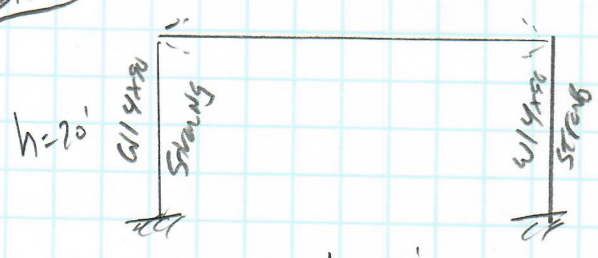
1 COEFFICIENT of THERMAL EXPANSION

[SERM PS 136] 9th Ed EX. 1.19

GIVEN



TEMP RISE = 50° F

COEF of EXPANSION = 6.5×10^{-6} IN/IN°F

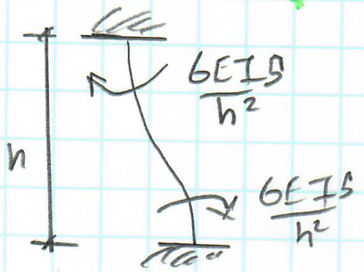
W14x90

$I = 999 \text{ in}^4$

BECAUSE of SYMMETRY of COLUMNS, THE LATERAL DISPLACEMENT of THE COLS IS 1/2 THE THERMAL EXPANSION of THE BEAM

SOLVE:

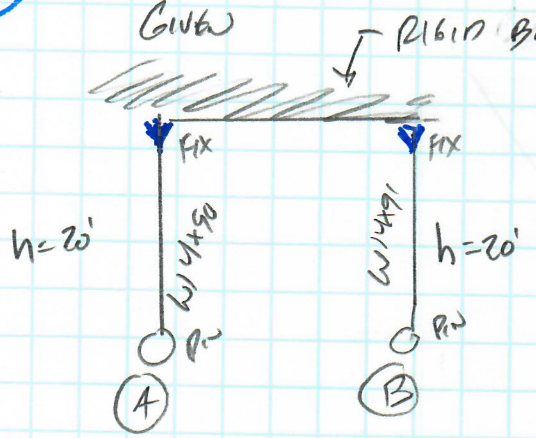
$$\Delta = \frac{\alpha T L}{2} = \left(6.5\times10^{-6}\ \frac{\text{in}}{\text{in}^\circ\text{F}}\right)(50^\circ\text{F})(30')\left(12\ \text{in}/\text{ft}\right) = 0.059\ \text{in}$$



$$M = \frac{6EI\Delta}{h^2} \qquad I = 999\ \text{in}^4$$

$$M = \frac{(6)(29000\ \text{ksi})(999\ \text{in}^4)(0.59\ \text{in})}{(20' \times 12'')^2} = \boxed{178\ \text{k-ft}}$$

2 GIVEN



SOLVE: RIGID BODY EXPANDS 1" LATERALLY

FIND: DETERMINE THE MOMENT AND OR REACTIONS AT THE BASE of THE COLUMN A & B

$I = 999\ \text{in}^4$, STRONG AXIS $= I_x$

HINT: FIND EXPLANATION SIMILAR TO

$$M = \frac{6EI\Delta}{h^2}$$

Gunnison River,
ural vibration pe-
: partially full and
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plex structure–soil and
hensive presentation of
However, an introduc-

ear systems and cannot
:thquakes if the ground
ıvolves numerical time-
ıods are also useful for
l force $p(t)$ or ground
y and is described only

1.11 STUDY OF SDF SYSTEMS: ORGANIZATION

We will study the dynamic response of linearly elastic SDF systems in free vibration (Chapter 2), to harmonic and periodic excitations (Chapter 3), to step and pulse excitations (Chapter 4), and to earthquake ground motion (Chapter 6). Because most structures are designed with the expectation that they will deform beyond the linearly elastic limit during major, infrequent earthquakes, the inelastic response of SDF systems is studied in Chapter 7. The time variation of response $r(t)$ to these various excitations will be of interest. For structural design purposes, the maximum value (over time) of response r contains the crucial information, for it is related to the maximum forces and deformations that a structure must be able to withstand. We will be especially interested in the peak value of response, or for brevity, *peak response*, defined as the maximum of the absolute value of the response quantity:

$$r_o \equiv \max_t |r(t)| \tag{1.11.1}$$

By definition the peak response is positive; the algebraic sign is dropped because it is usually irrelevant for design. Note that the subscript o attached to a response quantity denotes its peak value.

