Licensed copy: Lee Shau Kee Library, HKUST, Version correct as of 03/01/2015, (c) The British Standards Institution 2013

6.4.3 Punching shear calculation

(1)P The design procedure for punching shear is based on checks at the face of the column and at the basic control perimeter u_1. If shear reinforcement is required a further perimeter $u_{out,ef}$ (see figure 6.22) should be found where shear reinforcement is no longer required. The following design shear stresses (MPa) along the control sections, are defined:

$v_{Rd,c}$ is the design value of the punching shear resistance of a slab without punching shear reinforcement along the control section considered.

$v_{Rd,cs}$ is the design value of the punching shear resistance of a slab with punching shear reinforcement along the control section considered.

$v_{Rd,max}$ is the design value of the maximum punching shear resistance along the control section considered.

(2) The following checks should be carried out:

(a) At the column perimeter, or the perimeter of the loaded area, the maximum punching shear stress should not be exceeded:

AC1 $v_{Ed} \leq v_{Rd,max}$ AC1

(b) Punching shear reinforcement is not necessary if:

AC1 $v_{Ed} \leq v_{Rd,c}$ AC1

(c) Where v_{Ed} exceeds the value $v_{Rd,c}$ for the control section considered, punching shear reinforcement should be provided according to 6.4.5.

(3) Where the support reaction is eccentric with regard to the control perimeter, the maximum shear stress should be taken as:

$$v_{Ed} = \beta \frac{V_{Ed}}{u_i d} \qquad (6.38)$$

where

d is the mean effective depth of the slab, which may be taken as $(d_y + d_z)/2$ where:

d_y, d_z is the effective depths in the y- and z- directions of the control section

u_i is the length of the control perimeter being considered

β is given by:

$$\beta = 1 + k \frac{M_{Ed}}{V_{Ed}} \cdot \frac{u_1}{W_1} \qquad (6.39)$$

where

u_1 is the length of the basic control perimeter

k is a coefficient dependent on the ratio between the column dimensions c_1 and c_2: its value is a function of the proportions of the unbalanced moment transmitted by uneven shear and by bending and torsion (see Table 6.1).

W_1 corresponds to a distribution of shear as illustrated in Figure 6.19 and is a function of the basic control perimeter u_1:

AC1 $W_i = \int_0^{u_i} |e| dl$ AC1 (6.40)

Licensed copy: Lee Shau Kee Library, HKUST, Version correct as of 03/01/2015, (c) The British Standards Institution 2013

dl is a length increment of the perimeter
e is the distance of dl from the axis about which the moment M_{Ed} acts

Table 6.1: Values of *k* for rectangular loaded areas

c_1/c_2	≤ 0,5	1,0	2,0	≥ 3,0
k	0,45	0,60	0,70	0,80

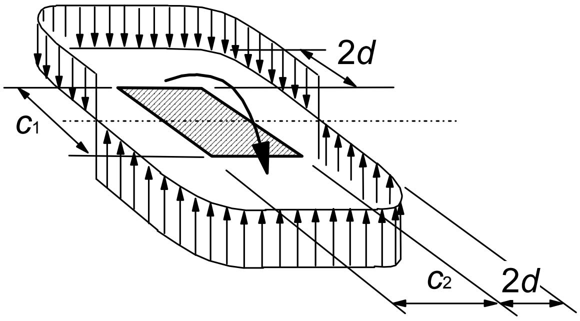


Figure 6.19: Shear distribution due to an unbalanced moment at a slab-internal column connection

For a rectangular column:

$$W_1 = \frac{c_1^2}{2} + c_1 c_2 + 4c_2 d + 16d^2 + 2\pi d c_1 \qquad (6.41)$$

where:

c_1 is the column dimension parallel to the eccentricity of the load
c_2 is the column dimension perpendicular to the eccentricity of the load

For internal circular columns β follows from:

$$\beta = 1 + 0{,}6\pi \frac{e}{D + 4d} \qquad (6.42)$$

where D is the diameter of the circular column
AC1 e is the eccentricity of the applied load $e = M_{Ed} / V_{Ed}$ AC1

For an internal rectangular column where the loading is eccentric to both axes, the following approximate expression for β may be used:

$$\beta = 1 + 1{,}8 \sqrt{\left(\frac{e_y}{b_z}\right)^2 + \left(\frac{e_z}{b_y}\right)^2} \qquad (6.43)$$

