$$\begin{bmatrix} n_x \\ n_y \\ n_{xy} \\ m_x \\ m_y \\ m_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}_0$$

Stiffness matrix for Stora Enso CLT

rethink.

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Disclaimer

The below presented documentation and structural analysis is an exemplified calculation. This calculation is solely a proposal for a design approach. This structural design proposal must be verified and approved regarding completeness and correctness by the project structural engineer in charge. Stora Enso Wood Products GmbH excludes all liability for the completeness or correctness of the analysis below. The project structural engineer is not allowed to use the calculation towards third parties. For further use in the project he has to produce his independent calculation. The below calculation does not constitute any warranty or representation for the product Cross-Laminated-Timber.

Stiffness matrix

1. General

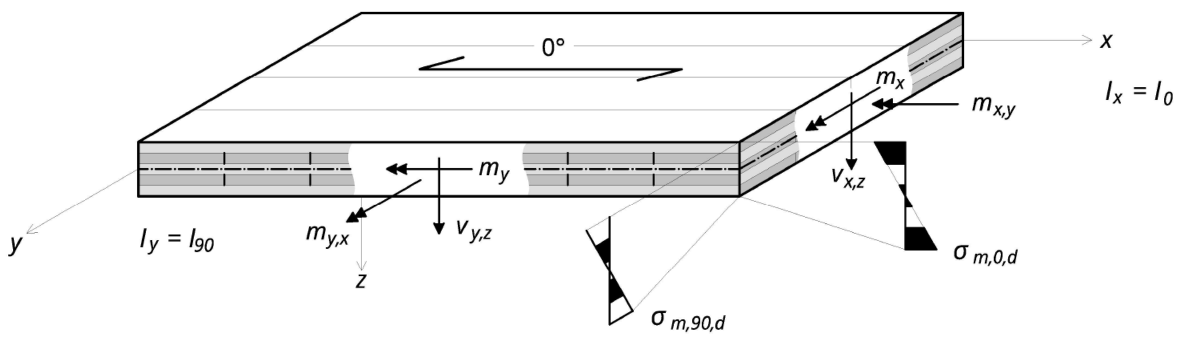
When analyzing a building structure from the structural point of view, the complexity of structures require/justify more and more the use of finite element models (FEM). For a numeric modeling of a structure, the building material with all its properties needs to be defined. For isotropic materials this can be rather simple. For orthotropic materials it can be a bit more challenging, even more when using a laminated surface. For laminated surfaces, the stiffness matrix also depends on the geometry of the laminate (thickness, orientation and location of layers).

Since Cross Laminated Timber (CLT) is a laminate and orthotropic at the same time, the stiffness matrix is different for each single type of CLT.

This document shall explain the stiffness matrix for CLT and provide the stiffness matrix for each individual type of CLT, produced by Stora Enso.

1.1. Definitions and nomenclature

It is assumed that the CLT panel (surface) has not edge gluing. This means that there are is no force transfer perpendicular to the grain, in the CLT-plane. Therefore the principal direction of a CLT panel shall be parallel to the grain of the top and bottom layer (usually the gain in top and bottom layer are oriented in the same direction). The wood grade for each layer shall be the same.



$E_{0,mean}$	Mean value of modulus of elasticity parallel to grain
$E_{90,mean}$	Mean value of modulus of elasticity perpendicular to grain
ν	Poisson ratio
$I_{0,net}$	Net value of moment of inertia parallel to grain (only layers in principal direction are taken into account)
K_{twist}	Reduction factor to reduce twisting rigidity According to Silly (2010): For cracked wood: K_{twist} = 0,65
i	The index i refers to the lamination layers of the net section (grain parallel to principal direction or span direction)
jj	The index jj refers to the lamination layers of the net section (grain perpendicular to principal direction or span direction)

Stiffness matrix

2. Stiffness matrix for orthotropic elements

$$\begin{Bmatrix} m_x \\ m_y \\ m_{xy} \\ v_x \\ v_y \\ n_x \\ n_y \\ n_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & D_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\ sym & & D_{33} & & & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{44} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & D_{55} & 0 & 0 & 0 \\ & & & 0 & 0 & D_{66} & 0 & 0 \\ & & sym & 0 & 0 & & D_{77} & 0 \\ & & & & & sym & & D_{88} \end{bmatrix} \cdot \begin{pmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \\ \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix}$$

The matrix above shows the applicable matrix for an orthotropic material, such as CLT.

