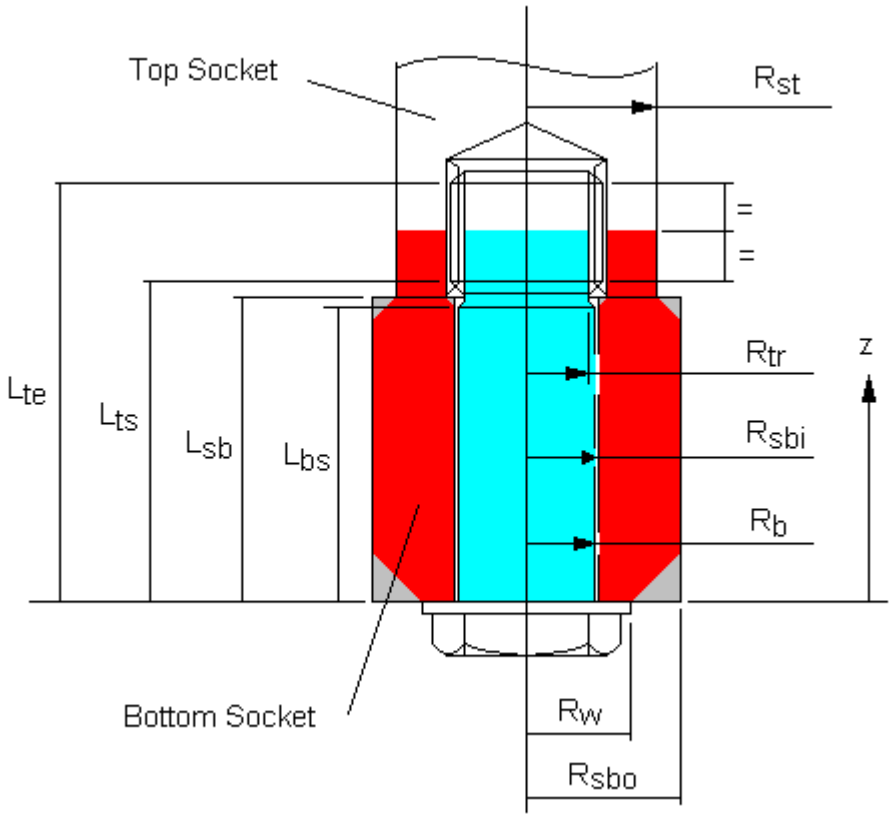
Preloaded Socket and Bolt Joint Under Applied External Loading



Input Data

Moduli ... $E_b := 210 \cdot GPa$ ... for bolt

$E_{sb} := 200 \cdot GPa$ ... for bottom socket

$E_{st} := 190 \cdot GPa$ ... for top socket

Lengths ... $L_{sb} := 100 \cdot mm$ ... bottom socket length

$L_{bs} := 97 \cdot mm$ ... bolt shank length

$L_{ts} := 103 \cdot mm$ ... length to start of thread

$L_{te} := 180 \cdot mm$ ... length to end of thread

Radii ... $R_b := 25\cdot mm$ ... bolt shank radius

$R_{tr} := 22.5\cdot mm$ ... bolt undercut radius

$R_{sbo} := 53\cdot mm$ ... bottom socket outer radius

$R_{sbi} := 25.5\cdot mm$ ... bottom socket inner radius

$R_{st} := 45\cdot mm$ ... top socket outer radius

$R_w := 40\cdot mm$ ... washer radius

$R_{br} := 22.9\cdot mm$ ... bolt male thread root radius, note $R_{br} > R_{tr}$

$R_{str} := 24.5\cdot mm$ ... top socket female thread root radius

Moment ... $M_{apl} := 11.50\cdot kN\cdot m$ ... applied BM at interface between top and bottom sockets

