International Journal of Structural Stability and Dynamics
Vol. 16, No. 8 (2016) 1550044 (16 pages)

DOI: 10.1142/S0219455415500443

Analytical Solutions of Lateral–Torsional Buckling of Castellated Beams

Boksun Kim*, Long-Yuan Li† and Ashley Edmonds

School of Marine Science and Engineering,
Plymouth University, Plymouth PL4 8AA, UK
**boksun.kim@plymouth.ac.uk*
†long-yuan.li@plymouth.ac.uk

Received 27 April 2015
Accepted 15 July 2015
Published 7 September 2015

The majority of the existing literature on the lateral stability of castellated beams deals with experimental and/or numerical studies. This paper presents a comprehensive analytical study of the lateral–torsional buckling of simply supported castellated beams subject to pure bending and/or a uniformly distributed load. Using the principle of total potential energy, analytical expressions for the critical buckling moments and loads are derived and applied for various beam lengths. The three different locations of the applied load are used: At the top flange, shear center and bottom flange. The results show that the influence of web openings on the critical buckling moments and loads are mainly due to the reduction of the torsional constant caused by the web openings. Web shear effects and web shear buckling become important only when the beam is short and the flange is wide. The critical moments and loads will be overestimated or underestimated if the full or reduced section properties are used. The accurate critical moment or load should be calculated based on the average torsional constant of the full and reduced sections rather than simply taking the average of the critical moments or loads calculated from the full and reduced section properties. The present analytical solutions are verified using 3D finite element analysis results.

Keywords: Castellated beams; lateral–torsional buckling; analytical solution; web openings; energy methods.

1. Introduction

Castellated beams have been used as structural members in structural steel frames.[1] An example is shown in Fig. 1. A castellated beam is fabricated from a standard universal beam or column by cutting the web on a half hexagonal line down the center of the beam. The two halves are moved across by a half unit of spacing and then rejoined by welding.[1] This process increases the depth of the beam and hence the bending strength and stiffness about the major axis without adding additional

*Corresponding author.

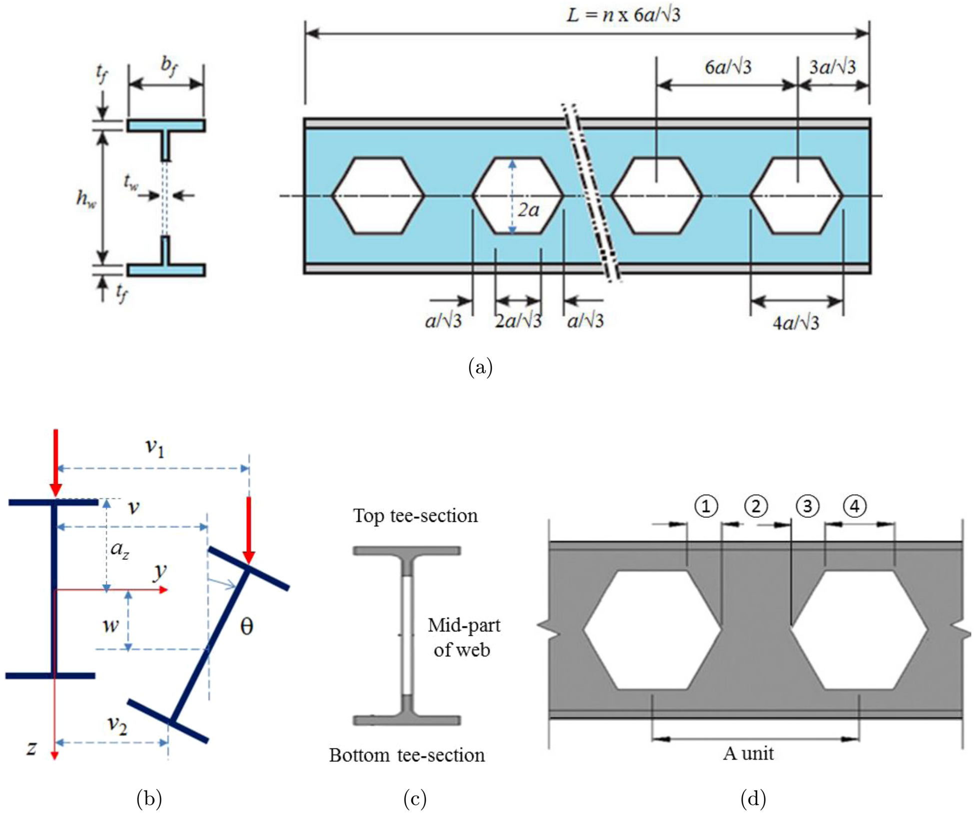


Fig. 1. A castellated beam. (a) Notations used in the beam. (b) Loading and displacements in web and flanges during lateral–torsional buckling. (c) Definition of three parts for calculation of strain energy. (d) Section properties of mid-part of web in four different regions. $I_{y3} = I^*_{y3}$, $I_{z3} = I^*_{z3}$, $J_3 = J^*_3$ in region ②, $I_{y3} = I_{z3} = J_3 = 0$ in region ④, section properties vary with x in regions ① and ③.

materials. This allows castellated beams to be used in long span applications with light or moderate loading conditions for floors and roofs. The fabrication process creates openings on the web, which can be used to accommodate services. Despite the increase in the beam depth, the overall building height can hence be reduced, compared with a solid web solution, where services are provided beneath the beam. This leads to savings in the cladding costs. Despite the increase in the fabrication costs caused by cutting and welding, the advantages outweigh the disadvantages. Design guidance on the strength and stiffness of castellated beams is available in some countries.[1–3]

Since castellated beams are made usually from I-beams or H-columns, they tend to be deep and slender and have reduced torsional stiffness of the web due to the openings in the web. Hence, they are more susceptible to lateral–torsional buckling. Some guidance for the determination of the lateral–torsional buckling moment of members with web openings is given in BS 5950.[4] Clause 4.15.4.5 of BS 5950[4] states that the same method used to determine the lateral buckling resistance moment of solid web beams can be used for beams with web openings using the section properties at the centerline of an opening, i.e. the reduced cross-section properties.