The stiffness matrix is arranged as follows:

- D_{11}, …, D_{33} are related to the flexural stiffness.
- D_{44} and D_{55} are related to the shear stiffness.
- D_{66}, …, D_{88} are related to the axial stiffness (in plane).

Actually all values outside the diagonal D_{11} – D_{88} are partially related to the stiffness out of plane which can be neglected, others are by default 0 and the rest of the values are so small that they can be neglected and therefore set 0.

D_{12} and D_{67} (and their symmetrical values) are 0, because it was chosen to assume the Poisson ratio $\nu = 0$. This assumption was taken to account for cracking parallel to the grain in a lamination layer or to account for the dry joints, when no edge gluing is being applied. Stora Enso CLT is actually edge glued, but the glue has no technical approval for load bearing purposes. Therefore the joint has to be assumed dry, as if there was no glue.

The individual components of the stiffness matrix will be described in the following chapters.

Stiffness matrix

2.1. Flexural stiffness components

$D_{11} = E_{0,mean} \cdot I_{0,net}$	[kNm²/m]	
$D_{22} = E_{0,mean} \cdot I_{90,net}$	[kNm²/m]	
$D_{33} = K_{twist} \cdot G_{0,mean} \cdot \frac{b \cdot d^3}{12}$	[kNm²/m]	In most literature and software the reduction factor for twisting stiffness is neglected. We do recommend to apply it. K_{twist} = 0,65 (constant)

2.2. Shear stiffness

$D_{44} = \kappa_x \cdot \sum G_{i,x} \cdot t_i$	[kN/m]	The corrective shear coefficient κ_x can be found in chapter 5.
$D_{55} = \kappa_y \cdot \sum G_{i,y} \cdot t_i$	[kN/m]	The corrective shear coefficient κ_y can be found in chapter 5.

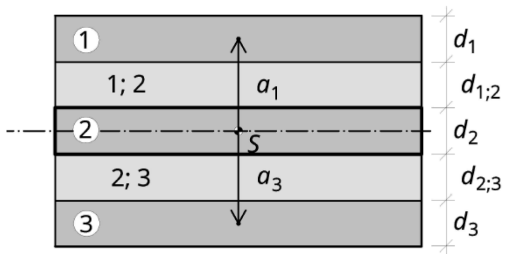
2.3. Axial stiffness in plane

$D_{66} = \sum_{i=1}^{n} h_{i,0,net} \cdot E_{0,mean}$	[kN/m]	
$D_{77} = \sum_{i=1}^{n} h_{i,90,net} \cdot E_{90,mean}$	[kN/m]	
$D_{88} = G_{0,mean} \cdot d \cdot K_{shear}$	[kN/m]	In most literature and software the reduction factor for shear rigidity is neglected. We do recommend to apply it. K_{shear} = 0,70 (constant)

3. Demonstrative example

In this chapter it will be demonstrated, how to obtain the values for the stiffness matrix for a given Stora Enso CLT section.

For demonstration a 1m² sample of CLT 100 L5s shall be chosen. All lamination layers are wood grade C24 (according to EN338). This is a CLT panel with 5 lamination layers – each layer has a thickness of 20 mm.



