Forces

Note that the above calculation for the horizontal joint shear stress does not include the effect of the axial column force from the weight of the CIP box girder superstructure. Compression across the horizontal plane of the cap beam due to this axial load would increase the joint's capacity to resist horizontal shear forces.

Design Step
10.3.3
(continued)Vertical Joint Shear Stress

Estimate the column's internal lever arm (jd_c) to determine the vertical forces T_c and C_c applied to the joint from the column's overstrength plastic moment capacity. If the distance from the centroid of the compression side is assumed equal to the distance to the centroid of the tension side, the internal lever arm would be equal to twice the z_{bar} dimension from Design Step 10.1.2 (see Figure 28).

$$z_{bar} := 13.07 \text{ in} \quad \text{Distance from tension side of reinforcement to center of column}$$

Then, the internal lever arm for the column forces becomes

$$jd_c := 2 \cdot z_{bar}$$

$$jd_c = 26 \text{ in}$$

The primary vertical shear forces acting on the joint are T_c and C_c from the column. The shears from the spans on either side of the joint, V_L and V_R , act to reduce the effective shear within the joint, as shown in Figure 30. That is,

$$V_{jv} := T_c - V_L \quad \text{or} \quad V_{jv} := C_c - V_R$$

If the shears from the span are conservatively neglected, the expression for the average vertical joint shear force reduces to

$$V_{jv} = T_c = C_c$$

where

$$T_c = C_c = M_{max_p} / jd_c$$

The horizontal joint shear force, V_{jh} , then reduces to

$$V_{jh} := \frac{M_{max_p}}{jd_c}$$

Design Step
10.3.3
(continued)

$$V_{jv} = 3163 \cdot \text{kip} \quad (\text{Conservatively})$$

Compute the average vertical shear stress using the dimensions of the cap beam.

$$h_b = 6 \cdot \text{ft} \quad \text{Cap beam depth}$$

$$b_b := 5 \cdot \text{ft} \quad \text{Cap beam width}$$

The average horizontal joint shear stress is

$$v_{jv} := \frac{V_{jv}}{h_b \cdot b_b}$$

$$v_{jv} = 0.732 \cdot \text{ksi}$$

The allowable joint shear stress is

$$v_{\text{allow}} := 12 \cdot \sqrt{f_c}$$

$$v_{\text{allow}} := .759 \cdot \text{ksi}$$

Greater than 0.732 ksi calculated,
say okay

Note that the above calculation for the vertical joint shear stress does not include the effect of post-tensioning of the CIP box girder superstructure. Compression across the width of the cap beam due to post-tensioning would increase the joint's capacity to resist vertical shear forces.

Calculation of joint shear stress levels is a routine part of seismic design for buildings, but has been generally ignored for bridges. To reduce the potential for damage and the possibility of collapse, bridge joint stresses are required to be checked. Shear stress levels in bridge joints have been addressed by Priestley, Seible, and Chai (1992).

Design Step
10.3.4

Connection of Column to Footing

The connection between the column base and the footing in this example is detailed and designed as a multidirectional pin using the Caltrans approach. The pinned connection type is shown in Figure 31.

Longitudinal hinge reinforcing is developed on either side of the column to footing interface, which has a smaller area than the column section.

An expansion joint filler is used around the interface area. The thickness of the expansion joint filler should allow maximum column deflection without crushing the edge of the column concrete against the footing. Caltrans recommends a minimum thickness of 1/2 inch in the *Bridge Design Details Manual* (1995).

A depressed shear key is often provided at the interface, though recent tests by Haroun et. al. (1993) suggest that the presence or absence of a depressed key has very little effect on the lateral resistance of the column. They also suggest that the ultimate shear strength of the tested columns and their pinned connections is determined by the strength of the columns themselves.

Shear and axial forces for design of the pinned column base are taken from Design Step 8.3.4.

$$P_{\max_p} := 1446 \cdot \text{kip} \quad (\text{without column weight})$$

$$P_{\min_p} := 272 \cdot \text{kip} \quad (\text{without column weight})$$

$$V_{\max_p} := 379 \cdot \text{kip}$$

$$V_u := V_{\max_p}$$

$$V_u = 379 \cdot \text{kip}$$

The Caltrans *Bridge Design Specifications* (BDS) contain Special Provisions for Column and Pier Wall Hinges (Article 8.16.4.5). The provisions state that the design shear strength shall be in accordance with Division I, Article 8.16.6.4, Shear Friction.

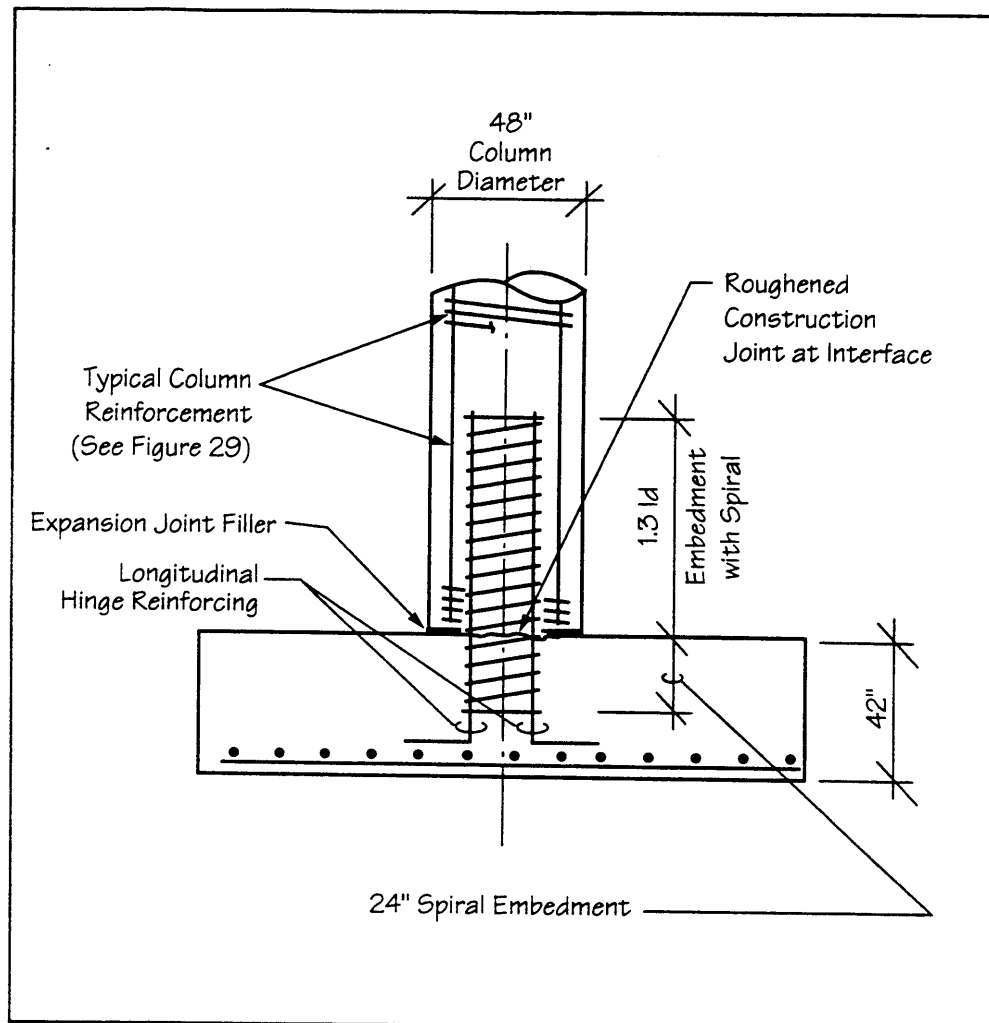
Design Step
10.3.4
(continued)

Figure 31 — Pinned Column Base

Compute the required area of contact at the column-to-footing interface using the shear stress limit for shear friction from Division I, Article 8.16.6.4.5.

$$f_c = 4000 \text{ psi}$$

Concrete compressive strength

$$\phi := 0.85$$

Strength reduction factor for shear

$$f_y := 60 \text{ ksi}$$

Yield strength of reinforcing

Design Step
10.3.4
(continued)

The required shear strength at the column-to-footing interface is

$$V_n := \frac{V_u}{\phi} \qquad V_n = 446 \cdot \text{kip}$$

The minimum contact area at interface for V_n is

$$A_{cvmin} := \frac{V_n}{0.2 \cdot f_c} \quad \text{Where } 0.2 \cdot f_c \text{ is less than or equal to } 0.8 \text{ ksi}$$

$$A_{cvmin} = 557 \cdot \text{in}^2$$

Assume a circular area for the pin contact interface. The minimum diameter of the area is

$$d_{min} := \sqrt{\frac{4 \cdot A_{cvmin}}{\pi}}$$

$$d_{min} = 26.6 \cdot \text{in} \quad \text{-----} > \quad \text{Try } d = 28 \text{ inches}$$

The actual contact area at the column to footing interface is

$$d := 28 \cdot \text{in} \qquad \text{Diameter of contact area at interface}$$

$$A_{cv} := \frac{\pi \cdot d^2}{4}$$

$$A_{cv} = 616 \cdot \text{in}^2 \qquad \text{Contact area at interface for 28-inch diameter}$$

Design Step
10.3.4
(continued)

Compute the required shear friction reinforcing area, A_{vf} , per Division I, Article 8.16.6.4.4.

$$\mu := 1.0$$

Coefficient of friction for normal weight concrete placed against hardened concrete with intentionally roughened surface

$$A_{vf} := \frac{V_n}{f_y \cdot \mu}$$

Division I
Eqn (8-56)

$$A_{vf} = 7.43 \cdot \text{in}^2$$

Minimum shear friction reinforcing required

Use 8 #9 bars longitudinally.

$$A_b := 1.0 \cdot \text{in}^2$$

Area of #9 bar

$$A_{vf} := 8 \cdot A_b$$

$$A_{vf} = 8 \cdot \text{in}^2$$

Area of shear friction reinforcing provided

Provisions of the Caltrans *Bridge Design Specifications*, in Article 8.16.4.5.2, use the following expression to compute the design axial compressive strength for Group VII loads only.

$$\phi P_0 = 0.85\phi f'_c (A_g - A_{st}) + A_{st}f_y$$

where

$$\phi = 0.90$$

P_u shall not exceed ϕP_0

- 1) f'_c shall be the strength of the supporting or supported member whichever is less.
- 2) A_g shall be the contact area at the interface ($= A_{cv}$).

Design Step
10.3.4
(continued)

- 3) A_{st} shall be the area, or the vertical component of the area of the longitudinal reinforcing steel crossing the interface and connecting the supporting and supported members.

The area of longitudinal hinge reinforcement, A_{st} , in excess of that required for design tensile axial load strength may be used for the required shear friction reinforcing area, A_{vf} .

