

## CHAPTER 8

### BEARINGS

**8-1. Beam Bearing Plates.** At the ends of beams supported by direct bearing on masonry or concrete, it is usually necessary to use bearing plates. The chief purpose of the plate is to distribute the beam reaction over a large enough area so that the masonry is not overstressed in com-

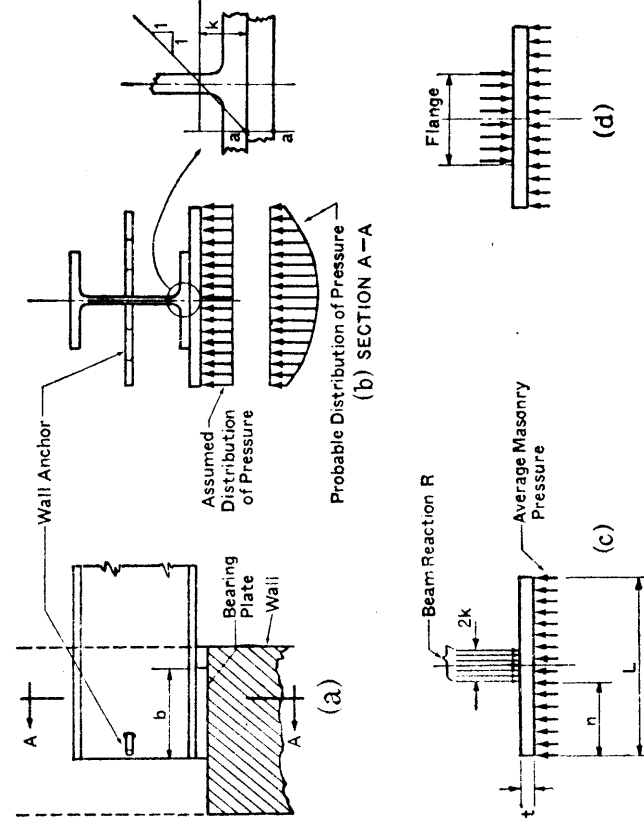


FIG. 8-1

pression. A bearing plate also facilitates erection by providing a smooth surface, easily adjusted to the desired elevation, on which the beam may be rested.

A typical bearing plate is illustrated by Fig. 8-1. If the masonry is extended over the beam (as indicated by dotted lines), some form of wall anchor is used. The type shown is called a "government anchor" and consists of a bent steel rod. A pair of short clip-angles, connected to the web, might be used instead of the rod. No structural strength is assumed to be provided by the anchor. Its purpose is to prevent longitudinal movement of the beam with respect to the wall. However, if longitudinal

loads of appreciable proportions are expected, anchor bolts extending vertically into the masonry and bonded to it would probably be used instead of the anchor shown.

The beam reaction causes pressure between the masonry and the bearing plate. The intensity of pressure  $p$  is assumed for design purposes to be uniform over the area of the plate. A small upward deflection of the outer edges of the plate does occur, of course, relieving the pressure near the outer edges and increasing the pressure near the center, as shown in Fig. 8-1(*b*). Because of deflection of the beam itself, there is an increase in pressure along the edge of the bearing plate nearest the center of the beam span and a corresponding decrease at the edge near the end of the beam. The allowable average unit compressive stresses in the masonry are low in order to compensate for these nonuniformities of pressure distribution. The design of bearing plates resulting from the assumption of uniform distribution of bearing stress is generally on the safe side.

Two methods of analysis for bearing plates are in general use. In the first method, the bearing pressure of the beam flanges on the plate and the bearing pressure of the plate on the concrete are both considered to be uniformly distributed. The critical sections of the bearing plate are at the edge of the beam flange and near the center of the beam. The larger bending moment at either section may control the bearing plate thickness. This method of design appears to assume that there is no deflection of the flange and bearing plate, and it would seem to be justified when web stiffeners are used over the bearing, thus distributing the load over the width of the flange. The bending moment at the center of the bearing plate is equal to one-eighth of the reaction times the difference between the plate width and the flange width. [See Fig. 8-1(*d*).]

When a stiffener is not placed over the bearing, the flange and the bearing plate act together to resist bending. If the flange and bearing plate are attached together by welding or riveting, the two will act as a unit to resist bending. However, if the bearing plate is not attached to the flange, the total resistance is equal to the sum of the resistances of the separate plate and flange. Assuming that the flange and the bearing plate bend to the same curve and that the radius of curvature is very large, the extreme fiber stresses in the two elements will be proportional to the thickness of the elements. In this method of design the thickness of the bearing plate is assumed, and the corresponding stress and bending moment in the flange are computed. The bearing plate must resist bending moment equal to the total moment at the critical section minus the moment computed for the beam flange. The required thickness  $t$  is computed. If the originally assumed thickness was not correct, a new computation may be necessary. This method of analysis provides a bearing

plate in fairly close agreement with that which would be obtained by the design method suggested by the *AISC Manual*.

In the *AISC* method it is assumed that the bearing plate provides the entire resistance to the bending moment and that the critical section is at a distance  $k$  from the center of the web. The value of  $k$  is shown as a property of the beam sections and is equal to the flange thickness plus the fillet radius. Fig. 8-1(c) shows the distribution of load on the bearing plate as assumed by the *AISC* method. The relation between the results obtained by the different methods can best be illustrated by an example:

**EXAMPLE 8-1.** A 16 WF 36 beam having an end reaction of 25 kips rests on a masonry wall. If the allowable bearing pressure is 300 psi on the masonry, design a bearing plate in accordance with the *AISC* specification.

Area of bearing required is  $25/0.3 = 83.3$  sq in.

The dimension  $b$  required to prevent web crippling (see Chap. 4) is

$$\frac{R}{24t(\text{web})} - k = \frac{25}{24 \times 0.299} - 0.94 = 2.54 \text{ in.}$$

A stiffener is not required to prevent web crippling if a bearing greater than 2.54 in. is provided, measured in the direction of the beam. A practical bearing dimension in the direction of the beam span would be 8 in. on a wall consisting of three rows of bricks.

If one dimension of the bearing plate is 8 in., the other dimension must be  $83.3/8 = 10.4$  in. A plate  $8 \times 0-10\frac{1}{2}$  would be satisfactory. The average masonry pressure would then be

$$\frac{25,000}{84} = 298 \text{ psi.}$$

The dimension  $k = 0.94$  in. and  $n = 4.31$  in. [Fig. 8-1(c)]. Hence, in accordance with the method suggested by the *AISC Manual*, the bending moment at the critical section is

$$298 \times 4.31 \times \frac{4.31}{2} = 2,770 \text{ in.-lb per linear in. of plate.}$$

The required section modulus for the plate is

$$\frac{2,770}{20,000} = 0.138 \text{ in.}^3$$

The required thickness of plate is then

$$\sqrt{6 \times 0.138} = 0.91 \text{ in.}$$

A  $\frac{15}{16}$ -in. PL could be used. A 1-in. PL would probably be used, however, since it is a more commonly stocked thickness.

If a design is made on the basis that both plate and flange take moment, the critical section will be at the edge of the fillet. The bending moment at this point will be

$$298 \times 4.60 \times 2.30 = 3,160 \text{ in.-lb.}$$

The section modulus for the flange per linear in. is

$$\frac{(0.428)^2}{6} = 0.0305 \text{ in.}^3$$

Assuming that a 1-in. bearing plate will be used, when the bending stress  $f$  in the plate is 20,000 psi,  $f$  for the flange will be  $20,000 \times 0.428/1.00 = 8,560$  psi. The bending moment for the flange per linear inch of beam is  $0.0305 \times 8,560 = 262 \text{ in.-lb.}$

The bending moment remaining for the plate to take is

$$3,160 - 262 = 2,898 \text{ in.-lb.}$$

The section modulus required for the plate is then

$$\frac{2,898}{20,000} = 0.1449 \text{ in.}^3,$$

and the thickness,

$$\sqrt{0.1449 \times 6} = 0.932 \text{ in.}$$

If web stiffeners were used over the bearing, it would be expected that the bearing pressure between flange and bearing plate would be practically uniform. This pressure would be

$$\frac{25,000}{7 \times 8} = 447 \text{ psi.}$$

The maximum bending moment in the bearing plate would occur at the centerline of the beam and would be

$$(298 \times 5.25 \times 2.625) - (447 \times 3.5 \times 1.75) = 1,367 \text{ in.-lb. per linear in.}$$

The required section modulus for the plate would be

$$\frac{1,367}{20,000} = 0.068 \text{ in.}^3,$$

and the thickness of the plate would be

$$\sqrt{6 \times 0.068} = 0.64 \text{ in.}$$

Hence, an 11/16-in. Pl may be used if web stiffeners are also used.

