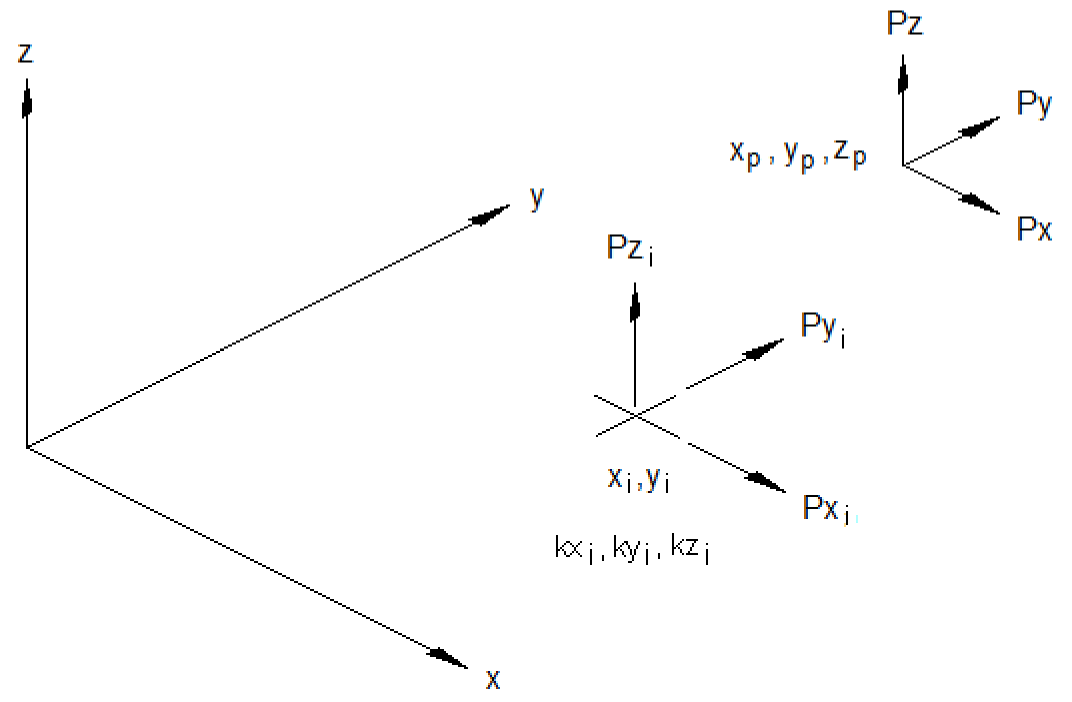
Bolt Group Analysis



Fastener positions are defined by x and y coordinates (x_i,y_i). A spring constant is given for each fastener, where kx_i, ky_i, and kz_i are spring constants in the x, y, and z directions. Spring constants are for fastener joint shear and axial deflections.

Fastener data ...