Intensive research on the lateral stability[5–14] of castellated beams started in the early 1980s. Experimental investigations[5–7,10,14] were carried out and finite elements methods[7–9,11–13] were also used to predict the lateral–torsional or distortional buckling behavior of such beams and/or to compare the predictions with the results from the experiments.[7,13] The effects of slenderness on the moment-gradient factor[8] and of elastic lateral bracing stiffness on the flexural–torsional buckling[9,14] of simply supported castellated beams were studied using 3D finite element analysis models. The failure modes[5–7,10–12] and the interaction of the buckling modes[11] of castellated beams were investigated. It was found that the web opening of castellated beams had little influence on lateral buckling behavior[5] and failure modes,[6] while web distortional buckling was likely when an effective lateral brace was provided at the midspan of the compression flange[10,14] and this type of failure reduced significantly the failure load[11] of slender castellated beams. Compared with the same sized I-beams, castellated beams have lower torsional rigidity due to the web openings and hence more likely fail by lateral–torsional buckling. In addition, the flexural–torsional buckling of thin-walled composite beams and Timoshenko beams have been discussed in papers.[15–18] The axially compressed buckling of battened columns, which have similar behavior to castellated beams, was investigated recently.[19]

The aforementioned survey shows that the majority of the existing literature on the lateral buckling stability of castellated beams deals with experimental and/or numerical studies. The only available analytical study is done by Pattanayak and Chesson.[20] They presented the lateral instability analysis of simply supported castellated beams using the energy method. In their analytical model, the two tee-sections and the webpost are treated separately for the calculation of the strain energy but not for the potential work of the applied loads. Furthermore, their analytical model was not validated either by experiments or by finite element analyses.

In this paper, a more comprehensive analytical study is presented to determine the critical buckling moments of simply supported castellated beams subject to a uniformly distributed load and/or a pure bending. Using the principle of the total potential energy, analytical expressions for the critical buckling moment of such a beam are derived. To demonstrate the present analytical solutions, finite element analyses are also conducted using the linear elastic analysis module build in the ANSYS software.

2. Analytical Study of Lateral–Torsional Buckling of Castellated Beam

Consider a simply supported castellated beam that is subjected to a uniformly distributed transverse load, q_z and a coupled bending moment, M_o, applied at the ends of the beam. Assume that when the beam has a lateral–torsional buckling, the shear center of the beam has a lateral displacement $v(x)$ and a transverse displacement $w(x)$, as shown in Fig. 1(b). The cross-section has an angle of twist $\phi(x)$. To determine the elastic critical moment when the lateral–torsional buckling occurs, one can use the variational method of the total potential energy function of the beam as follows[21,22]:

$$\Pi = \frac{1}{2}\int_{o}^{l}\left[EI_y\left(\frac{d^2w}{dx^2}\right)^2 + EI_z\left(\frac{d^2v}{dx^2}\right)^2 + EI_w\left(\frac{d^2\phi}{dx^2}\right)^2 + GJ\left(\frac{d\phi}{dx}\right)^2\right]dx - \int_{o}^{l}\left(M_y\frac{d^2w}{dx^2} + M_y\phi\frac{d^2v}{dx^2} + \frac{a_z q_z}{2}\phi^2\right)dx, \tag{1}$$

where E is the Young's modulus, G is the shear modulus, I_y and I_z are the second moments of the cross-sectional area about the y and z axes respectively, I_w is the warping constant, J is the torsional constant, M_y is the internal bending moment about the y axis, a_z is the z-coordinate of the loading point defining the vertical distance between the loading point and the shear centre of the beam and l is the length of the beam. The first integration in Eq. (1) represents the strain energy, whereas the second one is the loss of potential energy of the applied loads while the buckling occurs.

For a castellated beam, I_y, I_z, I_w and J are a function of x due to the existence of web openings. For convenience of calculation, the cross-section of the castellated beam is now decomposed into three parts, two of which represent the top and bottom tee-sections, one of which represents the mid-part of the web (see Fig. 1). Using the assumption of small displacements, the displacements at the shear centers of the top and bottom tee-sections can be expressed as follows (see Fig. 1(b)):

$$v_1 = v + \frac{h}{2}\sin\phi \approx v + \frac{h\phi}{2}, \tag{2}$$

$$w_1 = w + \frac{h}{2}(1 - \cos\phi) \approx w, \tag{3}$$

$$v_2 = v - \frac{h}{2}\sin\phi \approx v - \frac{h\phi}{2}, \tag{4}$$

$$w_2 = w + \frac{h}{2}(1 - \cos\phi) \approx w, \tag{5}$$

where v_1 and w_1 are the lateral and transverse displacements of the shear center of the top tee-section, v_2 and w_2 are the lateral and transverse displacements of the shear center of the bottom tee-section, h is the distance between the shear centers of top and bottom tee-sections.

For both tee and mid-part of the web, the warping constants are so small and thus can be ignored. Hence, the strain energy of the beam calculated based on the three parts may be expressed as follows:

$$U = \frac{1}{2}\int_{o}^{l}\left[EI_{y1}\left(\frac{d^2w_1}{dx^2}\right)^2 + EI_{z1}\left(\frac{d^2v_1}{dx^2}\right)^2 + GJ_1\left(\frac{d\phi}{dx}\right)^2\right]dx$$
$$+\frac{1}{2}\int_{o}^{l}\left[EI_{y2}\left(\frac{d^2w_2}{dx^2}\right)^2 + EI_{z2}\left(\frac{d^2v_2}{dx^2}\right)^2 + GJ_2\left(\frac{d\phi}{dx}\right)^2\right]dx$$
$$+\frac{1}{2}\int_{o}^{l}\left[EI_{y3}\left(\frac{d^2w}{dx^2}\right)^2 + EI_{z3}\left(\frac{d^2v}{dx^2}\right)^2 + GJ_3\left(\frac{d\phi}{dx}\right)^2\right]dx, \tag{6}$$

