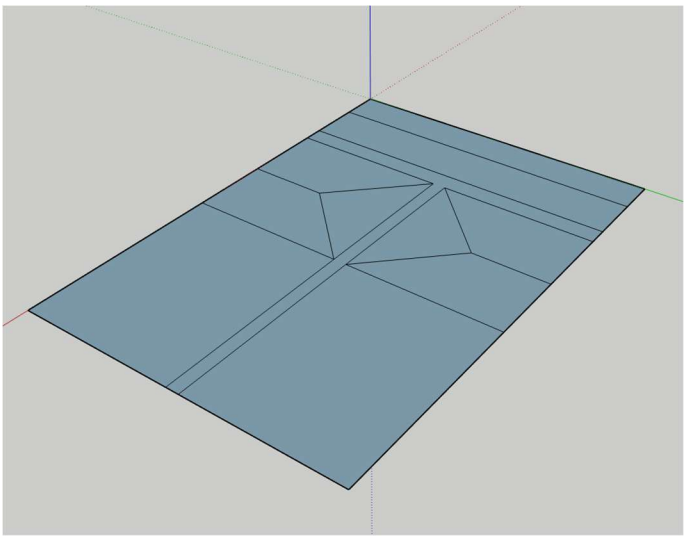
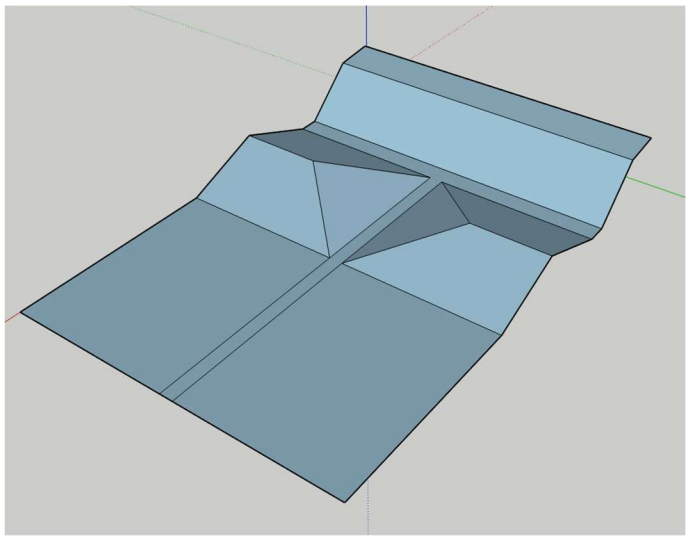
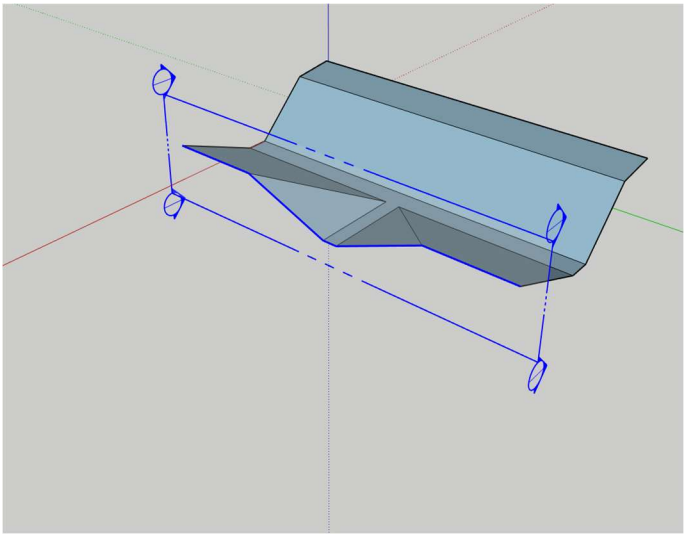
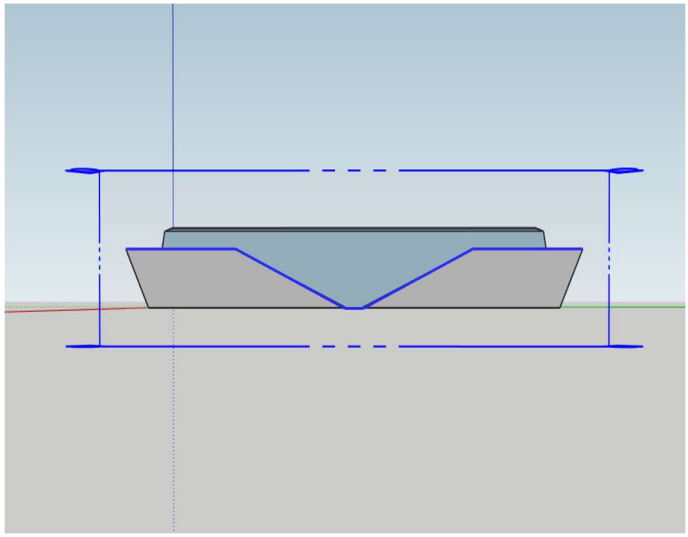