**8-2. Column Base Plates.** A typical column base plate detail is shown in Fig. 8-2. The plate may be welded to the column as illustrated in (b), or it may be riveted by means of connecting angles to the flanges or to the flanges and the web of the column. If the column end is milled, its connection to the plate need be strong enough only to hold the parts together. If the column is not milled, the connection is required to transmit the entire column load to the plate.

For an undeformed plate, the upward pressure  $p$  would be uniformly distributed over the entire area  $A \times B$ . Because of elastic deformation of the plate and the supporting material, there is actually a reduction of pressure near the edges of the plate and an increase near the portion beneath the column flanges and web. As with beam bearing plates, the

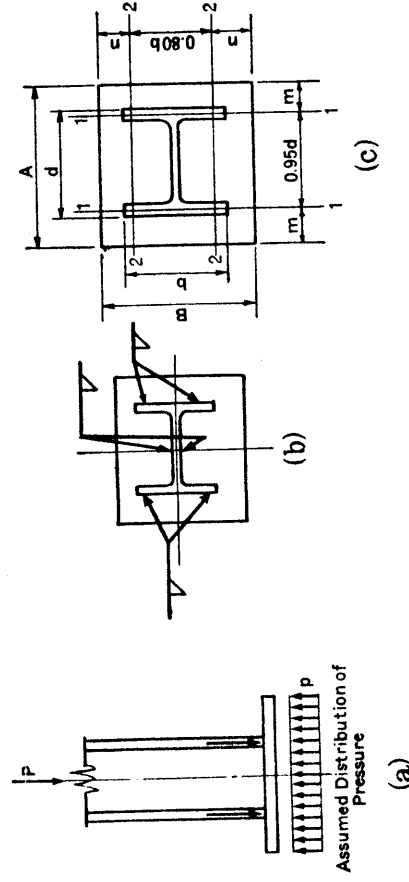


FIG. 8-2

allowable masonry bearing pressure is low to permit safe use of the assumption that the pressure distribution is uniform. The plate design based on this assumption is conservative.

The downward pressure  $P$  is concentrated on the area of contact between the column and the plate. The assumed uniform pressure  $p$  tends to bend upward those portions of plate which project beyond the outline of the column. The projecting portions are considered as uniformly loaded cantilever beams. The bending moment is maximum near the point where the flange contacts the plate. It is usually assumed that the maximum moment occurs on the sections marked 1-1 and 2-2 in Fig. 8-2(c).

The maximum moment would occur at a line of zero shear. This line would fall within the area of contact between the column and the plate. Since the unit pressure on this area is much greater than the unit pres-

sure on the masonry, the line of zero shear will be near the outer face of the column. The values 0.95*d* and 0.80*b* are arbitrary but are selected to meet average values.

For section 1-1,

$$M = \frac{pm^2}{2} \text{ per inch strip of plate,}$$

or  $M = \frac{Bpm^2}{2}$  for the entire overhanging portion of plate.

Since

$$f = \frac{M}{S}, \quad f = \frac{pm^2}{2} \times \frac{6}{t^2} = \frac{3pm^2}{t^2}.$$

Similarly, for section 2-2,

$$f = \frac{3pn^2}{t^2}.$$

For a satisfactory base design, the flexural stress *f* must not exceed the allowable for the material projecting from either section 1-1 or section 2-2.

Using the 1946 AISC allowable flexural stress of 20 kips per sq in., the above equations may be rewritten as follows:

$$t^2 = 0.15pn^2,$$

and

$$t^2 = 0.15pm^2,$$

in which *p* is the bearing pressure in kips per sq in.

The required thickness is the larger value indicated by these equations. (For allowable unit flexural stresses other than 20 kips per sq in., the equations are similar but have a constant other than 0.15.)

**EXAMPLE 8-2** Design a column base to develop the full capacity of a 10 W<sup>F</sup> 49 column, 18 ft 8 in. long, laterally supported at the top and bottom only. Use the allowable unit stresses of the 1946 AISC specification for steel and an allowable bearing stress of 500 psi on the concrete footing.

The maximum column  $L/r = 224/2.54 = 88.2$ . The allowable column stress is

$$\frac{P}{A} = 17,000 - 0.485 \times (88.2)^2 = 13,220 \text{ psi.}$$

The allowable column load is then

$$P = 13,220 \times 14.40 = 190,000 \text{ lb, or 190 kips.}$$



When the bearing pressure  $p = 0.5$  kips per sq in., the bearing area required is

$$A = \frac{190}{0.5} = 380 \text{ sq in.}$$

A plate measuring  $19 \times 20$  in. will provide the necessary area. The position of the plate will be that for which the plate bending moments are the least, unless the position is otherwise controlled by detail requirements, such as the location of anchor bolts. If the flange width is not greater than the depth of the column section, the better position of the plate is with its long dimension parallel to the web. In Fig. 8-2 it will be seen that

$$m = \frac{20}{2} - \left(0.95 \times \frac{10}{2}\right) = 5.25 \text{ in.,}$$

$$n = \frac{19}{2} - \left(0.80 \times \frac{10}{2}\right) = 5.50 \text{ in.}$$

Since  $n$  is greater than  $m$ , the maximum bending moment will occur along sections 2-2 on Fig. 8-2(c). Thus,

$$I^2 = 0.15 \times 0.5 \times (5.5)^2 = 2.27.$$

The required thickness is then

$$t = \sqrt{2.27} = 1.51 \text{ in.}$$

A plate  $20 \times 1\frac{1}{2} \times 1-7$  will be used.

**8-3. Column Bases to Develop End Moment.** Columns frequently are loaded eccentrically, or they have lateral loads causing end moments. The column bases are then required to carry end moments as well as direct stress. Fig. 8-3 illustrates a column base in which end moment is developed by means of anchor bolts. Several methods of attaching the anchor bolts to the column are shown in section A-A.

When the column is loaded with axial loads alone, the upward pressure of the foundation on the plate is assumed to be uniform, as shown in Fig. 8-3(a). The total downward load on the base is the sum of the column load and the initial tensile loads  $T_1$  in the anchor bolts. The unit upward pressure is the total downward load divided by the area of the base plate.

The effect of a small end moment  $M$  is to increase the pressure on one side of the center and to decrease it on the other, as shown in Fig. 8-3(b). The pressure at any point is equal to  $p = P/A \pm Mx/I$ , in which  $x$  is the distance of the point from the center of gravity line of the plate, normal to the plane of the moment  $M$ . By using this same equation for larger values of  $M$ , the distribution of pressure shown in Fig. 8-3(c) is

obtained. Such a distribution is not possible, however, because tensile stress cannot exist between the foundation material and the plate. The tensile force required to maintain equilibrium is provided by the anchor bolt near the left edge of the plate illustrated. The distribution of pres-