Load ... $P_{apl} := 0\cdot kN$ ... applied axial load

$P_{pl} := 600\cdot kN$ ... joint preload

Socket Inner Radius Profiling

$z_{si1} := L_{sb} - \left(\max\left(R_{str}, R_{sbi}\right) - \min\left(R_{str}, R_{sbi}\right)\right)$ $z_{si1} = 99.00\,mm$

$z_{si2} := 0.5\cdot\left(L_{ts} + L_{te}\right)$ $z_{si2} = 141.50\,mm$

$$Skt\_In := \begin{pmatrix} 0\cdot mm & R_{sbi} \\ z_{si1} & R_{sbi} \\ L_{sb} & R_{str} \\ z_{si2} & R_{str} \end{pmatrix}$$

$$Skt\_In = \begin{array}{c} \begin{matrix} z & r\ (mm) \end{matrix} \\ \begin{pmatrix} 0 & 25.5 \\ 99 & 25.5 \\ 100 & 24.5 \\ 141.5 & 24.5 \end{pmatrix} \end{array} mm$$

... interpolating socket inner radius

$r_{si}(z) := \text{linterp}\left(Skt\_In^{\langle 0\rangle}, Skt\_In^{\langle 1\rangle}, z\right)$ ... socket inner radius function

Socket Outer Radius Profiling

$z_{so1} := R_{sbo} - R_w$ $z_{so1} = 13.00\,mm$

$z_{so2} := L_{sb} - (R_{sbo} - R_{st})$ $z_{so2} = 92.00\,mm$

$z_{so3} := 0.5\cdot(L_{ts} + L_{te})$ $z_{so3} = 141.50\,mm$

$$Skt\_Out := \begin{pmatrix} 0\cdot mm & R_w \\ z_{so1} & R_{sbo} \\ z_{so2} & R_{sbo} \\ L_{sb} & R_{st} \\ z_{so3} & R_{st} \end{pmatrix}$$

$$Skt\_Out = \begin{array}{c} \begin{matrix} z & r\ (mm) \end{matrix} \\ \begin{pmatrix} 0 & 40 \\ 13 & 53 \\ 92 & 53 \\ 100 & 45 \\ 141.5 & 45 \end{pmatrix} \end{array} mm$$

... interpolating socket outer radius

$r_{so}(z) := linterp(Skt\_Out^{\langle 0 \rangle}, Skt\_Out^{\langle 1 \rangle}, z)$ ... socket outer radius function

Bolt Radius Profiling

$z_{b1} := L_{bs} + (R_b - R_{tr})$ $z_{b1} = 99.50\,mm$

$z_{b2} := L_{ts} - (R_{br} - R_{tr})$ $z_{b2} = 102.6\,mm$

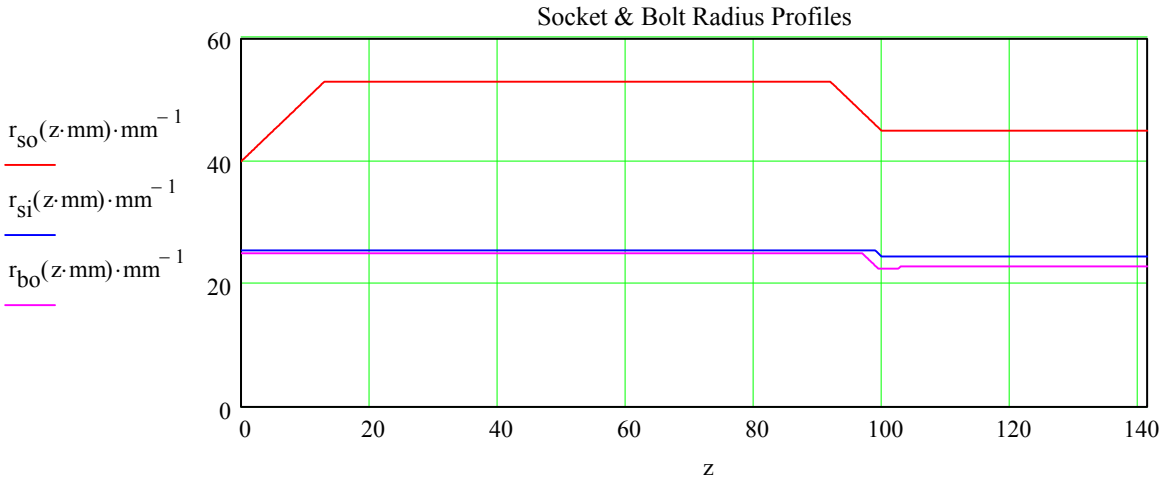
$z_{b3} := 0.5\cdot(L_{ts} + L_{te})$ $z_{b3} = 141.50\,mm$

