TECHNICAL NOTE / NOTE TECHNIQUE

Deflection calculation and control for reinforced concrete flexural members

W. Zhou and T. Kokai

Abstract: A methodology was proposed to calculate the incremental deflection of reinforced concrete flexural members for the purpose of protecting nonstructural elements. The methodology is based on the long-term deflection calculation equation provided in the current Canadian standard, CSA A23.3–04, and takes into account realistic load application sequence during construction. Based on the proposed methodology, simple equations for calculating the incremental deflections were derived. The use of the proposed equations is demonstrated through a realistic design example. Implications of construction load and camber for deflection calculation and control are also discussed.

Key words: reinforced concrete, long-term deflection, incremental deflection, nonstructural element, construction load, camber.

Résumé : Une méthode est proposée afin de calculer la flexion incrémentielle d'éléments en flexion en béton armé dans le but de protéger les éléments non structuraux. La méthode est basée sur l'équation de calcul de la flexion à long terme fournie dans le Code canadien de calcul des ouvrages en béton CSA A23.3–04 et tient compte d'une séquence réaliste d'application de charge durant la construction. Des équations simples de calcul des flexions incrémentielles sont dérivées de cette méthodologie proposée. L'utilisation des équations proposées est démontrée dans un exemple de calcul réaliste. Les implications de la charge durant la construction et de la cambrure sur le calcul et le contrôle de la flexion sont également abordées.

Mots-clés : béton armé, flexion à long terme, flexion incrémentielle, élément non structural, charge pendant la construction, cambrure.

[Traduit par la Rédaction]

Introduction

In practice, deflection limits rather than strength requirements often govern the minimum sizes of reinforced concrete (RC) flexural members, such as beams and slabs. The main objective of deflection calculation and control is to ensure that deformations of structural members do not damage the nonstructural elements being supported and are acceptable to human perception. Deflection calculations are also used to specify camber, which is commonly used in building constructions to control the visual appearance of floor systems.

Because of concrete creep and shrinkage, the long-term deflection of a reinforced concrete member due to a sustained load is significantly larger than the corresponding immediate deflection (MacGregor and Bartlett 2000). According to the Canadian standard *Design of concrete structures*, CSA A23.3–04 (CSA 2004), the total deflection of an RC member subjected to a sustained load with a duration t is calculated using the following equation:

$$[1] \qquad \Delta_t = \left(1 + \frac{S_t}{1 + 50\rho'}\right)\Delta_\mathrm{i}$$

where Δ_t is the total deflection at the end of the duration of the load; Δ_i is the immediate deflection; ρ' is the compression reinforcement ratio; S_t is the creep deflection factor, and the subscript t is used to emphasize that the value of S_t (and hence Δ_t) depends on the duration of the sustained load. The right-hand side of eq. [1] consists of two components: the immediate deflection, Δ_i, and the long-term deflection, $\frac{S_t}{1+50\rho'}\Delta_\mathrm{i}$. The values of S_t corresponding to different durations of the load are given in Table 1 (CSA 2004), which shows that S_t increases with the duration of the load and reaches its maximum value of 2 for $t \geq 5$ years.

To protect nonstructural elements from being damaged by excessive deformation, the deflection that takes place after the installation of nonstructural elements should be limited. Such a deflection is referred to as the incremental deflection

Received 23 March 2009. Revision accepted 6 August 2009. Published on the NRC Research Press Web site at cjce.nrc.ca on 21 January 2010.

W. Zhou.[1] Department of Civil & Environmental Engineering, The University of Western Ontario, London, ON N6A 5B9, Canada.
T. Kokai. NORR Limited, 175 Bloor Street East, North Tower, 15th Floor, Toronto, ON M4W 3R8, Canada.

Written discussion of this technical note is welcomed and will be received by the Editor until 31 May 2010.

[1]Corresponding author (e-mail: wzhou@eng.uwo.ca).

Table 1. Values of the creep deflection factor S_t.