where $I_{y1} = I_{y2}$ and $I_{z1} = I_{z2}$ are the second moments of the tee cross-sectional area about the y and z axes, $J_1 = J_2$ is the torsional constant of the tee-section, I_{y3} and I_{z3} are the second moments of the cross-sectional area of the mid-part of the web about the y and z axes respectively, and J_3 is the torsional constant of the mid-part of the web. Substituting Eqs. (2)–(5) into Eq. (6) yields

$$U = \frac{1}{2}\int_{o}^{l}\left[2EI_{y1}\left(\frac{d^2w}{dx^2}\right)^2 + 2EI_{z1}\left(\frac{d^2v}{dx^2}\right)^2 + \frac{h^2}{2}EI_{z1}\left(\frac{d^2\phi}{dx^2}\right)^2 + 2GJ_1\left(\frac{d\phi}{dx}\right)^2\right]dx$$
$$+\frac{1}{2}\int_{o}^{l}\left[EI_{y3}\left(\frac{d^2w}{dx^2}\right)^2 + EI_{z3}\left(\frac{d^2v}{dx^2}\right)^2 + GJ_3\left(\frac{d\phi}{dx}\right)^2\right]dx. \tag{7}$$

The first integration in Eq. (7) represents the strain energy of the two tee-sections, whereas the second one represents the strain energy of the mid-part of the web. Note that I_{y1}, I_{z1} and J_1 are constants while I_{y3}, I_{z3} and J_3 are a function of x, which depend upon the position of the web openings (see Fig. 1(d)). It can be found, by comparing Eqs. (7) and (1), that,

$$I_y = 2I_{y1} + I_{y3}, \tag{8}$$

$$I_z = 2I_{z1} + I_{z3}, \tag{9}$$

$$J = 2J_1 + J_3, \tag{10}$$

$$I_w = \left(\frac{h}{2}\right)^2 I_z \approx \frac{h^2}{2} I_{z1}. \tag{11}$$

Equation (11) indicates that, although the warping strain energy is negligible in the two individual tee-sections when using local displacement variables, it is not in the assembly of the two tee sections when the displacement compatibility is enforced. For a simply supported castellated beam where $v = w = \phi = 0$ at the two ends, the following displacement functions for $v(x)$, $w(x)$ and $\phi(x)$ can be assumed,

$$w(x) = A\sin\frac{\pi x}{l}, \tag{12}$$

$$v(x) = B\sin\frac{\pi x}{l}, \tag{13}$$

$$\phi(x) = C\sin\frac{\pi x}{l}, \tag{14}$$

where A, B, and C are constants to be determined. Substituting Eqs. (12)–(14) into Eq. (7) yields

$$U = \frac{l}{4}\left(\frac{\pi}{l}\right)^4\left[E(2I_{y1} + kI_{y3}^*)A^2 + E(2I_{z1} + kI_{z3}^*)B^2 + EI_wC^2 + G(2J_1 + kJ_3^*)\left(\frac{l}{\pi}\right)^2C^2\right], \tag{15}$$

where k is a constant between 0 and 1. I_{y3}^* and I_{z3}^* are the second moments of the cross-sectional area of the mid-part of the web with no openings about the y and z axes, and J_3^* is the torsional constant of the mid-part of the web with no openings. If $k = 1$, then the full section properties are used (i.e. the web openings are ignored). If $k = 0$, then the reduced section properties are used (i.e. the whole mid-part of the web is ignored and only the two tee-sections are taken into account). Physically, k represents the volume fraction of the solid in the mid-part of the web. Clearly, if the areas of the solid and voids in the mid-part of the web are identical, which is often used in castellated beams, then $k = 1/2$ can be taken.

For a simply supported beam subject to a uniformly distributed load and a pure bending where two equal moments in opposite directions are applied at the ends of the beam, the internal bending moment can be expressed as follows:

$$M_y(x) = M_o + \frac{1}{2}q_zx(l - x). \tag{16}$$

By substituting Eqs. (12)–(14) and Eq. (16) into the second integration of Eq. (1), the loss of potential energy of the applied loads during buckling may be expressed as

$$W = -2\left(\frac{\pi M_o}{l} + \frac{q_zl}{\pi}\right)A - \left(\frac{\pi^2}{2l}\right)\left[M_o + \frac{q_zl^2}{4}\left(\frac{1}{3} + \frac{1}{\pi^2}\right)\right]BC - \frac{q_za_zl}{4}C^2. \tag{17}$$

The second-order variation of the total potential energy functional, $\delta^2\prod = \delta^2(U + W)$, leads to the following eigenvalue equation:

$$A = 0, \tag{18}$$

$$E(2I_{z1} + kI_{z3}^*)\left(\frac{\pi}{l}\right)^2B = \left[M_o + \frac{q_zl^2}{4}\left(\frac{1}{3} + \frac{1}{\pi^2}\right)\right]C, \tag{19}$$

$$\left[\frac{\pi^2EI_w}{l^2} + G(2J_1 + kJ_3^*)\right]C = \left[M_o + \frac{q_zl^2}{4}\left(\frac{1}{3} + \frac{1}{\pi^2}\right)\right]B + \frac{q_za_zl^2}{\pi^2}C. \tag{20}$$

Eliminating B in Eqs. (19) and (20), it yields

$$\frac{\left[M_o + \frac{q_zl^2}{4}\left(\frac{1}{3} + \frac{1}{\pi^2}\right)\right]^2}{E\big(2I_{z1} + kI_{z3}^*\big)\left(\frac{\pi}{l}\right)^2} + \frac{q_za_zl^2}{\pi^2} = \left[\frac{\pi^2EI_w}{l^2} + G(2J_1 + kJ_3^*)\right]. \tag{21}$$

Equation (21) provides the relationship between the critical moment and critical uniform load when the lateral–torsional buckling of the castellated beam occurs. For a beam with pure bending, the critical moment of lateral–torsional buckling can be simplified from Eq. (21) as follows:

$$(M_o)_{\rm cr} = \pm\left(\frac{\pi}{l}\right)\sqrt{\left[\frac{\pi^2 EI_w}{l^2} + G(2J_1 + kJ_3^*)\right]E(2I_{z1} + kI_{z3}^*)}. \tag{22}$$