Yield Line Mechanism

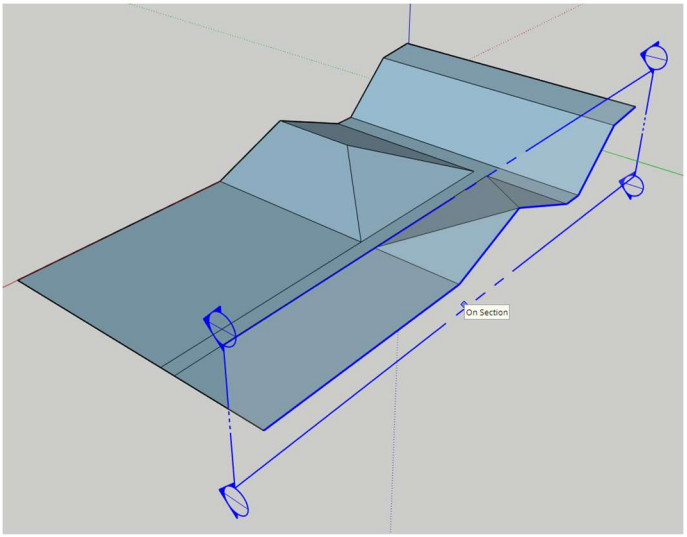


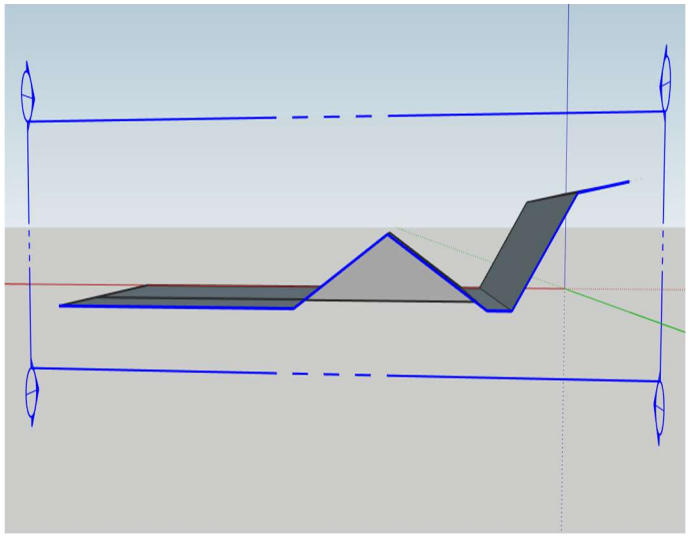
Yield Line Mechanism (Deformation from Virtual Rotation)



