

Designing Compact Gussets with the Uniform Force Method

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In 1991 an AISC task group endorsed the uniform force method (UFM) as the preferred method for determining the forces that exist at gusset interfaces. Since that time it has been included in the *AISC Manual of Steel Construction*. The UFM provides a standardized way to obtain economical, statically admissible force distributions for vertical bracing connections. One criticism of the method is that it sometimes results in oddly shaped or disproportionately large gusset plates. To overcome this perceived limitation of the UFM, designers have been seeking out alternate methods.

This paper demonstrates that removing one unnecessary geometrical constraint from the formulation of the UFM will allow greater freedom in gusset geometry, while maintaining the efficiencies that result from the method. A new formulation of the UFM is presented, and the strengths and weaknesses of other proposed design methods are also explored.

THE UNIFORM FORCE METHOD

The uniform force method has been included in the *AISC Manual of Steel Construction* since 1992. The UFM was originally proposed by Thornton (1991) and was based on observations from Richard's (1986) research. In the commonly accepted form, the UFM produces the following force distribution:

$$H_b = \frac{\alpha}{r} P \quad (1a)$$

$$V_b = \frac{e_b}{r} P \quad (1b)$$

$$H_c = \frac{e_c}{r} P \quad (1c)$$

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$$V_c = \frac{\beta}{r} P \quad (1d)$$

where

$$r = \sqrt{(\alpha + e_c)^2 + (\beta + e_b)^2} \quad (2)$$

$$\alpha = \tan \theta (\beta + e_b) - e_c \quad (3)$$

In order to satisfy the relationship between α and β , the designer is often forced to use either an oddly shaped or disproportionately large gusset plate. Alternately, moments can be introduced at the connection interfaces. Neither approach is ideal.

ALTERNATIVES TO THE UNIFORM FORCE METHOD

Any viable alternative sought to replace the UFM should meet the following criteria: (1) it must provide a clear procedure to satisfy equilibrium and conform to the basic assumptions made during the analysis and design of the main members (the most important criterion); (2) since the UFM readily accommodates a wide range of geometries and boundary conditions, any alternate method should also be able to accommodate such situations; and (3) it must result in economical designs.

Several alternatives to the UFM have been proposed. Chief among the alternatives are the KISS Method, the parallel force method and the truss analogy method. None of these methods suffer from the constrictive relationship between α and β that exists in the UFM. In other words, these methods can be used with any gusset geometry and do not force the use of oddly shaped or large gusset plates. The strengths and weaknesses of these methods will be explored. In all of the discussions the work-point of the brace is assumed to be located at the intersection of the centerlines of the beam and the column, since this is the typical case.

The KISS Method

KISS (Figure 1) is an acronym for “keep it simple stupid,” and the method is simple, as the name implies, and fool-proof, though uneconomical. The method involves delivering the entire horizontal brace component directly to the beam through the beam-to-gusset connection and the entire vertical brace component directly to the column through the column-to-gusset connection. To satisfy equilibrium, moments must be introduced. At the beam-to-gusset the moment is equal to He_b , and at the column-to-gusset the moment is equal to $V_e c_c$.

The KISS Method satisfies two of the three criteria for a viable alternative to the UFM. It satisfies equilibrium and the design and analysis assumptions, and it is universally applicable to all geometries and boundary conditions. However, the presence of the large moments at the connection interfaces makes it an uneconomical choice in practice.

The Parallel Force Method

In the parallel force method, sometimes referred to as the component method (Figure 2), the reactions of the gusset at the beam and column interfaces are assumed to act parallel to the brace force. Since the forces are parallel, they obviously do not intersect at a common point, as is the case with the UFM. Therefore, in order to maintain rotational equilibrium, two choices are available. Either the magnitude of the parallel forces are set so that they balance each other about the work-line of the brace, or moments are added at the beam and/or column interfaces. The additional moments, though lesser in magnitude than the KISS method, adversely impact the economy of the connection.

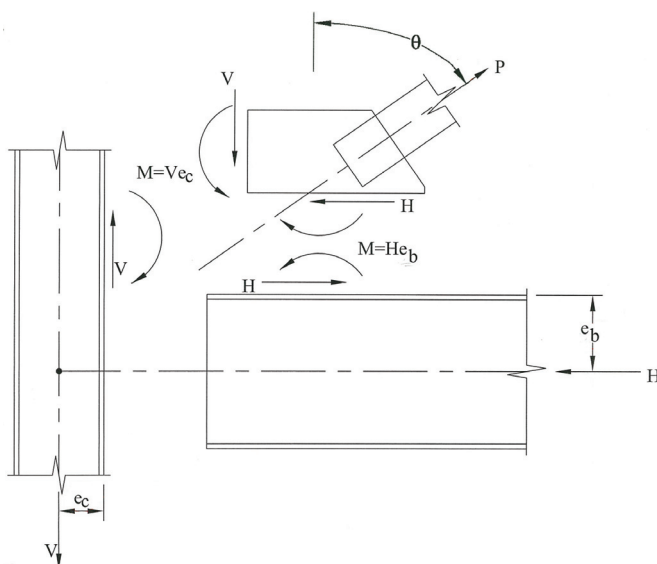


Fig. 1. KISS method.