There is no design tensile axial load because $P_{min_p} = 272$ kips (compression) without the column weight included. Therefore, all of the longitudinal hinge reinforcing, A_{st} , may be used as A_{vf} .

$$A_{st} := A_{vf}$$

$$A_{st} = 8 \cdot \text{in}^2 \quad \begin{array}{l} \text{Longitudinal hinge reinforcing,} \\ 8 \text{ \#9 bars} \end{array}$$

Check the contact area at the interface computed previously for shear.

$$A_g := A_{cv} \quad A_g = 616 \cdot \text{in}^2$$

Compute the design axial load. Include the weight of the column.

$$P_{max_p} = 1446 \cdot \text{kip}$$

$$H_{clr} := 20.0 \cdot \text{ft} \quad \text{Clear height of column}$$

$$A_{col} := \frac{\pi \cdot (4.0 \cdot \text{ft})^2}{4} \quad \text{Cross-sectional area of column}$$

$$\gamma_{conc} := .150 \cdot \frac{\text{kip}}{\text{ft}^3} \quad \text{Unit weight of concrete}$$

The weight of the column is

$$W_{col} := A_{col} \cdot H_{clr} \cdot \gamma_{conc} \quad W_{col} = 38 \cdot \text{kip}$$

Design Step
10.3.4
(continued)

Design the axial loads.

$$P_{\max_u} := P_{\max_p} + W_{\text{col}} \quad P_{\max_u} = 1484 \cdot \text{kip}$$

$$P_{\min_u} := P_{\min_p} + W_{\text{col}} \quad P_{\min_u} = 310 \cdot \text{kip}$$

The nominal compressive axial load strength from Caltrans is

$$P_o := 0.85 \cdot f_c \cdot (A_g - A_{st}) + A_{st} \cdot f_y$$

$$P_o = 2546 \cdot \text{kip}$$

With $\phi := 0.90$ the design compressive axial load strength is

$$\phi \cdot P_o = 2292 \cdot \text{kip} \quad \text{This is greater than } P_u (= 1484 \text{ kip}),$$

say okay

Per Caltrans' *Bridge Design Specifications*, Article 8.16.4.5.5, the longitudinal reinforcement should be centralized within the hinge to minimize the moment strength. It is not necessary to distribute the shear friction reinforcement over the interface area. Experimental study by Haroun et. al. (1994) strongly suggests that development of a shear friction failure at the pinned connection would not be expected since the ultimate shear strength of all tested columns was governed by direct shear failure.

The longitudinal reinforcement shall be developed on both sides of the hinge interface.

Development length for a #9 straight bar per Division I, Article 8.25, without the additional Caltrans requirement, gives

$$L_{db} := 38 \cdot \text{in} \quad \text{Basic development length, similar to Design Step 10.3.3(a)}$$

Design Step
10.3.4
(continued)

Modify the basic length per Caltrans' (1990) *Bridge Design Details Manual*.

$$1.3 \cdot L_{db} = 49.4 \cdot \text{in}$$

Extend #9 longitudinal pin reinforcing
50 inches above the top of the
footing

Development length for a #9 hooked bar per Division I, Article 8.29, without the additional requirements of Division I-A, gives

$$L_{hb} := 22 \cdot \text{in}$$

Basic development length for
standard hook

Modify the basic length for a bar cover greater than 2.5 inches per Division I, Article 8.29.3.2.

$$L_{dh} := 0.7 \cdot L_{hb}$$

$$L_{dh} = 15 \cdot \text{in}$$

Modify the length for probable steel yield stress, $1.25 f_y$
[Division I-A, Article 7.6.4]

$$L_{dh} := 1.25 \cdot L_{hb}$$

For final development length for a #9 hooked bar, include both of the above effects.

$$L_{dh} := (0.7) \cdot (1.25) \cdot L_{hb}$$

$$L_{dh} = 19 \cdot \text{in}$$

Final development required for #9 hook

The length available for development of the hooks is 42 inches, less cover and thickness of the bottom mat of steel in the footing. This is sufficient length for development. To ensure proper force transfer and to simplify construction, the hooks should be extended to the bottom mat of steel in the footing, as shown in Figure 31.

The longitudinal pin reinforcing is confined with a small diameter spiral to centralize the reinforcing within the hinge. The spiral should extend above and below the column to footing interface to the distance determined in Division I-A, Article 7.6.4, as a minimum.

Design Step
10.3.4
(continued)

The minimum extent for the spiral is a distance equal to one half of the maximum column dimension = 24 inches (but not less than 15 inches). Extend the spiral 24 inches below the top of the footing, and extend the spiral above the top of the footing to the end of the longitudinal hinge reinforcing, as shown in Figure 31.

The size and spacing of the spiral or tie reinforcing to confine the longitudinal pin reinforcing is not specifically addressed in Caltrans' *Bridge Design Specifications*, as the design shear strength for the pinned connection is computed from requirements for shear friction. For this example, #4 spiral at a 4-inch pitch is selected.

Check the actual pinned base plastic overstrength moment capacity against the pin reinforcing section shown in Figure 32.

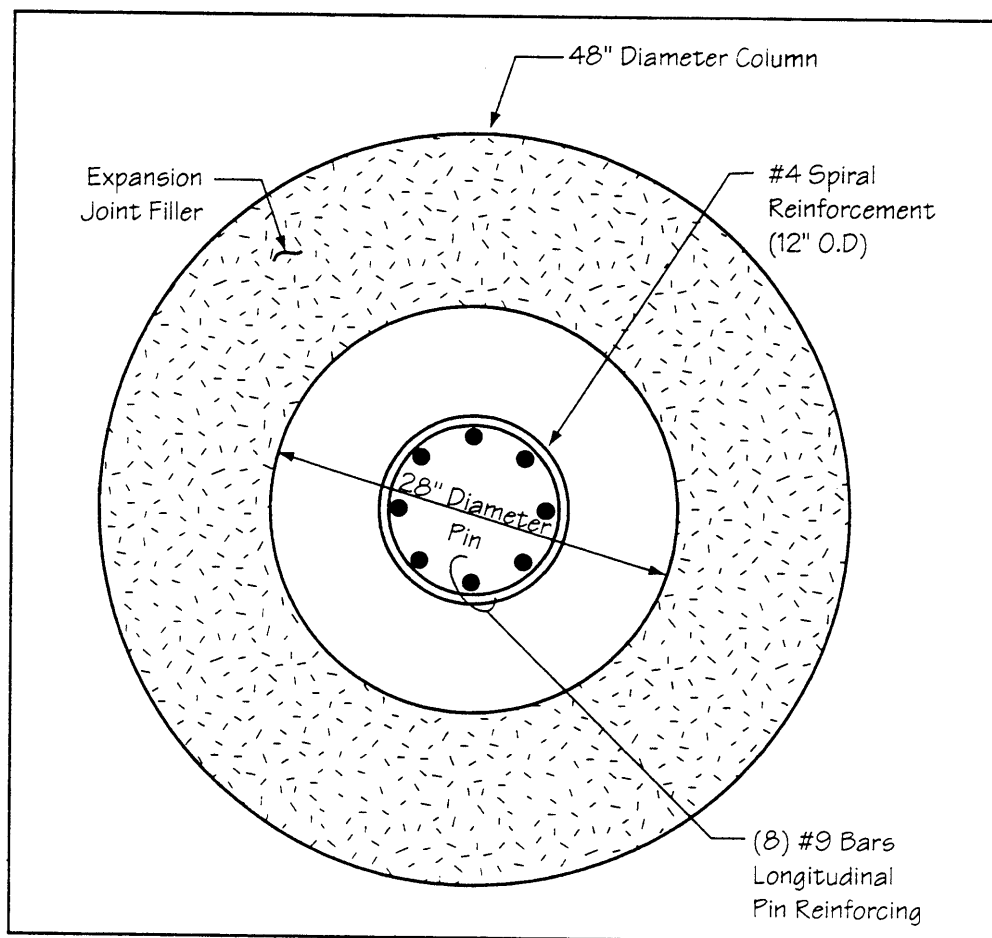


Figure 32 — Cross Section at Concrete Hinge

Design Step
10.3.4
(continued)

Use a #4 12-inch O.D. spiral at a 4-inch pitch to confine the longitudinal reinforcing in the hinge, as shown in Figure 32. Determine the overstrength plastic moment capacity of the pin section from the nominal moment interaction diagram shown in Figure 33, for axial loads equal to P_n for both maximum and minimum cases ($\phi = 0.90$).

Maximum Axial Load

$$P_{\max_n} := \frac{P_{\max_u}}{\phi} \qquad P_{\max_n} = 1649 \cdot \text{kip}$$

$$M_n := 485 \cdot \text{kip} \cdot \text{ft} \qquad \text{From Figure 33}$$

$$M_{\max_p} := 1.3 \cdot M_n$$

$$M_{\max_p} = 631 \cdot \text{kip} \cdot \text{ft} \qquad \text{Plastic overstrength capacity of pinned base column for } P_{\max_n}$$

Minimum Axial Load

$$P_{\min_n} := \frac{P_{\min_u}}{\phi} \qquad P_{\min_n} = 344 \cdot \text{kip}$$

$$M_n := 475 \cdot \text{kip} \cdot \text{ft} \qquad \text{From Figure 33}$$

$$M_{\min_p} := 1.3 \cdot M_n$$

$$M_{\min_p} = 618 \cdot \text{kip} \cdot \text{ft} \qquad \text{Plastic overstrength capacity of pinned base column for } P_{\min_n}$$

The computed plastic overstrength capacities of the pinned connection at the column base are close to the preliminary values used for computing the plastic hinging forces in Design Step 7.4. (Preliminary values were 10 percent of the column top capacity.) These plastic overstrength capacities are used for the design of the spread footings in Design Step 11.1.

Design Step
10.3.4
(continued)

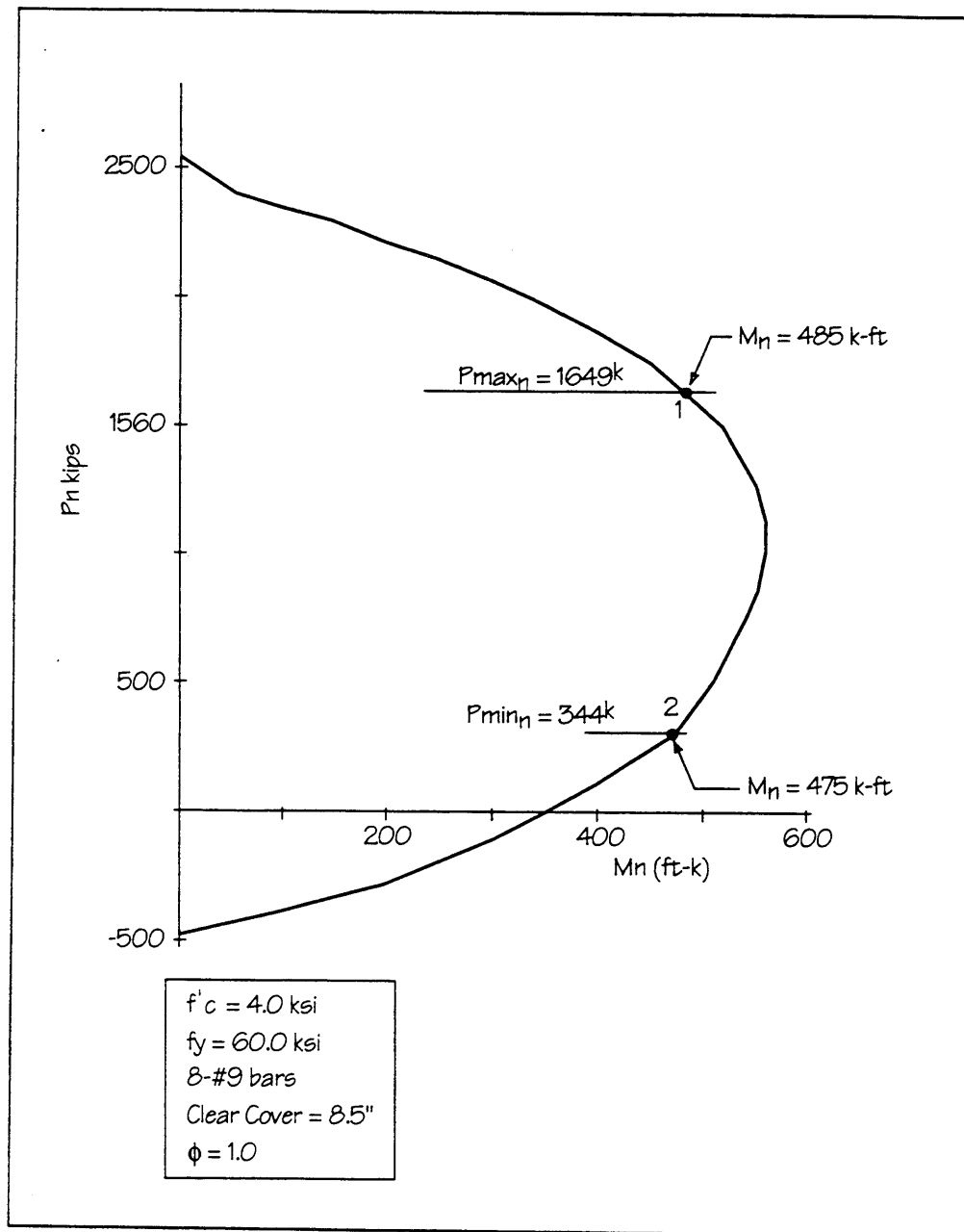


Figure 33 — Pinned Column Base Interaction Diagram

Design Step
10.4

Cap Beam Design

Not illustrated in this example.

