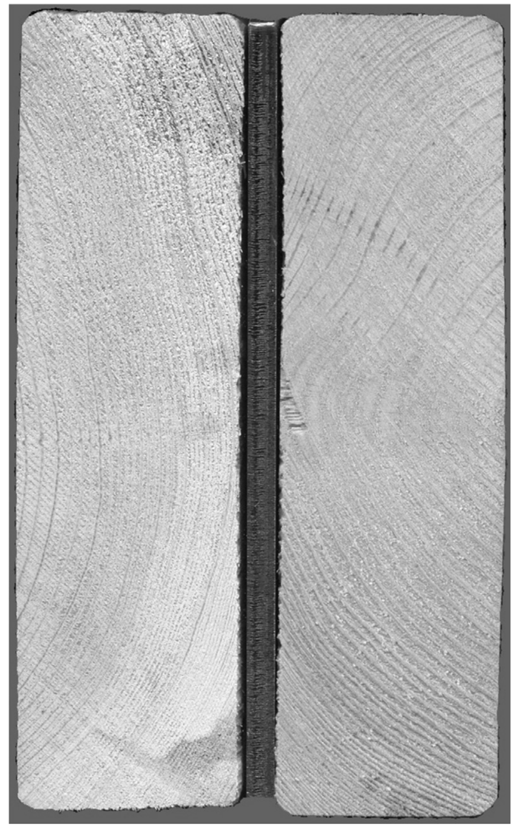
Architecture 544
Wood Structures

Flitched Beams

- Strain Compatibility
- Transformed Sections
- Flitched Beams

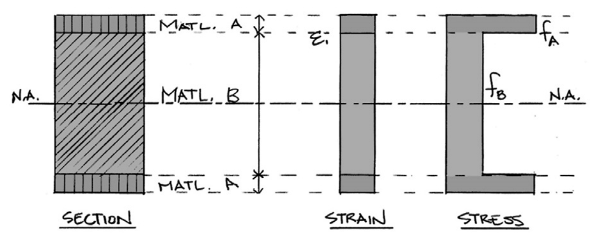


Strain Compatibility

With two materials bonded together, both will act as one, and the deformation in each is the same.

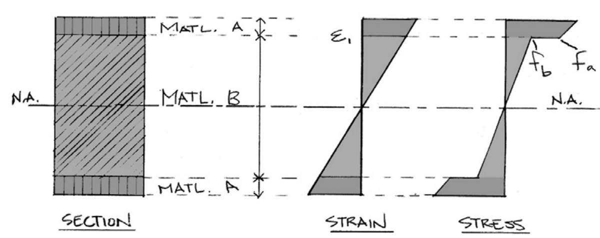
Therefore, the strains will be the same in each material under axial load.

In flexure the strains are the same as in a homogeneous section, i.e. linear.



Axial

In flexure, if the two materials are at the same distance from the N.A., they will have the same strain at that point because both materials share the same strain diagram. We say the strains are "compatible".

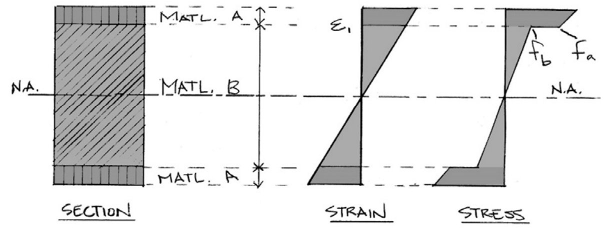


Flexure

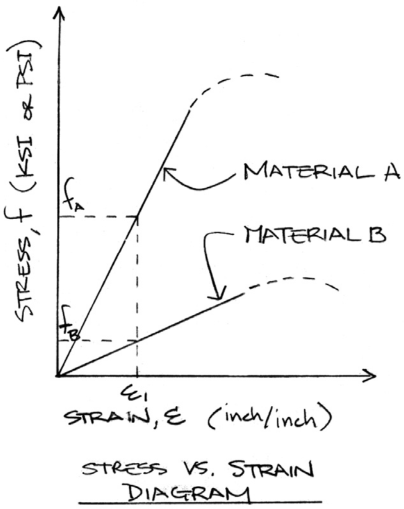
Strain Compatibility (cont.)