For a beam with a uniformly distributed load applied at the shear center (i.e. $a_z = 0$), the critical load can be simplified from Eq. (21) as follows:

$$\left(\frac{q_z l^2}{8}\right)_{\rm cr} = \pm\frac{3\pi^2}{2(\pi^2+3)}\left(\frac{\pi}{l}\right)\sqrt{\left[\frac{\pi^2 EI_w}{l^2} + G(2J_1 + kJ_3^*)\right]E(2I_{z1} + kI_{z3}^*)}. \tag{23}$$

For a beam with a uniformly distributed load applied on the top flange of the beam (i.e. $a_z = h_w/2$), the critical load can be simplified from Eq. (21) as follows:

$$\left(\frac{q_z l^2}{8}\right)_{\rm cr} = \frac{-\left(\frac{h_w}{2}+t_f\right) + \sqrt{\left(\frac{h_w}{2}+t_f\right)^2 + \left(\frac{\pi^2}{6}+\frac{1}{2}\right)^2\left[I_w + \frac{G(2J_1+kJ_3^*)l^2}{\pi^2 E}\right]\frac{1}{2I_{z1}+kI_{z3}^*}}}{\left(\frac{1}{3}+\frac{1}{\pi^2}\right)^2} \times \frac{E(2I_{z1}+kI_{z3}^*)}{l^2}. \tag{24}$$

For a beam with a uniformly distributed load applied on the bottom flange of the beam (i.e. $a_z = -h_w/2$), the critical load can be simplified from Eq. (21) as follows:

$$\left(\frac{q_z l^2}{8}\right)_{\rm cr} = \frac{\left(\frac{h_w}{2}+t_f\right) + \sqrt{\left(\frac{h_w}{2}+t_f\right)^2 + \left(\frac{\pi^2}{6}+\frac{1}{2}\right)^2\left[I_w + \frac{G(2J_1+kJ_3^*)l^2}{\pi^2 E}\right]\frac{1}{2I_{z1}+kI_{z3}^*}}}{\left(\frac{1}{3}+\frac{1}{\pi^2}\right)^2} \times \frac{E(2I_{z1}+kI_{z3}^*)}{l^2}. \tag{25}$$

For most castellated beams, $I_{z3}^* << 2I_{z1}$. Hence, I_{z3}^* can be ignored and Eqs. (22)–(25) can be further simplified as follows:

$$(M_o)_{\rm cr} = \pm\left(\frac{\pi}{l}\right)\sqrt{2EI_{z1}\left[\frac{\pi^2 EI_w}{l^2} + G(2J_1 + kJ_3^*)\right]}, \tag{26}$$

$$\left(\frac{q_z l^2}{8}\right)_{\rm cr} = \pm\frac{3\pi^2}{2(\pi^2+3)}\left(\frac{\pi}{l}\right)\sqrt{2EI_{z1}\left[\frac{\pi^2 EI_w}{l^2} + G(2J_1 + kJ_3^*)\right]}, \tag{27}$$

$$\left(\frac{q_z l^2}{8}\right)_{\rm cr} = \frac{-\left(\frac{h_w}{2}+t_f\right) + \sqrt{\left(\frac{h_w}{2}+t_f\right)^2 + \left(\frac{\pi^2}{6}+\frac{1}{2}\right)^2\left[\frac{I_w}{2I_{z1}} + \frac{G(2J_1+kJ_3^*)l^2}{2\pi^2 EI_{z1}}\right]}}{\left(\frac{1}{3}+\frac{1}{\pi^2}\right)^2} \times \frac{2EI_{z1}}{l^2}, \tag{28}$$

$$\left(\frac{q_z l^2}{8}\right)_{\text{cr}} = \frac{\left(\frac{h_w}{2}+t_f\right)+\sqrt{\left(\frac{h_w}{2}+t_f\right)^2+\left(\frac{\pi^2}{6}+\frac{1}{2}\right)^2\left[\frac{I_w}{2I_{z1}}+\frac{G(2J_1+kJ_3^*)l^2}{2\pi^2 EI_{z1}}\right]}}{\left(\frac{1}{3}+\frac{1}{\pi^2}\right)^2} \times \frac{2EI_{z1}}{l^2}. \tag{29}$$

It can be seen from Eqs. (26)–(29) that if I_w were neglected in the equation, the critical load would be largely underestimated. Also, Eqs. (26)–(29) indicate that the influence of web openings on the critical loads of lateral–torsional buckling is mainly due to the reduction of the torsional constant caused by the web openings. It can be seen from these equations that the use of the full section properties (i.e. $k = 1$) will lead to an overestimation of the critical load, whereas the use of the reduced section properties (i.e. $k = 0$) will lead to an underestimation of the critical load. The true critical load will be in between these two bounds but is not simply the average of them, since the relationship between the critical load and k-value is not linear.

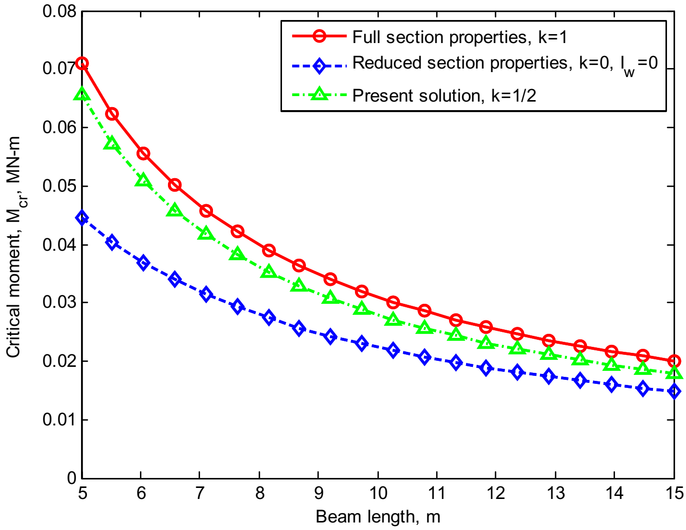
3. Parametric Study

Two castellated beams are considered. The cross-sectional dimensions of the beams are given in Table 1. For these two beams, the ratios of the second moment of the cross-sectional area of the mid-part of the web about the z axis to that of the two tee-sections are $I^*_{z3}/(2I_{z1}) = 0.77\%$ for Beam 1 and 0.029% for Beam 2, respectively. This demonstrates that the second moment of the cross-sectional area of the mid-part of the web about the z axis can be ignored in the calculation. In contrast, the ratios of the torsional constant of the cross-sectional area of the mid-part of the web to that of the two tee-sections are $J^*_3/(2J_1) = 73\%$ for Beam 1 and 31% for Beam 2, respectively. This indicates that the torsional constant of the cross-sectional area of the mid-part of the web is comparable to that of the two tee-sections and thus cannot be ignored.