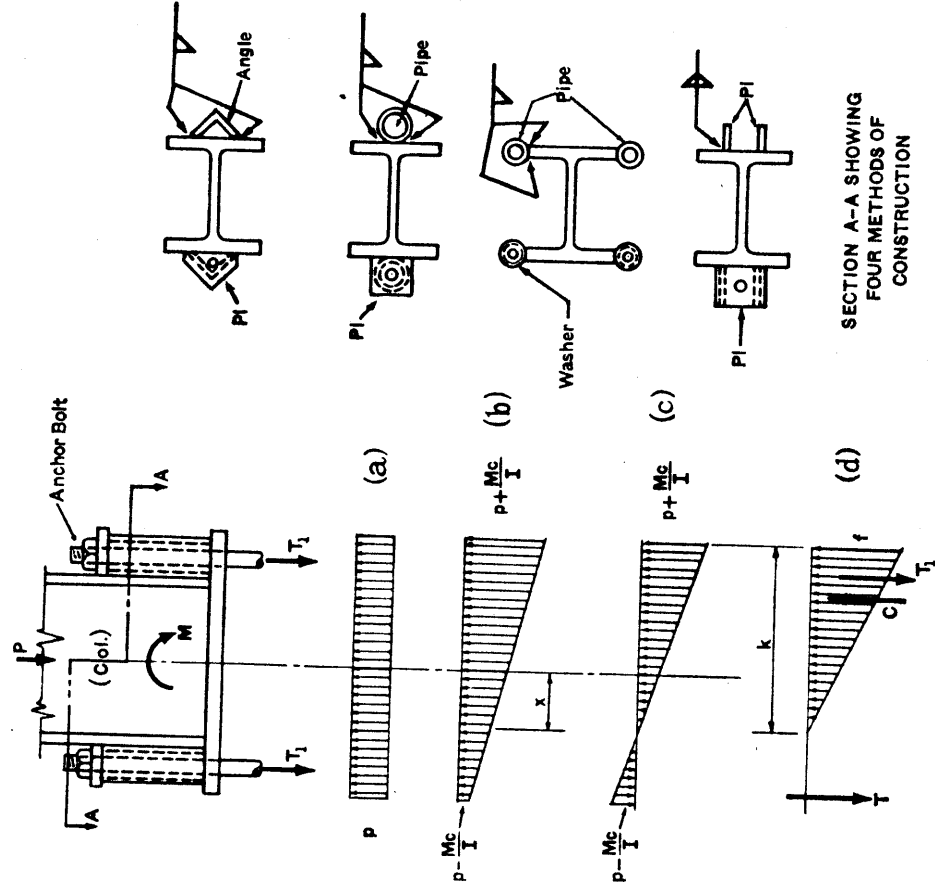


FIG. 8-3

sure is shown in Fig. 8-3(d). The base is subject to the simultaneous application of force  $P$  and the end moment  $M$  on the column, as well as to the initial tension in the bolts and to the force  $C$  (resultant of bearing pressures). As demonstrated in Art. 2-6, the tension in the bolts is not increased by the bending moment until the stress produced by the moment exceeds the initial stress.



Reference to Fig. 8-3(b) suggests that, when the end moment is not sufficient to produce a value of  $M_c/I$  greater than  $P/A$  or when the value

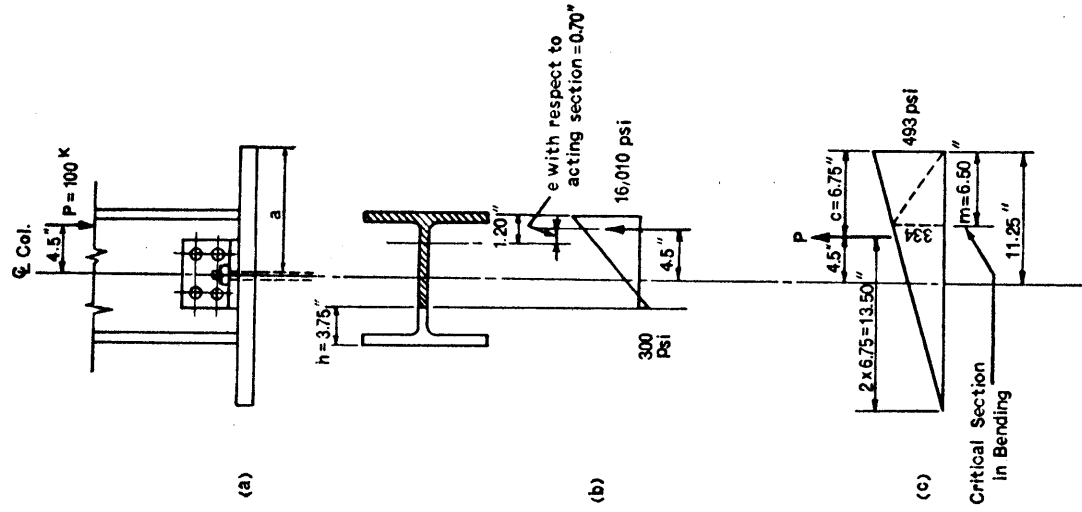


FIG. 8-4

of  $P/A + M_c/I$  is not greater than the allowable bearing pressure, anchor bolts are not necessary to develop end moment.

Fig. 8-4 illustrates a column resting on a base plate without the use of anchor bolts to develop end moment. The maximum end moment

that will be resisted by the column is that moment required to tip the column about its edge, or a moment equal to  $Pa$ , when  $a$  is one-half the column width. A safe moment value will be less than this amount because, when the column is on the point of tipping, the bearing between the column and the plate would be great enough to cause bearing failure. The design limit of end moment should then be the moment that will produce the allowable bearing pressure. Care should be taken in the design of such column bases that provision is made for possible uncertainties. It is recommended that designs should provide for at least one-third of the width to be in compression, when tension bolts are not provided.

When the end moment is such that bearing exists over the entire cross section of the column, the maximum bearing is  $p = P/A + Mc/I$ ; then the end moment would be  $M = (p - P/A) I/c$ , in which  $I/c$  is the section modulus of the cross section of the column. Similarly, the bearing of the plate on the masonry may control the limit of the end moment, and the section modulus of the acting section of the plate would be used in the calculation. When positive bearing exists over a part of the column section, only that part of the section is used in computing the section modulus.

**EXAMPLE 8-3.** Determine whether the type of base connection shown by Fig. 8-4 is suitable for a 10 W F 49 column having an axial load of 100 kips and an end moment of 450 in.-kips. Proportion the base plate required. Use the AISC allowable stresses for steel and an allowable bearing stress of 500 psi between steel and concrete. The anchor bolts are placed on the  $x$ -axis and are not effective in resisting the end moment.

A trial calculation is made first assuming the entire column cross section to be in bearing against the base plate. By this assumption  $A = 14.4$  sq in., and  $S = 54.6$  in.<sup>3</sup> Then,

$$\frac{P}{A} = \frac{100,000}{14.4} = 6,950 \text{ psi compression,}$$

$$\frac{M}{S} = \frac{450,000}{54.6} = 8,240 \text{ psi.}$$

The combined stresses are 1,290 psi tension at one side of the column and 15,190 psi compression at the other. Since the column is not welded to the base plate, the tensile stress does not actually exist. In the areas for which the calculations show a tensile stress, the surfaces separate slightly. The remaining area of actual contact becomes the active area.

For a second trial assume the separation to extend from the edge of the column to a distance of 2.0 in. from the same edge. (See dimension  $h$  of Fig. 8-4(b).) The properties of the assumed active section are now computed:

	$A$	$d$	$Ad$	$Ad^2$	$I_o$
Full section.....	14.40	5.00	72.0	360	273
-Flange.....	-5.58	9.73	-54.2	-528	...
-1.44 × 0.34.....	-0.49	8.72	-4.3	-37	...
	8.33 in. <sup>2</sup>		13.5	-205	273
				273	

$$\bar{d} = \frac{13.5}{8.33} = 1.62 \text{ in.}$$

$$-\bar{d}\Sigma(Ad) = -1.62 \times 13.50 = -22$$

$$I = 46 \text{ in.}^4$$

$$S_{\text{comp.}} = \frac{46}{1.62} = 28.4 \text{ in.}^3$$

$$S_{\text{tens.}} = \frac{46}{8.00 - 1.62} = 7.21 \text{ in.}^3$$

The eccentricity  $e$  with respect to the column center is  $M/P$  or 4.5 in. With respect to the center of gravity of the assumed active section,  $e = 1.12$  in. Thus,  $M$  for the acting section is  $1.12 \times 100,000$  or  $112,000$  in.-lb. Then

$$f_t = \frac{-100,000}{8.33} + \frac{112,000}{7.21} = 3,500 \text{ psi,}$$

$$f_c = \frac{100,000}{8.33} + \frac{112,000}{28.4} = 15,940 \text{ psi.}$$

The presence of a tensile stress indicates that the separation extends beyond the assumed 2.0 inches. The values of  $f_t$  and  $f_c$  obtained can be used as a guide in choosing the next trial section. If  $f_t$  and  $f_c$  were as computed, the neutral axis would occur at

$$8 \left( \frac{15.9}{15.9 + 3.5} \right) = 6.55 \text{ in.}$$

from the compression edge, or at about 3.5 in. from the other face of the column. Apparently an error of 1.5 in. was made in the assumed distance  $h$ , but a further trial will show separation slightly beyond the 3.5 in. estimated above. Therefore,  $h$  is assumed to be 3.75 in. in the next trial.