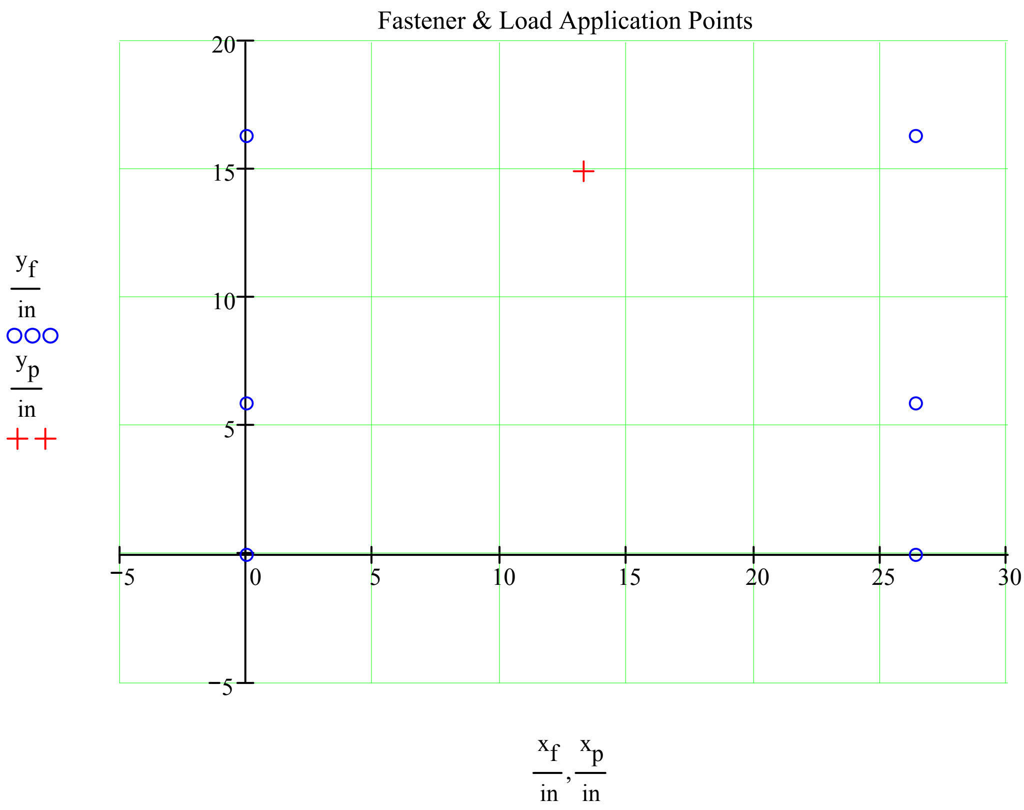
data :=

"Bolt No"	"X (in)"	"Y (in)"	"kx (lbf/in)"	"ky (lbf/in)"	"kz (lbf/in)"
1	0.000	0.000	100000	100000	100000
2	0.000	16.340	100000	100000	100000
3	26.375	0.000	100000	100000	100000
4	26.375	16.340	100000	100000	100000
5	0.000	5.906	100000	100000	100000
6	26.375	5.906	100000	100000	100000

X and Y are for the fastener coordinates.

kx, ky, and kz are the fastener spring constants in the X, Y, and Z directions respectively (lbf/in).

Applied forces ...	X axis direction ...	$Px := 3350 \cdot lbf$
	Y axis direction ...	$Py := 0 \cdot lbf$
	Z axis direction ...	$Pz := 0 \cdot lbf$
Acting at ...	X coordinate ...	$x_p := 13.281 \cdot in$
	Y coordinate ...	$y_p := 14.969 \cdot in$
	Z coordinate ...	$z_p := 14.500 \cdot in$
Applied moments ...	about X axis ...	$Mx := 0 \cdot lbf \cdot in$
	about Y axis ...	$My := 0 \cdot lbf \cdot in$
	about Z axis ...	$Mz := 0 \cdot lbf \cdot in$



Note: For calculation purposes, the following variables are used.

For fastener position ... $x_{f_i} = x_i$ ... and ... $y_{f_i} = y_i$

For fastener spring constants ... $kx_{f_i} = kx_i$ $ky_{f_i} = ky_i$ ... and ... $kz_{f_i} = kz_i$

Calculating Bolt Group Centroid for Moment about Z Axis

Z axis moment about centroid due to forces from δ_x unit displacement in the x direction to be zero.

Moment ... $\sum_i \left[kx_{f_i} \cdot \delta_x \cdot \left(y_{f_i} - y'_z \right) \right] = 0 \cdot lbf \cdot in$ ... where y'_z is the centroid y coordinate

Giving ... $y'_z := \sum_{i=0}^{n} \left(kx_{f_i} \cdot y_{f_i} \right) \cdot \left[\sum_{i=0}^{n} \left(kx_{f_i} \right) \right]^{-1}$ $y'_z = 7.4153\ in$

Z axis moment about centroid due to forces from δ_y unit displacement in the y direction to be zero.