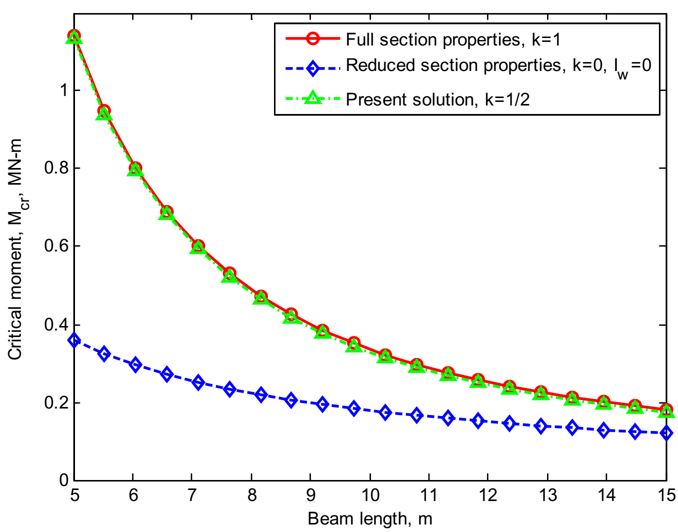
The critical moments of the lateral–torsional buckling of the two beams under pure bending are calculated using Eq. (26) for various beam lengths and corresponding results are plotted in Fig. 2, in which, for the purpose of comparison, the critical moments calculated based on the full section properties ($k = 1$) and reduced section properties ($k = 0$ and $I_w = 0$) are also superimposed. It can be seen from the figure that, for a castellated beam with wider flanges, the web openings have almost no influence on the critical moment of the beam, if the warping strain energy is properly considered. It is only the beam with narrow flanges for which the web

Table 1. Cross-sectional dimensions of the two castellated beams considered.

Beam	Flange width b_f (mm)	Flange thickness t_f (mm)	Web depth h_w (mm)	Web thickness t_w (mm)	Web opening $2a$ (mm)
1	117	10	350	10	247
2	350	10	350	10	247



(a)



(b)

Fig. 2. Critical moments of lateral–torsional buckling of beams under pure bending with flange width (a) $b_f = 117\,\text{mm}$ and (b) $b_f = 350\,\text{mm}$ (flange thickness $t_f = 10\,\text{mm}$, web depth $h_w = 350\,\text{mm}$, web thickness $t_w = 10\,\text{mm}$ and depth of web opening $2a = 247\,\text{mm}$, $k = 0.5$).

openings reduce the critical moment. This is partly because the ratio of the torsional constant of the cross-sectional area of the mid-part of the web to that of the two tee-sections is relatively large and partly because the ratio of the torsional constant of the whole section to the second moment of the whole section about the z axis becomes more significant. In this case, the reduction of the torsional constant due to the web openings will affect the critical moment significantly. The figure also shows that, in either section if the warping constant is neglected, then the critical moment will be significantly underestimated.

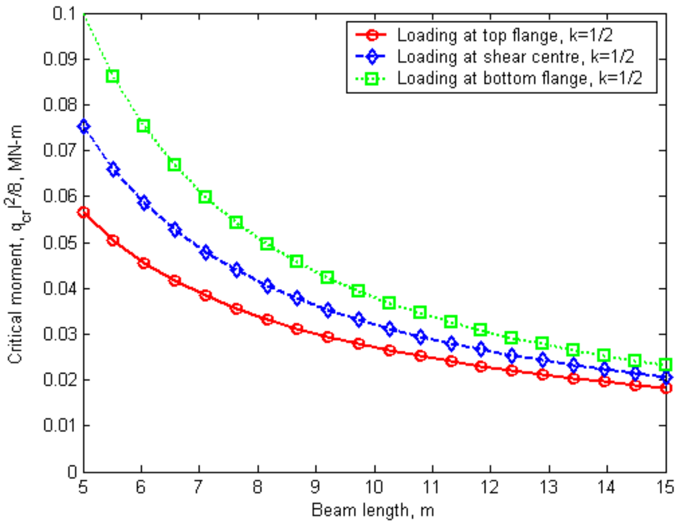
The critical load of lateral–torsional buckling of the castellated beam subjected to a uniformly distributed load applied at the shear center can be calculated using Eq. (27). However, the comparison of Eqs. (26) and (27) indicates that, in terms of the maximum moment, the critical moment of the beam with a uniformly distributed load is very similar to that of the beam under pure bending. The only difference between them is the pre-factor $1.5\pi^2/(3+\pi^2)$. This pre-factor appeared in the critical moment reflects the influence of the varying pre-buckling moment along the beam axis, which is similar to what has been reported for I-beams in the literature.[21,22]

Figure 3 presents the effect of the locations of the applied load on the critical loads of lateral–torsional buckling of the two castellated beams subject to a uniformly distributed load. Equations (27)–(29) are used for various beam lengths and corresponding results are plotted in Fig. 3. As expected, the critical load decreases as the beam length increases. It can be seen from the figure that, the loading position has a significant influence on the critical load. The shorter the beam, the greater the influence of the load position becomes. The comparison of the critical load curves between the two beams indicates that the wider the flange of the beam, the greater the influence of the load position becomes.