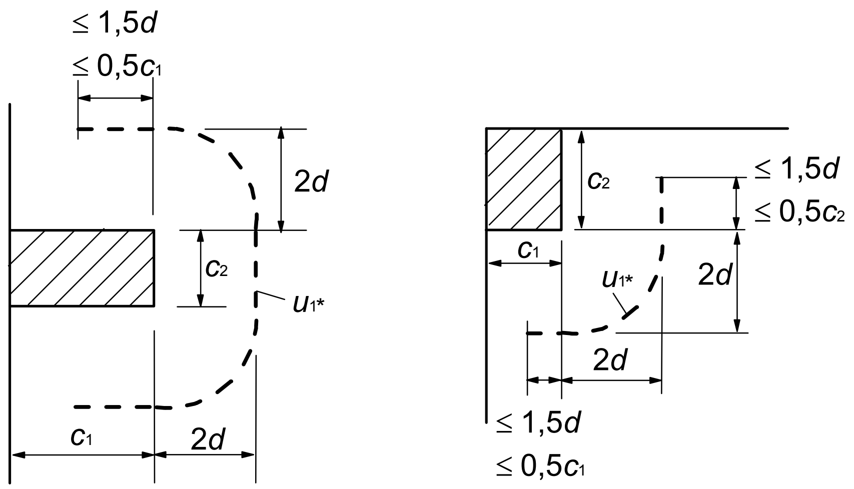
where:

e_y and e_z are the eccentricities M_{Ed}/V_{Ed} along y and z axes respectively
b_y and b_z is the dimensions of the control perimeter (see Figure 6.13)

Note: e_y results from a moment about the z axis and e_z from a moment about the y axis.

(4) For edge column connections, where the eccentricity perpendicular to the slab edge (resulting from a moment about an axis parallel to the slab edge) is toward the interior and there is no eccentricity parallel to the edge, the punching force may be considered to be uniformly distributed along the control perimeter u_{1^*} as shown in Figure 6.20(a).

Licensed copy: Lee Shau Kee Library, HKUST, Version correct as of 03/01/2015, (c) The British Standards Institution 2013



a) edge column **b) corner column**

Figure 6.20: Reduced basic control perimeter u_{1*}

Where there are eccentricities in both orthogonal directions, β may be determined using the following expression:

$$\beta = \frac{u_1}{u_{1*}} + k\frac{u_1}{W_1}e_{par} \tag{6.44}$$

where:

u_1 is the basic control perimeter (see Figure 6.15)
u_{1*} is the reduced basic control perimeter (see Figure 6.20(a))
e_{par} is the eccentricity parallel to the slab edge resulting from a moment about an axis perpendicular to the slab edge.
k may be determined from Table 6.1 with the ratio c_1/c_2 replaced by $c_1/2c_2$
W_1 is calculated for the basic control perimeter u_1 (see Figure 6.13).

For a rectangular column as shown in Figure 6.20(a):

$$W_1 = \frac{c_2^2}{4} + c_1c_2 + 4c_1d + 8d^2 + \pi dc_2 \tag{6.45}$$

If the eccentricity perpendicular to the slab edge is not toward the interior, Expression (6.39) applies. When calculating W_1 [AC1) the distance e should be measured from the centroid axis of the control perimeter.(AC1]

(5) For corner column connections, where the eccentricity is toward the interior of the slab, it is assumed that the punching force is uniformly distributed along the reduced control perimeter u_{1*}, as defined in Figure 6.20(b). The β-value may then be considered as:

$$\beta = \frac{u_1}{u_{1*}} \tag{6.46}$$

If the eccentricity is toward the exterior, Expression (6.39) applies.

Licensed copy: Lee Shau Kee Library, HKUST, Version correct as of 03/01/2015, (c) The British Standards Institution 2013

(6) For structures where the lateral stability does not depend on frame action between the slabs and the columns, and where the adjacent spans do not differ in length by more than 25%, approximate values for β may be used.

Note: Values of β for use in a Country may be found in its National Annex. Recommended values are given in Figure 6.21N.

