VALVE MINOR LOSSES

❖ Valve Head Loss Equations

Valve Coefficient, C_v: $\quad \boldsymbol{\Delta P = \left(\frac{Q}{C_v}\right)^2}$

Minor Loss Coefficient, k: $\quad \boldsymbol{H_L = k \cdot \frac{V^2}{2 \cdot g}}$

❖ Express the Valve Coefficient Equation in terms of $\boldsymbol{V^2}$ and $\boldsymbol{H_L}$

$$\Delta P = \left(\frac{Q}{C_v}\right)^2 \leftrightarrow \Delta P\left[\frac{lbf}{in^2}\right] = \left(\frac{Q\left[\frac{gal}{min}\right]}{C_v\left[\frac{gal}{min}\cdot\frac{in}{\sqrt{lbf}}\right]}\right)^2$$

$$\left(Q\left[\frac{gal}{min}\right]\right)^2 = \left(C_v\left[\frac{gal}{min}\cdot\frac{in}{\sqrt{lbf}}\right]\right)^2\cdot\Delta P\left[\frac{lbf}{in^2}\right]$$

$$Q\left[\frac{gal}{min}\right] = Q\left[\frac{ft^3}{s}\right]\cdot 7.4805\frac{gal}{ft^3}\cdot 60\frac{s}{min} = Q\left[\frac{ft^3}{s}\right]\cdot\frac{448.83\frac{gal}{min}}{\frac{ft^3}{s}}$$

$$\left(Q\left[\frac{ft^3}{2}\right]\cdot\frac{448.83\frac{gal}{min}}{\frac{ft^3}{s}}\right)^2 = \left(C_v\left[\frac{gal}{min}\cdot\frac{in}{\sqrt{lbf}}\right]\right)^2\cdot\Delta P\left[\frac{lbf}{in^2}\right]$$

$$Q = A\cdot V \leftrightarrow Q\left[\frac{ft^3}{s}\right] = A[ft^2]\cdot V\left[\frac{ft}{s}\right]$$

$$(A[ft^2])^2\cdot\left(V\left[\frac{ft}{s}\right]\right)^2\cdot\left(\frac{448.83\frac{gal}{min}}{\frac{ft^3}{s}}\right)^2 = \left(C_v\left[\frac{gal}{min}\cdot\frac{in}{\sqrt{lbf}}\right]\right)^2\cdot\Delta P\left[\frac{lbf}{in^2}\right]$$

$$\left(V\left[\frac{ft}{s}\right]\right)^2 = \frac{\left(C_v\left[\frac{gal}{min}\cdot\frac{in}{\sqrt{lbf}}\right]\right)^2\cdot\Delta P\left[\frac{lbf}{in^2}\right]}{(A[ft^2])^2\cdot\left(\frac{448.83\frac{gal}{min}}{\frac{ft^3}{s}}\right)^2}$$

$$A = \frac{1}{4}\cdot\pi\cdot d^2 \leftrightarrow A[ft^2] = \frac{1}{4}\cdot\pi\cdot(d[ft])^2 = \frac{1}{4}\cdot\pi\cdot\left(\frac{d[in]}{12\frac{in}{ft}}\right)^2$$

$$\Delta P\left[\frac{lbf}{in^s}\right] = H_L[ft]\cdot\frac{62.4\frac{lbf}{ft^3}}{\left(12\frac{in}{ft}\right)^2}$$

$$\left(V\left[\frac{ft}{s}\right]\right)^2 = \frac{\left(C_v\left[\frac{gal}{min}\cdot\frac{in}{\sqrt{lbf}}\right]\right)^2\cdot H_L[ft]\cdot\frac{62.4\frac{lbf}{ft^3}}{\left(12\frac{in}{ft}\right)^2}}{\left(\frac{1}{4}\cdot\pi\cdot\left(\frac{d[in]}{12\frac{in}{ft}}\right)^2\right)^2\cdot\left(\frac{448.83\frac{gal}{min}}{\frac{ft^3}{s}}\right)^2}$$