$$Blt\_Out := \begin{pmatrix} 0\cdot mm & R_b \\ L_{bs} & R_b \\ z_{b1} & R_{tr} \\ z_{b2} & R_{tr} \\ L_{ts} & R_{br} \\ z_{b3} & R_{br} \end{pmatrix}$$

$$Blt\_Out = \begin{array}{c} \begin{matrix} z & r\ (mm) \end{matrix} \\ \begin{pmatrix} 0 & 25 \\ 97 & 25 \\ 99.5 & 22.5 \\ 102.6 & 22.5 \\ 103 & 22.9 \\ 141.5 & 22.9 \end{pmatrix} \end{array} mm$$

... interpolating bolt outer radius

$r_{bo}(z) := linterp(Blt\_Out^{\langle 0 \rangle}, Blt\_Out^{\langle 1 \rangle}, z)$ ... bolt outer radius function



Socket Modulus

$$E_s(z) := \begin{cases} E_{sb} & \text{if } z < L_{sb} \\ E_{st} & \text{otherwise} \end{cases}$$ ... bottom and top socket modulus function

Socket Spring Stiffness

$$k_s := \frac{1}{\pi} \cdot \int_0^{z_{so3}} \frac{1}{E_s(z) \cdot \left(r_{so}(z)^2 - r_{si}(z)^2 \right)} dz \qquad k_s = 1.288 \times 10^{-7} \frac{mm}{N}$$

Equivalent Socket Modulus

$$E_{se} := \frac{z_{so3}}{k_s \cdot \pi \cdot \left(R_{st}^{\ 2} - R_{str}^{\ 2} \right)} \qquad E_{se} = 245.36\,GPa$$

Bolt Spring Stiffness

$$k_b := \frac{1}{\pi \cdot E_b} \cdot \int_0^{z_{so3}} \frac{1}{r_{bo}(z)^2} dz \qquad k_b = 3.637 \times 10^{-7} \frac{mm}{N}$$

Equivalent Bolt Modulus

$$E_{be} := \frac{z_{so3}}{k_b \cdot \pi \cdot R_{tr}^{\,2}} \qquad E_{be} = 244.61\ \text{GPa}$$

Section Axial Strain and Stress

$$\varepsilon_a(x, \varepsilon_1, \varepsilon_2) := \varepsilon_1 + \frac{1}{2} \cdot (\varepsilon_2 - \varepsilon_1) \cdot \left(1 + \frac{x}{R_{st}}\right) \qquad \text{... function for change in axial strain}$$

$$\sigma_{ab}(x, \varepsilon_1, \varepsilon_2) := \begin{cases} E_{be} \cdot \varepsilon_a(x, \varepsilon_1, \varepsilon_2) + \dfrac{P_{pl}}{\pi \cdot R_{tr}^{\,2}} & \text{if } -R_{tr} \le x \le R_{tr} \\ 0 \cdot \text{MPa} & \text{otherwise} \end{cases} \qquad \text{... bolt axial stress}$$

