#1: LATERAL STIFFNESS OF TAUT CABLE'S MIDPOINT

#2: CaseMode := Sensitive

#3: InputMode := Word

#4: L = distance between support points L0 = unstretched length of cable

#5: K = cable's end-to-end axial stiffness D = cable's midpoint lateral deflection

#6: L1 = cable's stretched length F = lateral force at cable's midpoint

#7: A = cable's deflection angle T = tension in (displaced) cable

#8: ---------------------------------------

#9: Geometry / trigonometry gives us formulae for L1 and SIN(A)

#10: $\sqrt{(L^2 + 4\cdot D^2)}$

#11: $\dfrac{2\cdot D}{L}$

#12: Elasticity, given the above formula for L1 (#10), gives us a formula for T

#13: $(\sqrt{(L^2 + 4\cdot D^2)} - L0)\cdot K$

#14: Equilibrium at the loaded point gives us a formula for F

#15: $2\cdot T\cdot SIN(A)$

#16: Substitute formula for T (#13) into this formula for F

#17: $2\cdot((\sqrt{L^2 + 4\cdot D^2} - L0)\cdot K)\cdot SIN(A)$

#18: Substitute formula for SIN(A) (#11) into this revised formula for F

#19: $2\cdot((\sqrt{L^2 + 4\cdot D^2} - L0)\cdot K)\cdot\dfrac{2\cdot D}{L}$

#20: The required lateral stiffness is the derivative of F wrt D

#21: $\dfrac{d}{dD}\left(2\cdot((\sqrt{L^2 + 4\cdot D^2} - L0)\cdot K)\cdot\dfrac{2\cdot D}{L}\right)$

#22: Evaluate this algebraically

#23: $$- \frac{4\cdot K\cdot(L0\cdot\sqrt{4\cdot D^2 + L^2} - 8\cdot D^2 - L^2)}{L\cdot\sqrt{4\cdot D^2 + L^2}}$$

#24: Expand this result into a Taylor's series

#25: $TAYLOR\left(- \dfrac{4\cdot K\cdot(L0\cdot\sqrt{4\cdot D^2 + L^2} - 8\cdot D^2 - L^2)}{L\cdot\sqrt{4\cdot D^2 + L^2}}, D, 0, 4\right)$

#26: $$- \frac{4\cdot K\cdot(10\cdot D^4 - 6\cdot D^2\cdot L^2 - L^4)\cdot SIGN(L)}{L^4} - \frac{4\cdot K\cdot L0}{L}$$

#27: Remove the SIGN(L) term since it is irrelevant in this context

#28: $$-\frac{4\cdot K\cdot(10\cdot D^4 - 6\cdot D^2\cdot L^2 - L^3\cdot(L - L0))}{L^4}$$

#29: This will simplify to the following, but 'Derive' won't express it in this form

#30: $$\frac{4\cdot K\cdot(L - L0)}{L} + 24\cdot K\cdot\left(\frac{D}{L}\right)^2 - 40\cdot K\cdot\left(\frac{D}{L}\right)^4$$

#31: Confirm that these two forms (#28 and #30) are the same by subtraction

#32: $$-\frac{4\cdot K\cdot(10\cdot D^4 - 6\cdot D^2\cdot L^2 - L^3\cdot(L - L0))}{L^4} - \left(\frac{4\cdot K\cdot(L - L0)}{L} + 24\cdot K\cdot\left(\frac{D}{L}\right)^2 - 40\cdot K\cdot\left(\frac{D}{L}\right)^4\right)$$

#33: This should simplify to zero

#34: $$0$$

#35: 'Quod erat demonstrandum' as the Romans used to say.