If the former approach is taken, rotational equilibrium of the beam and column will not generally be satisfied. A moment connection will then have to be added between the beam and the column. Since this will normally be a field-welded connection, this is considered to be an uneconomical alternative in most parts of the country.

In terms of applicability to a variety of boundary conditions, the parallel force method suffers from a major shortcoming. Since the forces at both the beam-to-gusset and the column-to-gusset interfaces are assumed parallel to the brace force, a horizontal component will always exist at the column-to-gusset connection. When framing to a column web, this presents a significant design challenge, which will usually be overcome by the addition of column stiffening local to the connection, further reducing the economy of the method.

The parallel force method only satisfies one of the three criteria for a viable alternative to the UFM. It satisfies equilibrium and the design and analysis assumptions, but it is not as economical as the UFM and is not suited to connections made to column webs.

The Truss Analogy Method

The truss analogy method (Figure 3) determines the force distribution on the gusset by modeling the interface forces as a pinned “truss” node located at the center of the brace-to-gusset connection. The truss analogy method suffers the same problem as the parallel force method when attaching to column webs. Additionally, the truss analogy method can result in counterintuitive and uneconomical force distributions. This is illustrated in Figure 3 where the gusset-

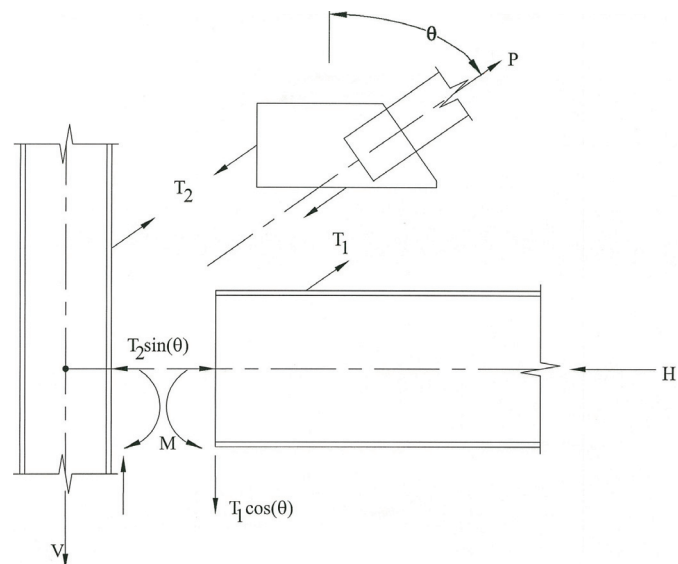


Fig. 2. Parallel force method.

to-column connection delivers only a horizontal component to the column. A formalized treatment of the equilibrium requirements for the beam and column has never been presented and is therefore left to the designer. Often moments are required at all of the connection interfaces in order to satisfy equilibrium.

The truss analogy method satisfies none of the criteria for a viable alternative to the UFM.

A GENERALIZED UFM

Since none of the alternatives investigated appear to provide better results than the UFM, it is advantageous to make adjustments to the formulation of the UFM to make it more applicable to compact gussets.

The goal of the UFM was to derive a procedure to obtain statically admissible force distributions, which would produce no moments at the connection interfaces and would be applicable to a wide range of geometries and boundary conditions. However, the procedure includes an additional constraint that unnecessarily limits its applicability. The force at the gusset-to-column interface, $\sqrt{V_c^2 + H_c^2}$, is forced to pass through a point that lies a distance, e_b , above the work-point.

Since there is a perceived problem with the UFM that can be overcome by removing this constraint, it is advantageous to eliminate it from the method. In order to do so, the problem must first be defined. There are essentially three elements involved: the beam, the column, and the gusset. The brace is neglected since it is assumed to carry only axial force and is not part of the indeterminate system. Each of the three members is subjected to three forces. In order for

moments to be eliminated from the interfaces the forces applied to each element must intersect at a single point. These points of intersection are referred to as control points.

The Beam

It is easiest to begin with the beam (Figure 4), since the location of its control point is evident. The three forces applied to the beam are the horizontal component of the brace, H , the beam-to-gusset force, $\sqrt{V_b^2 + H_b^2}$, and the beam-to-column force, $\sqrt{V_c^2 + H_c^2}$. The horizontal component of the brace is resisted along the centerline of the beam and intersects the beam-to-column force at the point $(e_c, 0)$. Therefore, the beam-to-gusset force must also pass through this point. From this we find that

$$\frac{V_b}{H_b} = \frac{e_b}{\alpha} \quad (4)$$

The Gusset

The three forces applied to the gusset (Figure 5) are the brace force, P , the beam-to-gusset force, $\sqrt{V_b^2 + H_b^2}$, and the gusset-to-column force, $\sqrt{V_c^2 + H_c^2}$. In order to eliminate moments at the interfaces, these three forces must intersect at a single point. Since the slope of the brace force, $1/\tan(\theta)$, and the slope of the beam-to-gusset force, e_b/α , are known, the intersection can be determined. The gusset control point is:

$$\left(\frac{e_b e_c \tan(\theta)}{e_b \tan(\theta) - \alpha}, \frac{e_b e_c}{e_b \tan(\theta) - \alpha} \right)$$