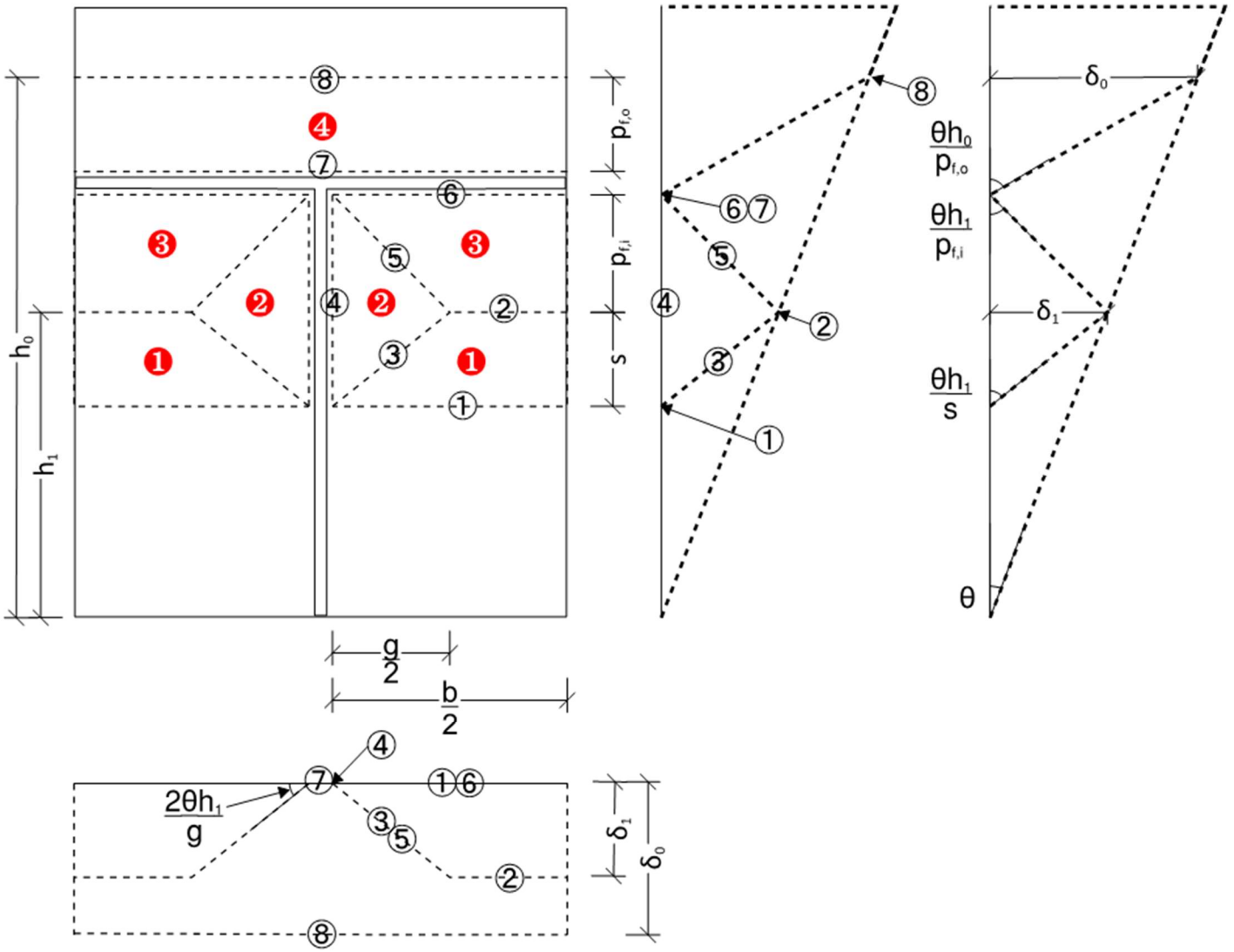


Section Cut (At Lower Bolts)





Section Cut (Side)



Yield Line Analysis	
Internal Work = External Work	$\sum W_i = \sum W_o$
External Work	$\sum W_o = M\theta$
Internal Work	$\sum W_i = \sum(ml\theta)$
Determine Angles of Yield Line Mechanism	$\theta = \frac{\delta_0}{h_0} = \frac{\delta_1}{h_1}$ $\therefore \quad \delta_0 = \theta h_0 \qquad \delta_1 = \theta h_1$ $\frac{\delta_0}{p_{f,o}} = \frac{h_0}{p_{f,o}}\theta \qquad \frac{\delta_1}{p_{f,i}} = \frac{h_1}{p_{f,i}}\theta \qquad \frac{\delta_1}{s} = \frac{h_1}{s}\theta \qquad \frac{\delta_1}{\frac{g}{2}} = \frac{2h_1}{g}\theta$

