$$R_a = \frac{1}{2}\cdot\omega\cdot L$$ ... support reaction load (vertical force equilibrium)

$$M(x) = R_a\cdot x - \frac{1}{2}\cdot\omega\cdot x^2$$ ... BM along beam length, as a function of x

$$M(x) = \frac{1}{2}\cdot\omega\cdot L^2\cdot\left[\frac{x}{L} - \left(\frac{x}{L}\right)^2\right]$$ ... BM, substituting for R_a

$$y_{xx}(x) = \frac{M(x)}{E\cdot I}$$

$$y_x(x_p) = y_{xA} + \frac{\omega\cdot L^2}{2\cdot E\cdot I}\cdot\int_0^{x_p} \frac{x}{L} - \left(\frac{x}{L}\right)^2 dx$$ ... gradient integral

$$y_x(x) = y_{xA} + \frac{\omega\cdot L^3}{12\cdot E\cdot I}\cdot\left[3\cdot\left(\frac{x}{L}\right)^2 - 2\cdot\left(\frac{x}{L}\right)^3\right]$$ ... gradient as function of x

$$y_x\left(\frac{L}{2}\right) = 0 = y_{xA} + \frac{\omega\cdot L^3}{24\cdot E\cdot I}$$ ... set gradient to zero at x = L/2

$$y_{xA} = -\frac{\omega\cdot L^3}{24\cdot E\cdot I}$$ ... same as Roark 7th Edition,
Table 8.1, Case 2e

$$y_x(x) = -\frac{\omega\cdot L^3}{24\cdot E\cdot I}\cdot\left[1 - 6\cdot\left(\frac{x}{L}\right)^2 + 4\cdot\left(\frac{x}{L}\right)^3\right]$$ ... gradient as function of x

$$y(x_p) = -\frac{\omega\cdot L^3}{24\cdot E\cdot I}\cdot\int_0^{x_p} 1 - 6\cdot\left(\frac{x}{L}\right)^2 + 4\cdot\left(\frac{x}{L}\right)^3 dx$$ ... displacement integral

$$y(x) = -\frac{\omega \cdot L^3 \cdot x}{24 \cdot E \cdot I} \cdot \left[1 - 2 \cdot \left(\frac{x}{L}\right)^2 + \left(\frac{x}{L}\right)^3\right]$$ ... displacement as function of x

$$y_{max} = y\left(\frac{L}{2}\right) = -\frac{5 \cdot \omega \cdot L^3}{384 \cdot E \cdot I}$$ ... same as Roark 7th Edition, Table 8.1, Case 2e

$$\delta L = \frac{1}{2} \cdot \int_0^L \left(\frac{d}{dx} y(x)\right)^2 dx$$ ... change in beam length

where ... $$\frac{d}{dx} y(x) = -\frac{\omega \cdot L^3}{24 \cdot E \cdot I} \cdot \left[1 - 6 \cdot \left(\frac{x}{L}\right)^2 + 4 \cdot \left(\frac{x}{L}\right)^3\right]$$

$$\delta L = \frac{1}{2} \cdot \left(\frac{\omega \cdot L^3}{24 \cdot E \cdot I}\right)^2 \cdot \int_0^L \left[1 - 6 \cdot \left(\frac{x}{L}\right)^2 + 4 \cdot \left(\frac{x}{L}\right)^3\right]^2 dx$$

$$\delta L = \frac{1}{2} \cdot \left(\frac{\omega \cdot L^3}{24 \cdot E \cdot I}\right)^2 \cdot \left(\frac{17}{35} \cdot L\right)$$

$L := 160 \cdot in$ $\quad$ $\omega := 2000 \cdot \frac{lbf}{L}$ $\quad$ $\omega = 12.5 \frac{lbf}{in}$

$E := 29 \cdot Mpsi$ $\quad$ $I := \frac{1}{12} \cdot (2 \cdot in)^4$

$\delta L := \frac{1}{2} \cdot \left(\frac{\omega \cdot L^3}{24 \cdot E \cdot I}\right)^2 \cdot \left(\frac{17}{35} \cdot L\right)$ $\quad$ $\delta L = 0.118\, in$