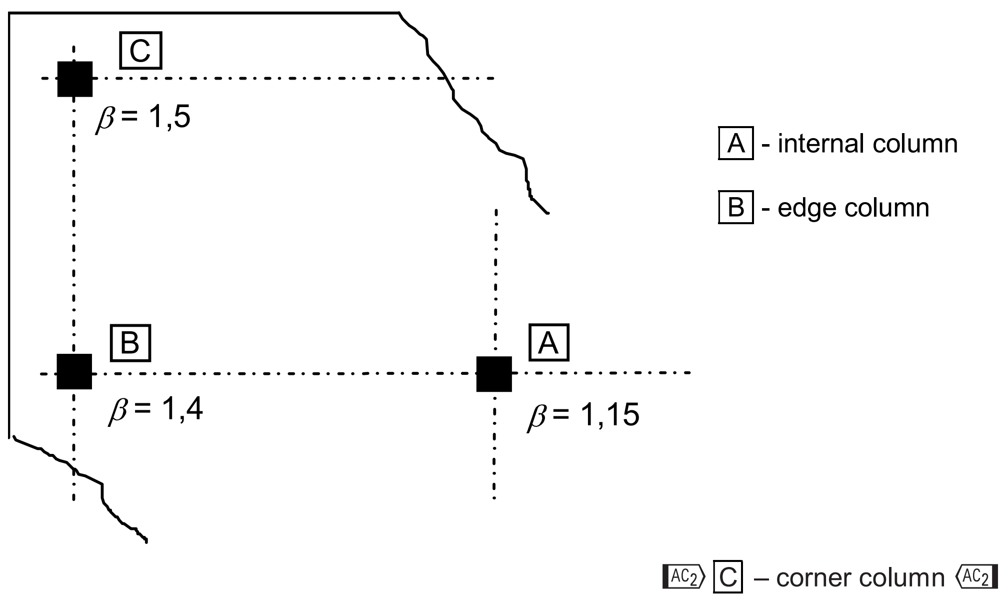


Figure 6.21N: Recommended values for β

(7) Where a concentrated load is applied close to a flat slab column support the shear force reduction according to 6.2.2 (6) and 6.2.3 (8) respectively is not valid and should not be included.

(8) The punching shear force V_{Ed} in a foundation slab may be reduced due to the favourable action of the soil pressure.

(9) The vertical component V_{pd} resulting from inclined prestressing tendons crossing the control section may be taken into account as a favourable action where relevant.

6.4.4 Punching shear resistance of slabs and column bases without shear reinforcement

(1) The punching shear resistance of a *slab* should be assessed for the basic control section according to 6.4.2. The design punching shear resistance [MPa] may be calculated as follows:

$$v_{Rd,c} = C_{Rd,c}\, k\, (100 \rho_l\, f_{ck})^{1/3} + k_1 \sigma_{cp} \geq \left(v_{min} + k_1 \sigma_{cp}\right) \tag{6.47}$$

where:

f_{ck} is in MPa

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2{,}0 \quad d \text{ in mm}$$

$$\rho_l = \sqrt{\rho_{ly} \cdot \rho_{lz}} \leq 0{,}02$$

ρ_{ly}, ρ_{lz} relate to the bonded tension steel in y- and z- directions respectively. The values ρ_{ly} and ρ_{lz} should be calculated as mean values taking into account a slab width

Licensed copy: Lee Shau Kee Library, HKUST, Version correct as of 03/01/2015, (c) The British Standards Institution 2013

equal to the column width plus 3d each side.

$\sigma_{cp} = (\sigma_{cy} + \sigma_{cz})/2$

where

σ_{cy}, σ_{cz} are the normal concrete stresses in the critical section in y- and z-directions (MPa, positive if compression):

$$\sigma_{c,y} = \frac{N_{Ed,y}}{A_{cy}} \text{ and } \sigma_{c,z} = \frac{N_{Ed,z}}{A_{cz}}$$

N_{Edy}, N_{Edz} are the longitudinal forces across the full bay for internal columns and the longitudinal force across the control section for edge columns. The force may be from a load or prestressing action.

A_c is the area of concrete according to the definition of N_{Ed}

Note: The values of $C_{Rd,c}$, v_{min} and k_1 for use in a Country may be found in its National Annex. The recommended value for $C_{Rd,c}$ is $0{,}18/\gamma_c$, for v_{min} is given by Expression (6.3N) and that for k_1 is 0,1.

(2) The punching resistance of column bases should be verified at control perimeters within 2d from the periphery of the column.

For concentric loading the net applied force is

$$V_{Ed,red} = V_{Ed} - \Delta V_{Ed} \tag{6.48}$$

where:

V_{Ed} is the applied shear force

ΔV_{Ed} is the net upward force within the control perimeter considered i.e. upward pressure from soil minus self weight of base.

$$v_{Ed} = V_{Ed,red}/ud \tag{6.49}$$

AC1 $$v_{Rd} = C_{Rd,c}k(100\,\rho_l f_{ck})^{1/3} \times 2d/a \geq v_{min} \times 2d/a \tag{6.50}$$ AC1

where

a is the distance from the periphery of the column to the control perimeter considered

$C_{Rd,c}$ is defined in 6.4.4(1)

v_{min} is defined in 6.4.4(1)

k is defined in 6.4.4(1)

For eccentric loading

$$v_{Ed} = \frac{V_{Ed,red}}{ud}\left[1 + k\frac{M_{Ed}\,u}{V_{Ed,red}W}\right] \tag{6.51}$$

Where k is defined in 6.4.3 (3) or 6.4.3 (4) as appropriate and W is similar to W_1 but for perimeter u.