Area ❶	Segment ①: $W_{①} = m\left(\frac{b}{2}\right)\left(\frac{\delta_1}{s}\right) = m\left(\frac{b}{2}\right)\left(\frac{h_1}{s}\right)\theta$ Segment ②: $W_{②} = m\left(\frac{b}{2} - \frac{g}{2}\right)\left(\frac{\delta_1}{s}\right) = m\left(\frac{b}{2} - \frac{g}{2}\right)\left(\frac{h_1}{s}\right)\theta$ Segment ③: $W_{③} = m\left(\frac{g}{2}\right)\left(\frac{\delta_1}{s}\right) = m\left(\frac{g}{2}\right)\left(\frac{h_1}{s}\right)\theta$ $\sum W_{❶} = W_{①} + W_{②} + W_{③}$ $\sum W_{❶} = m\left(\frac{b}{2}\right)\left(\frac{h_1}{s}\right)\theta + m\left(\frac{b}{2} - \frac{g}{2}\right)\left(\frac{h_1}{s}\right)\theta + m\left(\frac{g}{2}\right)\left(\frac{h_1}{s}\right)\theta$
Factor out $m\theta$ Grouping $\frac{b}{2}$ terms Grouping $\frac{g}{2}$ terms	$\sum W_{❶} = m\theta\left[\left(\frac{b}{2}\right)\left(\frac{h_1}{s}\right) + \left(\frac{b}{2} - \frac{g}{2}\right)\left(\frac{h_1}{s}\right) + \left(\frac{g}{2}\right)\left(\frac{h_1}{s}\right)\right]$ $\sum W_{❶} = m\theta\left[\left(\frac{b}{2}\right)\left(\frac{h_1}{s} + \frac{h_1}{s}\right) + \left(\frac{g}{2}\right)\left(\frac{h_1}{s} - \frac{h_1}{s}\right)\right]$ $\sum W_{❶} = m\theta\left[\left(\frac{b}{2}\right)\left(\frac{2h_1}{s}\right)\right]$
Area ❷	Segment ③: $W_{③} = m(s)\left(\frac{\delta_1}{\frac{g}{2}}\right) = m(s)\left(\frac{2h_1}{g}\right)\theta$ Segment ④: $W_{④} = m(s + p_{f,i})\left(\frac{\delta_1}{\frac{g}{2}}\right) = m(s + p_{f,i})\left(\frac{2h_1}{g}\right)\theta$ Segment ⑤: $W_{⑤} = m(p_{f,i})\left(\frac{\delta_1}{\frac{g}{2}}\right) = m(p_{f,i})\left(\frac{2h_1}{g}\right)\theta$ $\sum W_{❷} = W_{③} + W_{④} + W_{⑤}$ $\sum W_{❷} = m(s)\left(\frac{2h_1}{g}\right)\theta + m(s + p_{f,i})\left(\frac{2h_1}{g}\right)\theta + m(p_{f,i})\left(\frac{2h_1}{g}\right)\theta$
Factor out $m\theta$	$\sum W_{❷} = m\theta\left[(s)\left(\frac{2h_1}{g}\right) + (s + p_{f,i})\left(\frac{2h_1}{g}\right) + (p_{f,i})\left(\frac{2h_1}{g}\right)\right]$ $\sum W_{❷} = m\theta\left[(s)\left(\frac{2h_1}{g}\right) + (s + p_{f,i})\left(\frac{2h_1}{g}\right) + (p_{f,i})\left(\frac{2h_1}{g}\right)\right]$
Grouping $\frac{2}{g}$ terms	$\sum W_{❷} = m\theta\left[\left(\frac{2}{g}\right)(h_1)(s + s + p_{f,i} + p_{f,i})\right]$ $\sum W_{❷} = m\theta\left[\left(\frac{2}{g}\right)(h_1)(2s + 2p_{f,i})\right]$