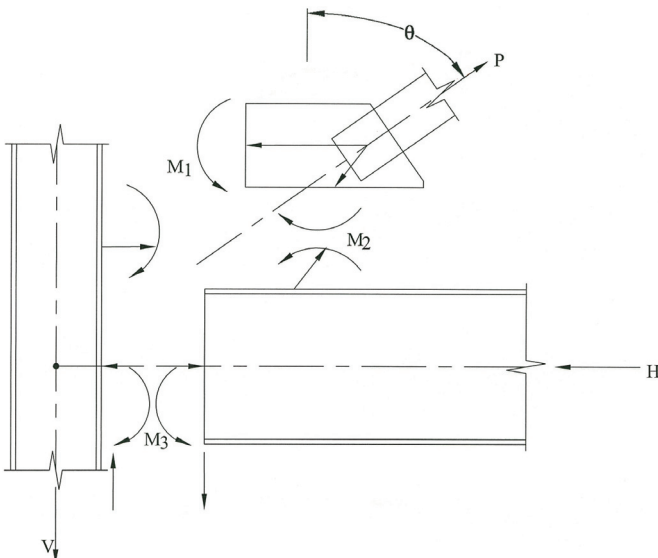


Fig. 3. Truss analogy method.

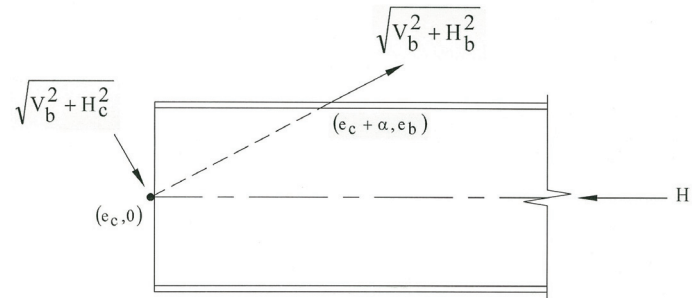


Fig. 4. Beam free body diagram.

The Column

The three forces applied to the column (Figure 6) are the vertical component of the brace, V , the column-to-gusset force, $\sqrt{V_c^2 + H_c^2}$, and the beam-to-column force, $\sqrt{V_b^2 + H_b^2}$. Knowing that the gusset-to-column force must pass through the gusset control point, the slope of the gusset-to-column force is:

$$\frac{V_c}{H_c} = \frac{\beta}{e_b} \left[\frac{e_b}{\beta} \left(1 - \frac{\tan(\theta)(e_b + \beta) - e_c}{\alpha} \right) + 1 \right] \quad (5)$$

From this, since the column-to-gusset force and the beam-to-column force must intersect at the centerline of the column, the slope of the beam-to-column force is:

$$\frac{V_b}{H_b} = \frac{e_b}{e_c} \left(\frac{\tan(\theta)(e_b + \beta) - e_c}{\alpha} \right) \quad (6)$$

The point of intersection of the column-to-gusset force and the beam-to-column force, the column control point, is:

$$\left(0, e_b \left(\frac{\tan(\theta)(e_b + \beta) - e_c}{\alpha} \right) \right)$$

Force Distribution

Having established the geometrical constraints required to eliminate moments at all connection interfaces, the forces at

the interfaces can be derived. Since the column must be in equilibrium, the following can be established:

$$\sum F_y = 0 = P \cos(\theta) - (V_b + V_c) \quad (7)$$

$$\sum F_x = 0 = H_c - H_b \quad (8)$$

$$\sum M = 0 = H_c(e_b + \beta) - P \cos(\theta)e_c \quad (9)$$

From this

$$H_c = \frac{\cos(\theta)e_c}{(e_b + \beta)} P \quad (10)$$

To satisfy the requisite geometry for the beam-to-gusset and beam-to-column forces, the following must be true:

$$V_b = \left[\frac{e_b(\sin(\theta)(e_b + \beta) - \cos(\theta)e_c)}{\alpha(e_b + \beta)} \right] P \quad (11)$$

The remaining forces are apparent:

$$H_b = P \sin(\theta) - H_c \quad (12)$$

$$V_c = P \cos(\theta) - V_b \quad (13)$$

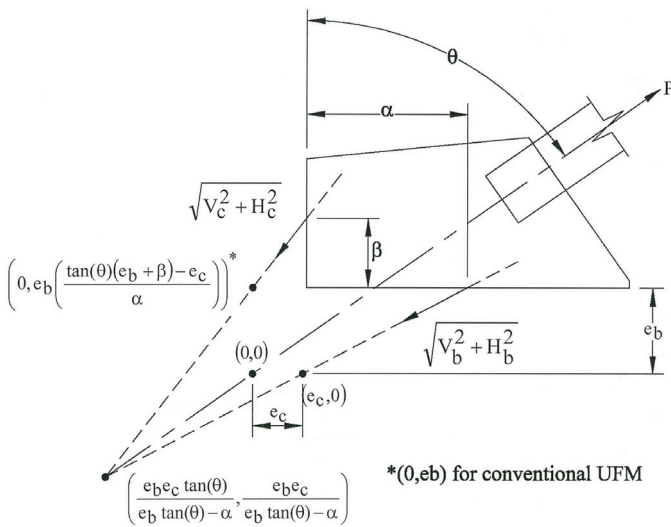


Fig. 5. Gusset free body diagram.