3.1. Flexural stiffness components

$D_{11} = E_{0,mean} \cdot I_{0,net} = 12.500\, N/mm^2 \cdot (6{,}6 \cdot 10^7) mm^4 = 826.136.363.636\, Nmm^2/m$	$I_{0,net} = 3 \cdot \frac{1.000\, mm \cdot (20\, mm)^3}{12} + 2 \cdot \left[(20\, mm \cdot 1.000\, mm) \cdot \left(\frac{20\, mm}{2} + 20\, mm + \frac{20\, mm}{2}\right)^2\right] = 66.000.000\, mm^4 = 6{,}6 \cdot 10^7$
$D_{22} = E_{0,mean} \cdot I_{90,net} = 12.500\, N/mm^2 \cdot (1{,}7\dot{3} \cdot 10^7) mm^4 = 216.666.666.666\, Nmm^2/m$	$I_{90,net} = 2 \cdot \frac{1.000\, mm \cdot (20\, mm)^3}{12} + 2 \cdot \left[(20\, mm \cdot 1.000\, mm) \cdot \left(\frac{20\, mm}{2} + \frac{20\, mm}{2}\right)^2\right] = 17.333.333\, mm^4 = 1{,}7\dot{3} \cdot 10^7$
$D_{33} = K_{twist} \cdot G_{0,mean} \cdot \frac{b \cdot d^3}{12} = 0{,}65 \cdot \underbrace{690\, N/mm^2 \cdot \frac{1000\, mm \cdot (100\, mm)^3}{12}}_{57.500.000.000} = 37.375.000.000$	In most literature and software the reduction factor for twisting stiffness is neglected. We do recommend to apply it. K_{twist} = 0,65 (constant)

3.2. Shear stiffness

$D_{44} = \kappa_x \cdot \sum G_{i,x} \cdot t_i = 0{,}184 \cdot (690 N/mm^2 \cdot 3 \cdot 20\, mm + 50 N/mm^2 \cdot 2 \cdot 20 mm) = 7.985{,}6\, N/m$	The corrective shear coefficient κ_x can be found in chapter 5.
$D_{55} = \kappa_y \cdot \sum G_{i,y} \cdot t_i = 0{,}145 \cdot (50 N/mm^2 \cdot 3 \cdot 20\, mm + 690 N/mm^2 \cdot 2 \cdot 20 mm) = 4.437\, N/m$	The corrective shear coefficient κ_y can be found in chapter 5.

3.3. Axial stiffness in plane

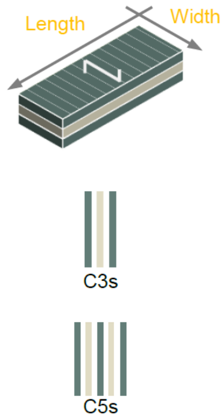
$D_{66} = \sum_{i=1}^{n} h_{i,0,net} \cdot E_{0,mean} = 3 \cdot (20\, mm \cdot 12.500\, N/mm^2) = 750.000\, N/m$	
$D_{77} = \sum_{jj=1}^{mm} h_{jj,90,net} \cdot E_{90,mean} = 2 \cdot (20\, mm \cdot 12.500\, N/mm^2) = 500.000\, N/m$	
$D_{88} = G_{0,mean} \cdot d \cdot K_{shear} = 690\, N/mm^2 \cdot 100\, mm \cdot 0{,}7 = 48.300\, N/m$	In most literature and software the reduction factor for shear rigidity is neglected. We do recommend to apply it. K_{shear} = 0,70 (constant)

Stiffness matrix

4. Stora Enso standard CLT elements

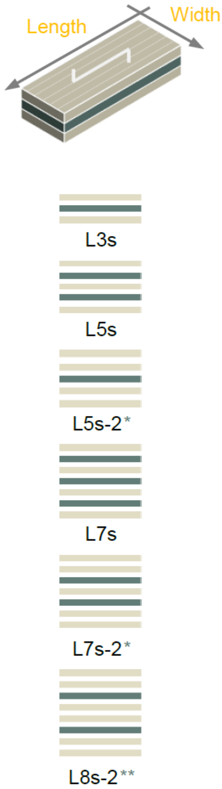
C panels

Nominal thickness [mm]	Designation [–]	Layers [–]	Lamella structure [mm]						
			C	L	C	L	C	L	C
60	C3s	3	20	20	20				
80	C3s	3	30	20	30				
90	C3s	3	30	30	30				
100	C3s	3	30	40	30				
120	C3s	3	40	40	40				
100	C5s	5	20	20	20	20	20		
120	C5s	5	30	20	20	20	30		
140	C5s	5	40	20	20	20	40		
160	C5s	5	40	20	40	20	40		