$$\sigma_{as}(x, \varepsilon_1, \varepsilon_2) := \begin{cases} \sigma_{at} \leftarrow E_{se} \cdot \varepsilon_a(x, \varepsilon_1, \varepsilon_2) - \dfrac{P_{pl}}{\pi \cdot \left(R_{st}^{\,2} - R_{str}^{\,2}\right)} \\ \sigma_a \leftarrow \sigma_{at} \quad \text{if } \sigma_{at} < 0 \cdot \text{MPa} \\ \sigma_a \leftarrow 0 \cdot \text{MPa} \quad \text{otherwise} \\ \sigma_a \quad \text{if } -R_{st} \le x \le R_{st} \\ 0 \cdot \text{MPa} \quad \text{otherwise} \end{cases} \qquad \text{... socket axial stress}$$

Cross Section Profiles

$$y_b(x) := \text{Re}\left(\sqrt{R_{tr}^{\,2} - x^2}\right) \qquad \text{... bolt y axis profile function}$$

$$y_{so}(x) := \text{Re}\left(\sqrt{R_{st}^{\,2} - x^2}\right) \qquad \text{... socket outer y axis profile function}$$

$$y_{si}(x, \varepsilon_1, \varepsilon_2) := \begin{cases} \text{Re}\left(\sqrt{R_{st}^{\,2} - x^2}\right) & \text{if } \sigma_{as}(x, \varepsilon_1, \varepsilon_2) \ge 0 \cdot \text{MPa} \\ \text{Re}\left(\sqrt{R_{str}^{\,2} - x^2}\right) & \text{otherwise} \end{cases} \qquad \text{... socket inner y axis profile function}$$

Section Axial Forces

$$P_{ab}(\varepsilon_1,\varepsilon_2) := 2\cdot\int_{-R_{st}}^{R_{st}}\int_{0}^{y_b(x)} \sigma_{ab}(x,\varepsilon_1,\varepsilon_2)\,dy\,dx \qquad \text{... bolt axial force}$$

$$P_{as}(\varepsilon_1,\varepsilon_2) := 2\cdot\int_{-R_{st}}^{R_{st}}\int_{y_{si}(x,\varepsilon_1,\varepsilon_2)}^{y_{so}(x)} \sigma_{as}(x,\varepsilon_1,\varepsilon_2)\,dy\,dx \qquad \text{... socket axial force}$$

$$P_a(\varepsilon_1,\varepsilon_2) := P_{ab}(\varepsilon_1,\varepsilon_2) + P_{as}(\varepsilon_1,\varepsilon_2) \qquad \text{... total axial force}$$

Section Moments

$$M_b(\varepsilon_1,\varepsilon_2) := -2\cdot\int_{-R_{st}}^{R_{st}}\int_{0}^{y_b(x)} \sigma_{ab}(x,\varepsilon_1,\varepsilon_2)\cdot x\,dy\,dx \qquad \text{... bolt moment}$$

$$M_s(\varepsilon_1,\varepsilon_2) := -2\cdot\int_{-R_{st}}^{R_{st}}\int_{y_{si}(x,\varepsilon_1,\varepsilon_2)}^{y_{so}(x)} \sigma_{as}(x,\varepsilon_1,\varepsilon_2)\cdot x\,dy\,dx \qquad \text{... socket moment}$$

$$M_a(\varepsilon_1,\varepsilon_2) := M_b(\varepsilon_1,\varepsilon_2) + M_s(\varepsilon_1,\varepsilon_2) \qquad \text{... total moment}$$

Solving for Unknown Strains

Given $P_a(\varepsilon_1,\varepsilon_2) = P_{apl}$ ... equating to applied axial load

$M_a(\varepsilon_1,\varepsilon_2) = M_{apl}$ ... equating to applied BM

Giving ... $\varepsilon_1 = 6.868\times 10^{-4}$ ... and ... $\varepsilon_2 = -6.804\times 10^{-4}$

Checks ...