The stress in each material is determined by using Young’s Modulus

$$\sigma = \mathrm{E}\varepsilon$$

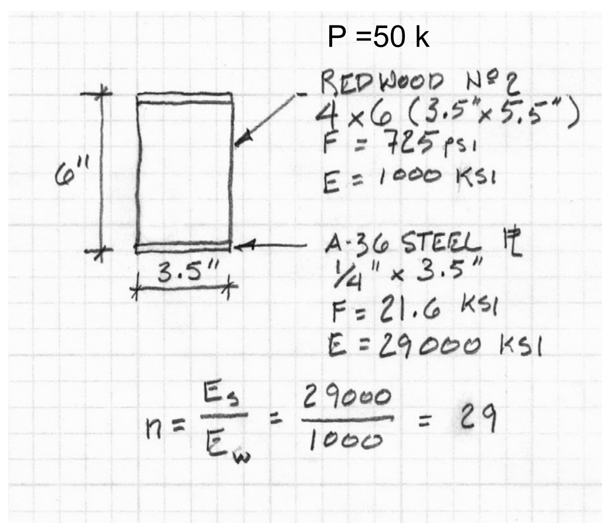


Care must be taken that the elastic limit of each material is not exceeded. The elastic limit can be expressed in either stress or strain.



Axial Compression
Stress Analysis:

Determine the safety (pass or fail) of the composite short pier under an axial load.



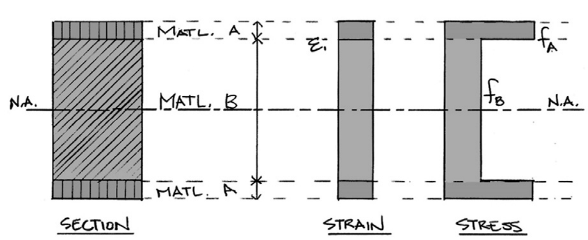
By Considère’s Law:

1. P = f A = fs As + fw Aw
   (the actual stresses are unknown)

2. ∈pier = ∈s = ∈w (equal strains)
   ∈ = f / E
   fs / Es = fw / Ew
   fs = (Es / Ew) fw

3. Substitute (Es / Ew) fw for fs into the original equation and solve for fw

4. Use second equation to solve fs



Example - Axial Compression

Pass/Fail Analysis:

Given: Load = 50 kips

Section: Aw = 19.25 in^2

Ew = 1000 ksi

As = 2x 0.875 = 1.75 in^2

Es = 29000 ksi

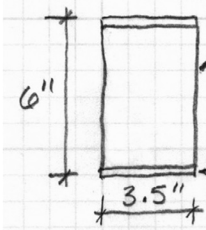
Braced against buckling.

Req'rd: actual stress in the material

1. P = f A = fs As + fw Aw
   (the actual stresses are unknown)

2. fs = (Es / Ew) fw

3. P = (Es / Ew) fw As + fw Aw



REDWOOD No 2
4 x 6 (3.5" x 5.5")
F = 725 psi
E = 1000 KSI

A-36 STEEL PL
1/4" x 3.5"
F = 21.6 KSI
E = 29000 KSI

$$n = \frac{E_s}{E_w} = \frac{29000}{1000} = 29$$

$P = fA$

$50^k = f_s(1.75 in^2) + f_w(19.25)$

$f_s = 29(f_w)$

$50 = 29(1.75) f_w + 19.25 f_w$

$50 = 70 f_w$

$f_w = 0.714\ ^k/_{in^2} = 714$ PSI

Example - Axial Compression

Pass/Fail Analysis:

4. Use second equation to solve for fs

$f_s = 29(f_w)$

$f_s = 29(.714) = 20.7$ KSI

WOOD

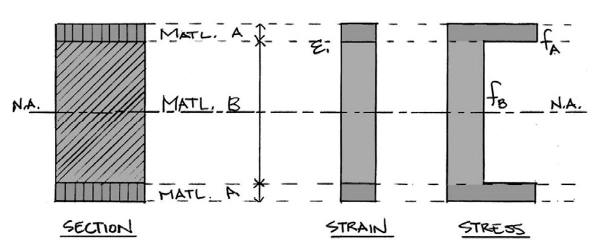
$F_c \geq f_c$

725 > 714 PSI ok

STEEL

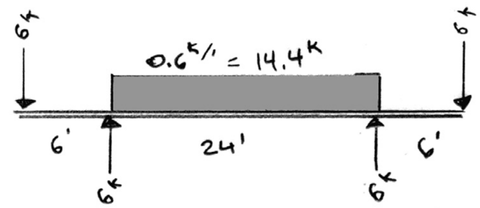
$F_c \geq f_c$

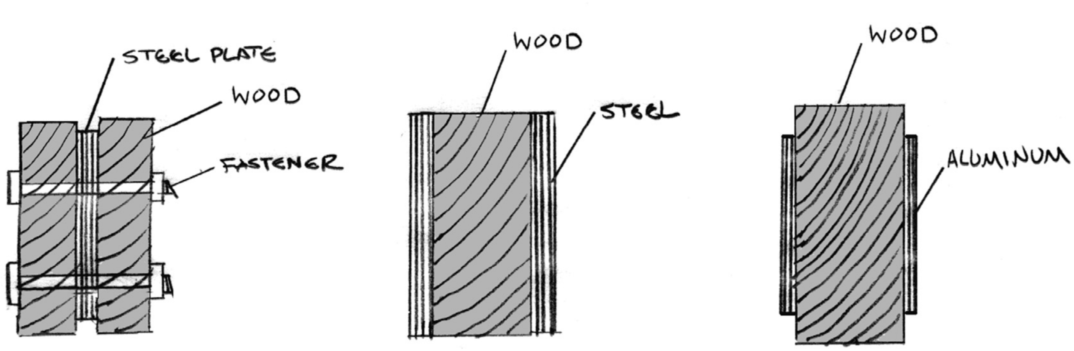
21.6 > 20.7 KSI ok



Flitched Beams & Scab Plates

- Compatible with the wood structure, i.e. can be nailed
- Lighter weight than a steel section
- Less deep than wood alone
- Stronger than wood alone
- Allow longer spans
- The section can vary over the length of the span to optimize the member (e.g. scab plates)
- The wood stabilizes the thin steel plate





Flexure Stress using Transformed Sections

In the basic flexural stress equation, I is derived based on a homogeneous section. Therefore, to use the stress equation one needs to “transform” the composite section into a homogeneous section.

$$f_b = \frac{Mc}{I}$$

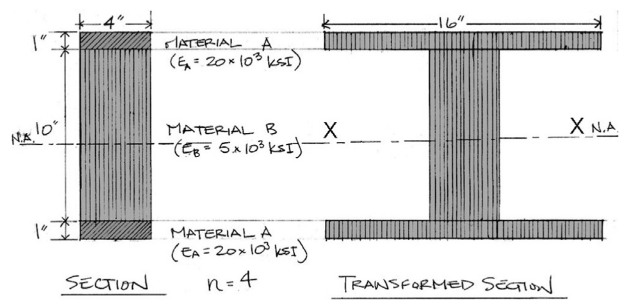
Homogeneous Section

$$f_b = \frac{Mc\ n}{I_{tr}}$$

Transformed Section

For the new “transformed section” to behave like the actual section, the stiffness of both would need to be the same.

Since Young’s Modulus, E, represents the material stiffness, when transforming one material into another, the area of the transformed material must be scaled by the ratio of one E to the other.



The scale factor is called the modular ratio, n.

$$n = \frac{E_A}{E_B}$$

In order to also get the correct stiffness for the moment of Inertia, I, only the width of the geometry is scaled. Using I from the transformed section (I_{TR}) will then give the same flexural stiffness as in the original section.