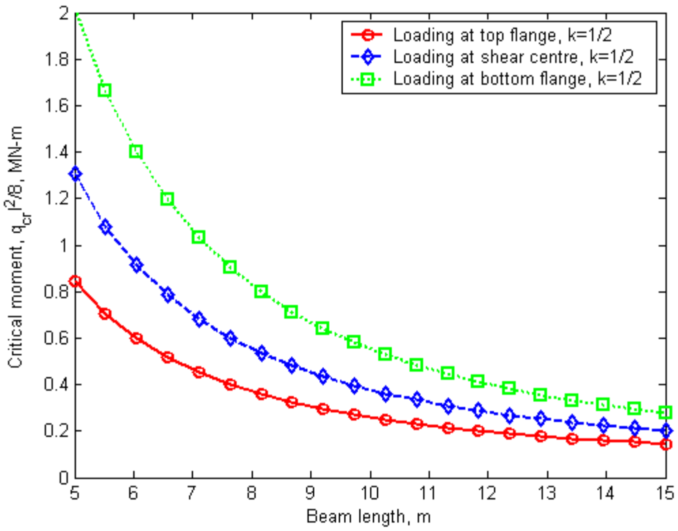
Figure 4 shows the critical loads of the lateral–torsional buckling of the two simply supported castellated beams of 7 m length, subjected to combined uniformly distributed load applied at the top flange and two equal bending moments but in opposite directions applied at the ends. This is for the case where the beam is subjected to an pure bending first, followed by an uniformly distributed load, or vice versa. The critical load of the uniformly distributed load is obviously dependent on the moment of the pure bending. The results are obtained by solving Eq. (21) for $q_{\rm cr}$ for a given $M_{\rm cr}$. As expected, $q_{\rm cr}$ decreases with the increase of $M_{\rm cr}$. The relationship between them is approximately linear. It is interesting to notice from Fig. 4 that, the horizontal critical moments, representing the lateral–torsional buckling strength under pure bending, are slightly higher than the vertical critical moments, representing the lateral–torsional buckling strength under a uniformly distributed load, since in the normal circumstance, the pure bending is a more severe loading condition. The reason for this is because the uniformly distributed load is applied at the top flange, which makes the beam less stable and thus, has a low critical value when compared to the load applied at the shear center or at the bottom flange.

4. Finite Element Analysis

In order to verify the analytical solution described in Sec. 2, finite element analyses were performed using the ANSYS software. Various lengths of the two castellated beams in Table 1 were modeled using 3D shell elements (shell 63) built in the



(a)



(b)

Fig. 3. Critical moments of lateral–torsional buckling of beams subjected to a uniformly distributed load with flange width (a) $b_f = 117\,\text{mm}$ and (b) $b_f = 350\,\text{mm}$ (flange thickness $t_f = 10\,\text{mm}$, web depth $h_w = 350\,\text{mm}$, web thickness $t_w = 10\,\text{mm}$ and depth of web opening $2a = 247\,\text{mm}$, $k = 0.5$).

software. For the material properties of the beam, Young's modulus of 210 GPa and Poisson's ratio of 0.3 were used. Owing to symmetry, only a half span was modeled. The boundary conditions of the beam were assumed to have zero transverse and lateral displacements ($v = w = 0$) for all nodes on one end section of the beam

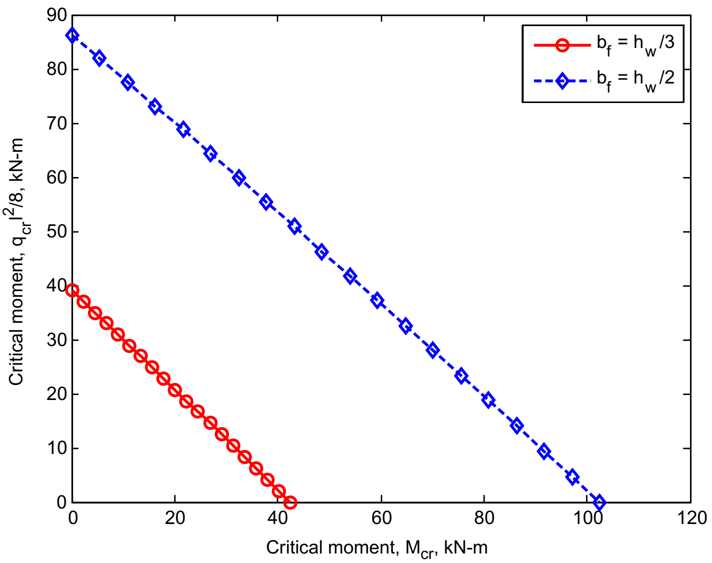
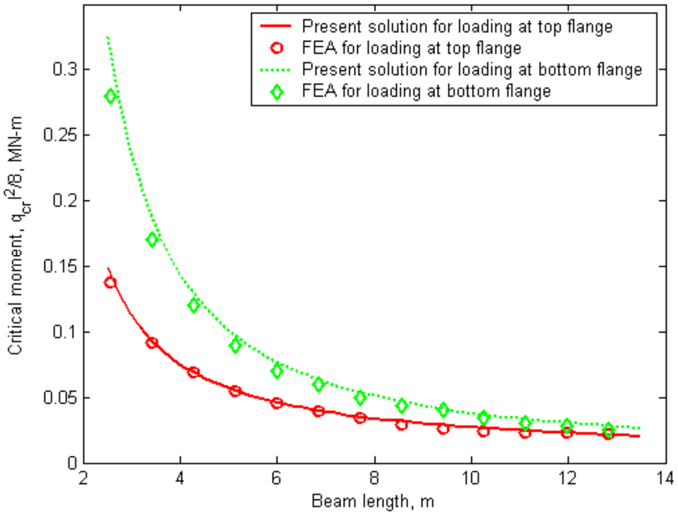