$P_{ab}(\varepsilon_1, \varepsilon_2) = 601253.7\,\text{N}$ ... bolt axial load

$P_{as}(\varepsilon_1, \varepsilon_2) = -601253.7\,\text{N}$ ... socket axial load

$P_a(\varepsilon_1, \varepsilon_2) = -0\,\text{N}$ ... total internal axial load, where ... $P_{apl} = 0\,\text{N}$

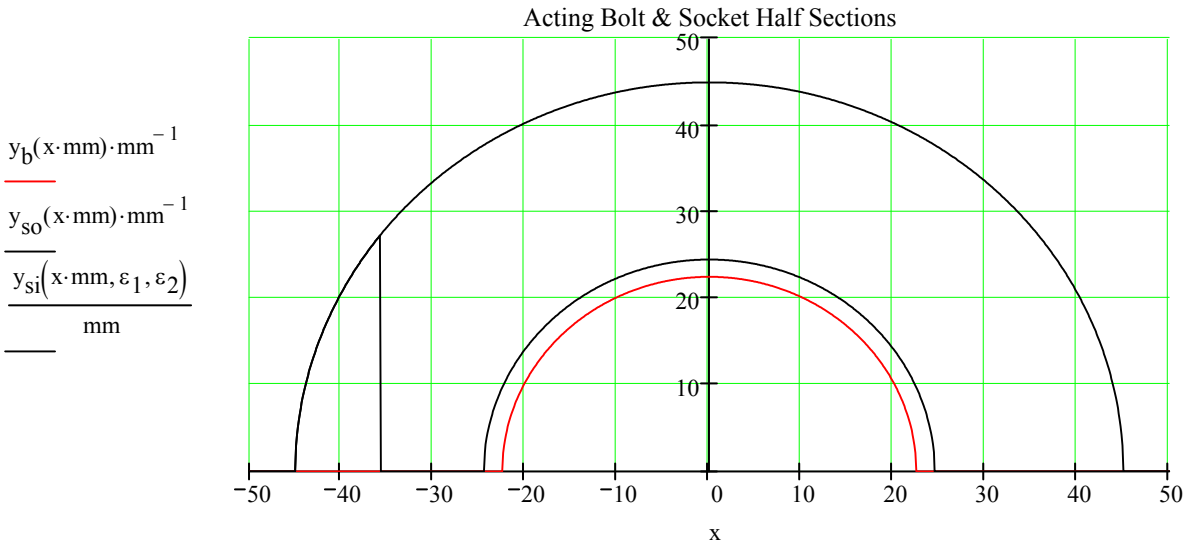
$M_b(\varepsilon_1, \varepsilon_2) = 747.9\,\text{N·m}$ ... bolt moment