	$A$	$d$	$Ad$	$Ad^2$	$I_o$
Full section.....	14.40	5.00	72.0	360	273
-Flange.....	-5.58	9.73	-54.2	-528	...
-3.19 × 0.34.....	-1.08	7.85	-8.5	-67	...
	7.74 in. <sup>2</sup>		9.3	-235	273
				273	

$$\bar{d} = \frac{9.3}{7.74} = 1.2 \text{ in.}$$

$$-\bar{d}\Sigma(Ad) = -1.2 \times 9.3 = -11$$

$$I = 27 \text{ in.}^4$$

$$S_{\text{comp.}} = \frac{27}{1.2} = 22.5 \text{ in.}^3$$

$$S_{\text{tens.}} = \frac{27}{10 - 3.75 - 1.2} = 5.3 \text{ in.}^3$$

The eccentricity  $e$  with respect to the center of gravity of the acting section is 0.7 in. The moment is  $0.7 \times 100,000$ , or 70,000 in.-lb. Thus

$$f_t = \frac{-100,000}{7.74} + \frac{70,000}{5.3} = 300 \text{ psi,}$$

$$f_c = \frac{100,000}{7.74} + \frac{70,000}{22.5} = 16,010 \text{ psi.}$$

The tensile stress computed is nearly zero. Therefore, it is assumed that the acting section has been determined. Since the bearing stress of 16,010 psi at the extreme fiber does not exceed the allowable of 30,000 psi for milled parts, the column base detail as shown is satisfactory.

A base plate may be designed on the basis of 500 psi allowable bearing pressure on the masonry and a load of 100,000 lb eccentric 4.5 in. from the center of the column. It is assumed that the eccentricity of the load may be in either direction from the center of the column; hence, the plate will be symmetrical.

Referring to Fig. 8-4(c), if the width of the plate parallel to the  $y$ -axis of the column is  $b$ , then, on the assumption that the pressure diagram is triangular, the maximum masonry pressure will be

$$500 \text{ psi} = \frac{100,000 \times 2}{b \times 3c}.$$

When  $b$  is 20 in.,

$$c = \frac{100,000 \times 2}{20 \times 500 \times 3} = 6.67 \text{ in.}$$

The location of the applied force  $P$  is known to be 4.5 in. from the centerline. The edge of the base plate must then be at least  $4.5 + 6.67 = 11.17$  in. from the centerline. Hence, a plate  $20 \times 22.5$  is assumed. The distance from the resultant force to the edge of the plate is  $11.25 - 4.5 = 6.75$  in. The pressure diagram will be triangular over a width of  $3 \times 6.75 = 20.25$  in. The maximum pressure

$$p = \frac{100,000 \times 2}{20 \times 20.25} = 493 \text{ psi.}$$

The thickness of the plate will be controlled by the bending moment along line 1-1 [Fig. 8-2(c)]. The distance  $m$  is 6.50 in., and the pressure diagram on the cantilevered strip is trapezoidal. Taking moments of the upward pressure on the strip about line 1-1,

$$334 \times \frac{6.50}{2} \times \frac{6.50}{3} = 2,350$$

$$493 \times \frac{6.50}{2} \times \frac{2 \times 6.50}{3} = 6,950$$

9,300 in.-lb (per inch width of plate).

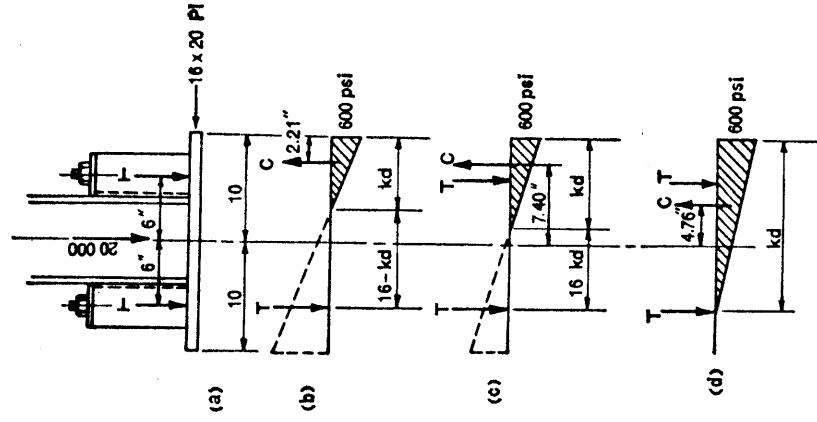


FIG. 8-5

The required section modulus is then  $9,300/20,000$  or  $0.465 \text{ in}^3$ , and the required thickness,

$$t = \sqrt{6 \times 0.465} = 1.67 \text{ in.}$$

A base plate measuring  $20 \times 1\frac{3}{4} \times 1-10\frac{1}{2}$  will be used.

EXAMPLE 8-4. A column base  $16 \times 20$  is shown in Fig. 8-5. Four  $1\frac{1}{2}$ -in. anchor bolts attach the column to the masonry, and the column has a direct load

of 20,000 lb. Determine how much end moment may be developed with a maximum unit bearing pressure of 600 psi, (a) when there is no initial stress in the anchor bolts, (b) when the initial stress in the anchor bolts is equal to the maximum tension due to bending, and (c) when the bolts have an initial tension of 20,000 psi. The modulus of elasticity of the concrete is assumed to be  $1/10$  of that of steel.

(a) Referring to Fig. 8-5(b) and using the same procedure as used in the analysis of a reinforced concrete beam,

$$C = 20,000 + T = 16 \times 600 \times \frac{kd}{2},$$

$$T = 4,800kd - 20,000.$$

Also,

$$T = 10 \times 1.386 \frac{(16 - kd)}{kd} \times 600.$$

(The net area of the two bolts acting is 1.386 sq in.)

Then,

$$4,800kd - 20,000 = 13.86 \frac{(16 - kd)}{kd} 600,$$

$$kd = 6.63 \text{ in.}$$

Thus

$$T = (4,800 \times 6.63) - 20,000 = 11,820 \text{ lb.}$$

$$C = 20,000 + 11,810 = 31,820 \text{ lb.}$$

The permissible end moment for the column is

$$(11,820 \times 6) + (31,820 \times 7.79) = 318,800 \text{ in.-lb.}$$

(b) When the initial tension in the anchor bolts is equal to the tension produced by the end moment, the forces will be as shown in Fig. 8-5(c).

Then,

$$C = 20,000 + 2T = 16 \frac{(600kd)}{2},$$

$$T = 2,400kd - 10,000.$$

Also,

$$T = 10 \times 1.386 \frac{(16 - kd)}{kd} 600,$$

or

$$2,400kd - 10,000 = 13.86 \frac{(16 - kd)}{kd} 600,$$



$$kd = 7.80 \text{ in.},$$

$$T = (2,400 \times 7.80) - 10,000 = 8,720 \text{ lb.},$$

$$C = 20,000 + 2(8,720) = 37,440 \text{ lb.}$$

The permissible end moment for the column is

$$37,440 \times 7.40 = 277,100 \text{ in.-lb.}$$

(c) When the unit stress in the anchor bolts is caused by initial tension and is 20,000 psi [Fig. 8-5(d)],

$$T = 20,000 \times 1.386 = 27,720 \text{ lb.},$$

$$C = 20,000 + 2(27,720) = 75,440 \text{ lb.}$$

Also, the bearing pressure is

$$p = \frac{2(75,440)}{16kd} = 600 \text{ psi,}$$

$$kd = 15.72 \text{ in.}$$

The end moment is then

$$75,440 \times 4.76 = 359,100 \text{ in.-lb.}$$

From Ex. 8-4 it will be seen that the values of end moment may be developed in the same base detail varying from 277,100 in.-lb to 359,100 in.-lb, depending on the amount of initial tension put in the anchor bolts when erected. The determination of the exact amount of initial tension in anchor bolts is very difficult, and too much dependence should not be placed on the value of initial tension. The condition of stress in anchor bolts is usually somewhere between that of cases (a) and (b). When the nuts are tightened by normal methods some initial tension will exist, but special methods must be used to load the anchor bolts accurately to a predetermined stress. It is safe to assume the case illustrated by (b), and it is likely that case (a) is usually approached.