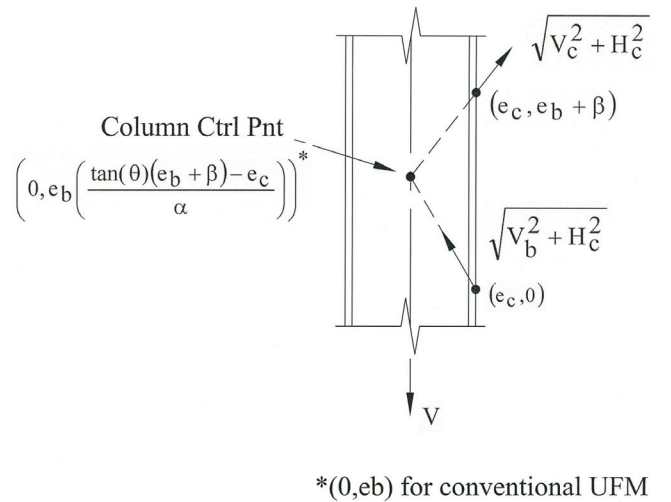


Fig. 6. Column free body diagram.

With the geometry and force distribution established, a new form of the UFM has been derived without the somewhat arbitrary constraint on the location of the column control point. Without this constraint, α and β can be set to any convenient values. This removes the need to consider the moments caused by $\bar{\alpha}$ and $\bar{\beta}$, where $\bar{\alpha}$ is the actual distance from the face of the column flange to the centroid of the gusset-to-beam connection, and $\bar{\beta}$ is the actual distance from the face of the beam flange to the centroid of the gusset-to-column connection.

However, there may still be a need to redistribute the vertical reaction delivered to the beam, V_b . This counteracting force is referred to as ΔV_b . ΔV_b can be introduced into this new formulation easily to produce the full spectrum of force distributions that can exist in the connection while maintaining column-to-gusset and beam-to-column connections free of moments. It is assumed that moments at the column-to-gusset and beam-to-column connections are uneconomical and therefore undesirable.

Of course the introduction of ΔV_b disrupts the established equilibrium and adjustments must be made. The adjustment involves introducing a moment at the beam-to-gusset interface. This moment can be calculated as:

$$M_b = H_b e_b - (V_b - \Delta V_b) \alpha \quad (14)$$

Column Moment

A moment gradient will exist in the column whether using the original formulation or the new formulation of the UFM presented in this paper. Using the original formulation, the moment will be zero at the intersection of the top of steel elevation and the centerline of the column. In the new formulation, the moment may be either positive or negative throughout the section of the column bounded by the connection or the moment may be zero at some section similar to the original formulation. In either case the maximum moment the column will be subjected to can be determined as:

$$M_c = \max \{ V_c e_c, (V_c e_c - H_c (e_b + \beta)) \} \quad (15)$$

Since the choice of column section will usually be governed by buckling and the column is restrained from buckling local to the brace connection, it is normal practice to neglect this moment. For this reason, the moment internal to the column is not mentioned in the AISC *Steel Construction Manual* (AISC, 2005) discussion of the UFM.

An Example

The forces on the connection shown in Figure 7 will be calculated to demonstrate the new formulation.

$$\alpha = \frac{27.75}{2} + 0.5 = 14.375 \text{ in.}$$

$$\beta = \frac{13}{2} = 6.5 \text{ in.}$$

$$H_c = \frac{\cos(55^\circ)(7)}{(12 + 6.5)} 100 = 21.7 \text{ kips}$$

$$V_b = \left[\frac{(12)(\sin(55^\circ)(12 + 6.5) - \cos(55^\circ)(7))}{(14.375)(12 + 6.5)} \right] (100) \\ = 50.3 \text{ kips}$$

$$H_b = 100 \sin(55^\circ) - 21.7 = 60.2 \text{ kips}$$

$$V_c = 100 \cos(55^\circ) - 50.3 = 7.06 \text{ kips}$$

Summing moments on the beam about the beam control point produces:

$$V_b \alpha - H_b e_b = 50.3(14.375) - 60.2(12) \approx 0 \text{ kip-in.}$$

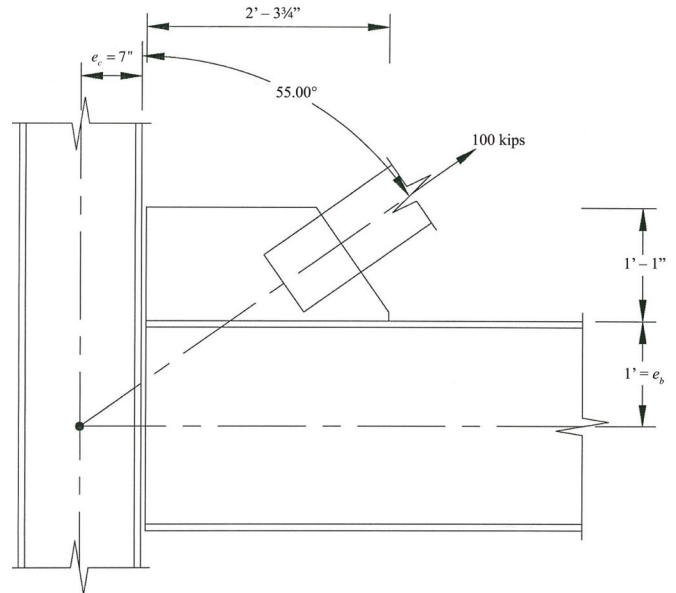


Fig. 7. Example.

Summing moments on the gusset about the work-point produces:

$$V_b(\alpha + e_c) - H_b e_b + V_c e_c - H_c(e_b + \beta) = 50.3(14.375 + 7) - 60.2(12) + 7.06(7) - 21.7(12 + 6.5) \approx 0 \text{ kip-in.}$$

Summing moments on the column about the beam-to-column connection produces:

$$P \cos(\theta) e_c - H_c(e_b + \beta) = 100 \cos(55^\circ)(7) - 21.7(12 + 6.5) \approx 0 \text{ kip-in.}$$

Note that:

$$\tan(\theta)(\beta + e_b) - e_c = \tan(55^\circ)(6.5 + 12) - 7 = 19.4 \neq \alpha$$

For completeness the vertical coordinate of the column control point can be calculated as:

$$y_{ccp} = e_b \left(\frac{\tan(\theta)(e_b + \beta) - e_c}{\alpha} \right) = (12) \left(\frac{\tan(55^\circ)(12 + 6.5) - 7}{14.375} \right) = 16.2 \text{ in.}$$

It may be noted that for this case the term V_b is significantly larger than would be obtained using the traditional UFM. As is the case with the traditional UFM, a ΔV_b can be introduced to manipulate the distribution of vertical force. Taking ΔV_b equal to 13.1 kips produces the same distribution of vertical force that is obtained from the UFM when all parameters except α are held constant.

As can be seen from Table 1, which presents a comparison of the traditional UFM to the modified UFM, each can be modified to produce identical results. This is to be expected since each must satisfy equilibrium. The primary advantage to the new formulation is that it eliminates the need for the modifiers $\bar{\alpha}$ and $\bar{\beta}$. Also the new formulation makes it easier to overcome the perceived limitations of the UFM.

Table 1. Comparison of the Traditional and the Modified Uniform Force Methods				
Parameters	Traditional UFM		Modified UFM	
	without $\bar{\alpha}$	with $\bar{\alpha}$	without ΔV_b	with ΔV_b
α	19.4	19.4	14.4	14.4
$\bar{\alpha}$	–	14.4	–	–
β	6.5	6.5	6.5	6.5
V_b	37.2	37.2	50.3	37.2
H_b	60.2	60.2	60.2	60.2
V_c	20.2	20.2	7.06	20.2
H_c	21.7	21.7	21.7	21.7
ΔV_b	–	–	–	13.1
M_b	–	188	–	188

OTHER PRACTICES THAT CAN REDUCE THE GUSSET PROFILE

Having eliminated the geometrical constraints on gusset size from the UFM, attention can be turned to other steps that can be taken to reduce the gusset profile.

The Whitmore Section

The Whitmore section is commonly accepted to be an area, which extends at a 30° angle from the edges of the brace-to-gusset connection along the length of the connection. The area beyond this section is assumed to be ineffective in terms of gross tension yielding and compression buckling of the gusset. It is common practice to try to include all of the allowed Whitmore section within the gusset, but it is not a requirement to do so. By allowing the edges of the gusset plate to encroach on the Whitmore section, the profile of the gusset can be reduced.

Weld Size

It is common practice to attempt to limit fillet weld sizes to those that can be applied in a single pass, usually $\frac{5}{16}$ in. This greatly enhances connection economy, since the number of passes required to complete a weld increases disproportionately with the leg size. To maintain a single pass weld, the gusset plate dimensions, particularly at the beam-to-gusset connection, are often increased. The gusset profile can be reduced by allowing multiple pass welds to be used, but only with increased fabrication costs.

Bolt Type

If reducing the gusset profile is of paramount concern, the strongest possible bolt configuration should be employed. Slip-critical connections should be avoided since they will require more bolts and therefore a larger gusset profile. Likewise, if the threads will be excluded from the shear plane, which is usually the case for heavily loaded bracing connections, then the “X-value” for the bolts should be used. Providing a detail that places the bolts in double shear at the brace-to-gusset connection also helps to reduce the gusset profile.