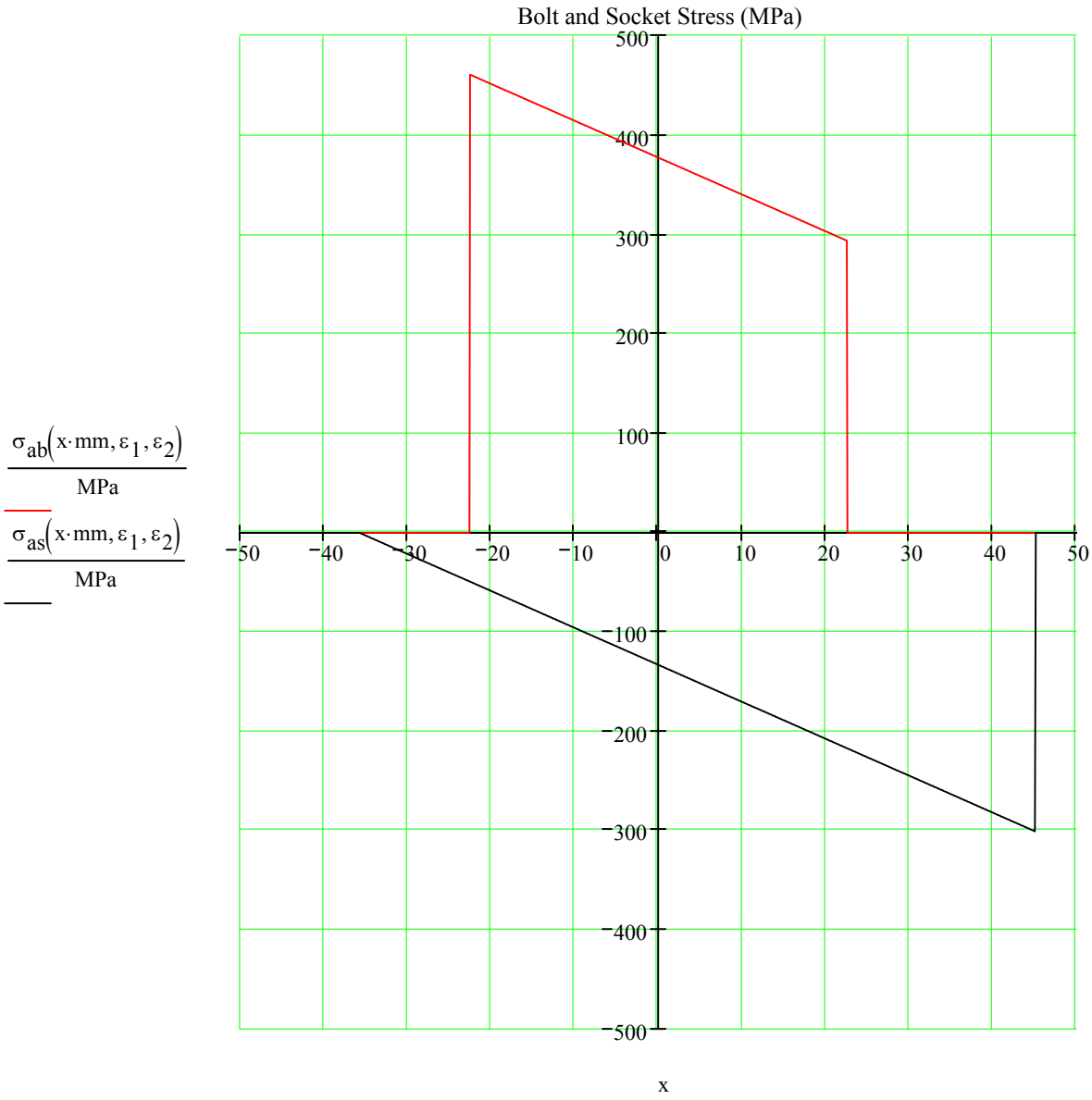
$M_s(\varepsilon_1, \varepsilon_2) = 10752.1\,\text{N·m}$ ... socket moment

$M_a(\varepsilon_1, \varepsilon_2) = 11500.0\,\text{N·m}$ ... total internal moment, where ... $M_{apl} = 11500\,\text{N·m}$

$\frac{M_b(\varepsilon_1, \varepsilon_2)}{M_{apl}} = 6.50\,\%$ ... percentage of BM seen by bolt

$\frac{M_a(\varepsilon_1, \varepsilon_2)}{M_{apl}} = 100.00\,\%$ ... internal BM to applied





Bolt stresses ... $\sigma_{ab}\left(-R_{tr}, \varepsilon_1, \varepsilon_2\right) = 461.6\,\text{MPa}$ ... thread undercut, LHS

$\sigma_{ab}\left(R_{tr}, \varepsilon_1, \varepsilon_2\right) = 294.4\,\text{MPa}$ ... thread undercut, RHS

Socket stress ... $\sigma_{as}\left(-R_{st}, \varepsilon_1, \varepsilon_2\right) = 0.0\,\text{MPa}$ ... top socket outer radius, LHS

$\sigma_{as}\left(R_{st}, \varepsilon_1, \varepsilon_2\right) = -301.0\,\text{MPa}$ ... top socket outer radius, RHS