**8-4. Fixed Bearings for Girders or Trusses.** Flat plate end bearings of the type illustrated by Fig. 8-1(a) are not suitable for long spans or for cases in which live load produces the principal reaction. Deflection of the span causes bearing pressures to become excessive and frequently results in serious damage to the masonry. As a result, bearings that provide for the possibility of deflection of the span structure without damage to the bearing are a necessity for many structures. Some type of pin detail is usually employed in these bearings.

The bearings are of two types—fixed and expansion. Several examples of fixed bearings are shown in Fig. 8-6. The term “fixed,” as used in the discussion of the “fixed bearing,” means only that the position of the bearing is fixed and not that the slope to the elastic curve is fixed, as defined in the study of statically indeterminate structures.

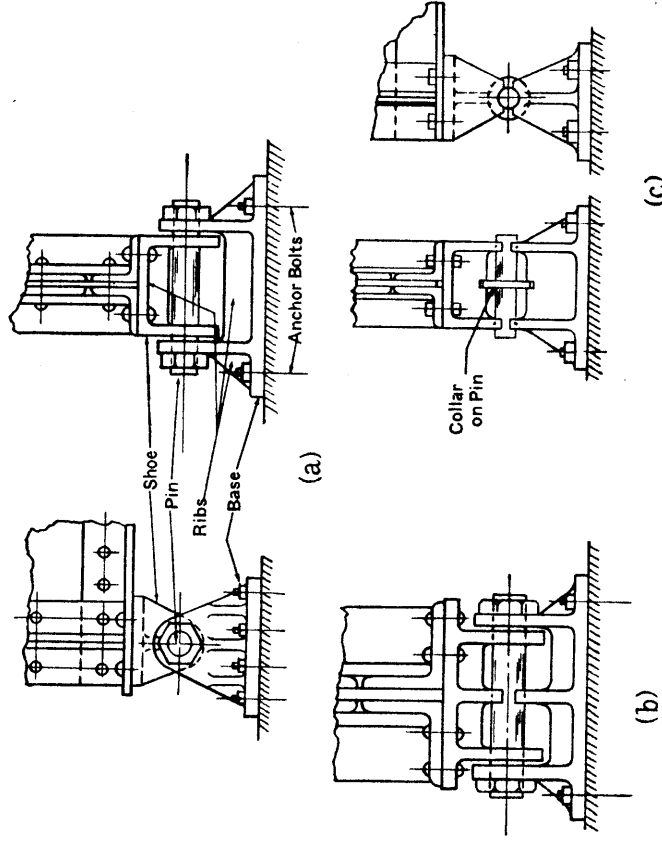


FIG. 8-6

The design of a fixed bearing, such as shown by Fig. 8-6(a), may be considered in steps, as follows:

- (1) Selection of a pin size with respect to the strength of the pin in shear and bending and the strength of the vertical plates in bearing against the pin.
- (2) Determination of the thickness of the vertical plates. This step is usually carried out simultaneously with the pin selection, since the plate thickness affects the bending moment on the pin.
- (3) Determination of the base size required to prevent excessive bearing stress on the supporting masonry.
- (4) Selection of a base plate of sufficient thickness to resist bending.

The procedure is similar to that used for beam and column base plates previously discussed.

(5) Selection of anchor bolts. These should hold the base firmly against the foundation so that the friction developed will prevent shifting under lateral or longitudinal load. They should also resist any expected uplift or negative reaction.

(6) Location of ribs and diaphragms to reinforce the vertical plates and the base plate. The selection of a base plate in step 4 may be affected by the location of such diaphragms.

The steps just given are usually first performed considering the vertical load only. The bearings may also be subject to lateral and longitudinal loads, and a complete design should include consideration of these loads as well as the vertical. Most specifications, however, permit an increase of 25 to 33 per cent in all allowable unit stresses when vertical load is combined with lateral or longitudinal loads. Since the vertical load on the bearing is usually many times the other loads, a bearing designed for vertical loads alone is usually satisfactory for combined loadings. A suggested procedure is to design for vertical loads at the given allowable unit stresses; then check the bearing so designed for the effect of simultaneously applied lateral and longitudinal loads, using the increased allowable unit stresses.

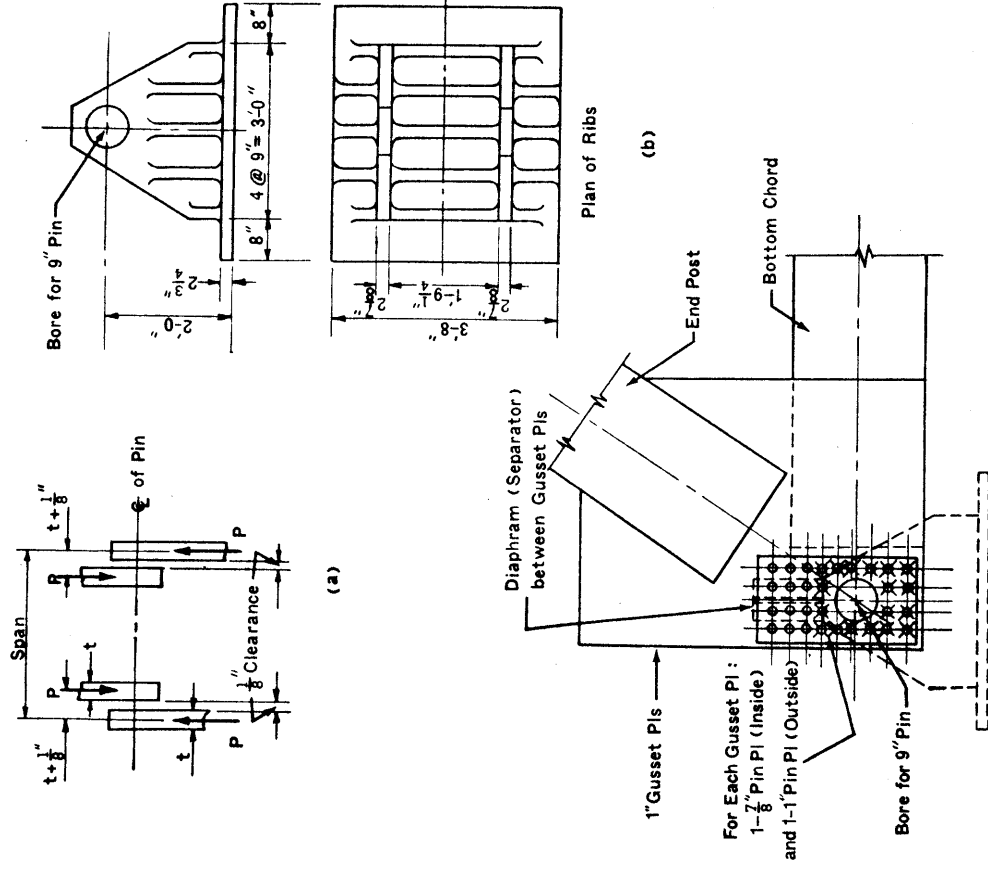
The shoe and pedestal may be of cast steel or of welded steel plate. The choice between the two depends on economy and on whether the applicable specification permits welding.

The bearings in parts (a) and (b) of Fig. 8-6 have vertical plates which completely surround the pin so that resistance to uplift is provided. The center plate of Fig. 8-6(b), however, does not completely encircle the pin, but contacts it on a surface which is slightly less than a half-cylinder. For a positive reaction, both the side plates and the center plates are effective; for a negative reaction, only the side plates are effective. The plates of Fig. 8-6(b) may be assumed to be loaded in proportion to their bearing areas on the pin.

Some specifications require that resistance to uplift be provided even though the design computations show that no uplift is expected. For such cases, details such as in Fig. 8-6(a) or (b) will be needed. If there is no requirement for resistance to uplift, a detail such as shown in Fig. 8-6(c) may be used, in which none of the plates surrounds the pin. In this type of detail, the upper and lower plates may be placed directly in line with each other, thus eliminating all bending moment and shear from the pin and possibly reducing its size. A collar on the pin engages slots in the upper and lower diaphragms to resist lateral loads.