FURTHER READING

Clough, R. W., and Penzien, J., *Dynamics of Structures*, McGraw-Hill, New York, 1993, Sections 4-3, 6-2, 6-3, and 12-6.

Humar, J. L., *Dynamics of Structures*, Prentice Hall, Englewood Cliffs, N.J., 1990, Chapter 9 and Section 13.5.

APPENDIX 1: STIFFNESS COEFFICIENTS FOR A FLEXURAL ELEMENT

To compute bending moments and shears in a flexural structural element—beam or column—the stiffness coefficients for the element are required. These are presented in Fig. A1.1 for a uniform element of length L, second moment of area I, and elastic

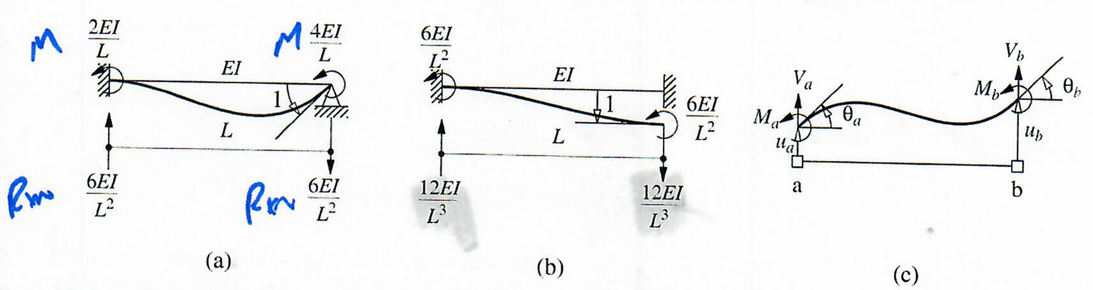


Figure A1.1

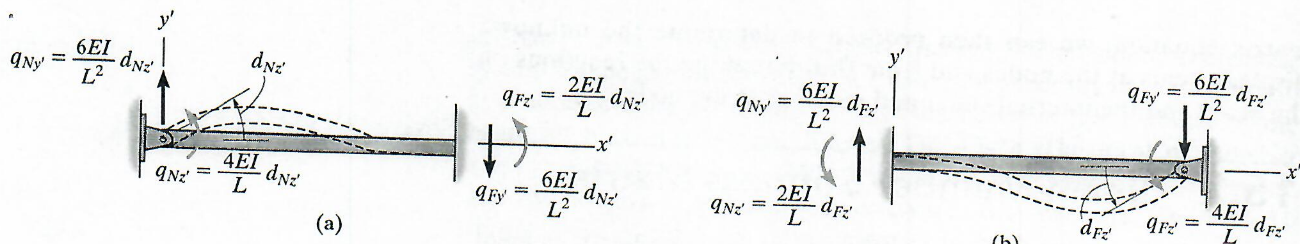


Fig. 15–6

z Rotations. If a positive rotation $d_{Nz'}$ is imposed while all other possible displacements are prevented, the required shear forces and moments necessary for the deformation are shown in Fig. 15–6*a*. In particular, the moment results have been developed in Sec. 11.2 as Eqs. 11–1 and 11–2. Likewise, when $d_{Fz'}$ is imposed, the resultant loadings are shown in Fig. 15–6*b*.

