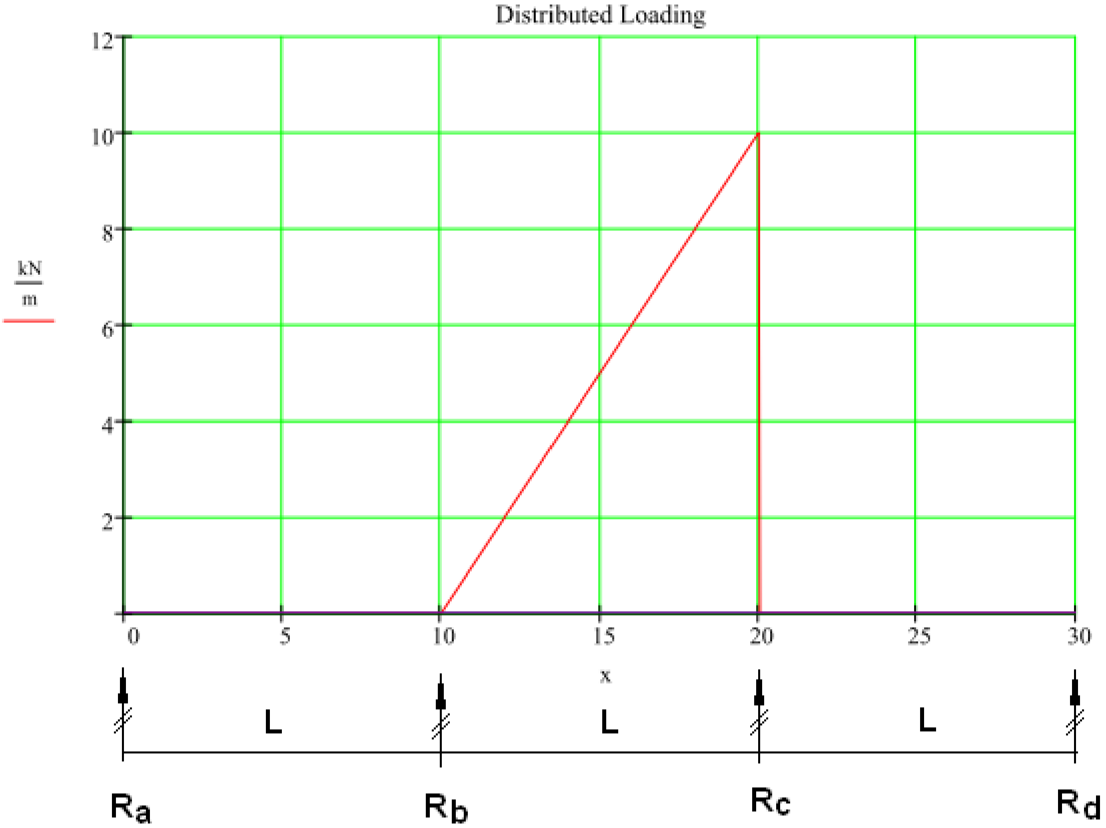
Beam Analysis



Inputs ... $E := 200 \cdot GPa$ $\quad I := 10^8 \cdot mm^4$ $\quad L := 10 \cdot m$ $\quad w := 10 \cdot \frac{kN}{m}$

Bending ... Moment

$$BM(x, R_a, R_b, R_c, R_d) := R_a \cdot x + \left[R_b \cdot (x - L) - \frac{w}{6 \cdot L} \cdot (x - L)^3 \right] \cdot (x \geq L) \ ...$$
$$+ \left[R_c \cdot (x - 2 \cdot L) + \frac{w}{6 \cdot L} \cdot (x - 2 \cdot L)^3 + \frac{w}{2} \cdot (x - 2 \cdot L)^2 \right] \cdot (x \geq 2 \cdot L)$$

$$y_{xx}(x, R_a, R_b, R_c, R_d) := \frac{1}{E \cdot I} \cdot BM(x, R_a, R_b, R_c, R_d)$$ ... 2nd displacement derivative

$$y_x(x_p, R_a, R_b, R_c, R_d, \theta_a) := \theta_a + \int_0^{x_p} y_{xx}(x, R_a, R_b, R_c, R_d) \, dx$$ ... 1st displacement derivative

$$y(x_p, R_a, R_b, R_c, R_d, \theta_a) := \int_0^{x_p} y_x(x, R_a, R_b, R_c, R_d, \theta_a) \, dx$$ ... displacement function