Duration of a sustained load	S_t
3 months	1.0
6 months	1.2
12 months	1.4
5 years or more	2.0

(CAC 2006). An incremental deflection calculation formula (referred to as the CAC formula) that takes into account the long-term load effects is suggested in the *Concrete design handbook* (CAC 2006) (see Section N9.8.2.5 of the "Explanatory notes on CSA A23.3–04"). However, the rationale for this formula is not given in the *Concrete design handbook*. Furthermore, the formula does not take into account the time of installation of nonstructural elements or the application sequence of the dead and live loads. Although it is suggested in the *Concrete design handbook* that different formulas be used to calculate the incremental deflection if the dead and live loads are applied at different times, no guidance is provided to the practitioners to derive the corresponding formulas.

The examples for deflection calculation are provided in the *Concrete design handbook* (Chapter 6 of CAC 2006). The approach used to calculate the incremental deflections in these examples is somewhat different from the CAC formula in that the installation time of nonstructural elements is taken into account. An implicit assumption in this approach is that the dead load and live load are applied simultaneously to the structure prior to the installation of nonstructural elements. However, this assumption is flawed because the self-weight of nonstructural elements is typically considered a dead load in the design, which implies that part of the dead load is applied at the time when nonstructural elements are installed. Furthermore, the live load is usually applied after the installation of nonstructural elements. It can then be inferred that this approach will underestimate the incremental deflection and lead to inadequate protection of nonstructural elements.

The objective of the study described in this technical note was to develop a methodology to calculate the incremental deflection of RC flexural members by taking account of realistic load application sequence during construction, and to provide a rational yet easy-to-use tool for design engineers to perform deflection calculations and checks.

Load categories

The following load categories were adopted:

(1) *Self-weight* of the structural members. The self-weight of the structural members is part of the total dead load and is a sustained load.

(2) *Superimposed dead load* is the self-weight of nonstructural elements, which may include mechanical and electrical services, ceiling, floor finish, partitions, and cladding. The superimposed dead load is also a sustained load and typically applied to the structure sometime after the application of the self-weight. The time interval between applications of the self-weight and the superimposed dead load depends to a large extent on the construction schedule. For typical office and residential buildings in Ontario, for example, cladding is installed about 7–8 weeks after the concrete floor is cast.

(3) Only the occupancy live load is considered in this study. For simplicity, the occupancy live load will be referred to as the *live load* throughout the paper. The live load is divided into the *transient live load* and *sustained live load*. The former is the "instantaneous" portion of the live load, and therefore has no long-term effects. The latter is the "permanent" part of the live load, and therefore will cause long-term deflection. Proportions of the sustained and transient live loads depend on the type and function of the structure and are also likely to vary with time. The sustained live load is often assumed to be between 20%–25% of the total live load for residential and office buildings in Canada.

Formulation

Let Δ_{iSW}, Δ_{iSD}, and Δ_{iLL} denote immediate deflections of an RC member due to the self-weight, the superimposed dead load, and the live load, respectively. The deflections, Δ_{iSW}, Δ_{iSD}, and Δ_{iLL}, can be calculated by using elastic deflection calculation methods and taking into account the impact of concrete cracking on member stiffness. Detailed calculation procedures can be found in many references (MacGregor and Bartlett 2000; CAC 2006) and are beyond the scope of this study. To calculate the incremental deflection, the following assumptions were adopted:

(1) Self-weight is applied to the structural member first, which is defined as the origin of the time for the long-term deflection calculation (i.e., T = 0).

(2) The superimposed dead load is applied at time $T = t_1$, $t_1 > 0$, after removal of the construction supporting assembly (i.e., shores and (or) re-shores). In other words, nonstructural elements are installed at time t_1, when the structural member is not subjected to the construction load, defined as the sum of all temporary loads (e.g., self-weights of formwork, fresh concrete and hardened concrete, and construction live load) applied during the construction.

(3) The live load is applied lastly at time $T = t_2$, $0 < t_1 < t_2$.

(4) The values of t_1 and t_2 are small compared with the time required (i.e., 5 years or more) for S_t to reach its maximum value.