By superposition, if the above results in Figs. 15–5 and 15–6 are added, the resulting four load-displacement relations for the member can be expressed in matrix form as

$$\begin{bmatrix} q_{Ny'} \\ q_{Nz'} \\ q_{Fy'} \\ q_{Fz'} \end{bmatrix} = \begin{array}{c} \begin{array}{cccc} \quad N_{y'} & \quad N_{z'} & \quad F_{y'} & \quad F_{z'} \end{array} \\ \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \end{array} \begin{bmatrix} d_{Ny'} \\ d_{Nz'} \\ d_{Fy'} \\ d_{Fz'} \end{bmatrix} \qquad (15\text{–}1)$$

These equations can also be written in abbreviated form as

$$\mathbf{q} = \mathbf{k}\mathbf{d} \qquad (15\text{–}2)$$

The symmetric matrix $\mathbf{k}$ in Eq. 15–1 is referred to as the *member stiffness matrix*. The 16 influence coefficients $\mathbf{k}_{ij}$ that comprise it account for the shear-force and bending-moment displacements of the member. Physically these coefficients represent the load on the member when the member undergoes a specified unit displacement. For example, if $d_{Ny'} = 1$, Fig. 15–5*a*, *while all other displacements are zero,* the member will be subjected only to the four loadings indicated in the first column of the $\mathbf{k}$ matrix. In a similar manner, the other columns of the $\mathbf{k}$ matrix are the member loadings for unit displacements identified by the degree-of-freedom code numbers listed above the columns. From the development, both equilibrium and compatibility of displacements have been satisfied. Also, it should be noted that this matrix is the *same* in both the local and global coordinates since the x', y', z' axes are parallel to x, y, z and, therefore, transformation matrices are not needed between the coordinates.

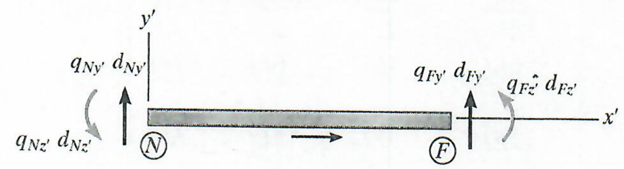
Displacements

matrix equation, we can then proceed to determine the unknown displacements at the nodes and from this determine the reactions on the beam and the internal shear and moment at the nodes.

15.2 Beam-Member Stiffness Matrix

In this section we will develop the stiffness matrix for a beam element or member having a constant cross-sectional area and referenced from the local x', y', z' coordinate system, Fig. 15–4. The origin of the coordinates is placed at the "near" end N, and the positive x' axis extends toward the "far" end F. There are two reactions at each end of the element, consisting of shear forces $q_{Ny'}$ and $q_{Fy'}$ and bending moments $q_{Nz'}$ and $q_{Fz'}$. These loadings all act in the positive coordinate directions. In particular, the moments $q_{Nz'}$ and $q_{Fz'}$ are positive *counterclockwise,* since by the right-hand rule the moment vectors are then directed along the positive z' axis, which is out of the page.

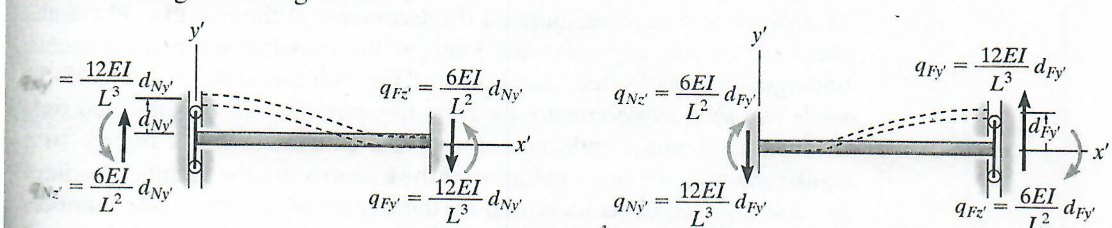
Linear and angular displacements associated with these loadings also follow this same positive sign convention. We will now impose each of these displacements separately and then determine the loadings acting on the member caused by each displacement.



positive sign convention

Fig. 15–4

Displacements. When a positive displacement $d_{Ny'}$ is imposed while other possible displacements are prevented, the resulting shear forces and bending moments that are created are shown in Fig. 15–5*a*. In particular, the moment has been developed in Sec. 11.2 as Eq. 11–5. Likewise, when $d_{Fy'}$ is imposed, the required shear forces and bending moments are given in Fig. 15–5*b*.



(a) y' displacements (b)

Fig. 15–5