Calculate the Transformed Section, I_{TR}

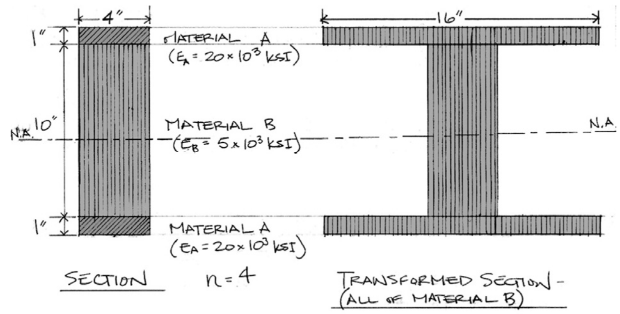
1. Use the ratio of the E modulus from each material to calculate a modular ratio, n.

$$n_A = \frac{E_A}{E_B} \quad and \quad n_B = \frac{E_B}{E_B}$$

Transformed material

Base material

2. Usually the softer (lower E) material is used as a base (denominator). Each material combination has a different n.



3. Construct a transformed section by scaling the width of each material by its modular, n.
4. I_{tr} is calculated about the N.A. using the transformation equation (parallel axis theorem) with the transformed section.
5. Separate transformed sections must be created for each axis (x-x and y-y)

$$I_{tr} = \sum I + \sum A d^2$$

Flitched Beam Analysis Procedure

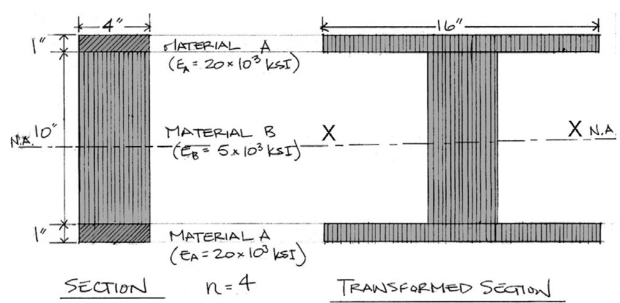
1. Determine the modular ratio(s). Usually the softer (lower E) material is used as a base (denominator). Each material has a different n.

$$n_A = \frac{E_A}{E_B} \quad and \quad n_B = \frac{E_B}{E_B}$$

Transformed material

Base material

2. Construct a transformed section by scaling the width of the material by its modular, n.



3. Determine the Centroid and Moment of Inertia of the transformed section.
4. Calculate the flexural stress in **each** material separately using:

$$f_b = \frac{Mc\ n}{I_{tr}}$$

$$I_{tr} = \sum I + \sum A d^2$$

Transformation equation or solid-void

Analysis Example:

For the composite section, find the maximum flexural stress level in each laminate material.

$$f_b = \frac{\mathrm{M}c\ n}{\mathrm{I}_{\mathrm{tr}}}$$

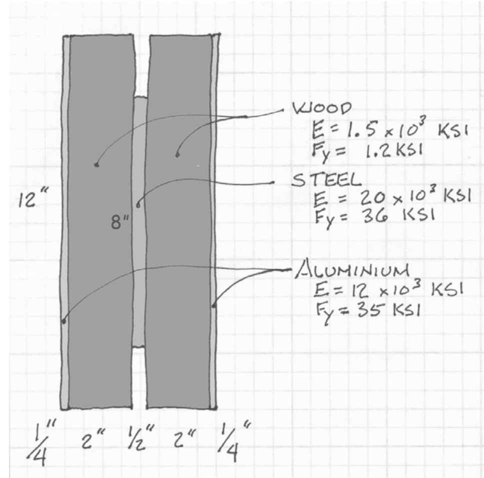
1. Determine the modular ratios for each material.

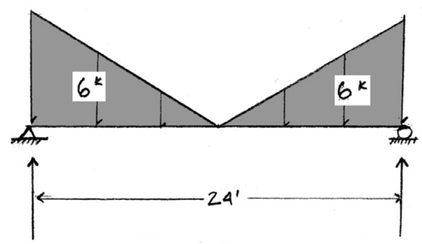
Use wood (the lowest E) as base material.

$$n_{WOOD} = \frac{1.5}{1.5} = 1.0$$

$$n_{AL} = \frac{12}{1.5} = 8.0$$

$$n_{ST} = \frac{30}{1.5} = 20.$$



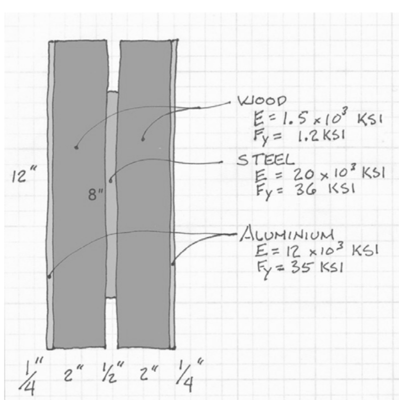


LOADING DIAGRAM

Analysis Example cont.:

2. Construct a transformed section.

   Determine the transformed width of each material.



$$n_{WOOD} = \frac{1.5}{1.5} = 1.0$$

$$n_{AL} = \frac{12}{1.5} = 8.0$$

$$n_{ST} = \frac{30}{1.5} = 20.$$

ALUM.