Based on eq. [1] as well as assumptions (1) and (2), the total deflection just before the application of the superimposed dead load, Δ_{t_1}, can be calculated as follows:

$$[2] \qquad \Delta_{t_1} = \left(1 + \frac{S_{t_1}}{1 + 50\rho'}\right)\Delta_{\mathrm{iSW}} + \Delta_{\mathrm{c}}$$

where Δ_{c} is the permanent deflection due to the construction load. It is assumed that the structural member does not reach its ultimate strength under the construction load. Note that the calculation of Δ_{c} is involved as it requires consideration of the construction practice (e.g., a shored system versus a shore–reshore method), loading age of concrete, extent of concrete cracking at various construction stages, and member stiffness under repeated loading (Sbarounis 1984; Gardner 1990). However, Δ_{c} does not affect the incremental deflection because of assumption (2), and therefore does not need to be calculated.

Consider the total deflection at the time when the long-term deflections due to sustained loads have reached their maximum values. Let ∞ and Δ_∞ denote such a point in time and the corresponding total deflection, respectively. Note that ∞ in reality represents a reasonably long time, i.e., more than 5 years, after the application of the self-weight. Based on the aforementioned assumptions, Δ_∞ can be evaluated as follows:

$$[3] \quad \Delta_\infty = \left(1+\frac{S_\infty}{1+50\rho'}\right)\Delta_{\mathrm{iSW}} + \left(1+\frac{S_{\infty-t_1}}{1+50\rho'}\right)\Delta_{\mathrm{iSD}} + \left(1+\frac{S_{\infty-t_2}}{1+50\rho'}\right)\alpha\Delta_{\mathrm{iLL}} + (1-\alpha)\Delta_{\mathrm{iLL}} + \Delta_{\mathrm{c}}$$

where α is the percentage of the sustained live load in the total live load, and $S_{\infty-t_1}$ and $S_{\infty-t_2}$ are the creep deflection factors for the superimposed dead load (with a duration of $\infty - t_1$) and live load (with a duration of $\infty - t_2$), respectively.

Given assumption (4), that is, $\infty >> t_1$ and $\infty >> t_2$, $S_{\infty-t_1} = S_{\infty-t_2} = S_\infty$. Equation [3] can be simplified as follows:

$$[4] \quad \Delta_\infty = \left(1+\frac{S_\infty}{1+50\rho'}\right)(\Delta_{\mathrm{iSW}}+\Delta_{\mathrm{iSD}}+\alpha\Delta_{\mathrm{iLL}}) + (1-\alpha)\Delta_{\mathrm{iLL}} + \Delta_{\mathrm{c}}$$

The incremental deflection, $\delta\Delta(t_1)$, is then evaluated according to its definition:

$$[5] \quad \delta\Delta\,(t_1) = \Delta_\infty - \Delta_{t_1} = \left(\frac{S_\infty - S_{t_1}}{1+50\rho'}\right)\Delta_{\mathrm{iSW}} + \left(1+\frac{S_\infty}{1+50\rho'}\right)\Delta_{\mathrm{iSD}} + \left(1+\frac{\alpha S_\infty}{1+50\rho'}\right)\Delta_{\mathrm{iLL}}$$

The notation $\delta\Delta(t_1)$ is used to emphasize that the incremental deflection is a function of t_1, i.e., the time when the superimposed dead load is applied. Note that the incremental deflection is independent of t_2 as a result of assumptions (3) and (4). Note also that the incremental deflection is independent of Δ_{c} as a result of assumption (2).

In some cases, it is reasonable to assume that nonstructural elements are not likely to be affected by the immediate deflection due to the superimposed dead load. A typical example is masonry wall partition. Since it takes a certain amount of time for the mortar in a masonry wall to harden, the immediate deflection due to the superimposed dead load can be "absorbed" by the masonry wall, and therefore considered not detrimental to the partition. Given this, the incremental deflection is calculated as follows:

$$[6] \quad \delta\Delta\,(t_1) = \Delta_\infty - \Delta_{t_1} - \Delta_{\mathrm{iSD}} = \left(\frac{S_\infty - S_{t_1}}{1+50\rho'}\right)\Delta_{\mathrm{iSW}} + \left(\frac{S_\infty}{1+50\rho'}\right)\Delta_{\mathrm{iSD}} + \left(1+\frac{\alpha S_\infty}{1+50\rho'}\right)\Delta_{\mathrm{iLL}}$$