EXAMPLE 8-5. Design a fixed bearing similar to the type shown in Fig. 8-6(a) for the support of a two-track, through-truss railway bridge of 160-ft span. The bridge rests on concrete abutments.

Use the AREA specification. The maximum reactions on one bearing are: vertical, 1,200 kips; longitudinal, 119 kips; and lateral, 10 kips due to nosing plus 36 kips due to wind.



(c)

FIG. 8-7

The end gusset plates and attached reinforcing plates are used in place of an upper shoe. The gusset plate thickness is 1 in. per plate, and the plates are spaced 18 in. center-to-center.

In Fig. 8-7(a) are shown the vertical loads acting on the pin. The pin is assumed to act as a simple beam supported at its ends by the vertical plates of the

shoe and loaded between by the gusset plates and any attached reinforcing plates. The span is unknown until the plate thickness  $t$  is determined; therefore, selection of the pin size will be by successive approximation.

The allowable unit stresses for the pin are as follows: shear, 13,500 psi; bearing on structural steel or cast steel, 24,000 psi; flexural stress, 27,000 psi. The required area for shear resistance is

$$\frac{600}{13.5}, \text{ or } 44.5 \text{ sq in.}$$

A pin with a  $7\frac{3}{4}$ -in. or more diam has sufficient cross-sectional area. Assuming a  $7\frac{3}{4}$ -in. pin,

$$t \text{ required for bearing} = \frac{600}{24 \times 7.75} = 3.23 \text{ in. (use } 3\frac{1}{4} \text{ in.)}$$

$$M = 600 (3\frac{1}{4} + \frac{1}{8}) = 2,025 \text{ in.-kips.}$$

The resisting moment of a solid pin,

$$M_r = \frac{fI}{r} = \frac{f\pi d^3}{32}$$

For  $M$  of 2,025 in.-kips,

$$d \text{ required} = \sqrt[3]{\frac{32 \times 2,025}{27\pi}} = 9.15 \text{ in.}$$

Therefore, a  $7\frac{3}{4}$ -in. pin is not satisfactory.

If the pin size is increased,  $t$  and  $m$  decrease. Therefore, the next pin size investigated will be slightly less than 9.15 in. in diam. For a pin with a 9-in. diam, by computations like those made for the first assumed size,

$$t \text{ required} = 2.78 \text{ in. (use } 2\frac{7}{8} \text{ in.)}$$

$$m = 1,800 \text{ in.-kips,}$$

$$d \text{ required for flexure} = 8.79 \text{ in.}$$

A 9-in. pin is satisfactory. Reinforcing plates will be added, as shown by Fig. 8-7(c), to make a total thickness of  $2\frac{7}{8}$  in. in bearing at each gusset plate.

The bearing area required to prevent crushing of the masonry will be determined for vertical loads alone and checked for the effect of simultaneous vertical and horizontal loads. The allowable bearing pressure of steel on concrete is given by the specification as 600 psi.

The area required =  $1,200/0.6 = 2,000$  sq in.

Sufficient area can be provided by a 45-in. square base.

The height of the pin above the base depends on many detail requirements peculiar to a particular structure and not necessarily controlled by strength. Assuming that a practical height for this case is 24 in., the maximum masonry bearing pressure may be computed. If the entire base is in compression,

$$f_{\max.} = \frac{P}{A} + \frac{Mc}{I} + \frac{M'c'}{I'}$$

in which  $M$ , moment of the longitudinal load about the base,  $= 24 \times 119 = 2,856$  in.-kips;  $M'$ , moment of the lateral load about the base,  $= 24 \times 46 = 1,104$  in.-kips; and  $I/c$  and  $I'/c'$  are values of section modulus of the base about the transverse and the longitudinal centerlines, respectively.

Solving for the maximum bearing pressure,

$$\begin{aligned} f_{\max} &= \frac{1,200}{2,025} + \frac{2,856}{15,200} + \frac{1,104}{15,200} \\ &= 0.592 + 0.188 + 0.073 = 0.853 \text{ kips per sq in.} \\ f_{\min} &= 0.592 - 0.188 - 0.073 = 0.331 \text{ kips per sq in.} \end{aligned}$$

Compression exists over entire area.

The AREA specification allows stresses caused by combinations of load, including longitudinal and lateral loads due to nosing or wind, to be 25 per cent greater than those permitted for other combinations of loading. The allowable bearing pressure for the loads being investigated is, therefore,  $1.25 \times 600$  or 750 psi. Since the maximum pressure computed above exceeds this allowable, a different base size must be used.

Similar computations for a base 52 in. long by 44 in. wide show the maximum bearing stress to be 733 psi and the minimum 313 psi. A  $52 \times 44$  in. base size is satisfactory.

The base thickness must be sufficient to resist bending. Transverse ribs are located so as to divide the base into narrow strips having small bending moments. For the rib arrangement shown in Fig. 8-7(b), the largest bending moment in the base occurs in the cantilevered edge strip. The critical section may reasonably be assumed to be at the face of the rib. (For other arrangements of ribs, an inside section of the base might be more highly stressed in bending than the cantilevered edge strip.)

Since the pressure resulting from combined vertical, lateral, and longitudinal loads exceeds that resulting from vertical loads alone by more than 25 per cent, it will be used in determining the required base thickness. The allowable unit stress for cast steel in flexure is three-quarters of that for rolled structural steel, and it is increased by 25 per cent when the loads include lateral and longitudinal loads.

A conservatively high value of the bending moment at the critical section can be computed by assuming a uniform pressure of 733 psi over the entire cantilevered strip. (If greater accuracy is desired, account of the pressure variation may be taken, as was done in Ex. 8-3.) Thus,

$$M = \frac{733 \times 8^2}{2} = 23,500 \text{ in.-lb per strip of 1-in. width,}$$

$$t \text{ required} = \sqrt{\frac{6 \times 23,500}{0.75 \times 18,000 \times 1.25}} = 2.89 \text{ in.}$$



The pressure used is conservatively high; therefore, a thickness of  $2\frac{3}{4}$  in. may be considered satisfactory.

Anchor bolts are required by the specification to be at least  $1\frac{1}{4}$  in. in diam. Since no uplift load is to be considered in this bearing, the selection of size and number of anchor bolts is based on the specification requirements and detail dimension requirements of the base. The lateral and longitudinal loads do not occur without simultaneous vertical loads; thus, friction may be depended on to transfer horizontal load to the footing. The anchor bolts, through their initial tension, aid in increasing the frictional resistance.

By using arrangements of ribs other than that used in this example, it is possible to design similar bases, some of which may be more economical than the one illustrated. A complete design should include a comparison, for economy, of several designs.

**8-5. Expansion Bearings.** Expansion bearings serve four purposes, as follows:

- (1) To permit free thermal expansion of the structure.
- (2) To permit free movement of the end of the structure as a result of length change caused by live load.
- (3) To permit shifting of abutments without causing additional stress in the structure.
- (4) To prevent the transfer of longitudinal loads to certain parts of the substructure.

Several types of expansion bearing used for bridges are illustrated in Fig. 8-8. For short spans, the simple sliding bearing shown in Fig. 8-8(a) is used. The sole plate is connected to the beam by welds or countersunk rivets. Uplift resistance is provided by anchor bolts which go through slotted holes in the sole plate and round holes in the base plate. During movement the sole plate slides against the base plate. The washer beneath the anchor-bolt nut also slides on the top plate. The plates may be lubricated to reduce the frictional force. To prevent roughening of the sliding surface by corrosion, the plates may be made of bronze. However, the amount of static friction to be overcome is not definite and cannot be well controlled. Also, dirt and corrosion may prevent free movement of the bearing, causing overstress or serious damage to either the metal structure or the supporting masonry. Thus for spans of greater length than about 60 or 80 ft, as limited by the specifications, one of the other types of bearing illustrated is used.

Fig. 8-8(b) shows a rocker-type bearing. The parts of this bearing are usually made of cast steel, although welded steel plates may be used if permitted by the specification. Longitudinal motion is permitted as the bearing rolls on the lower curved surface of the rocker. At the same time the washer of the anchor-bolt nut slides on the upper curved surface, or the anchor bolt bends slightly to follow the movement.