CONCLUSIONS

The UFM, as currently presented in the Manual, contains an unnecessary constraint on the location of the column control point. This constraint often gives designers the perception that the method is ill suited to the design of compact gusset plates.

By eliminating the unnecessary constraint in the new formulation, force distributions can be derived that consist of only shear and axial forces at the connection interfaces. The new formulation also simplifies the UFM by eliminating the need for $\bar{\alpha}$ and $\bar{\beta}$.

By manipulating the term ΔV_b , designers can obtain the full spectrum of force distributions that can exist in the connection while maintaining column-to-gusset and beam-to-column connections free of moments.

NOTATION

e_b	=	one-half the depth of the beam
e_c	=	one-half the depth of the column
y_{ccp}	=	vertical coordinate of the column control point
P	=	brace load
H	=	horizontal component of the brace load

H_b	=	shear force on the beam-to-gusset connection
H_c	=	axial force on the beam-to-column and gusset-to-column connections (assumes no transfer force)
M_b	=	moment on the beam-to-gusset connection
V	=	vertical component of the brace load
V_b	=	shear force on the beam-to-column connection and axial force on the beam-to-gusset connection
V_c	=	shear force on the gusset-to-column connection
ΔV_b	=	change in the distribution of vertical load
α	=	distance from the face of the column flange or web to the centroid of the gusset-to-beam connection
β	=	distance from the face of the beam flange to the centroid of the gusset-to-column connection
$\bar{\alpha}$	=	actual distance from the face of the column flange to the centroid of the gusset-to-beam connection (This term is not required in the new formulation.)
$\bar{\beta}$	=	actual distance from the face of the beam flange to the centroid of the gusset-to-column connection (This term is not required in the new formulation.)

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CLOSURE

Designing Compact Gussets with the Uniform Force Method

Paper by Larry S. Muir
(First Quarter, 2008)

Closure by Larry S. Muir

I would like to begin by thanking Mr. Arias for his interest in and comments regarding my paper. I believe that an open and vigorous discourse is the best way to advance our understanding and practice of engineering.

Mr. Arias addresses three separate issues, which I will try to restate:

1. I have found only one of a number of possible solutions to the problem.
2. I have applied arbitrary geometric constraints to the analysis, and my analysis does not reflect the behavior of the connection.
3. I present an alternative that uses ΔV_b to arbitrarily manipulate the distribution of vertical forces in the connection.

I agree with all three of Mr. Arias' points enumerated here. However, I disagree with the conclusions developed from these points. It is my understanding that Mr. Arias's main problem with the approach presented in my paper is that it is arbitrary and does not accurately reflect the true behavior of the connection. From this he concludes that the procedure may result in inadequate designs and that the traditional UFM more accurately reflects the behavior of the connection and therefore results in safer designs.

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I contend that no one—not Mr. Arias, not myself, not Dr. Thornton, the originator of the UFM—can accurately predict the behavior of any connection. That is why all connection design—and, in all likelihood, virtually all structural steel design—is accomplished based, either implicitly or explicitly, on the Lower Bound Theorem. The Lower Bound Theorem states that the applied external forces in equilibrium with the internal force field are less than or, at most, equal to the applied external force that would cause failure, provided that all the limit states are satisfied and sufficient ductility exists to allow redistribution of the forces. In other words, as long as sufficient ductility is present and all applicable limit states are satisfied, design can safely proceed based on any arbitrary distribution of forces, as long as the distribution satisfies equilibrium. If this was not true, designs would quickly grind to a halt as we constructed and calibrated, through physical testing, highly complex finite element models for every detail and possible load case for our designs.

Mr. Arias brings up many arguments that are certainly true. There will undoubtedly be some moment present in the physical connection at the beam-to-column interface. However, this moment will be limited to some value less than the ultimate strength of the beam-to-column connection. As the loads imposed on the connection approach the connection strength, the elements will begin to yield and therefore shed load to stiffer elements. As it turns out, neglecting the rotational stiffness of this connection and the resulting imposed moments in the analysis actually adds to, and not subtracts from, the safety of the connection. Any additional restraint will serve to strengthen, not weaken, the structure.

As Mr. Arias states, increasing the β dimension of the connection will tend to make it more rigid at the gusset-to-column interface. This will, as Mr. Arias asserts, draw moment from the gusset-to-beam interface. The prediction that no moment exists at the gusset-to-column interface is most certainly incorrect, as are all the other forces predicted by the proposed procedure. Some of the predicted forces are too

high, some are too low, but still the resulting design is safe and will carry the loads, or else the Lower Bound Theorem is wrong and so too are countless structures in service.

This same logic justifies the use of ΔV_b to manipulate the distribution of the forces in the connection. The use of ΔV_b predates my paper and has been present in the AISC *Manual* for many years. It is used primarily where the beam end connection is subjected to a high shear load due to gravity loads, so that it cannot resist the additional load imposed by the bracing with a typical connection. In some instances, the additional shear induced by the bracing may be such that the beam web itself is overstressed when subjected to the forces predicted by the UFM. If the beam and its connections maintained their stiffness throughout loading and then suddenly snapped like glass, it would be inappropriate to apply ΔV_b —but this is not how steel behaves.