DESIGN STEP 11**DESIGN FOUNDATIONS**

[Division I-A, Article 7.4.2]

**Design Step
11.1****Design Spread Footings**

The footings under Bents 1 and 2 are individual spread footings located under each of the two columns. The following soil properties are assumed to apply to this bridge site. The actual properties would be specified in the geotechnical report. For this example, geotechnical information is provided in Appendix A.

$q_{eq} := 20 \cdot \text{ksf}$ Ultimate soil pressure under seismic loads

$\gamma_{soil} := 125 \cdot \text{pcf}$ Unit weight of soil

$\mu := 0.6$ Sliding coefficient for seismic loads

$K_{PE} := 6.50$ Seismic passive pressure coefficient

**Design Step
11.1.1****Find Forces at Bottom of Footing**

The shear and axial forces at the top of the footing are taken from Design Step 8.6, and are associated with the plastic hinging forces at the bottom of the column. The plastic moments, M_{minp} and M_{maxp} , are equal to the plastic moment capacities of the pinned column base as determined in Design Step 10.3.4. The forces associated with both maximum and minimum axial loads will be considered in the design. The axial forces include the weight of the column as computed in Design Step 10.3.4.

Add the weight of the column (W_{col}) determined in Design Step 10.3.4 to the axial forces (P_{minp} and P_{maxp}) from Design Step 8.6.

$W_{col} := 38 \cdot \text{kip}$

Forces w/minimum axial force

$P_{min_u} := 272 \cdot \text{kip} + W_{col}$

$P_{min_u} = 310 \cdot \text{kip}$

Forces w/maximum axial force

$P_{max_u} := 1446 \cdot \text{kip} + W_{col}$

$P_{max_u} = 1484 \cdot \text{kip}$

Design Step
11.1.1
(continued)

$$M_{min_p} := 618 \cdot \text{kip} \cdot \text{ft}$$

$$M_{max_p} := 631 \cdot \text{kip} \cdot \text{ft}$$

$$V_{min_p} := 329 \cdot \text{kip}$$

$$V_{max_p} := 379 \cdot \text{kip}$$

The moment at the bottom of the footing, M_f , must include the moment due to V_p times the footing depth, d_f . Therefore, $M_f = M_p + V_p \cdot d_f$. See Figure 34.

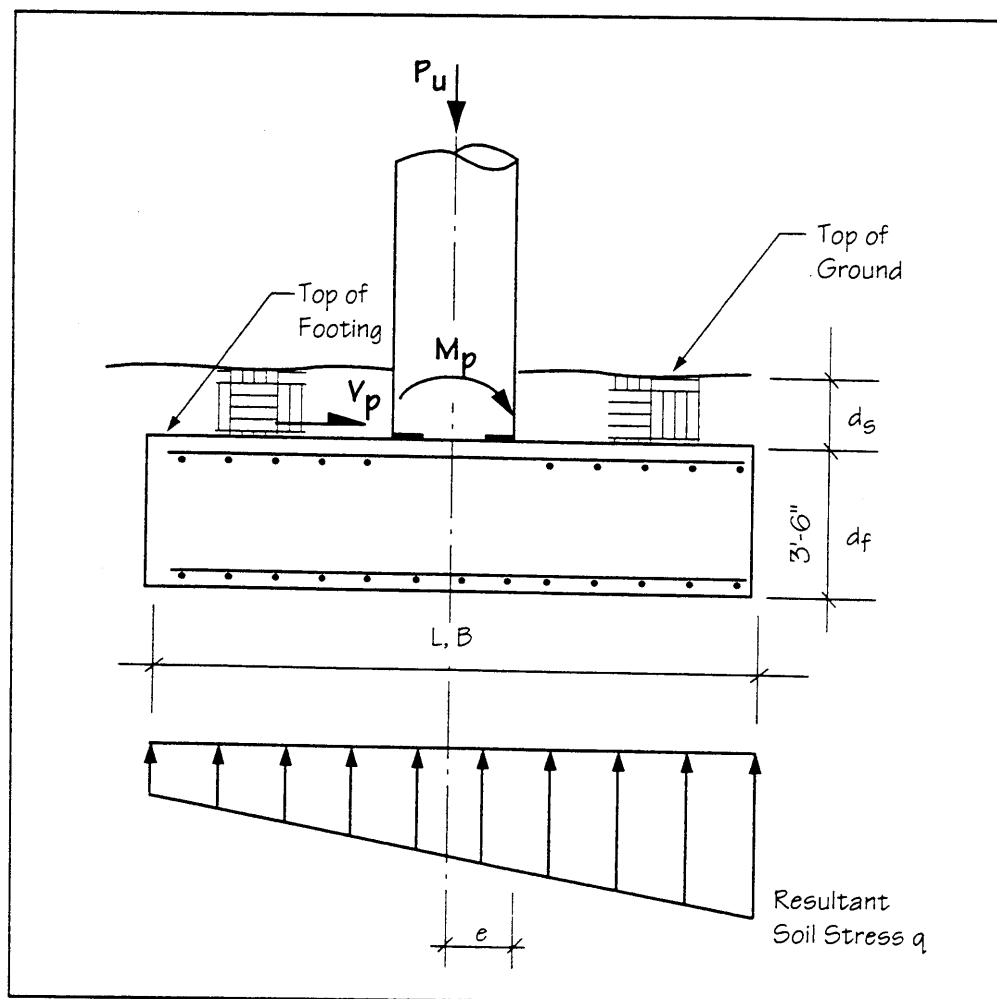


Figure 34 — Footing and Soil Pressure Distribution

Design Step
11.1.1
(continued)

An initial footing depth must be assumed in order to calculate M_f . Use the depth from preliminary design.

$$d_f := 3.5 \cdot \text{ft} \quad \text{Assumed footing depth}$$

Moment associated with $P_{\min U}$

$$M_{\min p} = 618 \cdot \text{ft} \cdot \text{kip}$$

$$V_{\min p} = 329 \cdot \text{kip}$$

$$M_{\min f} := M_{\min p} + V_{\min p} \cdot d_f \quad M_{\min f} = 1770 \cdot \text{kip} \cdot \text{ft}$$

Moment associated with $P_{\max U}$

$$M_{\max p} = 631 \cdot \text{ft} \cdot \text{kip}$$

$$V_{\max p} = 379 \cdot \text{kip}$$

$$M_{\max f} := M_{\max p} + V_{\max p} \cdot d_f \quad M_{\max f} = 1958 \cdot \text{kip} \cdot \text{ft}$$

The axial load at the bottom of the footing must include the dead weight of the soil and the self-weight of the footing. Assume a 2-foot minimum soil cover on top of the footing. Assume a 14-foot-square footing size for weight purposes. Refer to Figure 34.

$$L := 14 \cdot \text{ft} \quad \text{Length of footing}$$

$$B := 14 \cdot \text{ft} \quad \text{Width of footing}$$

$$d_s := 2 \cdot \text{ft} \quad \text{Depth of soil above footing}$$

$$d_f = 3.5 \cdot \text{ft} \quad \text{Thickness of footing}$$

Therefore, the axial dead load due to the self-weight of the footing, and the weight of the soil above the footing, is P_d . Using the following unit weights

$$\gamma_{\text{soil}} = 125 \cdot \text{pcf} \quad \gamma_{\text{conc}} := 150 \cdot \text{pcf}$$

Design Step
11.1.1
(continued)

$$P_d := \gamma_{\text{soil}}(L \cdot B \cdot d_s) + \gamma_{\text{conc}}(L \cdot B \cdot d_f)$$

$$P_d = 152 \cdot \text{kip}$$

Additional dead load from the weight of the footing and overlying soil

Therefore, the total axial load at the bottom of the footing, Q , is

Minimum Axial Load

$$P_{\min_U} = 310 \cdot \text{kip}$$

$$Q_{\min} := P_{\min_U} + P_d$$

$$Q_{\min} = 462 \cdot \text{kip}$$

Maximum Axial Load

$$P_{\max_U} = 1484 \cdot \text{kip}$$

$$Q_{\max} := P_{\max_U} + P_d$$

$$Q_{\max} = 1636 \cdot \text{kip}$$

Design Step
11.1.2

Find Stresses at Bottom of Footing

In addition to checking for maximum soil stresses, footing uplift must be considered. Per Division I-A, Article 7.4.2(B), the footing may have a separation of the soil up to one-half of the contact area of the foundation under seismic loading. Or stated another way, the footing may have up to one-half uplift. This is allowed only under foundations not susceptible to loss of strength under cyclic loading. Half uplift will probably be controlled by the load case with the minimum axial load.

The stresses under the footing can then be checked for forces associated with both maximum and minimum axial loads. In cases where the soil material has a large ultimate capacity, the soil stress limit will usually not control the design, and half uplift will control the size of the footing. For an average soil ultimate capacity, as with this example, the soil stress limit is likely to control for the maximum axial load side of the bent. See Figure 34 for the general soil stress.

In a two-column bent, meeting the requirements for half uplift and resistance to sliding for the minimum axial load side of the bent can be difficult. The designer may find that the footing size needed to meet the criteria is unusually large. For these cases, consideration should be given to pinning the base of the columns to reduce the moments at the column

Design Step
11.1.2
(continued)

base; use of a combined footing; or use of a grade beam between footings. A grade beam can be designed to transfer the shear from the minimum axial load side to the maximum axial load side, which has a much higher resistance to sliding. If the shear is transferred near the top of the footing, the effective moment at the base of the footing, M_f , will be reduced by the term $V_p \cdot d_f$.

a) Check Maximum Soil Pressure under Minimum Axial Load

The effective eccentricity, e , of the axial load is

$$M_{\min f} = 1770 \cdot \text{kip} \cdot \text{ft} \qquad Q_{\min} = 462 \cdot \text{kip}$$

$$e_{\min} := \frac{M_{\min f}}{Q_{\min}} \qquad e_{\min} = 3.8 \cdot \text{ft}$$

To ensure that there is no more than one-half uplift on the footing, the eccentricity must be less than $L/3$.

$$L = 14 \cdot \text{ft}$$

$$\frac{L}{3} = 4.7 \cdot \text{ft}$$

Because $L/3$ is greater than e_{\min} ($4.7 > 3.8$ feet), a 14-foot-square footing is large enough to resist half uplift.