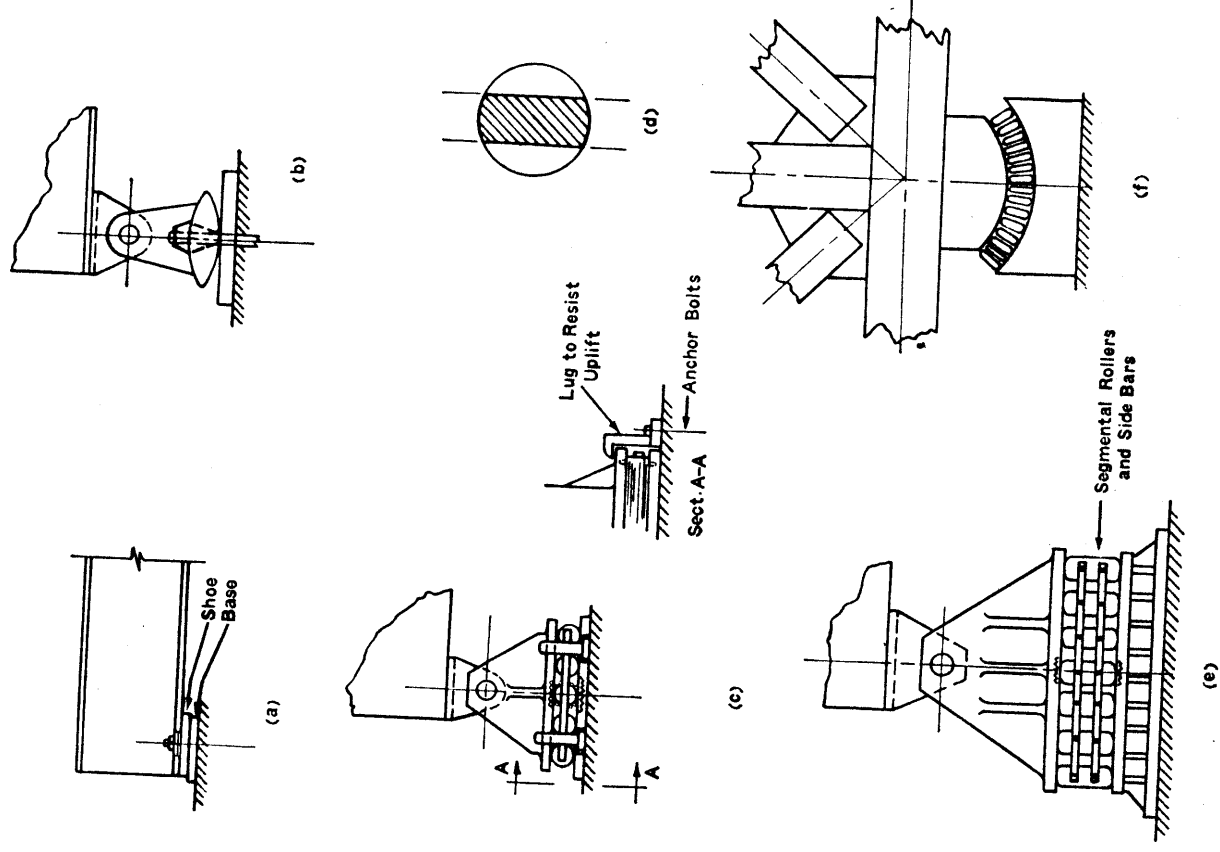


FIG. 8-8

Fig. 8-8(c) shows a roller bearing. The side bars connecting the rollers insure a constant and predetermined spacing between them. One or more of the rollers has teeth which engage the moving upper plate and the fixed lower plate so as to resist lateral forces and to prevent the nest of rollers from creeping or shifting in position. Uplift can be resisted in this type of bearing by lugs which are anchored to the masonry and have projections extending over the upper plate. When new, this type of bearing provides for very easy longitudinal motion, but, because small rollers may be difficult to clean, it may quickly become less effective. The allowable bearing stress per inch of roller length varies directly with the roller diameter. Since the roller diameter is necessarily small, a large number of rollers may be required. For larger loads the detail may be impractical or too expensive.

To reduce the number of rollers, the roller diameter could be increased, but to do so would not appreciably change the overall size of the upper and lower plates. However, since the amount of movement is not large, only small portions of the surface of rollers of larger diameter are effective as bearing surfaces. If rollers of larger diameter are used, their sides may be trimmed, as shown in Fig. 8-8(d); and the remaining portions (shaded) may be placed closer together to give a more economical bearing, as shown in Fig. 8-8(e). This segmental-roller bearing has the advantage of higher allowable bearing stress resulting from the use of large diameter rollers and the advantage of more compactness than is possible with cylindrical rollers.

Resistance to uplift, lateral forces, and creeping of the roller nest is provided in the same manner for the segmental roller as for the roller bearing of Fig. 8-8(c). Two side bars are required at each end of the segmental rollers to keep them parallel to each other and properly spaced.

The segmental roller may be used for a fixed bearing by making the base curved, as shown by Fig. 8-8(f). Such a detail might be useful where the pin sizes or plate thicknesses of details, such as those of Fig. 8-6, are impractical because of large reactions. The well-known cantilever railway bridge over the Ohio River at Beaver, Pennsylvania, has this type of bearing at the tower bases.

**8-6. Design of Expansion Bearings.** The first consideration in the design of any expansion bearing is the amount of movement required. For short spans, the movement required for elastic deformation is negligible. For larger trussed structures it may be determined by computation of the change of length of the bottom chord due to stress.

The allowance for thermal expansion is often limited by the specification. For example, the AREA specification requires provisions for

a change of span length, resulting from temperature change, of 1 in. per 100 ft of span.

The 1946 *Specifications for Design of Highway Structures*, of the Ohio State Highway Department, controls in greater detail the provision for thermal expansion, as follows:

86. . . . . For steel beam structures (of not more than 150 feet overall length) with timber substructures, no expansion devices need be provided; for bridges of this type with overall length greater than 150 feet, provision shall be made for a change of length (in each direction) of  $\frac{1}{8}$  inch for each 30 feet of length.

87. For other superstructures which are free to become longer or shorter with changes of temperature, such as the usual statically-determinate slab, beam, girder or truss which is partially supported on rockers, rollers, sliding plates, flexible columns or flexible piers, of either single or multiple span, provision shall be made for expansion and contraction due to the following temperature variations, in degrees Fahrenheit:

	Rise	Fall
Reinforced concrete superstructure . . . . .	40	75
Structural steel with concrete floor . . . . .	45	85
Structural steel with metal floor . . . . .	50	95

Elsewhere in the specification just quoted is the requirement that to allow for slight tilting or sliding of masonry abutments (except on rock or shale) after the backfill has been placed, there shall be provided, between superstructure and abutment, an opening (in addition to that required for elongation of superstructure) of at least  $\frac{1}{2}$  inch and preferably  $\frac{1}{4}$  inch for each foot of height (measured from bottom of footing to roadway surface) for each abutment.

Before deformation of the bed or base plate and of the roller or rocker, there is only a line of contact between the plate and the rocker. The allowable stress in bearing between the roller and plate is given in pounds per linear inch of contact.

Experiments at the University of Illinois show that the allowable bearing stress is affected by the roller diameter and by the tensile yield points of the roller and plate materials. The AREA specification considers both of these variables in its formulas. Most other specifications show the diameter as the only variable, although the constant of any one formula includes the effect of the yield point for the materials covered by that specification.

Using the allowable bearing values, satisfactory combinations of diameter and length of rollers may be determined. From these combinations, a choice between rollers and rockers may be made.

In a rocker or segmental-roller design, the next step is to determine the dimensions necessary to allow the required movement. For a rocker

of known radius, the degrees of arc for the upper and lower surfaces to permit both full expansion and full contraction must be computed. For a segmental roller, the width of the segments must provide enough arc length on the upper and lower surfaces to permit both full expansion and full contraction. The spacing between the segments must be enough so that clearance between the segments exists even in the extreme expanded or contracted positions. Since no factor of safety exists in the specified amounts of movement to be provided for, it is the practice to provide a capacity for some movement beyond the specified amount. For example a segmental roller is not designed so that under the full specified movement the segment is in bearing along a line at the edge of the curved surface.

