$$R_a = \frac{1}{2} \cdot \omega \cdot L \qquad \text{... support reaction load (vertical force equilibrium)}$$

$$M(x) = R_a \cdot x - \frac{1}{2} \cdot \omega \cdot x^2 \qquad \text{... BM along beam length, as a function of x}$$

$$M(x) = \frac{1}{2} \cdot \omega \cdot L^2 \cdot \left[\frac{x}{L} - \left(\frac{x}{L} \right)^2 \right] \qquad \text{... BM, substituting for } R_a$$

$$y_{xx}(x) = \frac{M(x)}{E \cdot I}$$

$$y_x\left(x_p\right) = y_{xA} + \frac{\omega \cdot L^2}{2 \cdot E \cdot I} \cdot \int_0^{x_p} \frac{x}{L} - \left(\frac{x}{L} \right)^2 \, dx \qquad \text{... gradient integral}$$

$$y_x(x) = y_{xA} + \frac{\omega \cdot L^3}{12 \cdot E \cdot I} \cdot \left[3 \cdot \left(\frac{x}{L} \right)^2 - 2 \cdot \left(\frac{x}{L} \right)^3 \right] \qquad \text{... gradient as function of x}$$

$$y_x\left(\frac{L}{2} \right) = 0 = y_{xA} + \frac{\omega \cdot L^3}{24 \cdot E \cdot I} \qquad \text{... set gradient to zero at x = L/2}$$

$$y_{xA} = -\frac{\omega \cdot L^3}{24 \cdot E \cdot I} \qquad \text{... same as Roark 7th Edition,}$$
$$\text{Table 8.1, Case 2e}$$

$$y_x(x) = -\frac{\omega \cdot L^3}{24 \cdot E \cdot I} \cdot \left[1 - 6 \cdot \left(\frac{x}{L} \right)^2 + 4 \cdot \left(\frac{x}{L} \right)^3 \right] \qquad \text{... gradient as function of x}$$

$$y\left(x_p\right) = -\frac{\omega \cdot L^3}{24 \cdot E \cdot I} \cdot \int_0^{x_p} 1 - 6 \cdot \left(\frac{x}{L} \right)^2 + 4 \cdot \left(\frac{x}{L} \right)^3 \, dx \qquad \text{... displacement integral}$$

$$y(x) = -\frac{\omega \cdot L^3 \cdot x}{24 \cdot E \cdot I} \cdot \left[1 - 2 \cdot \left(\frac{x}{L}\right)^2 + \left(\frac{x}{L}\right)^3 \right]$$ ... displacement as function of x

$$y_{max} = y\left(\frac{L}{2}\right) = -\frac{5 \cdot \omega \cdot L^4}{384 \cdot E \cdot I}$$ ... same as Roark 7th Edition,
Table 8.1, Case 2e

$$\delta L = \frac{1}{2} \cdot \int_0^L \left(\frac{d}{dx} y(x)\right)^2 dx$$ ... change in beam length

where ... $$\frac{d}{dx} y(x) = -\frac{\omega \cdot L^3}{24 \cdot E \cdot I} \cdot \left[1 - 6 \cdot \left(\frac{x}{L}\right)^2 + 4 \cdot \left(\frac{x}{L}\right)^3 \right]$$

$$\delta L = \frac{1}{2} \cdot \left(\frac{\omega \cdot L^3}{24 \cdot E \cdot I}\right)^2 \cdot \int_0^L \left[1 - 6 \cdot \left(\frac{x}{L}\right)^2 + 4 \cdot \left(\frac{x}{L}\right)^3 \right]^2 dx$$

$$\delta L = \frac{1}{2} \cdot \left(\frac{\omega \cdot L^3}{24 \cdot E \cdot I}\right)^2 \cdot \left(\frac{17}{35} \cdot L\right)$$

$$L := 160 \cdot in \qquad \omega := 2000 \cdot \frac{lbf}{L} \qquad \omega = 12.5 \frac{lbf}{in}$$

$$E := 29 \cdot Mpsi \qquad I := \frac{1}{12} \cdot (2 \cdot in)^4$$

$$\delta L := \frac{1}{2} \cdot \left(\frac{\omega \cdot L^3}{24 \cdot E \cdot I}\right)^2 \cdot \left(\frac{17}{35} \cdot L\right) \qquad \delta L = 0.1183 \ in$$