L panels

Nominal thickness [mm]	Designation [–]	Layers [–]	Lamella structure [mm]						
			L	C	L	C	L	C	L
60	L3s	3	20	20	20				
80	L3s	3	30	20	30				
90	L3s	3	30	30	30				
100	L3s	3	30	40	30				
120	L3s	3	40	40	40				
100	L5s	5	20	20	20	20	20		
120	L5s	5	30	20	20	20	30		
140	L5s	5	40	20	20	20	40		
160	L5s	5	40	20	40	20	40		
180	L5s	5	40	30	40	30	40		
200	L5s	5	40	40	40	40	40		
160	L5s-2*	5	60	40	60				
180	L7s	7	30	20	30	20	30	20	30
200	L7s	7	20	40	20	40	20	40	20
240	L7s	7	30	40	30	40	30	40	30
220	L7s-2*	7	60	30	40	30	60		
240	L7s-2*	7	80	20	40	20	80		
260	L7s-2*	7	80	30	40	30	80		
280	L7s-2*	7	80	40	40	40	80		
300	L8s-2**	8	80	30	80	30	80		
320	L8s-2**	8	80	40	80	40	80		



* *Cover layers consisting of 2 lengthwise layers*
** *Cover layers and inner layer consisting of 2 lengthwise layers*

Status: 04/2012

Width (Charged widths): *245 cm, 275 cm, 295 cm*
Length (Production lengths): *From minimum production length of 8.00 m per charged width up to max. 16.00 m (in 10 cm increments).*

5. Corrective shear coefficients

The corrective shear coefficient κ is dependent on the orientation of the lamination, geometrical layout of the CLT (number and thickness of layers) and mechanical properties (shear modulus and rolling shear modulus) of the base material (C24 timber). The derivation of the coefficient can be found in various literature, among others also in the Brettsperrholz (BSP) Handbuch, TU Graz (Schickhofer, Bogensperger, Moosbrugger), Nov. 2009.

CLT-Type	Shear coeff.	
	κ_0	κ_{90}
CLT 60 L3s	0,156	0,728
CLT 80 L3s	0,172	0,685
CLT 90 L3s	0,156	0,728
CLT 100 L3s	0,153	0,752
CLT 120 L3s	0,156	0,728
CLT 160 L5s	0,172	0,685
CLT 100 L5s	0,184	0,145
CLT 120 L5s	0,178	0,156
CLT 140 L5s	0,179	0,136
CLT 160 L5s	0,208	0,140
CLT 180 L5s	0,189	0,139
CLT 200 L5s	0,184	0,145
CLT 180 L7s	0,203	0,175
CLT 200 L7s	0,212	0,289
CLT 240 L7s	0,198	0,608
CLT 220 L7s	0,188	0,152
CLT 240 L7s	0,226	0,125
CLT 260 L7s	0,194	0,128
CLT 280 L7s	0,179	0,136
CLT 300 L8s	0,229	0,146
CLT 320 L8s	0,208	0,140
CLT 60 C3s	0,155	0,728
CLT 80 C3s	0,172	0,685
CLT 90 C3s	0,156	0,728
CLT 100 C3s	0,153	0,752
CLT 120 C3s	0,156	0,728
CLT 100 C5s	0,184	0,145
CLT 120 C5s	0,178	0,156
CLT 140 C5s	0,179	0,136
CLT 160 C5s	0,208	0,140