$t = 1/4''$

$t_{tr} = 1/4 \times n_{AL}$

$= 1/4\,(8.0) = 2.0''$

STEEL

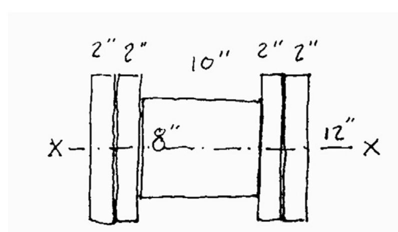
$t = 1/2''$

$t_{tr} = 1/2 \times n_{ST}$

$= 1/2\,(20) = 10.''$

WOOD

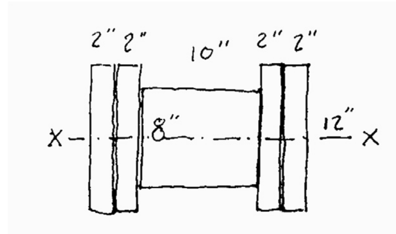
$t = t_{tr} = 2''$



Transformed Section

Analysis Example cont.:

2. Construct a transformed section.

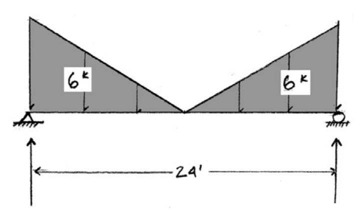


3. Calculate the Centroid and the Moment of Inertia for the transformed section.

$$I_{tr} = \frac{8\ 12^3}{12} + \frac{10\ 8^3}{12}$$
$$= 1152 + 426$$
$$= 1578\ in^4$$

Analysis Example cont.:

Find the maximum moment.



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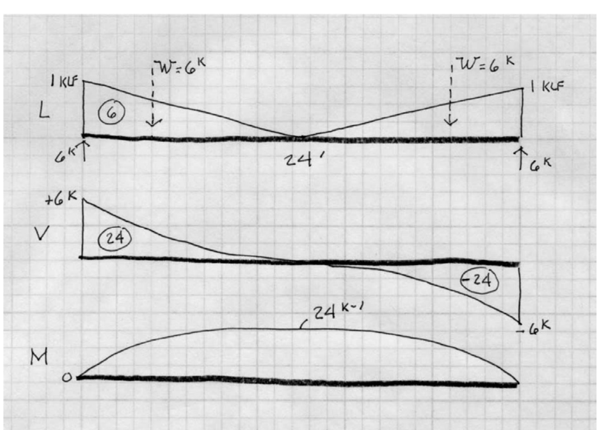
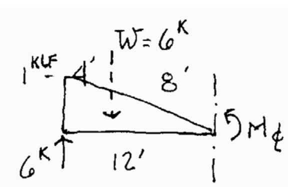




$$\Sigma M_{@¢}: 0 =$$
$$6(12') - 6(8') - M_¢ = 0$$
$$M_¢ = 24^{K-1}$$

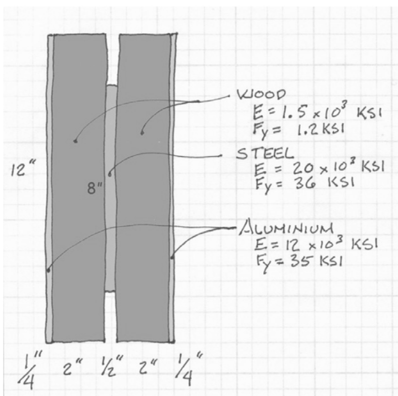
By diagrams or by summing moments

Analysis Example cont.:

4. Calculate the stress for each material using stress equation with the transformed moment of inertia.

$$f_b = \frac{Mc\ n}{I_{tr}}$$

Compare the stress in each material to limits of yield stress or the safe allowable stress.