Moment ... $\sum_i \left[ky_{f_i} \cdot \delta_y \cdot \left(x_{f_i} - x'_z \right) \right] = 0 \cdot lbf \cdot in$ ... where x'_z is the centroid x coordinate

Giving ... $x'_z := \sum_{i=0}^{n} \left(ky_{f_i} \cdot x_{f_i} \right) \cdot \left[\sum_{i=0}^{n} \left(ky_{f_i} \right) \right]^{-1}$ $x'_z = 13.1875\ in$

Calculating Bolt Group Centroid for Moments about X and Y Axis

X axis moment about centroid due to forces from δ_z unit displacement in the z direction is to be zero.

Moment ... $\sum_i \left[kz_{f_i} \cdot \delta_z \cdot \left(y_{f_i} - y'_{xy} \right) \right] = 0 \cdot lbf \cdot in$ ... where y'_{xy} is the centroid y coordinate

Giving ... $y'_{xy} := \sum_{i=0}^{n} \left(kz_{f_i} \cdot y_{f_i} \right) \cdot \left[\sum_{i=0}^{n} \left(kz_{f_i} \right) \right]^{-1}$ $y'_{xy} = 7.4153\, in$

Y axis moment about centroid due to forces from δ_z unit displacement in the z direction is to be zero.

Moment ... $\sum_i \left[kz_{f_i} \cdot \delta_z \cdot \left(x_{f_i} - x'_{xy} \right) \right] = 0 \cdot lbf \cdot in$ ... where x'_{xy} is the centroid x coordinate

Giving ... $x'_{xy} := \sum_{i=0}^{n} \left(kz_{f_i} \cdot x_{f_i} \right) \cdot \left[\sum_{i=0}^{n} \left(kz_{f_i} \right) \right]^{-1}$ $x'_{xy} = 13.1875\, in$

Applied Moments About Axes

Moment about X axis ... $\Sigma Mx := Mx + Pz \cdot \left(y_p - y'_{xy} \right) - Py \cdot z_p$ $\Sigma Mx = 0\, lbf \cdot in$

Moment about Y axis ... $\Sigma My := My + Px \cdot z_p - Pz \cdot \left(x_p - x'_{xy} \right)$ $\Sigma My = 48575\, lbf \cdot in$

Moment about Z axis ... $\Sigma Mz := Mz + Py \cdot \left(x_p - x'_z \right) - Px \cdot \left(y_p - y'_z \right)$ $\Sigma Mz = -25305\, lbf \cdot in$

Applied Displacements

X axis displacement ... $\delta_x := Px \cdot \left(\sum_{i=0}^{n} kx_{f_i} \right)^{-1}$ $\delta_x = 5.5833 \times 10^{-3}\ in$

Y axis displacement ... $\delta_y := Py \cdot \left(\sum_{i=0}^{n} ky_{f_i} \right)^{-1}$ $\delta_y = 0.0000\ in$

Z axis displacement ... $\delta_z := Pz \cdot \left(\sum_{i=0}^{n} kz_{f_i} \right)^{-1}$ $\delta_z = 0.0000\ in$

Calculating Z axis forces due to moments about X and Y axes

Z axis displacement ... $z_i = Rx \cdot \left(y_{f_i} - y'_{xy} \right) - Ry \cdot \left(x_{f_i} - x'_{xy} \right)$

Fastener Z axis force ... $Pz_i = kz_{f_i} \cdot z_i$

$$Pz_i = kz_{f_i} \cdot \left[Rx \cdot \left(y_{f_i} - y'_{xy} \right) - Ry \cdot \left(x_{f_i} - x'_{xy} \right) \right]$$

Fastener X axis moment ... $Mx_i = Pz_i \cdot \left(y_{f_i} - y'_{xy} \right)$

$$Mx_i = kz_{f_i} \cdot \left[Rx \cdot \left(y_{f_i} - y'_{xy} \right)^2 - Ry \cdot \left(x_{f_i} - x'_{xy} \right) \cdot \left(y_{f_i} - y'_{xy} \right) \right]$$

