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N_{bal} design axial load capacity of a balanced section; for symmetrically- reinforced rectangular sections, it may be taken as $0.25 f_{cu} bd$.

3.8.3 Deflection induced moments in solid slender columns

3.8.3.1 *Design*

In general, a cross-section may be designed by the methods given for a short column (see **3.8.4**) but in the design, account has to be taken of the additional moment induced in the column by its deflection.

The deflection of a rectangular or circular column under ultimate conditions may be taken to be:

$$a_u = \beta_a K h \qquad \text{equation 32}$$

In this expression β_a has the value obtained from Table 3.21 or, alternatively, from equation 34 from which the table is derived, where K is a reduction factor that corrects the deflection to allow for the influence of axial load. K is derived from the following equation:

$$K = \frac{N_{uz} - N}{N_{uz} - N_{bal}} \leq 1 \qquad \text{equation 33}$$

where $N_{uz} = 0.45 f_{cu} A_c + 0.95 f_y A_{sc}$ (including allowances, as appropriate for γ_m).

The appropriate values of K may be found iteratively, taking an initial value of 1. Alternatively, it will always be conservative to assume that $K = 1$.

Table 3.21 is derived from the following equation:

$$\beta_a = \frac{1}{2000}\left(\frac{l_e}{b'}\right)^2 \qquad \text{equation 34}$$

NOTE b' is generally the smaller dimension of the column (but see **3.8.3.6** for biaxial bending).

The deflection induces an additional moment given by:

$$M_{add} = N a_u \qquad \text{equation 35}$$

3.8.3.2 *Design moments in braced columns bent about a single axis*

Figure 3.20 shows the distribution of moments assumed over the height of a typical braced column. It may be assumed that the initial moment at the point of maximum additional moment (i.e. near mid-height of the column) is given by:

$$M_i = 0.4 M_1 + 0.6 M_2 \geq 0.4 M_2 \qquad \text{equation 36}$$

where

- M_1 is the smaller initial end moment due to design ultimate loads;
- M_2 is the larger initial end moment due to design ultimate loads.

Assuming the column is bent in double curvature, M_1 should be taken as negative and M_2 positive.

It will be seen from Figure 3.20 that the maximum design moment for the column will be the greatest of a) to d):

a) M_2;
b) $M_i + M_{add}$;
c) $M_1 + M_{add}/2$;
d) $e_{min} N$.

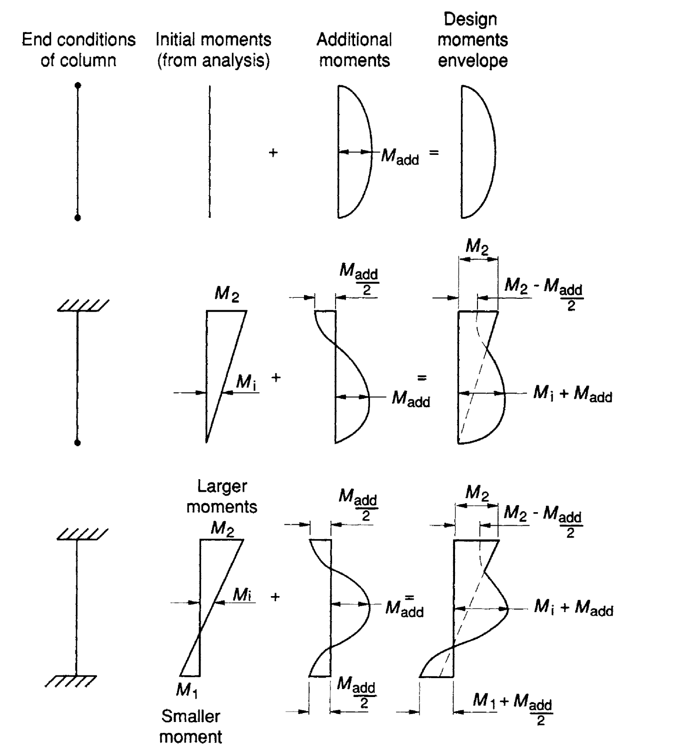

Figure 3.20 — Braced slender columns

Table 3.21 — Values of β_a

l_e/b'	12	15	20	25	30	35	40	45	50	55	60
β_a	0.07	0.11	0.20	0.31	0.45	0.61	0.80	1.01	1.25	1.51	1.80

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10.10.6 — Moment magnification procedure — Nonsway

Compression members shall be designed for factored axial force P_u and the factored moment amplified for the effects of member curvature M_c as

$$M_c = \delta_{ns} M_2 \qquad (10\text{-}11)$$

where

$$\delta_{ns} = \frac{C_m}{1 - \dfrac{P_u}{0.75 P_c}} \geq 1.0 \qquad (10\text{-}12)$$

and

$$P_c = \frac{\pi^2 EI}{(k\ell_u)^2} \qquad (10\text{-}13)$$

R10.10.6 — Moment magnification procedure — Nonsway

The ϕ-factors used in the design of slender columns represent two different sources of variability. First, the stiffness reduction ϕ_K-factor accounts for the variability in the stiffness EI and the moment magnification analysis. Second, the strength reduction ϕ-factor for tied and spiral columns accounts for the variability of the strength of the cross section. Studies reported in Reference 10.41 indicate that the stiffness reduction factor ϕ_K and the cross-sectional strength reduction ϕ-factors do not have the same values. These studies suggest the stiffness reduction factor ϕ_K for an isolated column should be 0.75 for both tied and spiral columns. The 0.75 factor in Eq. (10-12) is the stiffness reduction factor ϕ_K. The factor is based on the probability of understrength of a single isolated slender column. In the case of a multistory frame, the column and frame deflections depend on the average concrete strength, which is higher than the strength of the concrete in the critical single understrength column. For this reason, the value of ϕ_K in 10.10.4 is 0.875.