$$f_{AL} = \frac{Mc(n)}{I_{tr}} = \frac{24(12)(6'')8}{1578} = 8.76 \text{ KSI} \quad (f_y \approx 35 \text{ KSI})$$

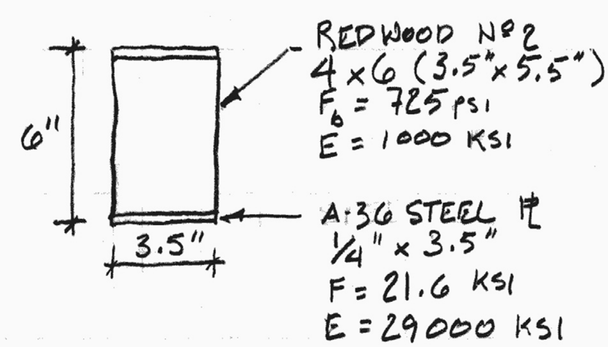
$$f_{ST} = \frac{Mc(n)}{I_{tr}} = \frac{24(12)(4'')20}{1578} = 14.6 \text{ KSI} \quad (f_y \approx 36 \text{ KSI})$$

$$f_{WD} = \frac{Mcn}{I_{tr}} = \frac{24(12)(6'')1.0}{1578} = 1.09 \text{ KSI} \quad (f_y \approx 1.5)$$

Capacity Analysis (ASD) Flexure

Given
- Dimensions
- Material

Required
- Load capacity



1. Determine the modular ratio. It is usually more convenient to transform the stiffer material.

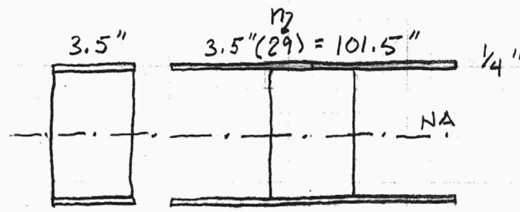
$$n = \frac{E_s}{E_w} = \frac{29000}{1000} = 29$$

Capacity Analysis (cont.)

2. Construct the transformed section. Multiply all widths of the transformed material by n. The depths remain unchanged.

3. Calculate the transformed moment of inertia, I_{tr} .

$$I_{tr} = \sum I + \sum Ad^2$$



$$I_w = \frac{3.5(5.5)^3}{12} = 48.53 \text{ in}^4$$

$$I_s = 2\left[\frac{101.5(0.25)^3}{12} + 25.375(2.875)^2\right]$$

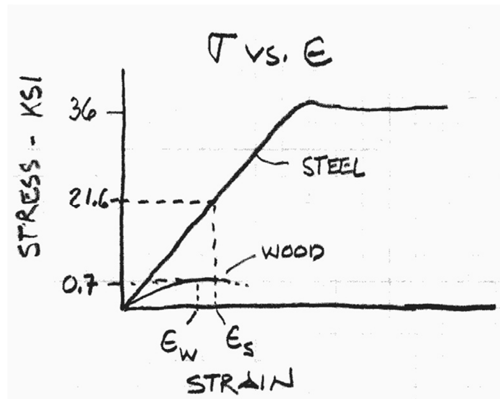
$$I_s = 2\left[0.132 + 209.74\right] = 419.7 \text{ in}^4$$

$$I_{TR} = 48.83 + 419.7 = 468.3 \text{ in}^3$$

Capacity Analysis (cont.)

4. Calculate the allowable strain based on the allowable stress for the material.

$$\varepsilon_{allow} = \frac{F_{allow}}{E}$$



$$\epsilon = \frac{\sigma}{E}$$

$$\epsilon_W = \frac{725}{1000000} = 0.000725$$

$$\epsilon_S = \frac{21.6}{29000} = 0.000745$$

Capacity Analysis (cont.)