Finally, Mr. Arias suggests that the traditional UFM is inherently superior to the procedure presented in the paper. Based on his previous arguments regarding the generalized UFM presented in the paper, this implies he feels the

traditional UFM is less arbitrary than the generalized method. In fact, it could be argued that the traditional UFM is actually more arbitrary in the constraints it chooses to impose on the force distribution. When he derived the traditional UFM, Dr. William Thornton arbitrarily chose to pass the forces V_c and H_c through a point at the intersection of the top of steel and the face of the column. This ensured that no moment would exist in a section cut through the column at the top of steel. This choice was based in part on figures shown in Blodgett's *Design of Welded Structures*. It resulted in more elegant-appearing equations for the interface forces than my proposed generalized method, but actually contained one additional arbitrary geometric constraint than the generalized procedure.

In conclusion, the procedure presented in my paper was never intended to accurately predict the forces present in the connection. It was intended instead as an improvement to an existing tool by which an admissible force distribution can be obtained that has been proven through use to produce safe and economical designs.

On the Analysis and Design of Bracing Connections



William A. Thornton

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Dr. William A. Thornton is chief engineer of Cives Steel Company and president of Cives Engineering Corporation, which are both located in Roswell, Georgia. He is responsible for all structural design originated by the company and is a consultant to the five divisions of Cives Steel Company in matters relating to connection design and fabrication practices. Dr. Thornton has 30 years experience in teaching, research, consulting and practice in the area of structural analysis and design, and is a registered professional engineer in 22 states.

He has frequently served as an invited lecturer at the American Institute of Steel Construction sponsored seminars on connection design and is author or co-author of a number of recently published papers on connection design and related areas. He is a member of the American Society of Civil Engineers, American Society of Mechanical Engineers, American Society for Testing Metals, American Welding Society, and the Research Council on Structural Connections.

Dr. Thornton currently serves as a member of technical committees of the American Institute of Steel Construction, American Society of Civil Engineers, American Welding Society, Research Council on Structural Connections and as chairman of the American Institute of Steel Construction Committee on Manuals, Textbooks and Codes.

Summary

Bracing connections constitute an area in which there has been much disagreement concerning a proper method for design. This paper considers three methods for design considered acceptable by the American Institute of Steel Construction Task Group on Heavy Bracing Connections and shows that these methods satisfy first principles from a limit analysis point of view, and are consistent with the results of extensive research performed on this problem since 1981. The paper includes a number of worked out examples to demonstrate the application of the methods to actual situations.

On the Analysis and Design of Bracing Connections

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INTRODUCTION

For many years, the methods for the analysis of bracing connections for heavy construction have been a source of controversy between engineers and steel fabricators. Beginning in 1981, the American Institute of Steel Construction sponsored extensive computer oriented research at the University of Arizona⁽¹⁾ to develop a rational analysis method. Since 1981 physical testing has been performed by Bjorhovde⁽²⁾ and Gross⁽³⁾ on full size models of gusset, beam, and column. The results of this work have not yet been distilled into a consistent method of design. It is the purpose of this paper to do so.

The AISC has formed a task group with ASCE to propose a design method (or design methods) for this problem. The recommendations arrived at by this Task Group at a meeting in Kansas City, Missouri, on March 13, 1990 are contained in Appendix A. This paper will attempt to justify the recommended design methods based on Models 2A, 3, and 4, and will include discussion of certain other possible models, such as Models 1 and 5. It will be noted that the author is co-chairman of this Task Group. This paper, however, is not the work of the Task Group and the author is solely responsible for its content

EQUILIBRIUM MODELS FOR DESIGN - CONCENTRIC CONNECTIONS

An equilibrium model for concentric connections is defined here as a model of the beam, column, gusset and brace(s) which make up the connection in which the connection interface forces provide equilibrium for the beam, column, gusset, and brace with no forces in the beam and column other than those that would be present in an ideally pin connected braced frame. In other words, there are no couples induced in the beam and column due to the connection components. Figs. 1 through 5 show the interface forces for equilibrium Models 1, 2A, 3, 4, and 5 respectively. Equilibrium models apply to concentric connections, i.e. those for which all member gravity axes meet at a common "working" point, and to eccentric connections, i.e. those for which all member gravity axes do not meet at a common point. In this latter case, couples are induced in the frame members which must be considered in the design of these members.

Model 1 - KISS

This is the simplest possible model which is still an equilibrium model. It has been referred to as the "keep it simple, stupid!" model, or the KISS model. It is simple with respect to calculations but it yields very conservative designs as will be shown. Thus it is easy to use and safe, but yields cumbersome looking and expensive connections. This method is not recommended by the AISC/ASCE Task Group and is included here for comparison purposes.

Model 2A - AISC

This model is a bit more complex computationally but yields less cumbersome designs than Model 1, which are still conservative. It is a generalization of the method presented in the AISC book Engineering for Steel Construction⁽⁴⁾ and hence will be referred to as the AISC Model. In Ref. 4, only connections to column webs are considered. This was intentional because the AISC Manual and Textbook Committee, which oversees the production of this book, could not (in 1983) agree on a proper method for connections to column flanges. Model 2A is a generalization suggested by the author. It will be shown to be conservative.