The maximum soil pressure at the toe of the footing, allowing for uplift of the footing, is q . (The minimum soil pressure is zero.)

$$q := \frac{2 \cdot Q_{\min}}{3 \cdot B \cdot \left(\frac{L}{2} - e_{\min} \right)} \qquad \begin{array}{l} \text{Division I} \\ \text{Eqn (4.4.71.1.1-6)} \end{array}$$

$$q = 6.9 \cdot \text{ksf}$$

Because q is less than q_{eq}
($6.9 \text{ ksf} < 20 \text{ ksf}$), say okay

Design Step
11.1.2
(continued)*b) Check Maximum Soil Pressure under Maximum Axial Load*

The effective eccentricity, e , of the axial load is

$$M_{\max f} = 1958 \cdot \text{kip} \cdot \text{ft}$$

$$Q_{\max} = 1636 \cdot \text{kip}$$

$$e_{\max} := \frac{M_{\max f}}{Q_{\max}}$$

$$e_{\max} = 1.2 \cdot \text{ft}$$

There is no more than one-half uplift on the footing, because the eccentricity, e_{\max} , is less than $L/3$. Check if the eccentricity is less than or equal to $L/6$; then there is no uplift on the footing.

$$L = 14 \cdot \text{ft}$$

$$\frac{L}{6} = 2.3 \cdot \text{ft}$$

Because e_{\max} is less than $L/6$ ($1.2 < 2.3$ feet), a 14-foot-square footing is large enough to ensure no uplift for the maximum axial load case.

Because the footing is square, and plastic hinging of the column could occur in either of the footing's orthogonal directions, increase the overturning moment contribution to the maximum pressure to account for a direction along the diagonal axis of the footing.

The maximum pressure is essentially computed as $Q/A + Mc/I$. For a square, "I" is the same for both orthogonal axes about its center. For diagonal axes about its center, the increase in pressure from the applied moment is proportional to the increase in "c," which is taken along the diagonal.

The maximum soil pressure at the corner of the footing is q_{\max} .

$$q_{\max} := \frac{Q_{\max}}{B \cdot L} + \frac{(M_{\max f}) \cdot 6 \cdot \sqrt{2}}{B \cdot L^2} \quad (\text{Bowles, 1988})$$

Design Step
11.1.2
(continued)

$$q_{\max} = 14.4 \text{ ksf}$$

Because q_{\max} is less than

$$q_{eq}, (14.4 < 20 \text{ ksf}), \text{ okay}$$

The resulting footing pressures indicate that the footing size may be reduced. Although not performed for this example, a recheck of a reduced footing size could be made per parts (a) and (b) of this step.

c) Check for Sliding Beneath the Footing, Excluding Passive Pressure

$$V_{\min_p} = 329 \text{ kip}$$

Plastic hinging shear force

$$\mu = 0.6$$

Sliding coefficient

$$Q_{\min} = 462 \text{ kip}$$

Minimum axial load

The capacity against sliding under the footing, $\mu \cdot P_{\min_f}$, must be greater than V_{\min_p} .

$$\mu \cdot Q_{\min} = 277 \text{ kip}$$

This figure is less than V_{\min_p}

Recheck sliding with passive pressure resistance.

Compute the passive pressure resistance. Ignore the contribution of the soil overlying the top of the footing, as shown in Figure 35.

$$\gamma_{\text{soil}} = 125 \text{ pcf}$$

Unit weight of soil

$$K_{PE} = 6.5$$

Seismic passive pressure coefficient

$$d_s = 2 \text{ ft}$$

Depth of soil above footing

$$d_f = 3.5 \text{ ft}$$

Thickness of footing

$$B = 14 \text{ ft}$$

Width of footing

The assumed effective passive pressure diagram acting along the depth of the footing is shown in Figure 35. The trapezoidal diagram can be computed as the sum of a rectangle and a triangle.

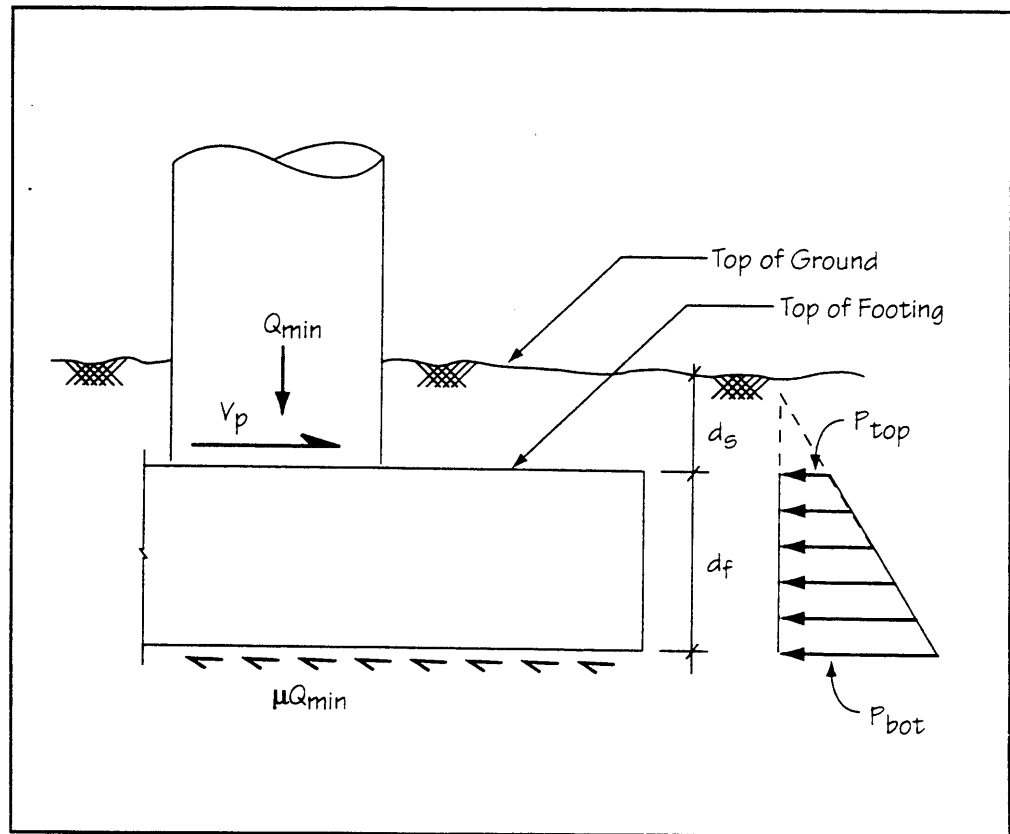
Design Step
11.1.2
(continued)

Figure 35 — Assumed Passive Pressure Diagram and Resistance to Sliding

The pressure at the top of the footing is

$$p_{top} := K_{PE} \cdot \gamma_{soil} \cdot d_s$$

$$p_{top} = 1.625 \cdot \text{ksf}$$

The pressure at the bottom of the footing is

$$p_{bot} := K_{PE} \cdot \gamma_{soil} \cdot (d_s + d_f)$$

$$p_{bot} = 4.469 \cdot \text{ksf}$$

Design Step
11.1.2
(continued)

Calculate the lateral soil passive pressure force associated with the assumed pressure diagram and computed pressures.

$$F_p := \left[(p_{top} \cdot d_f) + \left(\frac{1}{2} \right) \cdot d_f \cdot (p_{bot} - p_{top}) \right] \cdot B$$

$$F_p = 149 \cdot \text{kip} \quad \text{Lateral resistance from passive pressure}$$

Recheck sliding with the passive pressure resistance.

$$\mu \cdot Q_{min} + F_p = 426 \cdot \text{kip} \quad \text{Greater than } V_{min_p} (= 329 \text{ kip}),$$

say okay

As discussed previously, there are several ways to meet sliding requirements for the minimum axial load case. Among these are

- Use of a grade beam to transfer all of the lateral load to the maximum axially loaded side of the bent
- Use of a shear key to engage passive pressure to resist sliding
- Use of a combined footing

Design Step
11.1.3

Finalize Footing Size

Because the maximum soil pressure under a seismic load combination, $q = 14.4 \text{ ksf}$, is less than the allowable limit of 20 ksf , use the 14-foot-square footing.

Using the soil pressures computed above, the designer can now design the footing for flexure and shear using Division I of AASHTO. Note that top layer reinforcement should be included in the footing in order to support the weight of the soil above the footing due to the uplift condition.

Both beam shear (one-way) and punching shear (two-way) should be examined for the footing, and shear reinforcement should be provided if required by calculation or by detailing practice of the agency (i.e., Department of Transportation).

DESIGN STEP 12

DESIGN ABUTMENTS

This example problem concentrates on the development of seismic forces for the abutment shear key. Unlike the special seismic design and detailing requirements for the columns specified in Division I-A, the design and detailing requirements for the abutment reinforcement are the typical ones specified in Division I.

Design Step
12.1

Design the Abutment Shear Key

Figure 36 shows the transverse shear force transmitted from the superstructure into the top of the abutment seat through the shear keys. In the longitudinal direction, the superstructure is free to slide. In the transverse direction, the concrete shear key does not allow the bridge to move transversely. Because this critical connection is very stiff and does not allow any dissipation of energy through yielding, the shear key is designed for a force greater than the full elastic seismic shear computed in the analysis.

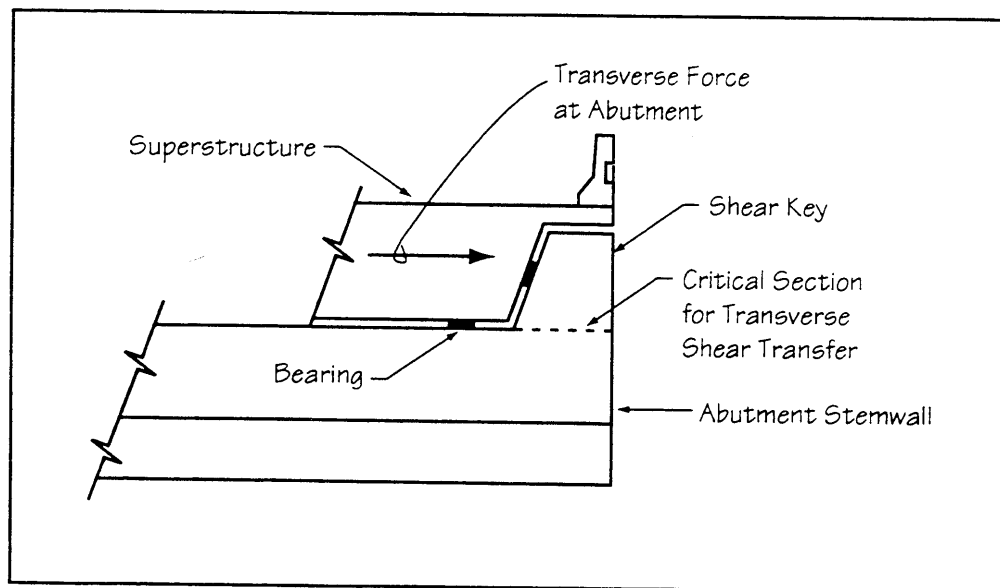


Figure 36 — Abutment Shear Key

The shear friction provisions of Division I, Article 8.16.6.4, will be used to calculate the required shear friction reinforcement for transfer of this horizontal force.

**Design Step
12.1**
(continued)

Given that

$$f_c := 4000 \cdot \text{psi} \quad \text{Concrete compressive strength}$$

$$f_y := 60 \cdot \text{ksi} \quad \text{Yield strength of reinforcing}$$

$$\phi := 0.85 \quad \text{Strength reduction factor per Division I, Article 8.16.1.2.2}$$

As discussed in Design Step 6.2.2, the structure was modeled with two different transverse translational spring stiffnesses and with a full transverse translational restraint at the abutments in order to provide an upper bound for the transverse response. The fully restrained condition resulted in the highest values for the transverse shear at the abutment, which were approximately 5 percent higher than the value used here. To be conservative, the designer could use the higher value, but for this example all design forces used are from the same analysis.