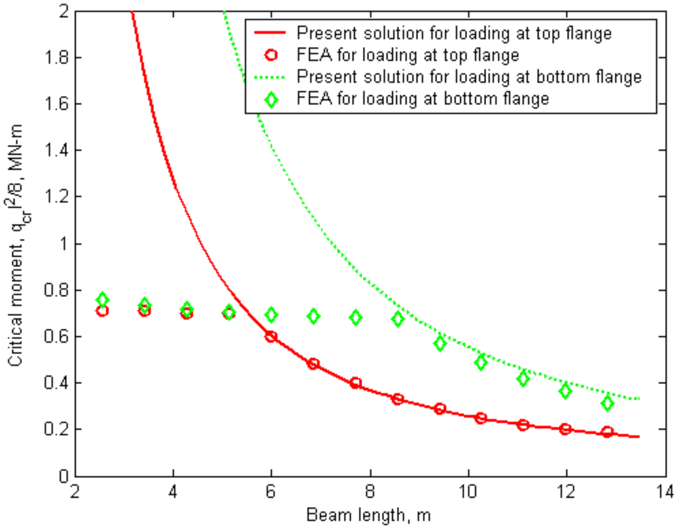
Fig. 4. Critical loads of lateral–torsional buckling of beams subjected to a uniformly distributed load applied at the top flange and a coupled bending moments applied at the ends (flange thickness $t_f = 10\,\text{mm}$, web depth $h_w = 350\,\text{mm}$, web thickness $t_w = 10\,\text{mm}$, depth of web opening $2a = 247\,\text{mm}$, beam length $l = 7000\,\text{mm}$, $k = 0.5$).

$(x = 0)$ and zero rotations about the transverse and lateral axes $(\phi_y = \phi_z = 0)$ and zero axial displacement $(u = 0)$ for all nodes on the other end section (symmetric section at $x = 1/2$). The uniformly distributed transverse load was applied at the intersection line between the top flange and web. A linear buckling analysis was employed to obtain the lowest positive and lowest negative eigenvalues, which represent the critical loads of the beam when the loading was applied at the top and bottom flanges, respectively. The maximum element size used in the analysis is 20 mm. This is based on the trial in which the obtained eigenvalues associated with the first two lowest buckling modes have almost no change with any further reduction in element sizes.

Figure 5 compares the critical loads of the two castellated beams obtained from Eqs. (28) and (29) and those obtained from the finite element analyses. It can be seen from the comparison that the present analytical solutions agree very well with the results obtained from the finite element analyses for the beam with narrow flanges. However, for the beam with wide flanges, the present analytical solution is valid only when the beam is longer than 5.5 m for upper flange loading and 8.5 m for lower flange loading. For short beams, the analytical solutions are much higher than those predicted by the finite element analyses. The reason for this is because when the beam is short and its flanges are also wide, the lowest buckling mode of the beam is no longer the lateral–torsional buckling. This is demonstrated by the buckling modes shown in Fig. 6. Figure 6(a) shows the half of the first buckling mode of the beam at



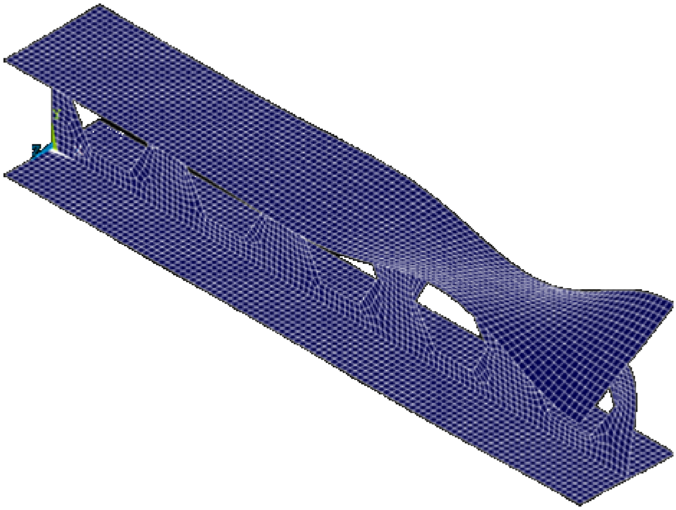
(a)



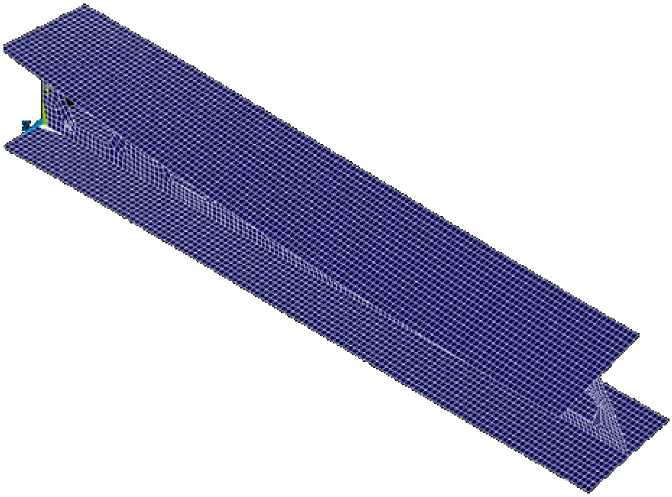
(b)

Fig. 5. Comparison between the analytical solution and FEA for the critical loads of beams subjected to a uniformly distributed load. (a) $b_f = 117\,\text{mm}$ and (b) $b_f = 350\,\text{mm}$ (flange thickness $t_f = 10\,\text{mm}$, web depth $h_w = 350\,\text{mm}$, web thickness $t_w = 10\,\text{mm}$ and depth of web opening $2a = 247\,\text{mm}$, $k = 0.5$).

length $L = 4.278\,\text{m}$ when it is subjected to a uniformly distributed load applied at top flange. It can be seen from this buckling mode that there is substantial distortion involved in the web and compressed flange. This web-flange distortion could be due to the widening of flanges and also the shear weakness of the web owing to the web



(a)



(b)

Fig. 6. Buckling modes associated with the critical load of a beam when the load is applied at the top flange obtained from the FEA. (a) $L = 4.278\,\text{mm}$ and (b) $L = 6.845\,\text{m}$ (flange width $b_f = 350\,\text{mm}$, flange thickness $t_f = 10\,\text{mm}$, web depth $h_w = 350\,\text{mm}$, web thickness $t_w = 10\,\text{mm}$, and depth of web opening $2a = 247\,\text{mm}$).

opening.[23–25] However, with the increase of beam length, this distortional buckling mode is overtaken by the lateral–torsional buckling mode as is demonstrated in Fig. 6(b). It should be pointed out that the dimensions chosen here represent two extreme cases, i.e. very narrow and very wide castellated beams. In practice, the wide flanged castellated beams are usually used for long span buildings in which case the lateral–torsional buckling is always dominant.