Design example

The incremental deflection of a RC flat slab needs to be limited to protect the cladding supported by the slab. Assume that the superimposed dead load (including the cladding load) is applied 1.5 months after the formwork is removed, i.e., t_1 = 1.5 months, and that 25% of the live load is the sustained live load, i.e., α = 0.25. Further assume that the cladding is susceptible to the immediate deflection due to the superimposed dead load. For simplicity, ignore compression reinforcement (ρ' = 0). From Table 1, the value of S_{t_1} is found to be 0.5 using interpolation; S_∞ equals 2.0. The incremental deflection, $\delta\Delta(t_1 = 1.5)$, is obtained by substituting S_{t_1}, S_∞, and α into eq. [5]:

$$[7] \quad \delta\Delta(t_1 = 1.5) = (2-0.5)\Delta_{\mathrm{iSW}} + 3\Delta_{\mathrm{iSD}} + (1+0.25\times 2)\Delta_{\mathrm{iLL}} = 1.5\Delta_{\mathrm{iSW}} + 3\Delta_{\mathrm{iSD}} + 1.5\Delta_{\mathrm{iLL}}$$

It is of interest to compare the incremental deflection calculated in eq. [7] with those calculated using the CAC formula and the approach illustrated in Chapter 6 of the *Concrete design handbook* (CAC 2006). Based on the CAC formula, the incremental deflection is calculated as follows:

$$[8] \quad \delta\Delta\,(t_1 = 1.5) = \left(\frac{S_\infty}{1+50\rho'}\right)(\Delta_{\mathrm{iSW}}+\Delta_{\mathrm{iSD}}+\alpha\Delta_{\mathrm{iLL}}) + (1-\alpha)\Delta_{\mathrm{iLL}} = 2\Delta_{\mathrm{iSW}} + 2\Delta_{\mathrm{iSD}} + 1.25\Delta_{\mathrm{iLL}}$$

Based on the approach in Chapter 6 of the *Concrete design handbook* (CAC 2006), the incremental deflection is calculated as follows:

$$[9] \quad \delta\Delta\,(t_1 = 1.5) = \left(\frac{S_\infty - S_{t_1}}{1+50\rho'}\right)(\Delta_{\mathrm{iSW}}+\Delta_{\mathrm{iSD}}+\alpha\Delta_{\mathrm{iLL}}) + (1-\alpha)\Delta_{\mathrm{iLL}} = 1.5\Delta_{\mathrm{iSW}} + 1.5\Delta_{\mathrm{iSD}} + 1.13\Delta_{\mathrm{iLL}}$$

Note that the incremental deflection given in eq. [8] is independent of t_1. A comparison between eq. [7] and eq. [8] indicates that the CAC formula can be either conservative or nonconservative for this example, depending on the relative magnitudes of Δ_{iSW}, Δ_{iSD}, and Δ_{iLL}. However, a comparison between eq. [7] and eq. [9] indicates that the approach illustrated in Chapter 6 of the *Concrete design handbook* (CAC 2006) underestimates the incremental deflection, as a direct result of the implicit assumption in the approach as pointed out in the Introduction.

Discussions

Camber of structural members

The purpose of upward cambering of structural members is to reduce the perceptible deflection (i.e., to mask the deflection) by changing the datum from which the deflection starts. It is important to note that camber does not reduce the total or incremental deflection of a structural member but only changes the appearance of the member. Therefore, camber cannot mitigate or reduce the deflection of a structural member. Given this, the authors disagree with the fourth note under Table 9.3 of CSA A23.3–04 (CSA 2004), which suggests that camber can be used to reduce the total (or incremental) deflection for the purpose of meeting the specified deflection limit.