The transverse shear force on the shear key was found in Design Step 8.7. This transfer assumed an R Factor of 0.8.

$$V_u := 1003 \cdot \text{kip} \quad \text{Seismic transverse shear}$$

Required shear capacity

$$V_n := \frac{V_u}{\phi} \quad \begin{array}{l} \text{Division I} \\ \text{Eqn (8-46)} \end{array}$$

$$V_n = 1180 \cdot \text{kip}$$

Check Division I, Article 8.16.6.4.5 for the minimum plan area of concrete to resist the shear transfer force.

$$a) \quad A_{cv} := \frac{V_n}{0.2 \cdot f_c} \quad A_{cv} = 1475 \cdot \text{in}^2$$

$$b) \quad A_{cv} := \frac{V_n}{800 \cdot \text{psi}} \quad A_{cv} = 1475 \cdot \text{in}^2$$

**Design Step
12.1
(continued)**

Therefore, assuming that there is one shear key on each side of the superstructure, each key must have this area. Based on a 3-foot 6-inch-thick abutment wall, each shear key must have a length of

$$L_s := \frac{A_{cv}}{3.5 \cdot \text{ft}} \qquad L_s = 35.1 \cdot \text{in}$$

Use a shear key length of $L_s := 5.5 \cdot \text{feet}$ as shown in Figure 37.

It is not advisable to place an additional shear key at the centerline of the roadway section, due to the unpredictability of how much force each shear key will resist. A single key or stop could be placed at the centerline of the bridge, but damage in this area might be harder to detect, and could be very difficult and costly to repair. Keeping the transverse stops at the outside faces of the abutment is generally a good practice. The stemwall can either be increased in thickness locally to reduce the required length of the shear key as calculated above, or can be extended out beyond the width of the bridge to provide for the required shear key area.

The shear friction reinforcement required for each shear key is based on Division I Article 8.16.6.4.4.

$$\mu := 1.0 \qquad \text{For an intentionally roughened surface}$$

$$A_{vf} := \frac{V_n}{f_y \cdot \mu} \qquad A_{vf} = 19.7 \cdot \text{in}^2$$

Because the shear area is 3 feet 6 inches in length, the minimum required shear friction reinforcement per foot is

$$A_{vf} := \frac{A_{vf}}{3.5 \cdot \text{ft}} \qquad A_{vf} = 5.6 \cdot \frac{\text{in}^2}{\text{ft}}$$

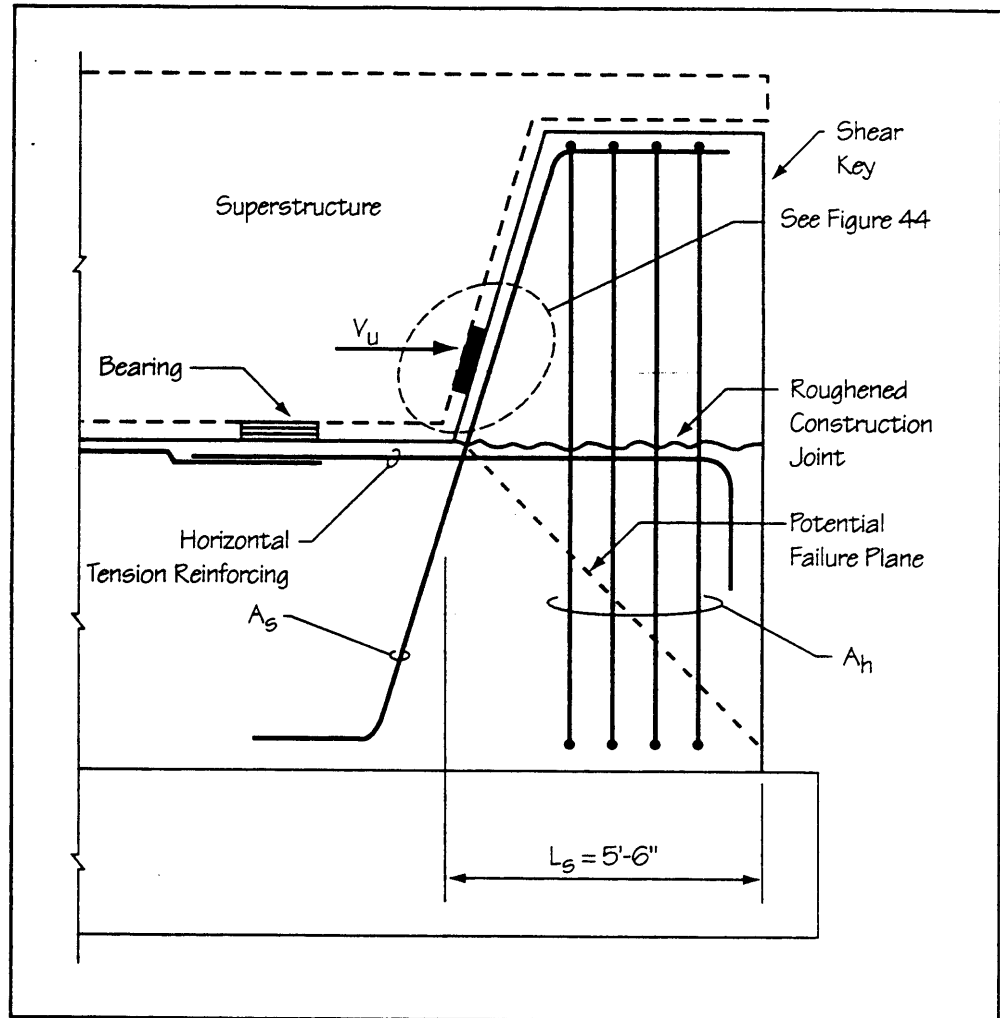
Design Step
12.1
(continued)

Figure 37 — Detail at Shear Key

For selection and distribution of the vertical reinforcement across the construction joint, the designer may wish to follow the Special Provisions for Brackets and Corbels from Division I, Article 8.16.6.8. As shown in Figure 37, the transfer of the transverse shear force (V_U) at the key is much like a vertical reaction to a corbel. (See Division I, Figure 8.16.6.8.) The vertical location of the transverse force, V_U , could have a significant lever arm which must be considered in the design of the reinforcement. Per Division I-A, Article 8.16.6.8, the primary tension reinforcement (A_S) is calculated from the shear friction reinforcement (A_{vf}) and direct tension reinforcement (A_n). The area of closed stirrups or ties (A_h) is calculated from A_S and A_n . Note that the horizontal tension reinforcing in the abutment stemwall acts as “hanger” reinforcement to resist V_U , and

Design Step
12.1
(continued)

prevents failure along the potential failure plane shown in Figure 37. This “hanger” reinforcement can be significant, depending on the magnitude of V_u , and may require more than one layer of reinforcing. For this example, the reinforcement is not designed in detail.

The transverse shear transferred to the abutment through the shear key is resisted by a combination of soil bearing against the wingwall and sliding resistance of the abutment. Sliding resistance is provided by soil friction under the abutment footing and along the backwall of the abutment as shown in Figure 38. The connection of the wingwall to the abutment backwall should be designed to resist the level of expected force provided by the soil bearing along the wingwall. This can be an iterative process, depending upon how much of the total force is considered being resisted by the wingwall versus how much of the total force is provided by resistance to sliding of the abutment.

A conservative check for this bridge would be to take the entire force transferred to the abutment at the shear key (V_u) and compute a resulting soil bearing pressure diagram along the wingwall (as shown in Figure 38). The wingwall pressure diagram could then be used to compute a prying moment at the back corner of the wingwall, where it connects to the abutment backwall. Providing a fillet at the corners, as shown, is recommended if the wingwall participates in the force resistance. This measure is required by some DOTs as a detailing practice. Another practice is to exclude the soil bearing resistance from the wingwall, detailing the connection of the wingwall to be “sacrificial” and expect it to be damaged but repairable. However, wingwalls can become sacrificial by default if the level of force exceeds that of the connection design.

**Design Step
12.1
(continued)**

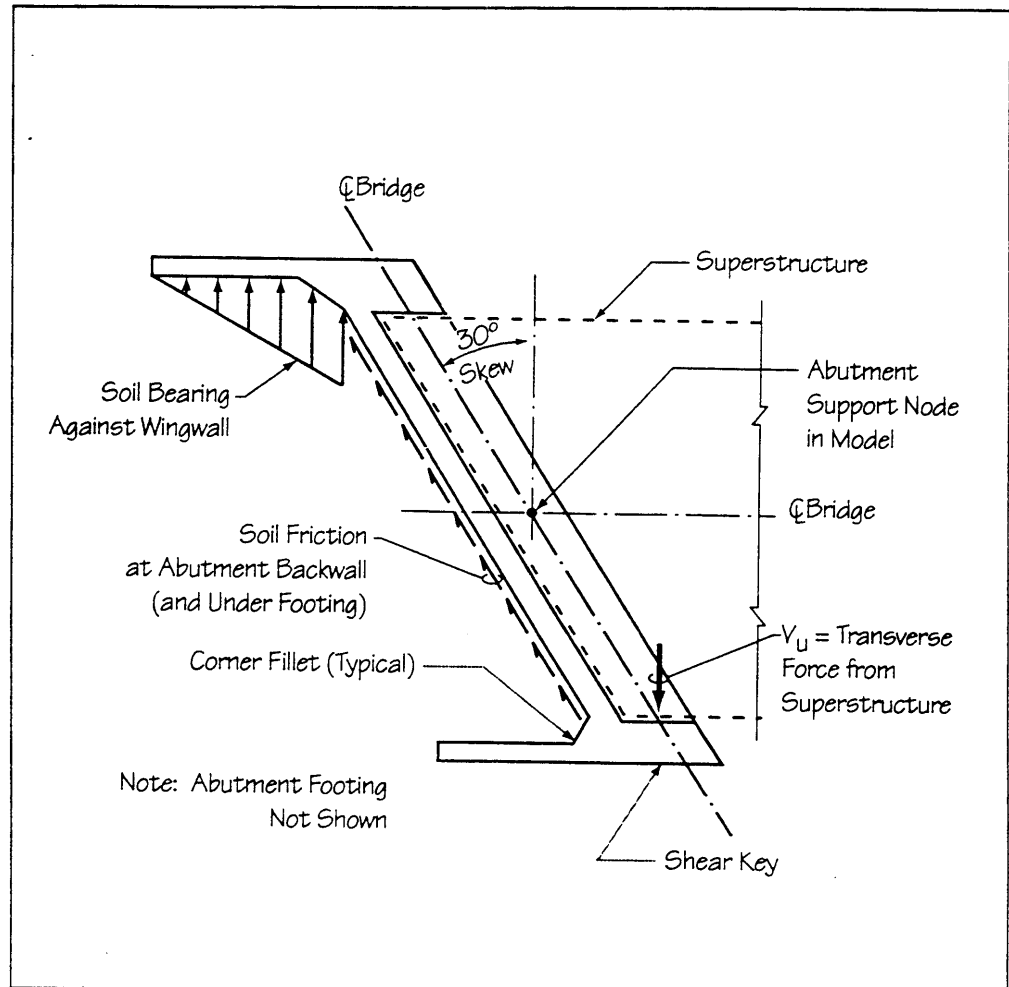


Figure 38 — Plan Detail at Abutment

DESIGN STEP 13

DESIGN SETTLEMENT SLABS

Not applicable.

DESIGN STEP 14

REVISE STRUCTURE

Not required.

DESIGN STEP 15**DETAILS
SUMMARY****SEISMIC DETAILS**

A number of details emphasizing the seismic issues discussed in this example are included within this section. These details, extracted from actual bridge plans, relate to the code requirements of Division I, Division I-A, and local agency requirements.

Footing Detail (Figure 39)

Both top and bottom reinforcement is required. Top reinforcement is required to support the weight of the soil above the footing during uplift conditions. Bottom reinforcement is required to resist forces due to soil bearing pressure. Shear reinforcement permits a reduction in footing thickness, and is required by some agencies. Note that the column and pin reinforcing are not shown.