Stiffness matrix

6. Stiffness Matrix for Stora Enso CLT

CLT-Type		Stiffness matrix							
		D_{11} kNm²/m	D_{22} kNm²/m	D_{33} kNm²/m	D_{44} kN/m	D_{55} kN/m	D_{66} kN/m	D_{77} kN/m	D_{88} kN/m
Cover layers in longitudinal direction (typ. for floor elements)	CLT 60 L3s	216,7	8,3	8,1	4.462	11.502	500.000	250.000	28.980
	CLT 80 L3s	525,0	8,3	19,1	7.293	11.508	750.000	250.000	38.640
	CLT 90 L3s	731,3	28,1	27,2	6.692	17.254	750.000	375.000	43.470
	CLT 100 L3s	975,7	66,7	37,4	6.640	23.011	750.525	500.000	48.300
	CLT 120 L3s	1.733,0	66,7	64,6	8.923	23.005	1.000.000	500.000	57.960
	CLT 160 L5s-2	4.200,0	66,7	153,1	14.586	23.016	1.500.000	500.000	77.280
	CLT 100 L5s	825,0	216,7	37,4	7.986	4.437	750.000	500.000	48.300
	CLT 120 L5s	1.583,0	216,7	64,6	10.182	4.930	1.000.000	500.000	57.960
	CLT 140 L5s	2.640,9	216,7	102,6	12.709	4.434	1.250.000	500.000	67.620
	CLT 160 L5s	3.800,0	466,7	153,1	17.638	4.704	1.500.000	500.000	77.280
	CLT 180 L5s	5.100,0	975,0	217,9	16.216	6.589	1.500.000	750.000	86.940
	CLT 200 L5s	6.600,0	1.733,0	299,0	15.971	8.874	1.500.000	1.000.000	96.600
	CLT 180 L7s	4.800,0	1.275,0	217,9	17.417	8.295	1.500.000	750.000	86.940
	CLT 200 L7s	4.533,0	3.800,0	299,0	12.974	25.085	1.000.000	1.500.000	96.600
	CLT 240 L7s	9.300,0	5.100,0	516,7	17.582	53.990	1.500.000	1.500.000	115.920
	CLT 220 L7s-2	10.115,9	975,0	398,0	21.319	7.509	2.000.000	750.000	106.260
	CLT 240 L7s-2	13.933,0	466,7	516,7	31.640	4.700	2.500.000	500.000	115.920
	CLT 260 L7s-2	17.333,0	975,0	656,5	27.354	6.579	2.500.000	750.000	125.580
	CLT 280 L7s-2	21.133,0	1.733,0	820,3	25.418	8.867	2.500.000	1.000.000	135.240
	CLT 300 L8s-2	25.800,0	2.325,0	1.008,8	38.609	7.796	3.000.000	750.000	144.900
	CLT 320 L8s-2	30.400,0	3.733,0	1.224,6	35.277	9.408	3.000.000	1.000.000	154.560
Cover layers in cross direction (typ. for wall elements)	CLT 60 C3s	8,3	216,7	8,1	2.449	20.821	250.000	500.000	28.980
	CLT 80 C3s	8,3	525,0	19,1	2.890	29.044	250.000	750.000	38.640
	CLT 90 C3s	28,1	731,3	27,2	3.697	31.231	375.000	750.000	43.470
	CLT 100 C3s	66,7	975,0	37,4	4.682	32.637	500.000	750.000	48.300
	CLT 120 C3s	66,7	1.733,0	64,6	4.930	41.642	500.000	1.000.000	57.960
	CLT 100 C5s	216,7	825,0	37,4	5.630	6.293	500.000	750.000	48.300
	CLT 120 C5s	216,7	1.583,0	64,6	5.625	8.923	500.000	1.000.000	57.960
	CLT 140 C5s	216,7	2.640,9	102,6	5.835	9.656	500.000	1.250.000	67.620
	CLT 160 C5s	466,7	3.800,0	153,1	6.989	11.872	500.000	1.500.000	77.280

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