6.4.5 Punching shear resistance of slabs and column bases with shear reinforcement

(1) Where shear reinforcement is required it should be calculated in accordance with Expression (6.52):

$$v_{Rd,cs} = 0{,}75\, v_{Rd,c} + 1{,}5\,(d/s_r)\, A_{sw}\, f_{ywd,ef}\,(1/(u_1 d))\,\sin\alpha \tag{6.52}$$

Licensed copy: Lee Shau Kee Library, HKUST, Version correct as of 03/01/2015, (c) The British Standards Institution 2013

where

- A_{sw} is the area of one perimeter of shear reinforcement around the column [mm^2]
- s_r is the radial spacing of perimeters of shear reinforcement [mm]
- $f_{ywd,ef}$ is the effective design strength of the punching shear reinforcement, according to $f_{ywd,ef}$ = 250 + 0,25 $d \leq f_{ywd}$ [MPa]
- d is the mean of the effective depths in the orthogonal directions [mm]
- α is the angle between the shear reinforcement and the plane of the slab

If a single line of bent-down bars is provided, then the ratio d/s_r in Expression (6.52) may be given the value 0,67.

(2) Detailing requirements for punching shear reinforcement are given in 9.4.3.

(3) Adjacent to the column the punching shear resistance is limited to a maximum of:

$$v_{Ed} = \frac{\beta V_{Ed}}{u_0 d} \leq v_{Rd,max} \tag{6.53}$$

where

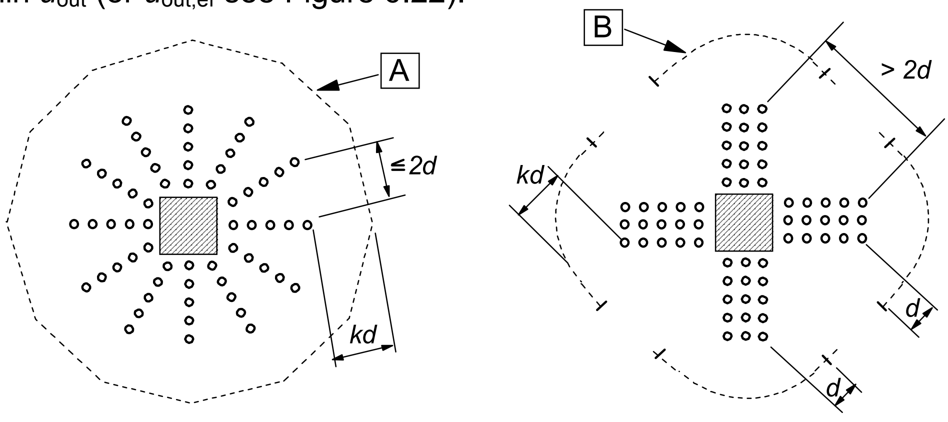
- u_0 for an interior column [AC2) u_0 = enclosing minimum periphery [mm] (AC2]
  for an edge column $u_0 = c_2 + 3d \leq c_2 + 2c_1$ [mm]
  for a corner column $u_0 = 3d \leq c_1 + c_2$ [mm]
- c_1, c_2 are the column dimensions as shown in Figure 6.20
  [AC2) *Texts deleted* (AC2]
- β see 6.4.3 (3), (4) and (5)

[AC2) **Note:** The value of $v_{Rd,max}$ for use in a Country may be found in its National Annex.The recommended value is 0,4 νf_{cd} where ν is given in Expression (6.6N).(AC2]

(4) The control perimeter at which shear reinforcement is not required, u_{out} (or $u_{out,ef}$ see Figure 6.22) should be calculated from Expression (6.54):

$$u_{out,ef} = \beta V_{Ed} / (v_{Rd,c}\, d) \tag{6.54}$$

The outermost perimeter of shear reinforcement should be placed at a distance not greater than kd within u_{out} (or $u_{out,ef}$ see Figure 6.22).



A Perimeter u_{out} B Perimeter $u_{out,ef}$

Figure 6.22: Control perimeters at internal columns

Note: The value of k for use in a Country may be found in its National Annex. The recommended value is 1,5.