Given $y(L, R_a, R_b, R_c, R_d, \theta_a) = 0 \cdot mm$ ... setting displacement at R_b to zero

$y(2 \cdot L, R_a, R_b, R_c, R_d, \theta_a) = 0 \cdot mm$ ... setting displacement at R_c to zero

$y(3 \cdot L, R_a, R_b, R_c, R_d, \theta_a) = 0 \cdot mm$ ... setting displacement at R_d to zero

$R_a + R_b + R_c + R_d - 0.5 \cdot w \cdot L = 0 \cdot N$ ... vertical force equilibrium

$BM(3 \cdot L, R_a, R_b, R_c, R_d) = 0 \cdot N \cdot mm$ ... zero BM at end of beam

Solving gives ... $R_a = -2222\,N$ $\quad \frac{R_a}{w \cdot L} = -0.02222$

$R_b = 18333\,N$ $\quad \frac{R_b}{w \cdot L} = 0.18333$

$R_c = 36667\,N$ $\quad \frac{R_c}{w \cdot L} = 0.36667$

and ... $R_d = -2778\,N$ $\quad \frac{R_d}{w \cdot L} = -0.02778$

Checks ... $y(L, R_a, R_b, R_c, R_d, \theta_a) = 0.0000\,mm$ $\quad R_a + R_b + R_c + R_d - 0.5 \cdot w \cdot L = -0.0000\,N$

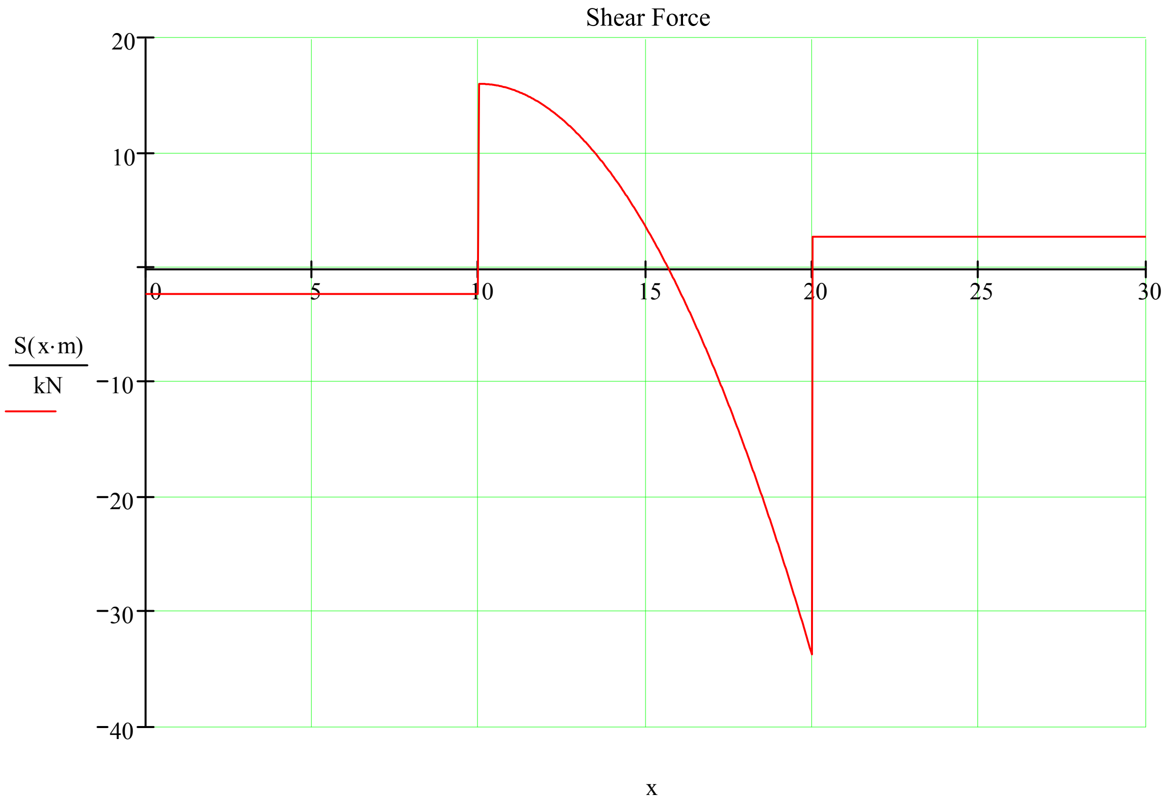
$y(2 \cdot L, R_a, R_b, R_c, R_d, \theta_a) = 0.0000\,mm$ $\quad BM(3 \cdot L, R_a, R_b, R_c, R_d) = 0.0000\,N \cdot mm$