Summing ... $\Sigma Mx = Rx \cdot \sum_{i=0}^{n} \left[kz_{f_i} \cdot \left(y_{f_i} - y'_{xy} \right)^2 \right] - Ry \cdot \sum_{i=0}^{n} \left[kz_{f_i} \cdot \left(x_{f_i} - x'_{xy} \right) \cdot \left(y_{f_i} - y'_{xy} \right) \right]$

Fastener Y axis moment ... $My_i = -Pz_i \cdot \left(x_{f_i} - x'_{xy}\right)$

$$My_i = -kz_{f_i} \cdot \left[Rx \cdot \left(x_{f_i} - x'_{xy}\right) \cdot \left(y_{f_i} - y'_{xy}\right) - Ry \cdot \left(x_{f_i} - x'_{xy}\right)^2 \right]$$

Summing ... $\Sigma My = Ry \cdot \sum_{i=0}^{n} \left[kz_{f_i} \cdot \left(x_{f_i} - x'_{xy}\right)^2 \right] - Rx \cdot \sum_{i=0}^{n} \left[kz_{f_i} \cdot \left(x_{f_i} - x'_{xy}\right) \cdot \left(y_{f_i} - y'_{xy}\right) \right]$

Let ... $Ixx := \sum_{i=0}^{n} \left[kz_{f_i} \cdot \left(y_{f_i} - y'_{xy}\right)^2 \right]$ $\quad Ixx = 27382986.1\ lbf \cdot in$

$Iyy := \sum_{i=0}^{n} \left[kz_{f_i} \cdot \left(x_{f_i} - x'_{xy}\right)^2 \right]$ $\quad Iyy = 104346093.7\ lbf \cdot in$

and ... $Ixy := \sum_{i=0}^{n} \left[kz_{f_i} \cdot \left(x_{f_i} - x'_{xy}\right) \cdot \left(y_{f_i} - y'_{xy}\right) \right]$ $\quad Ixy = -0.0\ lbf \cdot in$

Moment equations in ... matrix format

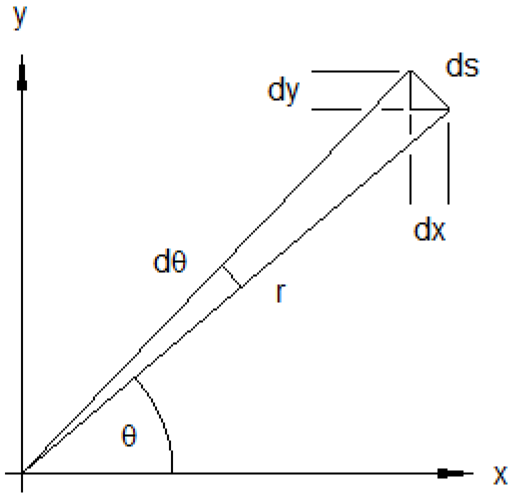
$$\begin{pmatrix} Ixx & -Ixy \\ -Ixy & Iyy \end{pmatrix} \cdot \begin{pmatrix} Rx \\ Ry \end{pmatrix} = \begin{pmatrix} \Sigma Mx \\ \Sigma My \end{pmatrix}$$

$$\begin{pmatrix} Rx \\ Ry \end{pmatrix} = \begin{pmatrix} Ixx & -Ixy \\ -Ixy & Iyy \end{pmatrix}^{-1} \cdot \begin{pmatrix} \Sigma Mx \\ \Sigma My \end{pmatrix}$$

$$\begin{pmatrix} Rx \\ Ry \end{pmatrix} := \begin{pmatrix} \dfrac{Iyy \cdot \Sigma Mx + Ixy \cdot \Sigma My}{Ixx \cdot Iyy - Ixy^2} \\ \dfrac{Ixy \cdot \Sigma Mx + Ixx \cdot \Sigma My}{Ixx \cdot Iyy - Ixy^2} \end{pmatrix} \qquad \begin{pmatrix} Rx \\ Ry \end{pmatrix} = \begin{pmatrix} 0 \\ 4.65518 \times 10^{-4} \end{pmatrix}$$

Fastener Z axis forces ... $Pz_i := kz_{f_i} \cdot \left[\delta_z + Rx \cdot \left(y_{f_i} - y'_{xy}\right) - Ry \cdot \left(x_{f_i} - x'_{xy}\right)\right]$