Application sequence of the superimposed dead load

The incremental deflection calculation equations (i.e., eqs. [5] and [6]) are derived based on the assumption that the superimposed dead load is applied simultaneously. In practice, different components of the superimposed dead load are likely to be applied at different times. For example, the cladding system is usually installed first, followed by the installation of mechanical and electrical services and then the attachment of partitions and floor finishes. In this case, the incremental deflection equation for protecting the cladding will be different from that for protecting the partition. The derivation of these equations should account for the application sequence of the superimposed dead load, and can be easily carried out by design engineers using the methodology described in this note.

Effects of loading age

Implicit in eq. [2] is that the long-term deflection factor, S_t, is independent of the age of concrete at the time of load application (i.e., the loading age). For example, an RC member subject to a sustained load applied at the concrete age of 28 d will have the same Δ_∞ as that of the same member subject to the same sustained load applied at the concrete age of 90 d. However, it has been widely recognized that RC members loaded at older ages have significantly less creep deflections than those of same members loaded at younger ages (ACI Committee 209 1992). A loading age factor was proposed by ACI Committee 209 to take into account this beneficial effect. The proposed loading age factor decreases with the increase of the loading age and is less than or equal to 1.0. The loading age factor is then applied to the creep coefficient (under the "standard" condition), which is similar to S_t. However, this methodology is not recommended in CSA A23.3–04 (CSA 2004); therefore, the loading age effect is ignored in the formulation.

Immediate deflection calculation

The immediate deflections due to the self-weight, superimposed dead load, and live load should be calculated by taking into account the extent of concrete cracking that is commensurate with the corresponding load level. For example, the deflection due to the superimposed dead load should be calculated as the total deflection due to the combined self-weight and superimposed dead load minus the deflection due to the self-weight only. Furthermore, the impact of construction load should be considered. Previous research results (Sbarounis 1984; Liu and Chen 1987) have indicated that the maximum load on flat slabs in multistory RC buildings during construction may exceed twice the slab self-weight. In this case, the extent of concrete cracking caused by construction load has a significant impact on immediate deflections. Interested readers are referred to the examples provided in Chapter 6 of the *Concrete design handbook* (CAC 2006) for detailed procedures to calculate immediate deflections.

Conclusions

This technical note presents a methodology to calculate the incremental deflection, i.e., the deflection that takes place after the installation of nonstructural elements, for RC flexural members. Based on the long-term deflection calculation equation given in CSA A23.3–04 (CSA 2004), this methodology differentiates different load categories, such as the self-weight, superimposed dead load, and live load, and also takes into account the impact of load application sequence on the incremental deflection. The methodology provides design engineers with a rational and easy-to-use tool to perform deflection checks to protect nonstructural elements and can be easily adapted to deal with various scenarios in terms of load application sequence. A design example is provided to illustrate the use of this methodology. The impact of construction load on the incremental and immediate deflections is discussed in the note. It is also pointed out that camber can only mask the deflection but cannot reduce the total or incremental deflection.

Acknowledgement

Financial support provided to the first author by the Faculty of Engineering at the University of Western Ontario is acknowledged. The authors also wish to thank Professor F.M. Bartlett for reviewing the manuscript and the anonymous reviewers for providing comments that improved this note.

References

ACI Committee 209. 1992. Prediction of creep, shrinkage, and temperature effects in concrete structures. American Concrete Institute, Mich.

CAC. 2006. Concrete design handbook. Canadian Cement Association, Ottawa, Ont.

CSA. 2004. Design of concrete structures. CSA Standard A23.3–04. Canadian Standard Association, Mississauga, Ont.

Gardner, N.J. 1990. Design and construction interdependence. Concrete International, **12**(11): 32–38.

Liu, X.-L., and Chen, W.-F. 1987. Probability distribution of maximum wooden shore loads in multistory R.C. buildings. Structural Safety, **4**(3): 197–215. doi:10.1016/0167-4730(87)90013-0.

MacGregor, J.G., and Bartlett, F.M.B. 2000. Reinforced concrete — mechanics and design. First Canadian ed. Prentice Hall Canada Inc., Scarborough, Ont.

Sbarounis, J.A. 1984. Multistory flat plate buildings: effect of construction loads on long-term deflections. Concrete International, **6**(4): 62–70.