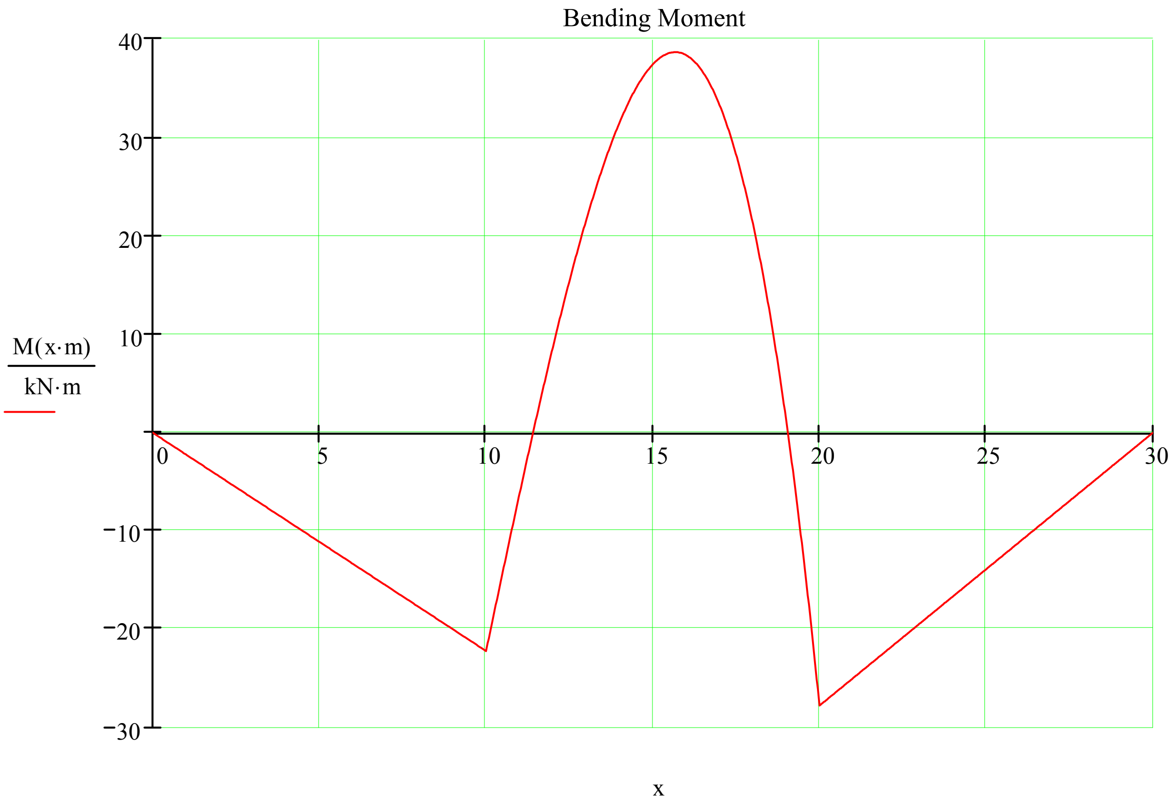
$y(3 \cdot L, R_a, R_b, R_c, R_d, \theta_a) = 0.0000\,mm$

Shear Force ...

$$S(x) := R_a + \left[R_b - \frac{w}{2 \cdot L} \cdot (x - L)^2\right] \cdot (x \ge L) \ldots$$
$$+ \left[R_c + \frac{w}{2 \cdot L} \cdot (x - 2 \cdot L)^2 + w \cdot (x - 2 \cdot L)\right] \cdot (x \ge 2 \cdot L)$$

Bending Moment $\quad M(x) := BM(x, R_a, R_b, R_c, R_d)$


Shear Force
S(x·m)
kN
20
10
−10
−20
−30
−40
0
5
10
15
20
25
30
x


Bending Moment
M(x·m)
kN·m
40
30
20
10
−10
−20
−30
0
5
10
15
20
25
30
x