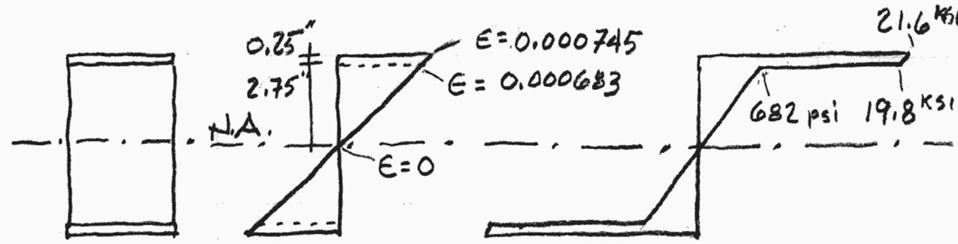
5. Construct a strain diagram to find which of the two materials will reach its limit first. The diagram should be linear, and neither material may exceed its allowable limit.

Allowable Strains:

$$\epsilon = \frac{\sigma}{E}$$

$$\epsilon_w = \frac{725}{1000000} = 0.000725$$

$$\epsilon_s = \frac{21.6}{29000} = 0.000745$$



$$\frac{\epsilon}{c} : \frac{0.000745}{3.0''} : \frac{\epsilon_w}{2.75''}$$

$$\epsilon_w = 0.000683$$

$$\sigma = E\epsilon$$

$$f_w = 1000000(0.000683)$$

$$f_w = 682 \text{ psi}$$

$$f_s = 29000(0.000683)$$

$$f_s = 19.8 \text{ KSI}$$

Capacity Analysis (cont.)

6. The allowable moments (load capacity) may now be determined based on the stress of either material. Either stress should give the same moment if the strain diagram from step 5 is compatible with the stress diagram (they align and allowables are not exceeded).

$$M_s = \frac{f_s I_{TR}}{c\,n} = \frac{21.6\,(468.3)}{3\,(29)} = 116.2^{\,K-''}$$

$$M_w = \frac{f_w I_{TR}}{c} = \frac{0.682\,(468.3)}{2.75''} = 116.1^{\,K-''}$$

7. Alternatively, the controlling moment can be found without the strain investigation by using the maximum allowable stress for each material in the moment-stress equation. The **lower momen**t will be the first failure point and the controlling material.

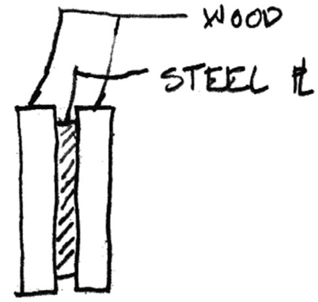
$$M_s = \frac{F_s I_{TR}}{c\,n} = \frac{21.6\,(468.3)}{3\,(29)} = 116.2^{\,K-''} \leftarrow$$

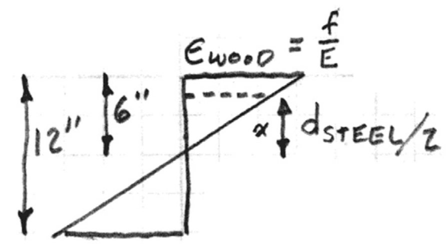
$$M_w = \frac{F_w I_{TR}}{c} = \frac{.725(468.3)}{2.75''} = 123.5^{\,K-''}$$

Design Procedure:

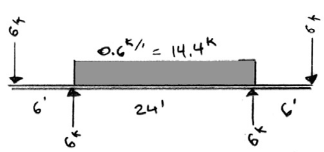
Given: Span and load conditions
Material properties
Wood dimensions
Req'd: Steel plate dimensions

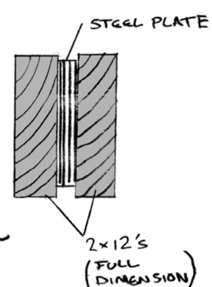
1. Determine the required moment.
2. Find the moment capacity of the wood.
3. Determine the required capacity for steel.
4. Based on strain compatibility with wood, find the largest d for steel where $\epsilon_s < \epsilon_{allow}$.
5. Calculate the required section modulus for the steel plate.
6. Using d from step 4. calculate b (width of plate).
7. Choose final steel plate based on available sizes and check total capacity of the beam.