Pinned Column Base Details

The connection of the column to footing was designed in Design Step 10.3.4 and special details for this connection are shown in Figures 31 and 32.

Spiral Details (Figures 40 and 41)

Spiral splice details include lap splice and welded splice options as shown. Welded splices are permitted in all column regions. Lap splices are permitted only within the center half of the column height. Refer to Division I-A, Article 7.6.2.

Column-to-Bent Joint Details (Figures 42 and 43)

Critical to the column-to-bent connection is the transverse and spiral reinforcement. End region requirements are specified in Division I-A, Article 7.6.2. Spiral embedment in the bent is specified in Division I-A, Article 7.6.4. Both a continuous spiral option and a discontinuous spiral option at the construction joint are detailed. The discontinuous spiral option is not addressed in AASHTO, but is recommended by some agencies as an alternative that is easier than the continuous spiral to construct.

**DETAILS
SUMMARY**

(continued)

Transverse Stop Detail at Shear Key (Figure 44)

Typically, the shear key is designed to resist transverse seismic forces, yet it must allow longitudinal movement due to shrinkage and contraction/expansion. The transverse stop detail at the shear key illustrates the concept of providing Ultra High Molecular Weight (UHMW) polyethylene plus a neoprene filler to provide a low-friction contact surface. As an alternate to adhesive bonding, a thicker sheet of UHMW polyethylene could be used, with countersunk screws tapped directly into the embedded plate in the shear key. In this case, the neoprene pad would be omitted. Note that the steel plates protect both the superstructure and the shear key to minimize damage during a seismic event.

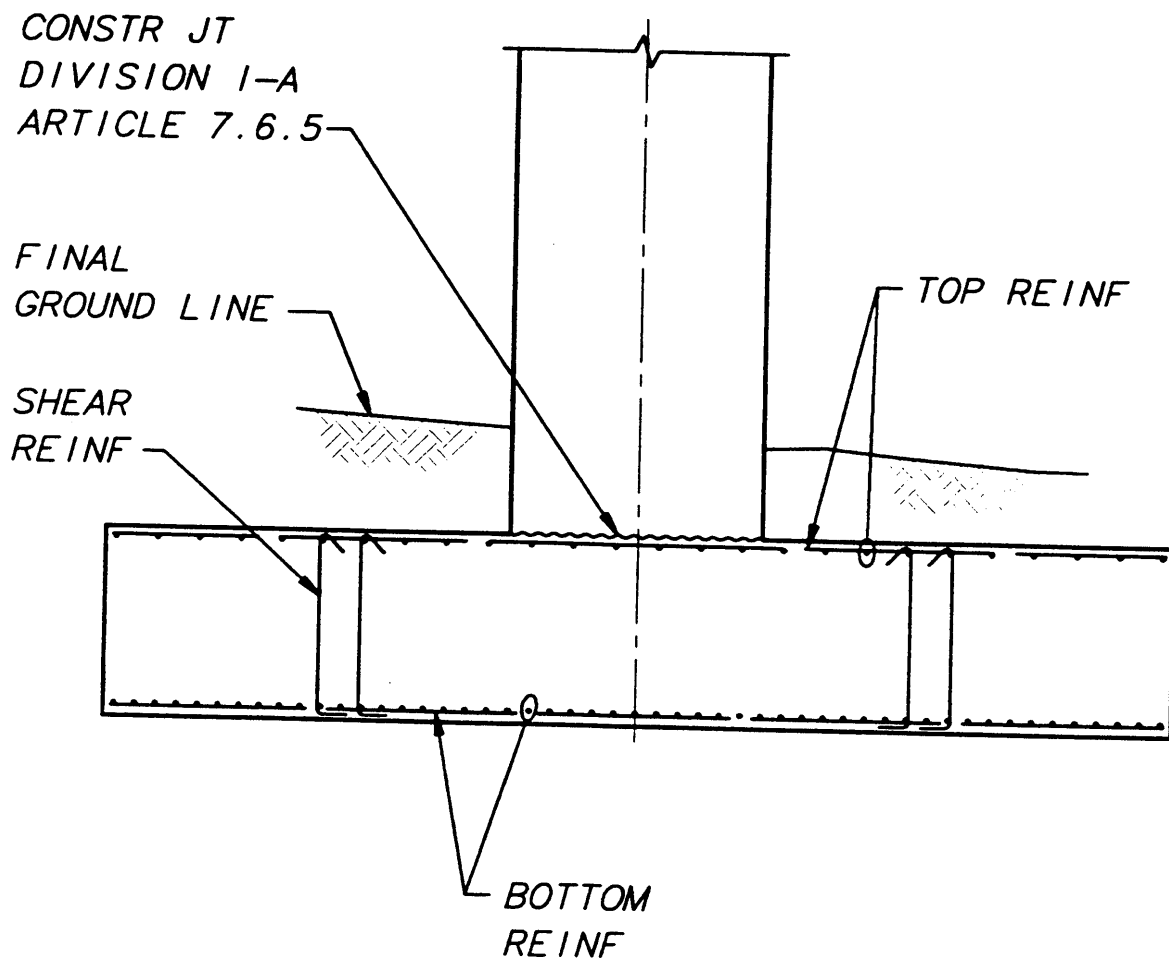
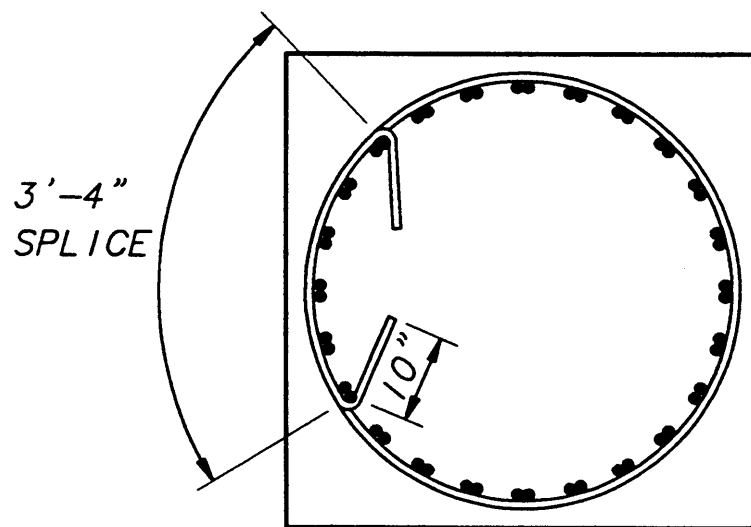