$$\left(V\left[\frac{ft}{s}\right]\right)^2 = \frac{\left(\frac{62.4}{12^2}\right)}{\left(\frac{\pi^2\cdot 448.83^2}{4^2\cdot 12^4}\right)}[units]\cdot\frac{{C_v}^2\cdot H_L[ft]}{(d[in])^4} = \frac{62.4\cdot 48^2}{\pi^2\cdot 448.83^2}[units]\cdot\frac{{C_v}^2\cdot H_L[ft]}{(d[in])^4}$$

- Solve the Minor Loss Coefficient Equation for $\boldsymbol{k}$ and Substitute in the Valve Coefficient Equation for $\boldsymbol{V^2}$

$$H_L = k\cdot\frac{V^2}{2\cdot g} \leftrightarrow H_L[ft] = k\cdot\frac{\left(V\left[\frac{ft}{s}\right]\right)^2}{2\cdot g\left[\frac{ft}{s^2}\right]}$$

$$k = \frac{2\cdot g\left[\frac{ft}{s^2}\right]\cdot H_L[ft]}{\left(V\left[\frac{ft}{s}\right]\right)^2} \rightarrow k = \frac{2\cdot 32.174\frac{ft}{s^2}\cdot H_L[ft]}{\left(V\left[\frac{ft}{s}\right]\right)^2}$$

$$k = \frac{2\cdot 32.174\frac{ft}{s^2}\cdot H_L[ft]}{\left(\frac{\left(C_v\left[\frac{gal}{kin}\cdot\frac{in}{\sqrt{lbf}}\right]\right)^2\cdot H_L[ft]\cdot\frac{62.4\frac{lbf}{ft^3}}{\left(12\frac{in}{ft}\right)^2}}{\left(\frac{1}{4}\cdot\pi\cdot\left(\frac{d[in]}{12\frac{in}{ft}}\right)^2\right)^2\cdot\left(\frac{448.83\frac{gal}{min}}{\frac{ft^3}{s}}\right)^2}\right)}$$

$$k = \frac{2\cdot 32.174}{\left(\frac{62.4\cdot 48^2}{\pi^2\cdot 448.83^2}\right)}[units]\cdot\frac{(d[in])^4}{{C_v}^2} = 889.88008967589348\ldots\cdot\frac{(d[in])^4}{{C_v}^2}$$

USE: $\boldsymbol{k = \frac{889.88\cdot(d[in])^4}{{C_v}^2}}$

❖ Notes

- I calculated the above coefficient using a SwissMicros DM42 calculator, which has a precision of 34 digits (overkill for this application). Excel, with 15 digits of precision (also overkill), agreed out to the 12th decimal place, as expected, but could provide no more.
- A coefficient of 890.4 appears in the published $k \leftrightarrow C_v$ conversion equations I have seen. Rounding g to $32.2\,ft/s^2$ (as is commonly done) produces a coefficient of 890.5992, which is close to the published value, but not quite. To get to 890.4 also requires using a specific weight of water, $\gamma_w = 62.414\,lbf/ft^3$ ($= \gamma_w\ @\ T{\sim}8.2°C$, which is not a standard temperature). However, it makes no sense to round g to one decimal place, but use three decimal places for γ_w, which varies with temperature. For comparison, at $T = 68°F$ (one of the common standard temperatures), $\gamma_w = 62.314\_\,lbf/ft^3$. I suspect either limited precision or a little sloppiness was behind the calculation of the slightly wrong published coefficient. On the other hand, both k and C_v are empirical values with significant error bars, so the published coefficient being off by well less 0.1% is not worth worrying about.
- Using a more exact conversion for gallons ($7.48051948\,gal/ft^3$), results in a flow rate conversion of $448.8311688\,gpm/cfs$ and a coefficient of 889.88472812. Rounded to two decimal places, this is the same as the value produced with simpler conversion factors.