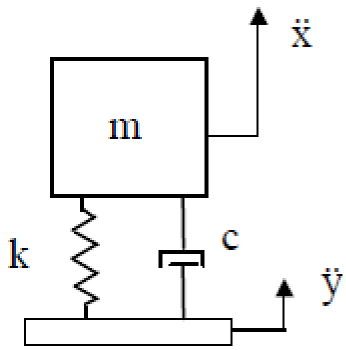
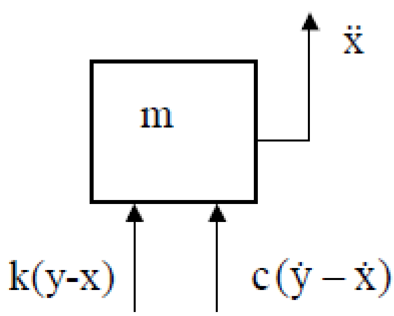
An ODE that looks real simple but....

Consider a simple single degree-of-freedom spring-mass-dashpot dynamic system with spring rate *k*, mass *m*, and the viscous damping coefficient *c*. *x* is the absolute displacement of the mass. The base input translation is *y*. A dot notation indicates differentiation with respect to time, *t*.



A free body diagram of the mass is:



and the base input acceleration is a half-sine pulse of magnitude *A* and duration *T*:

$$\ddot{y} = \begin{cases} A\sin\left(\frac{\pi t}{T}\right), & 0 \le t \le T \\ 0, & t > T \end{cases}$$

Determine the displacement *x* as a function of time *t*.
This problem is actually a realistic simulation of the response of a single DOF suspension system from an impact of duration *T*.
To take this a little further,

If $z = x - y$, then $\dot{z} = \dot{x} - \dot{y}$ and $\ddot{z} = \ddot{x} - \ddot{y}$
After making these substitutions and simplifying,

$$m\ddot{z} + c\dot{z} + kz = -m\ddot{y}$$

To put this in more traditional form,

$$\ddot{z} + \frac{c}{m}\dot{z} + \frac{k}{m}z = -\ddot{y}$$

These ratios of constants appear often in vibration theory and they are related to

the natural frequency ω_n in radians/sec,
and the critical damping ratio Σ as follows:

$$\frac{c}{m} = 2\Sigma\omega_n$$

$$\frac{k}{m} = \omega_n^2$$

then the ODE looks like this after substituting:

$$\ddot{z} + 2\Sigma\omega_n\dot{z} + \omega_n^2 z = -2\Sigma\omega_n\ddot{y}$$

then the final form of the ODE is:

$$\ddot{z} + 2\Sigma\omega_n\dot{z} + \omega_n^2 z = -2\Sigma\omega_n\begin{cases} Asin\left(\frac{\pi t}{T}\right), & 0 \le t \le T \\ 0, & t > T \end{cases}$$

This is as far as I'm taking it. Take this the rest of the way and let me know how it works out for you ☺.