Model 3 - Thornton

This method was developed by the author and is capable of producing uniform stress distributions on all connection interfaces. For this reason, it will always produce the greatest capacity for a given connection or the smallest connection for given loads. In the sense of the Lower Bound Theorem of Limit Analysis, it comes closest to giving the true force distribution among the connection interfaces. It will be shown to come extremely close to predicting exactly the failure load of the Chakraborti and Bjorhovde⁽²⁾ tests. Of the three models thus far considered, it is the most complex computationally, but will yield the most economic and least cumbersome connections. Further discussion of this model can be found in Appendix B of Ref. 3.

Model 4 - Ricker

This method was developed by David Ricker, Vice President of Engineering of Berlin Structural Steel Company of Berlin, Connecticut and a member of the AISC/ASCE Task Group. As shown in Fig. 4, the forces at the centroids of the gusset edges are always assumed to be parallel to the brace force. The method is fairly complex computationally, as can be seen from Fig. 4a, probably about as complex as Model 3. The moment M is required because the resultant force on the gusset to beam interface does not necessarily pass through the centroid of the beam to column connection causing a moment M on this connection which must be considered in design for this to be a true equilibrium model. Note that the moment M , because it is a free vector, can be applied either to the beam to column interface or the gusset to beam and gusset to column interfaces. The choice is the designer's option.

A weakness of this model lies in the "rigidity" of the assigned directions of the gusset interface forces. When the connection is to a column web, the gusset to column interface force is still parallel to the brace force. This means there is a force component on this interface perpendicular to the column web. Since the column web is very flexible in this direction, this model may require that the column web be stiffened to accommodate the force component perpendicular to the web. It will be shown later that the results of Gross⁽³⁾ test 3A can not be predicted by this model because the web is not stiffened.

Model 5 - Modified Richard

Of the five models presented here, this is the only one which is not solely based on first principles, but rather contains empirical coefficients derived by Richard⁽¹⁾ from extensive computer analysis. As originally presented by Richard, this is an equilibrium model only if the

force resultants act at some points on the gusset to beam and gusset to column interfaces other than their centroids. Richard has not defined the interface points where his interface forces act. Since it is standard practice in connection design to refer all forces to the centroid of the connection under consideration, the author has done so and called this method the "Modified Richard" method. The moments M_B and M_C of Fig. 5 are required on the gusset edges to transport the Richard interface forces to the interface centroids.

As is the case with Model 1, this model was not recommended by the AISC/ASCE Task Group, but is included here for purposes of comparison.

ECCENTRIC CONNECTIONS

Eccentric Connections are those with member gravity axes which do not intersect at a common working point. Instead the working point is usually assumed at the face of the flange of the beam or column or both as shown in Fig. 6. This working point is chosen to allow more compact connections to result. Fig. 7 shows the gusset interface forces usually assumed. These are shears on the gusset edges. Because these shears intersect the brace line at a common point equilibrium of the gusset can be enforced, and it is a true equilibrium model only if the couples induced in the beam and column are considered in the design of the beam and column. Figs. 6 and 7 call this the "classical" case because it was a very commonly used method in the past but presently is rejected by many engineers because of the induced beam and column couples. One of the objects of this paper is to investigate the consequences of use of this method.

It should be noted that models 1, 2A, and 3 all reduce to the classical case if $e_B = e_C = 0$. Models 4 and 5 do not reduce to the classical case.

COMPARISON OF MODEL PREDICTIONS WITH PHYSICAL TEST RESULTS

Two sets of data for full scale and $3/4$ scale physical tests are available to assess the accuracy of failure prediction of the five equilibrium models discussed in the previous section. These are the tests of Chakrabarti and Bjorhovde⁽²⁾ and those of Gross⁽³⁾.

Chakrabarti and Bjorhovde Tests

A set of six tests were performed by Chakrabarti and Bjorhovde⁽²⁾ on the specimens of Fig. 8. Fig. 8 was replicated six times, i.e. for each of two gusset thicknesses ($1/8$ and $3/8$) and three brace angles from the horizontal ($\phi = 30^\circ, 45^\circ$, and 60°). Only the $1/8$ gusset is treated here because the $3/8$ gusset specimens exceeded the capacity of the testing frame. In the test frame, the specimen was oriented with column horizontal and bolted to the test frame which was in turn bolted to the laboratory floor, and the beam was vertical with top end free. Thus, this setup is roughly equivalent to a situation in a real building where the brace horizontal component ($P \cos \phi$ of Fig. 8) is passed to an adjacent bay. The force $P \cos \phi$ is referred to as a "transfer force" and denoted T_f . In the calculations to predict capacity using the five models, the transfer force T_f for the Chakrabarti/Bjorhovde tests is $P \cos \phi$ and this is made up of H_C from the gusset to column connection and H_B from the beam to column connection. Thus, the beam to column connection for all models will be subjected to H_B (axial) and V_B (shear).