Calculating X and Y axis forces due to moment about Z axis



radial displacement ... $ds = r \cdot d\theta$

x direction component ... $dx = \sin(\theta) \cdot ds$

$dx = \sin(\theta) \cdot r \cdot d\theta$

y direction component ... $dy = \cos(\theta) \cdot ds$

$dy = \cos(\theta) \cdot r \cdot d\theta$

Within the analysis, the constant of proportionality $d\theta$ will be substituted for Rz.

fastener radius ... $r_i = \sqrt{\left(x_{f_i} - x'_z\right)^2 + \left(y_{f_i} - y'_z\right)^2}$

cosine function ... $\cos(\theta)_i = \dfrac{\left(x_{f_i} - x'_z\right)}{\sqrt{\left(x_{f_i} - x'_z\right)^2 + \left(y_{f_i} - y'_z\right)^2}}$

sine function ... $\sin(\theta)_i = \dfrac{\left(y_{f_i} - y'_z\right)}{\sqrt{\left(x_{f_i} - x'_z\right)^2 + \left(y_{f_i} - y'_z\right)^2}}$

x direction force ...

$$Px_i = kx_{f_i} \cdot dx_i$$

$$Px_i = kx_{f_i} \cdot \sin(\theta)_i \cdot r_i \cdot Rz$$

$$Px_i = -kx_{f_i} \cdot \left(y_{f_i} - y'_z\right) \cdot Rz$$ ... for a +ve Rz

y direction force ...

$$Py_i = ky_{f_i} \cdot dy_i$$

$$Py_i = ky_{f_i} \cdot \cos(\theta)_i \cdot r_i \cdot Rz$$

$$Py_i = ky_{f_i} \cdot \left(x_{f_i} - x'_z\right) \cdot Rz$$

Moment about Z ...

$$Mz_i = Py_i \cdot \left(x_{f_n} - x'_z\right) - Px_i \cdot \left(y_{f_i} - y'_z\right)$$

$$Mz_i = \left[ky_{f_i} \cdot \left(x_{f_i} - x'_z\right)^2 + kx_{f_i} \cdot \left(y_{f_i} - y'_z\right)^2\right] \cdot Rz$$

$$\Sigma Mz = \sum_{i=0}^{n} \left[kx_{f_i} \cdot \left(y_{f_i} - y'_z\right)^2 + ky_{f_i} \cdot \left(x_{f_i} - x'_z\right)^2\right] \cdot Rz$$

Constant ...

$$Rz := \frac{\Sigma Mz}{\sum_{i=0}^{n} \left[kx_{f_i} \cdot \left(y_{f_i} - y'_z\right)^2 + ky_{f_i} \cdot \left(x_{f_i} - x'_z\right)^2\right]}$$ $$Rz = -1.92097 \times 10^{-4}$$

X direction force ... $Px_i := kx_{f_i} \cdot \left[\delta_x - \left(y_{f_i} - y'_z \right) \cdot Rz \right]$ $\sum Px = 3350.00\ lbf$

Y direction force ... $Py_i := ky_{f_i} \cdot \left[\delta_y + \left(x_{f_i} - x'_z \right) \cdot Rz \right]$ $\sum Py = -0.00\ lbf$

Resultant ... $Ps_i := \sqrt{\left(Px_i\right)^2 + \left(Py_i\right)^2}$ $\sum Pz = -0.00\ lbf$

result =

"Bolt No"	"X (in)"	"Y (in)"	"Px (lbf)"	"Py (lbf)"	"Ps (lbf)"	"Pz (lbf)"
1	0	0	415.89	253.33	486.97	613.9
2	0	16.34	729.77	253.33	772.49	613.9
3	26.38	0	415.89	-253.33	486.97	-613.9
4	26.38	16.34	729.77	-253.33	772.49	-613.9
5	0	5.91	529.34	253.33	586.84	613.9
6	26.38	5.91	529.34	-253.33	586.84	-613.9