Ref. Roark, 7th Edition
Table 8.1 - Shear, moment, slope, and deflection formula for elastic beams
Case 2e - Simply supported beam, partial distributed load

Settings ... $E := 200\cdot GPa$ $I := 10^{11}\cdot mm^4$ ... and ... $L := 10\cdot m$

Unit load ... $w := 10\cdot\frac{kN}{m}$

Loading 1 - Distributed loading from R_b to R_d

$$R_{a1} := \frac{4}{9}\cdot w\cdot L$$

$$\theta_{a1} := \frac{-26}{45}\cdot\frac{w\cdot L^3}{E\cdot I}$$

$$y_1(x) := \theta_{a1}\cdot x + \frac{R_{a1}\cdot x^3}{6\cdot E\cdot I} - \frac{2\cdot w}{120\cdot E\cdot I\cdot 2\cdot L}\cdot(x - L)^5\cdot(x > L)$$

Loading 2 - Distributed loading from R_c to R_d

$$R_{a2} := \frac{2}{9}\cdot w\cdot L$$

$$\theta_{a2} := \frac{-19}{60}\cdot\frac{w}{E\cdot I}\cdot L^3$$

$$y_2(x) := \theta_{a2}\cdot x + \frac{R_{a2}\cdot x^3}{6\cdot E\cdot I} - \frac{w}{24\cdot E\cdot I}\cdot(x - 2\cdot L)^4\cdot(x > 2L) - \frac{w}{120\cdot E\cdot I\cdot L}\cdot(x - 2\cdot L)^5\cdot(x > 2\cdot L)$$

Displacement ... $y(x) := y_1(x) - y_2(x)$

$y_{Rb} := y(L)$ $y_{Rb} = -1.1204\,mm$ ... displacement at R_b

$y_{Rc} := y(2L)$ $y_{Rc} = -1.1713\,mm$ ... displacement at R_c

Ref. Roark, 7th Edition
Table 8.1 - Shear, moment, slope, and deflection formula for elastic beams
Case 1c - Simply supported beam, point load

$$R_{Ab} = \frac{R_b}{3L}\cdot(3L - L)$$ ... reaction due to load at R_b

$$\theta_{Ab} = \frac{-R_b\cdot L}{6\cdot E\cdot I\cdot(3L)}\cdot[2\cdot(3L) - L]\cdot[(3L) - L]$$

$$y_{Rb}(x) = \theta_{Ab}\cdot x + \frac{R_{Ab}\cdot x^3}{6\cdot E\cdot I} - \frac{R_b}{6\cdot E\cdot I}\cdot(x - L)^3\cdot(x > L)$$

$$y_{Rb}(x) = -\frac{5}{9}\cdot\frac{R_b\cdot L^2\cdot x}{E\cdot I} + \frac{1}{9}\cdot\frac{R_b\cdot x^3}{E\cdot I} - \frac{R_b}{6\cdot E\cdot I}\cdot(x - L)^3\cdot(x > L)$$ ... substituting and simplifying

$$k_{Rbb} := \frac{-4}{9}\cdot\frac{L^3}{E\cdot I} \qquad k_{Rbb} = -2.222\times 10^{-5}\,\frac{mm}{N}$$ ... deflection at R_b due to R_b

$$k_{Rbc} := \frac{-7}{18}\cdot\frac{L^3}{E\cdot I} \qquad k_{Rbc} = -1.944\times 10^{-5}\,\frac{mm}{N}$$ ... deflection at R_c due to R_b

$$k_{Rcb} := \frac{-7}{18}\cdot\frac{L^3}{E\cdot I} \qquad k_{Rcb} = -1.944\times 10^{-5}\,\frac{mm}{N}$$ ... deflection at R_b due to R_c

$$k_{Rcc} := \frac{-4}{9}\cdot\frac{L^3}{E\cdot I} \qquad k_{Rcc} = -2.222\times 10^{-5}\,\frac{mm}{N}$$ ... deflection at R_c due to R_c

$$\begin{pmatrix} k_{Rbb} & k_{Rcb} \\ k_{Rbc} & k_{Rcc} \end{pmatrix}\cdot\begin{pmatrix} R_b \\ R_c \end{pmatrix} = \begin{pmatrix} y_{Rb} \\ y_{Rc} \end{pmatrix}$$ ... displacement equations in matrix format

$$\begin{pmatrix} R_b \\ R_c \end{pmatrix} := \begin{pmatrix} k_{Rbb} & k_{Rcb} \\ k_{Rbc} & k_{Rcc} \end{pmatrix}^{-1}\cdot\begin{pmatrix} y_{Rb} \\ y_{Rc} \end{pmatrix}$$ ... solving for R_b and R_c reaction forces

$$R_b \cdot L + R_c \cdot (2 \cdot L) + R_d \cdot (3 \cdot L) - \frac{1}{2} \cdot w \cdot L \cdot \left(L + \frac{2}{3} \cdot L \right) = 0$$

... taking moments about R_a

Giving ... $R_d := \frac{5}{18} \cdot w \cdot L - \frac{1}{3} \cdot R_b - \frac{2}{3} \cdot R_c$

$$R_a + R_b + R_c + R_d - \frac{1}{2} \cdot w \cdot L = 0$$

... force equilibrium

Giving ... $R_a := \frac{1}{2} \cdot w \cdot L - \left(R_b + R_c + R_d \right)$

Reaction results ... $R_a = -2222\,\mathrm{N}$

$R_b = 18333\,\mathrm{N}$

$R_c = 36667\,\mathrm{N}$

$R_d = -2778\,\mathrm{N}$