Figure 39 — Footing Detail



*HOOKS SHALL BE PLACED TO AVOID
VERTICAL REINF. LAP SPLICES NOT
PERMITTED IN COLUMN END REGIONS*

Figure 40 — Lap Splice Spiral Detail

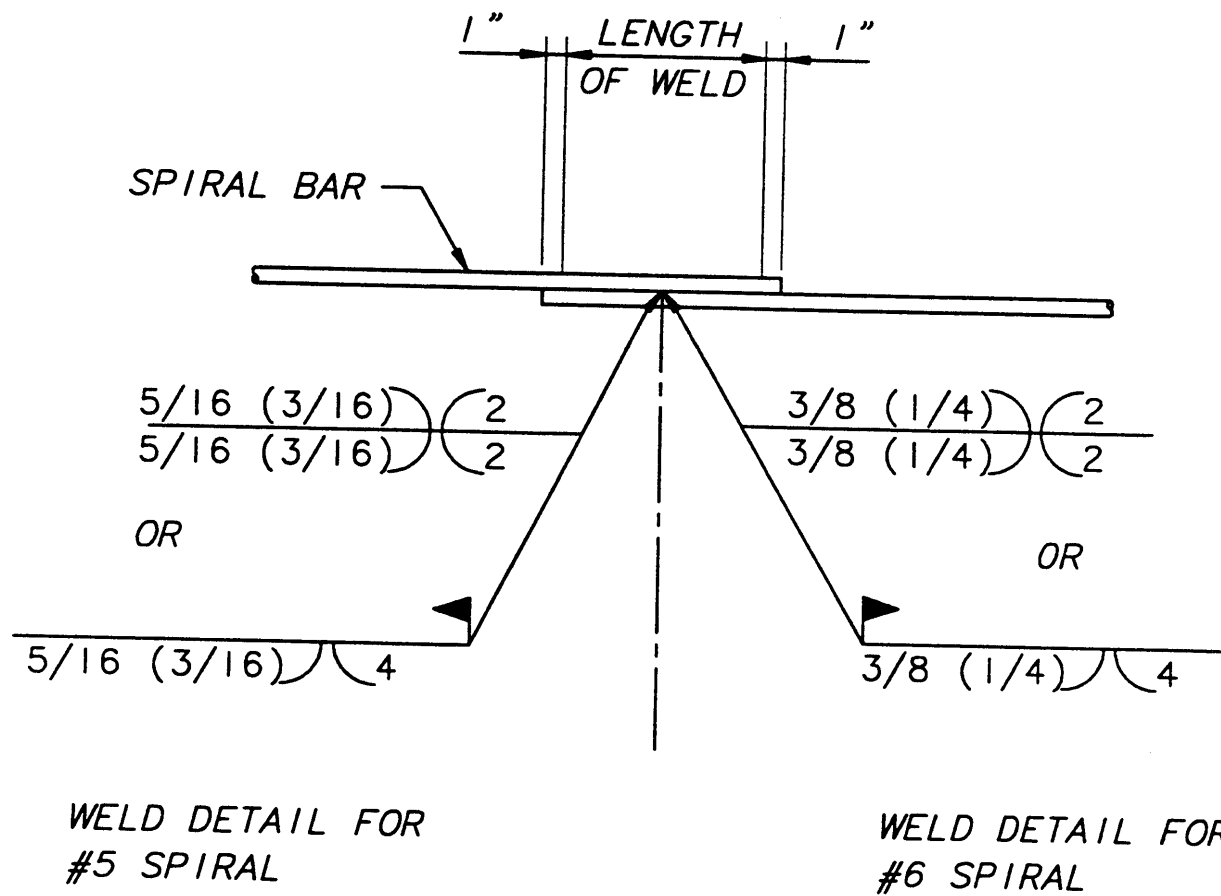


Figure 41 — Welded Splice Spiral Detail

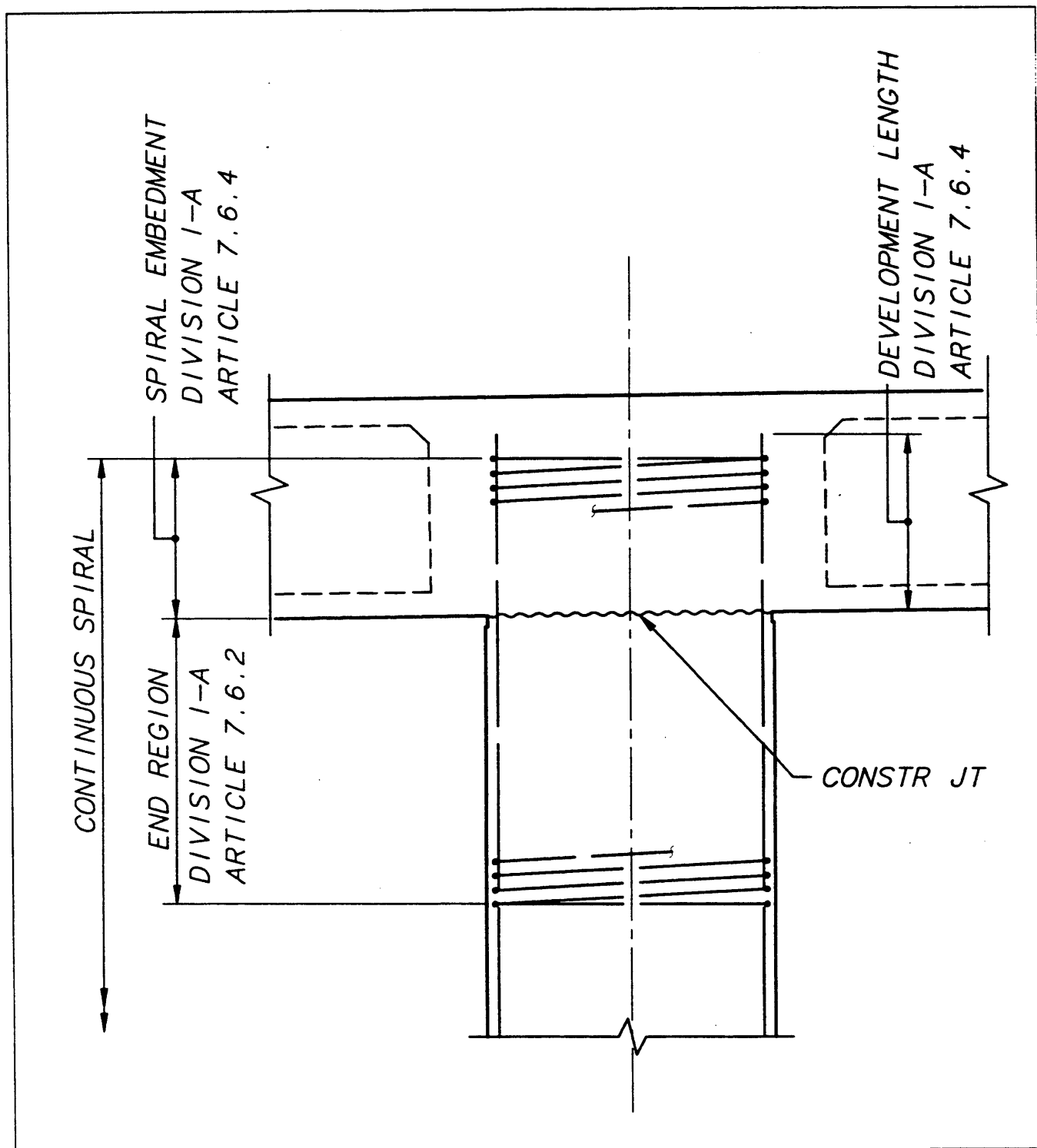


Figure 42 — Column-to-Bent Joint Detail
(Continuous Spiral Option)

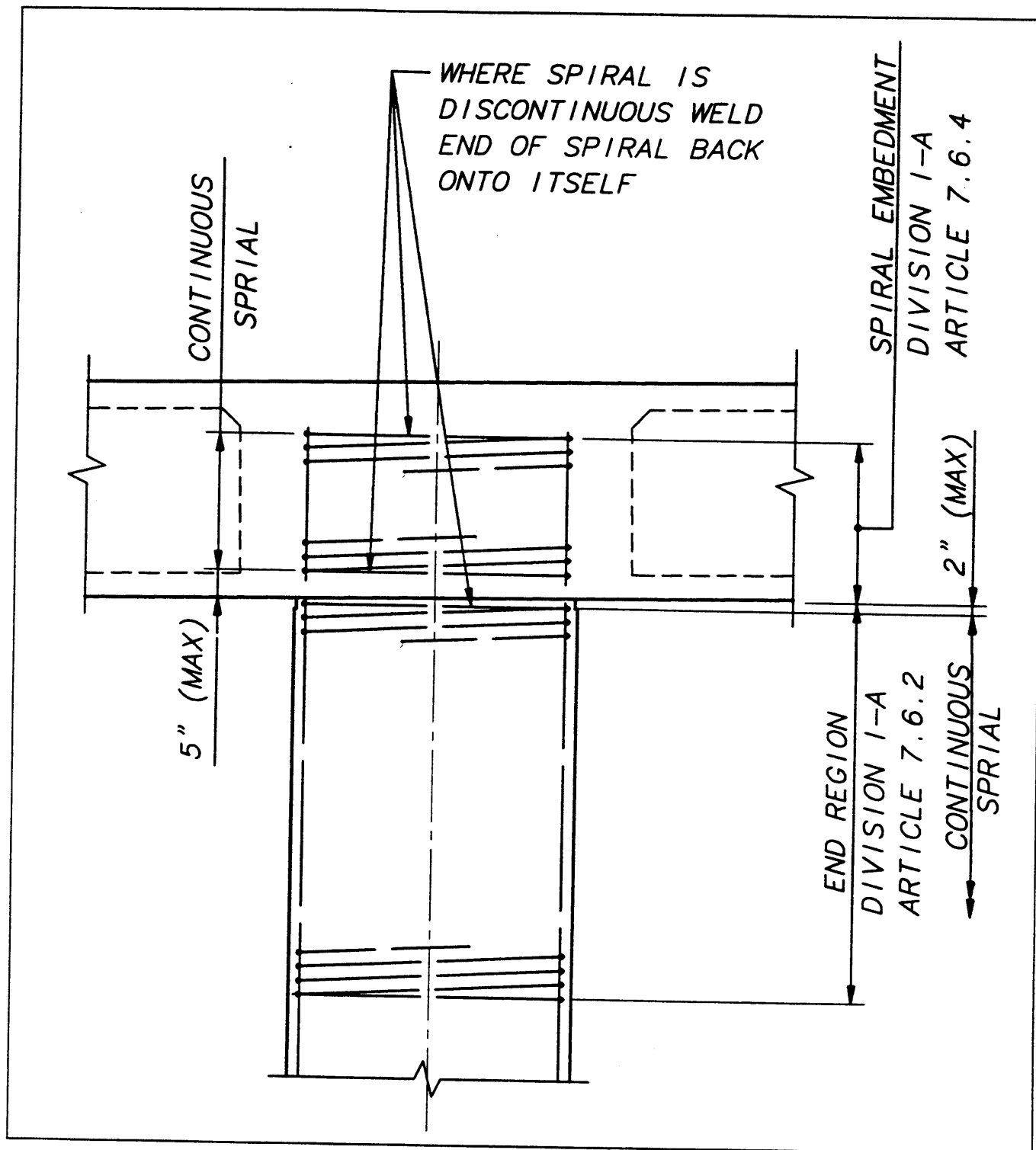


Figure 43 — Column-to-Bent Joint Detail
(Discontinuous Spiral Option)

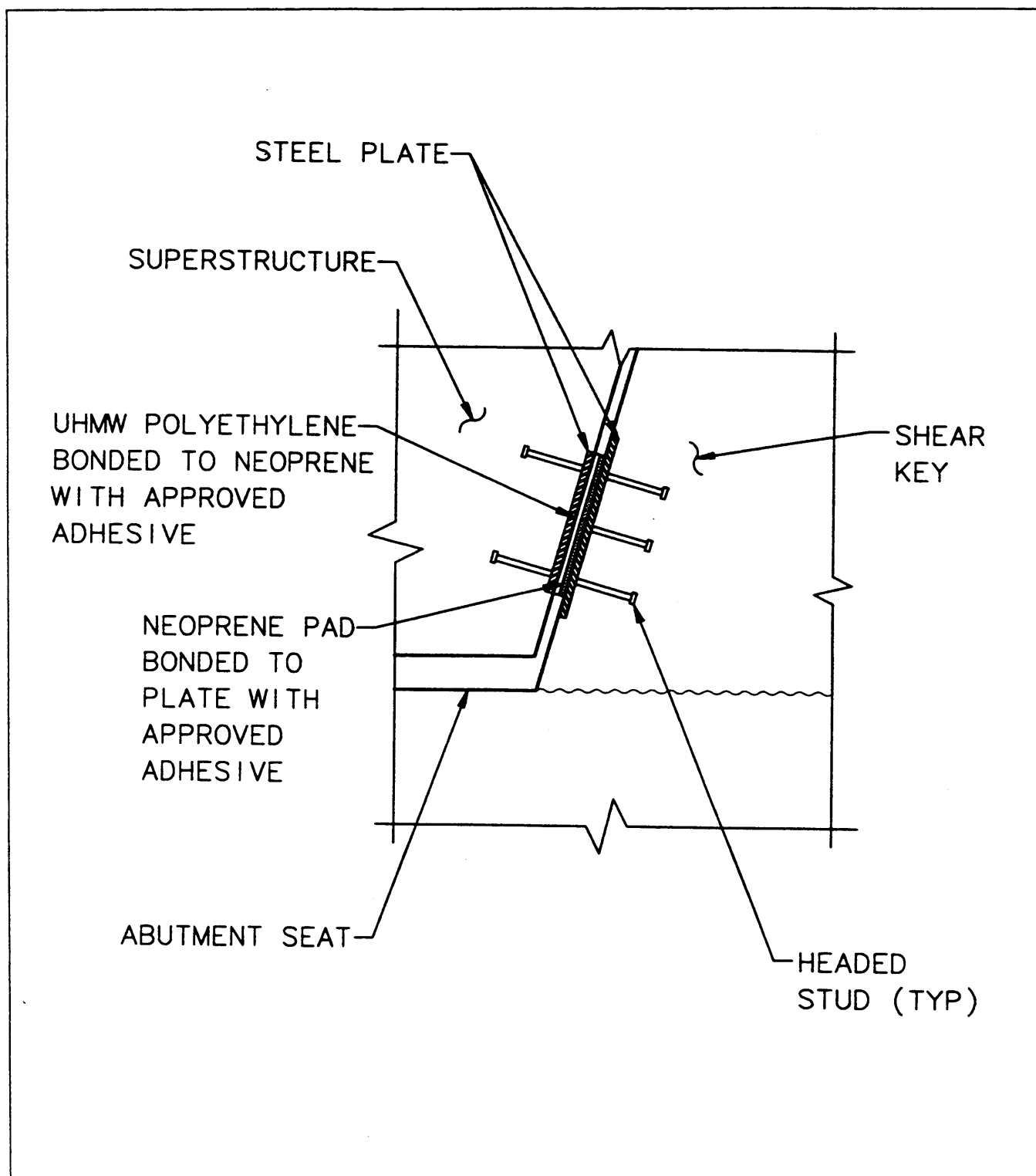


Figure 44 — Transverse Stop Detail at Shear Key

Section IV
Closing Statements

SECTION IV

CLOSING STATEMENT

Modeling of this structure for multimode analysis was fairly straightforward, because the structure is fully continuous from superstructure to substructure. Therefore, no special modeling details for connection of the superstructure to the bents were required. Because the bent columns are pinned at the base, the overall response of the structure is not likely to show as much sensitivity to stiffness changes of the foundation springs as columns having rigid connections to the spread footings. As discussed above, variation of transverse translational stiffness at the abutment did not significantly change design forces from transverse seismic loading. The effects of the structure's skew were accounted for by orientation of the intermediate bent elements, but the skew effect relating to transverse seismic response at the abutments was not specifically modeled.

It was shown that simple hand calculations of fundamental periods of the structure for both longitudinal and transverse directions were quite close to the multimode analysis results. Though such steps were not provided in the example, the periods from the hand calculation checks were used to obtain preliminary design seismic shears in order to check the sizes of substructure components reported in Design Step 1.

The structure was shown to develop plastic hinges in both directions for the design seismic event. Plastic hinging shear forces were less than the full elastic seismic combination forces, and were used in design of the structural components and foundations.