**EXAMPLE 8-6.** Select a possible size and length of segmental rollers in an expansion bearing for the truss of Ex. 8-5.

The bearing stress allowed by the AREA specification is given, for rollers up to 25-in. diam, by the expression

$$\frac{p - 13,000}{20,000} \times 600d \text{ (pounds per linear inch),}$$

in which  $d$  = roller diameter in inches, and  $p$  = tensile yield point of the steel in the roller or in the base, whichever is smaller.

Elsewhere in the same specification is the requirement that forged steel be used for the rollers and that it must have a yield point of not less than 33,000 psi. Substituting 33,000 for  $p$ , the expression reduces to

$$f_{brz} \text{ allowable} = 600d,$$

which is the equation given by the AISC and other specifications.

In the selection of a roller size and length, a practical diameter is assumed, the allowable bearing stress per inch computed, and the required length for that size bearing computed. The process is repeated until a satisfactory or economical combination of diameter and length is determined.

Assuming a 6-in. roller diam,

$$f_{brz} = 600 \times 6 = 3,600 \text{ lb per in.},$$

$$L \text{ required} = 1,200 \div 3.6 = 333 \text{ in.}$$

Ten rollers, 6 in. in diam by 33 in. long, would provide the necessary bearing strength. Similarly computed, ten rollers of 8-in. diam and 25-in. length, or eight rollers of 10-in. diam and 25-in. length would satisfy the strength requirements.

Design of the shoe and base castings or assemblies is similar to that of Ex. 8-5, although it is not affected by longitudinal loads.

**8-7. Grillage Footings.** Fig. 8-9 shows a typical grillage footing. The grillage consists of two or more tiers of beams arranged so as to spread a

concentrated load over a large area, thus preventing excessive bearing pressures against the supporting soil. A permanent grillage is usually filled with and encased in concrete. At one time grillage footings were extensively used. Now, however, footings of reinforced concrete are made, which are equally satisfactory and less expensive. Although many grillage footings still exist, new ones are seldom constructed except for underpinnings or for temporary footings.

In the design of a grillage footing, four types of possible failure are considered. They are the failure of the soil by bearing pressure, and shear, crippling, and flexural failures of the beams. Vertical buckling of the web is not considered, since the web is laterally supported by the encasing concrete and by the separators and bolts which space the beams.

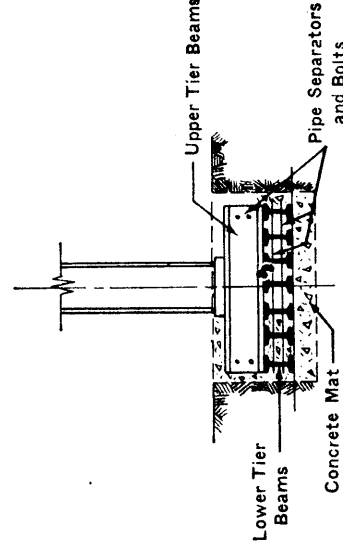


FIG. 8-9

Because the beams are of short length, shear often controls the selection of the section.

In design, the soil pressure is considered to be distributed uniformly over the base. The effective base area is assumed to extend beyond the edges of the beams for a distance equal to about one-half of the flange width, provided the concrete mat extends that far. Thus, in the average case, the entire area of the concrete mat may be used as the effective base area. In computing the bearing pressure on the soil, the entire load (column load plus the footing weight) is considered. In computing the shear and bending moment in the beams, it is assumed that the pressure against the beam is distributed uniformly along the length of the beam. The weight of the encasing concrete does not load the beams; therefore, the column load only is used in designing the beams. (For greater accuracy, the weight of the upper tier might be added to the load used in designing the lower tier beams.)

Detail requirements include a spacing between the beam flanges sufficient to permit the placing and tamping or vibrating of the concrete. A



2-in. space is usually sufficient, although this varies with the size of aggregate to be used. To allow bond of the concrete to the steel, the beams are left unpainted. On beams of not more than 8 in. in depth, a single row of pipe spacers and bolts is used. On deeper beams two rows are used. The spacers are placed at intervals of 5 ft or less.

**EXAMPLE 8-7.** Design a grillage footing to support a column load of 400 kips on a soil having an allowable bearing pressure of 4,000 lb per sq ft. The column has a base plate measuring 24 in. square. Use the AISC specification.

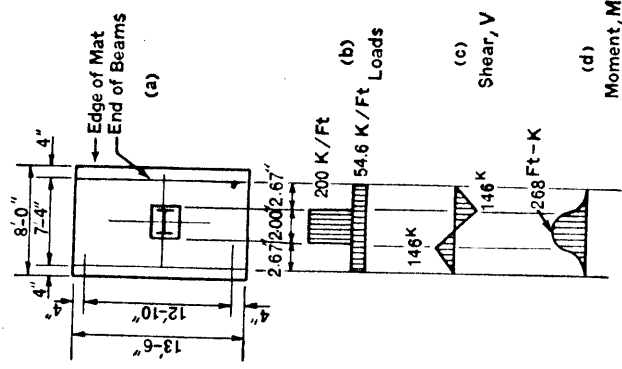


Fig. 8-10

The weight of the footing will be assumed at 30 kips, making the total load 430 kips. The soil bearing area required  $= 430/4 = 107.5$  sq ft.

Try a footing size of 8-0  $\times$  13-6. A footing having these dimensions is shown by Fig. 8-10. Assuming an encasement of concrete of 4-in. thickness, the lengths of the beams are 7 ft 4 in. for the top tier and 12 ft 10 in. for the lower tier. The upper tier beams will be selected first.

The loads on the upper tier are shown in Fig. 8-10(b). From these loads the shear and moment values are computed. Fig. 8-10(c) and (d) shows, respectively, the shear and bending moment diagrams.

$S$  required for the entire tier  $= (268 \times 12,000)/20,000 = 161$  in.<sup>3</sup>

The flexural strength needed can be provided by three beams, 16 WF 36 ( $S = 3 \times 56.3$ ). Checking these beams for shear and web crippling,

$$v = \frac{146,000}{3 \times 15.85 \times 0.299} = 10,270 \text{ psi,}$$

$$f_{brg} = \frac{146,000}{3 \times 0.299 (24 + 1.88)} = 6,270 \text{ psi.}$$

Since neither of the above stresses exceeds the allowable values, the use of three 16 WF 36 sections is satisfactory. As the flange width of one of these beams is 7 in., it will be necessary to extend the flange of the outer beams  $\frac{1}{2}$  in. beyond the edge of the column base plate in order to maintain a clear spacing of 2 in. between the flanges.

By similar computations, the values obtained for the lower tier are

$$V = 169 \text{ kips; } M = 542 \text{ ft-kips; required } S = 325 \text{ in.}^3$$

Ten 12 WF 27 sections will satisfy the flexural requirements. For these sections,

$$v = \frac{169,000}{10 \times 11.95 \times 0.24} = 5,900 \text{ psi,}$$

which is less than the allowable value.

Web crippling is checked by using the width of the flange of an upper tier beam as the length in bearing against the lower tier beams. Since the flange of the upper beam is not rigid, advantage is not taken of the  $k$  distance when checking the lower sections. Thus computed,

$$f_{brg} = \frac{400,000}{6.99 \times 30 \times 0.24} = 7,950 \text{ psi,}$$

which is less than the allowable of 24,000 psi. (The number 30 is the number of intersections of upper and lower beams.)

The footing will consist of an upper tier of three 16 WF 36 beams and a lower tier of ten 12 WF 27 beams. The entire footing will rest on a concrete mat from 6 to 9 in. thick, according to the specification requirements. If used for a permanent footing, the grillage will be encased in concrete.

## PROBLEMS

8-1. Design bearing plates for beams as follows:

(a) For a 16 WF 36 with an end reaction of 25 kips bearing on a brick wall. Use the AISC specification.

(b) For a 30 WF 116 with an end reaction of 100 kips bearing on a concrete abutment. Use the AREA specification.

8-2. Select base plates for columns as follows:

(a) For an 8 WF 31 having an axial load of 92 kips and supported by a concrete footing. Use the AISC specification.

(b) Same as above, but for a 14 WF 87 having an axial load of 400 kips.