5. Conclusion

An analytical solution is presented to determine the critical moment of the lateral–torsional buckling of simply supported castellated beams subjected to a uniformly distributed load and/or a coupled bending moment applied at the ends of the beam. The analytical solution is verified using the finite element analysis. From the present study, the following conclusions may be drawn:

(1) For most castellated beams, the reductions in both warping rigidity and lateral flexural rigidity caused due to the web openings are very small and can generally be ignored in the calculation.
(2) The main effect of the web openings on the lateral–torsional buckling of the castellated beam is due to the reduction in torsional rigidity caused by the web openings. The critical load of the lateral–torsional buckling of castellated beams thus should be calculated using the average torsional constant of the full and reduced section properties.
(3) For a castellated beam with wider flanges, the web openings have almost no influence on the critical load of the lateral–torsional buckling of the beam. For a castellated beam with narrow flanges, the web openings can marginally reduce the critical load of the lateral–torsional buckling of the beam.
(4) Since the web openings have only marginal influence on the lateral–torsional buckling behavior of castellated beams, most features found in I-beams can be also applied to castellated beams.
(5) It should be noted that the analytical solution derived here is only for simply supported castellated beams. However the principle proposed could be applied to beams with other boundary conditions.

References

1. C. Harper, *Design in Steel 4: Castellated and Cellular Beams* (British Steel, Port Talbot, UK, 1991).
2. P. R. Knowles, *Design of Castellated Beams* (Constrado, London, UK, 1985).
3. P. R. Knowles, Castellated beams, *Proc. Institution of Civil Engineers: Structural and Building Board, Part 1*, Vol. 90 (1991), pp. 521–536.
4. BSI. BS 5950-1:2000 Structural use of steelwork in buildings — Part 1: Code of practice for design — Rolled and welded sections, London: BSI (2006).
5. D. Nethercot and D. Kerdal, Lateral-torsional buckling of castellated beams, *The Struct. Eng.* **60** (1982) 53–61.
6. D. Kerdal and D. Nethercot, Failure modes for castellated beams, *J. Construct. Steel Res.* **4**(4) (1984) 295–315.
7. R. Redwood, W. Zaarour and J. Megharief, Web post buckling in castellated beams, in *Advances in Steel Structures (ICASS '96)* S. L. Chan and J. G. Teng (eds.), (Pergamon, Oxford, 1996), pp. 67–71.

8. A. Mohebkhah, The moment-gradient factor in lateral–torsional buckling on inelastic castellated beams, *J. Construct. Steel Res.* **60**(10) (2004) 1481–1494.
9. A. Mohebkhah and H. Showkati, Bracing requirements for inelastic castellated beams, *J. Construct. Steel Res.* **61**(10) (2005) 1373–1386.
10. T. Zirakian and H. Showkati, Distortional buckling of castellated beams, *J. Construct. Steel Res.* **62**(9) (2006) 863–871.
11. E. Ellobody, Interaction of buckling modes in castellated steel beams, *J. Construct. Steel Res.* **67**(5) (2011) 814–825.
12. S. Gholizadeh, A. Pirmoz and R. Attarnejad, Assessment of load carrying capacity of castellated steel beams by neural networks, *J. Construct. Steel Res.* **67**(5) (2011) 770–779.
13. M. R. Soltani, A. Bouchaïr and M. Mimoune, Nonlinear FE analysis of the ultimate behaviour of steel castellated beams, *J. Construct. Steel Res.* **70** (2012) 101–114.
14. H. Showkati, T. G. Ghazijahani, A. Noori and T. Zirakian, Experiments on elastically braced castellated beams, *J. Construct. Steel Res.* **77** (2012) 163–172.
15. N. Kim, D. K. Shin and M. Y. Kim, Improved flexural–torsional stability analysis of thin-walled composite beam and exact stiffness matrix, *Int. J. Mech. Sci.* **49**(8) (2007) 950–969.
16. E. Ghandi, B. Rafezy and W. P. Howson, On the bi-planar motion of a Timoshenko beam with shear resistant in-fill, *Int. J. Mech. Sci.* **57**(1) (2012) 1–8.
17. S. Schnabl and I. Planinc, Inelastic buckling of two-layer composite columns with non-linear interface compliance, *Int. J. Mech. Sci.* **53**(12) (2011) 1077–1083.
18. W. R. Chen, Parametric instability of spinning twisted Timoshenko beams under compressive axial pulsating loads, *Int. J. Mechan. Sci.* **52**(9) (2010) 1167–1175.
19. J. K. Chen and L. Y. Li, Elastic axially compressed buckling of battened columns, *Int. J. Mech. Sci.* **77** (2013) 1–7.
20. U. C. Pattanayak and E. Chesson, Lateral instability of castellated beams, *AISC Eng. Journal* **11**(3) (1974) 73–79.
21. S. P. Timoshenko and J. M. Gere, *Theory of Elastic Stability*, 2nd edn. (McGraw-Hill Book Company, New York, 1961).
22. C. H. Yoo and S. C. Lee, *Stability of Structures, Principles and Applications* (Elsevier, Oxford, 2011).
23. W. B. Yuan, B. Kim and L. Y. Li, Buckling of axially loaded castellated steel columns, *J. Construct. Steel Res.* **92** (2014) 40–45.
24. K. El-Sawy, A. Sweedan and M. Martini, Major-axis elastic buckling of axially loaded castellated steel columns, *Thin-Walled Struct.* **47**(11) (2009) 1295–1304.
25. J. K. Chen, B. Kim and L. Y. Li, Analytical approach for transverse vibration analysis of castellated beams, *Int. J. Struct. Stab. Dynam.* **14**(3) (2014) 1–13 (1350071).