Area 3	Segment ②: $W_{②} = m\left(\frac{b}{2} - \frac{g}{2}\right)\left(\frac{\delta_1}{p_{f,i}}\right) = m\left(\frac{b}{2} - \frac{g}{2}\right)\left(\frac{h_1}{p_{f,i}}\right)\theta$ Segment ⑤: $W_{⑤} = m\left(\frac{g}{2}\right)\left(\frac{\delta_1}{p_{f,i}}\right) = m\left(\frac{g}{2}\right)\left(\frac{h_1}{p_{f,i}}\right)\theta$ Segment ⑥: $W_{⑥} = m\left(\frac{b}{2}\right)\left(\frac{\delta_1}{p_{f,i}}\right) = m\left(\frac{b}{2}\right)\left(\frac{h_1}{p_{f,i}}\right)\theta$ $\sum W_3 = W_{②} + W_{⑤} + W_{⑥}$ $\sum W_3 = m\left(\frac{b}{2} - \frac{g}{2}\right)\left(\frac{h_1}{p_{f,i}}\right)\theta + m\left(\frac{g}{2}\right)\left(\frac{h_1}{p_{f,i}}\right)\theta + m\left(\frac{b}{2}\right)\left(\frac{h_1}{p_{f,i}}\right)\theta$
Factor out $m\theta$	$\sum W_3 = m\theta\left[\left(\frac{b}{2} - \frac{g}{2}\right)\left(\frac{h_1}{p_{f,i}}\right) + \left(\frac{g}{2}\right)\left(\frac{h_1}{p_{f,i}}\right) + \left(\frac{b}{2}\right)\left(\frac{h_1}{p_{f,i}}\right)\right]$
Grouping $\frac{b}{2}$ terms Grouping $\frac{g}{2}$ terms	$\sum W_3 = m\theta\left[\left(\frac{b}{2}\right)\left(\frac{h_1}{p_{f,i}} + \frac{h_1}{p_{f,i}}\right) + \left(\frac{g}{2}\right)\left(\frac{h_1}{p_{f,i}} - \frac{h_1}{p_{f,i}}\right)\right]$
	$\sum W_3 = m\theta\left[\left(\frac{b}{2}\right)\left(\frac{2h_1}{p_{f,i}}\right)\right]$
Area 4	Segment ⑦: $W_{⑦} = m(b)\left(\frac{\delta_0}{p_{f,o}}\right) = m(b)\left(\frac{h_0}{p_{f,o}}\right)\theta$ Segment ⑧: $W_{⑧} = m(b)\left(\frac{\delta_0}{p_{f,o}} - \theta\right) = m(b)\left(\frac{h_0}{p_{f,o}} - 1\right)\theta$ $\sum W_4 = W_{⑦} + W_{⑧}$ $\sum W_4 = m(b)\left(\frac{h_0}{p_{f,o}}\right)\theta + m(b)\left(\frac{h_0}{p_{f,o}} - 1\right)\theta$
Factor out $m\theta$	$\sum W_4 = m\theta\left[(b)\left(\frac{h_0}{p_{f,o}}\right) + (b)\left(\frac{h_0}{p_{f,o}} - 1\right)\right]$
Grouping b terms	$\sum W_4 = m\theta\left[(b)\left(\frac{h_0}{p_{f,o}} + \frac{h_0}{p_{f,o}} - 1\right)\right]$
	$\sum W_4 = m\theta\left[(b)\left(\frac{2h_0}{p_{f,o}} - 1\right)\right]$

Sum of Internal Work

$$\sum W_i = 2W_{\textcircled{1}} + 2W_{\textcircled{2}} + 2W_{\textcircled{3}} + W_{\textcircled{4}}$$

$$\sum W_i = 2m\theta\left[\left(\frac{b}{2}\right)\left(\frac{2h_1}{s}\right)\right] + 2m\theta\left[\left(\frac{2}{g}\right)(h_1)(2s+2p_{f,i})\right] + 2m\theta\left[\left(\frac{b}{2}\right)\left(\frac{2h_1}{p_{f,i}}\right)\right] + m\theta\left[(b)\left(\frac{2h_0}{p_{f,o}}-1\right)\right]$$

