Pressure in isentropic choked flow

The energy equation for one-dimensional steady state, adiabatic, non-viscous, frictionless flow is:

$\frac{1}{2}u^2 + h = constant$, where u is fluid speed and h is enthalpy.

This condition means the "total" enthalpy remains constant, but in fact is a specific form of energy conservation.

For a perfect gas, we can write:

$$\frac{1}{2}u^2 + C_pT = \frac{1}{2}u^2 + \frac{kR}{k-1}T = \frac{1}{2}u^2 + \frac{a^2}{k-1} = constant$$

where $a^2 = kRT = k\frac{p}{\rho}$

When the flow has the particularity *Mach=1* we are talking about "choked" conditions and we note this status with subscript *. That means in this flow status, the flow speed equals the "sonic" speed as defined bellow:

$$u_* = a_* = \sqrt{kRT_*} = \sqrt{k\frac{p_*}{\rho_*}}$$

One can calculate the sonic speed and choked parameters T_*, p_* based on the assumption that there is an evolution between "no flow" condition to this "sonic" condition and the total enthalpy remains constant. So we can imagine the fluid flow as evolution between several conditions. State 0 would be defined as stagnation since is "reservoir status" with flow velocity zero. Flow can reach choked condition during the evolution. May be one or several points where flow is isentropic choked (isentropic assumption gives the possibility to count the sound speed as above).

For an isentropic evolution from stagnation point to any choked point we can write:

$$\frac{a_0^2}{k-1} = \frac{1}{2}u^2 + \frac{a_*^2}{k-1}\bigg|_{u=a_*} = \frac{1}{2}\frac{k+1}{k-1}a_*^2$$

i.e.

$$\frac{2a_0^2}{k+1} = a_*^2$$

or

$$a_* = \sqrt{\frac{2}{k+1}}a_0 = \sqrt{\frac{2}{k+1}}\sqrt{kRT_0} = \sqrt{2R_u}\sqrt{\frac{kT_0}{M(k+1)}}$$

and one may note that this is the fluid velocity in isentropic choked status.

By the other hand, by definition $a_* = \sqrt{k\frac{p_*}{\rho_*}}$, so

$$p_* = \frac{\rho_* a_*^2}{k}$$

Since

$$\rho_* = \frac{W}{a_* A_{choked}}$$

we can write

$$p_* = \frac{\frac{W}{a_* A_{choked}}a_*^2}{k} = \frac{W}{kA_{choked}}a_* = \frac{W}{kA_{choked}}\sqrt{2R_u}\sqrt{\frac{kT_0}{M(k+1)}} = \sqrt{2R_u}\frac{W}{A_{choked}}\sqrt{\frac{T_0}{Mk(k+1)}}$$

This is the pressure in isentropic choked status, as function of W - the flowrate, T_0 the stagnation temperature, A_{choked}- area where flow is choked, etc. The above relations are related to isentropic evolution to choked points and are not specifically linked to that assumption for the rest of evolutions. The relations are valid only under the assumptions made- perfect gas in one-dimensional steady state flow, adiabatic, frictionless evolution.