10.10.6.1 — EI shall be taken as

$$EI = \frac{(0.2 E_c I_g + E_s I_{se})}{1 + \beta_{dns}} \qquad (10\text{-}14)$$

or

$$EI = \frac{0.4 E_c I_g}{1 + \beta_{dns}} \qquad (10\text{-}15)$$

Alternatively, EI shall be permitted to be computed using the value of I from Eq. (10-8) divided by $(1 + \beta_{dns})$.

R10.10.6.1 — In defining the critical load, the main problem is the choice of a stiffness EI that reasonably approximates the variations in stiffness due to cracking, creep, and nonlinearity of the concrete stress-strain curve. Either Eq. (10-14) or (10-15) may be used to compute EI. Equation (10-14) was derived for small eccentricity ratios and high levels of axial load where slenderness effects are most pronounced. Equation (10-15) is a simplified approximation to Eq. (10-14) and is less accurate.[10.42] For improved accuracy, EI can be approximated using the suggested E and I values provided by Eq. (10-8) divided by $(1 + \beta_{dns})$.

10.10.6.2 — The term β_{dns} shall be taken as the ratio of maximum factored axial sustained load to maximum factored axial load associated with the same load combination, but shall not be taken greater than 1.0.

R10.10.6.2 — Creep due to sustained load will increase the lateral deflections of a column and hence the moment magnification. This is approximated for design by reducing the stiffness EI used to compute P_c and hence δ by dividing EI by $(1 + \beta_{dns})$. Both the concrete and steel terms in Eq. (10-14) are divided by $(1 + \beta_{dns})$ to reflect the premature yielding of steel in columns subjected to sustained load. For simplification, it can be assumed that $\beta_{dns} = 0.6$. In this case, Eq. (10-15) becomes

$$EI = 0.25 E_c I_g$$

10.10.6.3 — The effective length factor, k, shall be permitted to be taken as 1.0.

R10.10.6.3 — The effective length factor for a compression member considering braced behavior ranges from 0.5 and 1.0. While lower values can be justified, it is recommended that a k value of 1.0 be used. If lower values are used, the calculation of k should be based on analysis of the frame using E_c and I values given in 10.10.4. The Jackson and Moreland Alignment Charts (Fig. R10.10.1.1) can be used to estimate lower values of k.[10.4,10.30]

10.10.6.4 — For members without transverse loads between supports, C_m shall be taken as

$$C_m = 0.6 + 0.4\frac{M_1}{M_2} \qquad (10\text{-}16)$$

where M_1/M_2 is positive if the column is bent in single curvature, and negative if the member is bent in double curvature. For members with transverse loads between supports, C_m shall be taken as 1.0.

R10.10.6.4 — The factor C_m is a correction factor relating the actual moment diagram to an equivalent uniform moment diagram. The derivation of the moment magnifier assumes that the maximum moment is at or near midheight of the column. If the maximum moment occurs at one end of the column, design should be based on an equivalent uniform moment $C_m M_2$ that would lead to the same maximum moment when magnified.[10.31]

In the case of compression members that are subjected to transverse loading between supports, it is possible that the maximum moment will occur at a section away from the end of the member. If this occurs, the value of the largest calculated moment occurring anywhere along the member should be used for the value of M_2 in Eq. (10-11). C_m is to be taken as 1.0 for this case.

10.10.6.5 — Factored moment, M_2, in Eq. (10-11) shall not be taken less than

$$M_{2,min} = P_u(0.6 + 0.03h) \qquad (10\text{-}17)$$

about each axis separately, where 0.6 and h are in inches. For members in which $M_{2,min}$ exceeds M_2, the value of C_m in Eq. (10-16) shall either be taken equal to 1.0, or shall be based on the ratio of the computed end moments, M_1/M_2.

R10.10.6.5 — In the Code, slenderness is accounted for by magnifying the column end moments. If the factored column moments are very small or zero, the design of slender columns should be based on the minimum eccentricity given in this section. It is not intended that the minimum eccentricity be applied about both axes simultaneously.

The factored column end moments from the structural analysis are used in Eq. (10-16) in determining the ratio M_1/M_2 for the column when the design should be based on minimum eccentricity. This eliminates what would otherwise be a discontinuity between columns with computed eccentricities less than the minimum eccentricity and columns with computed eccentricities equal to or greater than the minimum eccentricity.