Use of a pinned base column allowed design of a smaller-sized footing. However, pinning a column base increases the fixed end moment at the column top, so this effect must be evaluated for all load cases. Additional longitudinal reinforcing may be required for an increased column top moment, resulting in a higher plastic moment capacity at the column top. Because the plastic shear is computed from the sum of the column's overstrength plastic moment capacities at the top and the bottom, the plastic shears will be different for pinned and fixed base conditions. Generally, pinning the column bases for multicolumn bents is a good practice. The structure's response to seismic loading is then softened by the reduced stiffness provided at the foundation level, resulting in longer periods and reduced seismic forces. The biggest advantage of pinning the column base is a reduction in the required foundation size.

Section V

References

SECTION V

REFERENCES

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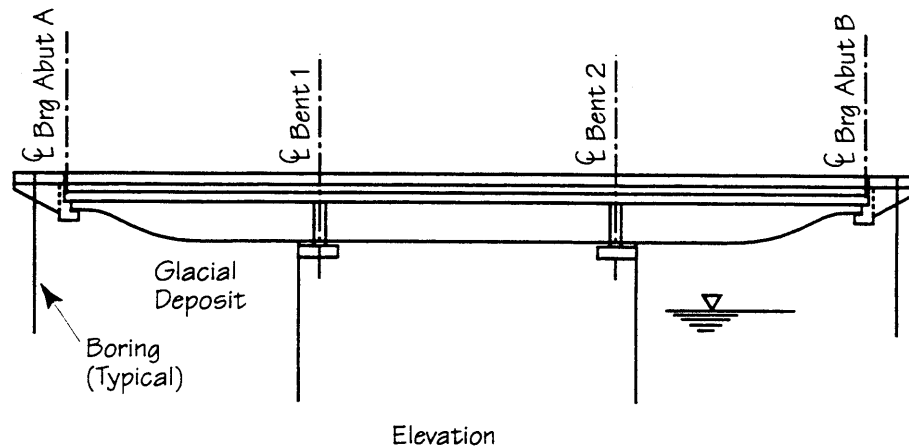
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Appendix A

Geotechnical Data

| APPENDIX A | GEOTECHNICAL DATA |
|---|--|
| SUBSURFACE CONDITIONS | Subsurface conditions were derived from four borings drilled along the bridge alignment. As shown in Figure A1, the site is underlain by dense to very dense sand and gravel. Based on other borings in the area, the depth to rock is estimated to be 250 feet. The water table is located at a depth of 50 feet below the ground surface under the bridge. |
| SOIL PROPERTIES | Soil properties for the subsurface materials encountered in the explorations are shown in Figure A1. These properties were estimated from correlations with the standard penetration test blow counts (N values). |
| SOIL PROFILE TYPE | Type II — Stable deposits of sands and gravels over 200 feet in depth. |
| SITE ACCELERATION | 0.3g — Taken from AASHTO seismicity map. |
| FOUNDATION DESIGN PARAMETERS | <p>Spread footings bearing on glacial deposits.</p> <p>Axial</p> <p>Ultimate Bearing Pressure = 20 ksf Ultimate Coefficient of Friction along Footing Base, $M = 0.6$</p> <p>Lateral Resistance</p> <p>$\phi = 38^\circ$ $c = 0$ $\gamma = 125$ pcf $\delta = \phi/3 = 13^\circ$ (coefficient of wall friction) $k_h = A/2 = 0.15g$ (horizontal acceleration coefficient) $k_v = 0$ (vertical acceleration coefficient) $k_{PE} = 6.5$ (seismic passive pressure coefficient using Mononobe-Okabe equation)</p> |
| OTHER CONCERNS | Liquefaction is not likely to occur at this site, because of the presence of dense to very dense soils. Similarly, because of these deposits, the abutment slopes should be stable during earthquake shaking. |



SOIL PROPERTIES

| Type | Depth (ft) | Description | N (bpf) | ϕ (deg) | c (psf) | γ (pcf) | ν | G (ksi) |
|-----------------|-----------------------|--|------------|-----------------|------------|-------------------|-------|------------|
| Glacial Deposit | Full depth of borings | Dense to very dense, clean sand and gravel | 50 | 38 | 0 | 125 | 0.35 | 10 |

Where:

- N average standard penetration resistance (blows per foot)
 ϕ internal angle of friction (degrees)
 c cohesion (pounds per square foot)
 γ total unit weight (pounds per cubic foot)
 ν Poisson's ratio
 G shear modulus (kips per square inch)

Figure A1 — Subsurface Conditions

Appendix B
SAP90 V6.0 Beta Input

FHWA BRIDGE NO 4 / SAP90 (BETA VERSION) INPUT FILE

SYSTEM

PAGE=LINES LINES=67 LENGTH=FT FORCE=KIP

COORDINATE

NAME=PIER X=100.00 Y=0 Z=0.0
X=106.50 Y=0 Z=11.26
X=120.00 Y=0 Z=0.0

JOINT

701 X= 0.00 Y= 3.38 Z= 0.00
702 X= 25.00 Y= 3.38 Z= 0.00
703 X= 50.00 Y= 3.38 Z= 0.00
704 X= 75.00 Y= 3.38 Z= 0.00
711 X= 100.00 Y= 3.38 Z= 0.00
712 X= 130.00 Y= 3.38 Z= 0.00
713 X= 160.00 Y= 3.38 Z= 0.00
714 X= 190.00 Y= 3.38 Z= 0.00
721 X= 220.00 Y= 3.38 Z= 0.00
722 X= 245.00 Y= 3.38 Z= 0.00
723 X= 270.00 Y= 3.38 Z= 0.00
724 X= 295.00 Y= 3.38 Z= 0.00
731 X= 320.00 Y= 3.38 Z= 0.00

;

; PIER 1

611 X= 93.50 Y= 3.38 Z= -11.26
511 X= 93.50 Y= 0.00 Z= -11.26
411 X= 93.50 Y= -20.00 Z= -11.26
311 X= 93.50 Y= -23.50 Z= -11.26

;

612 X= 106.50 Y= 3.38 Z= 11.26
512 X= 106.50 Y= 0.00 Z= 11.26
412 X= 106.50 Y= -20.00 Z= 11.26
312 X= 106.50 Y= -23.50 Z= 11.26

;

; PIER 2

621 X= 213.50 Y= 3.38 Z= -11.26
521 X= 213.50 Y= 0.00 Z= -11.26
421 X= 213.50 Y= -20.00 Z= -11.26
321 X= 213.50 Y= -23.50 Z= -11.26

;

622 X= 226.50 Y= 3.38 Z= 11.26
522 X= 226.50 Y= 0.00 Z= 11.26
422 X= 226.50 Y= -20.00 Z= 11.26
322 X= 226.50 Y= -23.50 Z= 11.26

LOCAL

ADD=311 CSYS=PIER
ADD=312 CSYS=PIER
ADD=321 CSYS=PIER
ADD=322 CSYS=PIER

RESTRAINT

ADD=701,731,30 DOF=UY,RX

SPRING

CSYS=PIER

ADD=311,312,1 U1=1.03E5 U2=9.44E4 U3=1.03E5 R1=7.12E6 R2=1.15E7 R3=7.12E6

ADD=321,322,1 U1=1.03E5 U2=9.44E4 U3=1.03E5 R1=7.12E6 R2=1.15E7 R3=7.12E6

CSYS=0

ADD=701,731,30 U1=0.0

U3=3.22E4

R2=0.0

R3=0.0

MASS

; ADD 15.5 KIPS AT EACH OF THE THREE INTERMEDIATE DIAPHRAGMS

ADD=703,723,10 UX=15.5/32.2 UY=15.5/32.2 UZ=15.5/32.2

MATERIAL

; WEIGHT OF SUPERSTRUCTURE = $(0.15\text{K/FT}^3) \times (72.74\text{FT}^2) = 10.91\text{ K/FT}$, SUPERIMPOSED
DEAD LOAD = 2.35 K/FT; ==> TOTAL DEAD LOAD = $10.91 + 2.35 = 13.26\text{ K/FT}$ ==> $13.26/72.74 = 0.182\text{ K/FT}^3$

NAME=SUPER TYPE=ISO M=0.182/32.2 W=0.182 IDES=C

E=519000 U=0.18 A=6.0E-06

NAME=SUB TYPE=ISO M=0.150/32.2 W=0.150 IDES=C

E=519000 U=0.18 A=6.0E-06

NAME=RIGID TYPE=ISO M=0 W=0 IDES=C

E=519000 U=0.18 A=6.0E-06

SECTION

NAME=SUPER MAT=SUPER A=72.74 I=401,9697 J=1177

NAME=M6 MAT=SUB A=27.00 I=100000,100000 J=100000

NAME=M5 MAT=RIGID SH=P T=400.

NAME=M4 MAT=SUB SH=P T=4.0

NAME=M3 MAT=SUB SH=R T=14.0,14.0

FRAME

CSYS=0

701 J=701,702 SEC=SUPER PLANE13=+Z

702 J=702,703 SEC=SUPER PLANE13=+Z

703 J=703,704 SEC=SUPER PLANE13=+Z

704 J=704,711 SEC=SUPER PLANE13=+Z

711 J=711,712 SEC=SUPER PLANE13=+Z

712 J=712,713 SEC=SUPER PLANE13=+Z

713 J=713,714 SEC=SUPER PLANE13=+Z

714 J=714,721 SEC=SUPER PLANE13=+Z

721 J=721,722 SEC=SUPER PLANE13=+Z

722 J=722,723 SEC=SUPER PLANE13=+Z

723 J=723,724 SEC=SUPER PLANE13=+Z

724 J=724,731 SEC=SUPER PLANE13=+Z

611 J=611,711 SEC=M6 PLANE12=+Y

612 J=612,711 SEC=M6 PLANE12=+Y

621 J=621,721 SEC=M6 PLANE12=+Y

622 J=622,721 SEC=M6 PLANE12=+Y

CSYS=PIER

311 J=311,411 SEC=M3 PLANE13=+Z

411 J=411,511 SEC=M4 IREL=R2,R3 PLANE13=+Z

511 J=511,611 SEC=M5 PLANE13=+Z

Appendix B — SAP90 V6.0 Beta Input**Design Example No. 4
Bridge with Two-Column Bents**

```
312 J=312,412 SEC=M3          PLANE13=+Z
412 J=412,512 SEC=M4 IREL=R2,R3 PLANE13=+Z
512 J=512,612 SEC=M5          PLANE13=+Z

321 J=321,421 SEC=M3          PLANE13=+Z
421 J=421,521 SEC=M4 IREL=R2,R3 PLANE13=+Z
521 J=521,621 SEC=M5          PLANE13=+Z

322 J=322,422 SEC=M3          PLANE13=+Z
422 J=422,522 SEC=M4 IREL=R2,R3 PLANE13=+Z
522 J=522,622 SEC=M5          PLANE13=+Z
```

LOAD

```
CSYS=0
NAME=DL
TYPE=GRAVITY ELEM=FRAME
ADD=* UY=-1
```

```
NAME=TL
TYPE=TEMPERATURE ELEM=FRAME
ADD=701,704,1,721,10 T=10
```

MODES

```
TYPE=EIGEN N=12 ; 3 SPANS AND 4 MODES PER SPAN
```

FUNCTION

```
NAME=S2 NPI=1
0.0 2.50
0.44 2.50
0.5 2.29
0.6 2.02
0.7 1.83
0.8 1.67
0.9 1.54
1.0 1.44
1.2 1.28
1.4 1.15
1.6 1.05
1.8 0.97
2.0 0.91
2.5 0.78
3.0 0.69
3.5 0.62
4.0 0.57
10.0 0.31
100.0 0.07
```

SPEC

```
CSYS=0
NAME=EQLONG MODC=CQC DAMP=0.05
ACC=X FUNC=S2 SF=32.2*0.30*1.0
NAME=EQTRAN MODC=CQC DAMP=0.05
ACC=Z FUNC=S2 SF=32.2*0.30*1.0
```