Factor out $2m\theta$

$$\sum W_i = 2m\theta\left[\left(\frac{b}{2}\right)\left(\frac{2h_1}{s}\right) + \left(\frac{2}{g}\right)(h_1)(2s+2p_{f,i}) + \left(\frac{b}{2}\right)\left(\frac{2h_1}{p_{f,i}}\right) + \left(\frac{b}{2}\right)\left(\frac{2h_0}{p_{f,o}}-1\right)\right]$$

Grouping $\frac{b}{2}$ terms

Grouping $\frac{2}{g}$ terms

$$\sum W_i = 2m\theta\left[\left(\frac{b}{2}\right)\left(\frac{2h_1}{s}+\frac{2h_1}{p_{f,i}}+\frac{2h_0}{p_{f,o}}-1\right) + \left(\frac{2}{g}\right)(h_1)(2s+2p_{f,i})\right]$$

$$\sum W_i = 2m\theta\left[\left(\frac{b}{2}\right)\left(2h_1\left(\frac{1}{s}+\frac{1}{p_{f,i}}\right)+2h_0\left(\frac{1}{p_{f,o}}\right)-1\right) + \left(\frac{2}{g}\right)(2h_1)(s+p_{f,i})\right]$$

Equate External Work to Internal Work

$$\sum W_o = \sum W_i$$

$$M\theta = 2m\theta\left[\left(\frac{b}{2}\right)\left(2h_1\left(\frac{1}{s}+\frac{1}{p_{f,i}}\right)+2h_0\left(\frac{1}{p_{f,o}}\right)-1\right) + \left(\frac{2}{g}\right)(2h_1)(s+p_{f,i})\right]$$

$$M = 2m\left[\left(\frac{b}{2}\right)\left(2h_1\left(\frac{1}{s}+\frac{1}{p_{f,i}}\right)+2h_0\left(\frac{1}{p_{f,o}}\right)-1\right) + \left(\frac{2}{g}\right)(2h_1)(s+p_{f,i})\right]$$

Set $M = M_n$

Set $m = \frac{t^2F_y}{4}$

Set $b = b_p$

$$M_n = 2\left(\frac{t^2F_y}{4}\right)\left[\left(\frac{b_p}{2}\right)\left(2h_1\left(\frac{1}{s}+\frac{1}{p_{f,i}}\right)+2h_0\left(\frac{1}{p_{f,o}}\right)-1\right) + \left(\frac{2}{g}\right)(2h_1)(s+p_{f,i})\right]$$

$$M_n = \left(\frac{t^2F_y}{2}\right)\left[\left(\frac{b_p}{2}\right)\left(2h_1\left(\frac{1}{s}+\frac{1}{p_{f,i}}\right)+2h_0\left(\frac{1}{p_{f,o}}\right)-1\right) + \left(\frac{2}{g}\right)(2h_1)(s+p_{f,i})\right]$$

$$M_n = \left(t^2F_y\right)\left[\left(\frac{b_p}{2}\right)\left(h_1\left(\frac{1}{s}+\frac{1}{p_{f,i}}\right)+h_0\left(\frac{1}{p_{f,o}}\right)-\frac{1}{2}\right) + \left(\frac{2}{g}\right)(h_1)(s+p_{f,i})\right]$$

Rearranging terms

$$M_n = \left(t^2F_y\right)\left[\left(\frac{b_p}{2}\right)\left(h_1\left(\frac{1}{p_{f,i}}+\frac{1}{s}\right)+h_0\left(\frac{1}{p_{f,o}}\right)-\frac{1}{2}\right) + \left(\frac{2}{g}\right)(h_1)(p_{f,i}+s)\right]$$

AISC Design Guide 16 Table 4.2. Equation (for Four-Bolt Extended Unstiffened Moment End-Plate)

$$\phi M_n = \phi_b F_{py}\, {t_p}^2 Y$$

$$Y = \frac{b_p}{2}\left[h_1\left(\frac{1}{p_{f,i}}+\frac{1}{s}\right)+h_0\left(\frac{1}{p_{f,o}}\right)-\frac{1}{2}\right] + \frac{2}{g}\left[h_1(p_{f,i}+s)\right]$$

$$s = \frac{1}{2}\sqrt{b_p\, g}$$