Design Example:

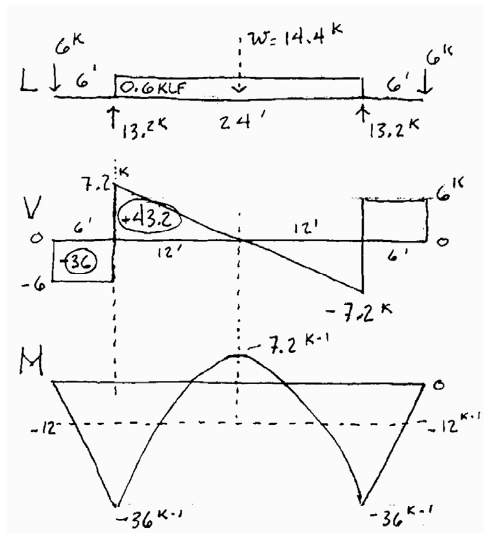




WOOD: E = 2000 K.S.I f_{all} = 1.5 K.S.I
STEEL: E = 30000 K.S.I f_{all} = 18 K.S.I

(A) DETERMINE THE DIMENSIONS OF THE STEEL PLATE REQUIRED FOR NEGATIVE MOMENT.

(B) DETERMINE THE LENGTH OF THE PLATES REQUIRED FOR NEGATIVE AND POSITIVE MOMENT.



1. Determine the required moment.
2. Find the moment capacity of the wood.
3. Determine the required capacity for steel.

WOOD

$b = 2''$ $d = 12''$

$S_x = \frac{bd^2}{6} = \frac{2(144)}{6} = 48 \text{ in}^3$

$\times$ 2 pcs. $S_{WOOD} = 96 \text{ in}^3$

$M_{WOOD} = F_b S_x$

$= 1.5^{KSI}\ 96 \text{ in}^3 = 144^{K-''}$

$= 12^{K-'}$

$M_{TOTAL} = M_{WOOD} + M_{STEEL} = 36^{K-'}$

$M_{STEEL} = 36^{K-'} - 12^{K-'} = 24^{K-'}$

Design Example cont:

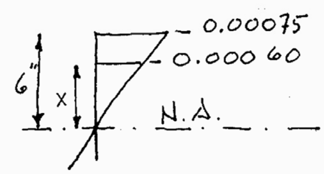
4. Based on strain compatibility with wood, find the largest d for steel where $\in_s \le \in_{ALLOW.}$

ALLOWABLE STRAINS

$$\epsilon_w = \frac{f}{E} = \frac{1.5}{2000} = 0.00075$$

$$\epsilon_s = \frac{f}{E} = \frac{18}{30000} = 0.00060$$

STRAIN DIAGRAM

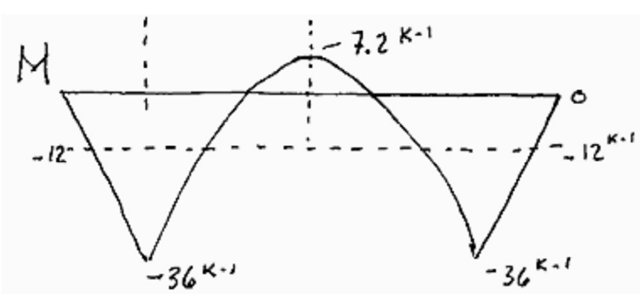


$$\frac{75}{6} : \frac{60}{x} \quad x = 4.8''$$

$\therefore$ d STEEL PLATE = 9.6"

Design Example cont:

5. Calculate the required section modulus for the steel plate.
6. Using d from step 4. calculate b (width of plate).
7. Choose final steel plate based on available sizes and check total capacity of the beam.



STEEL

$M_{STEEL} = 24^{K\text{-}'} = 288^{K\text{-}''}$

$F_{ST} = 18^{KSI}$ (GIVEN)

$$S_x = \frac{M}{F} = \frac{288}{18} = 16\ in^3$$

STEEL PLATE

$$S_x \text{ REQ'D} = 16\ in^3 = \frac{bd^2}{6}$$

$$b = \frac{S_x\, 6}{d^2} = \frac{16(6)}{9.6^2} = 1.042''$$

ROUND TO $\frac{1}{8}$" = $1\frac{1}{8}$" (SAFE)

$\therefore$ USE

$9.5'' \times 1\frac{1}{8}''$

$S_x = 16.9\ in^3$

Design Example cont:

8. Determine required length and location of plate.

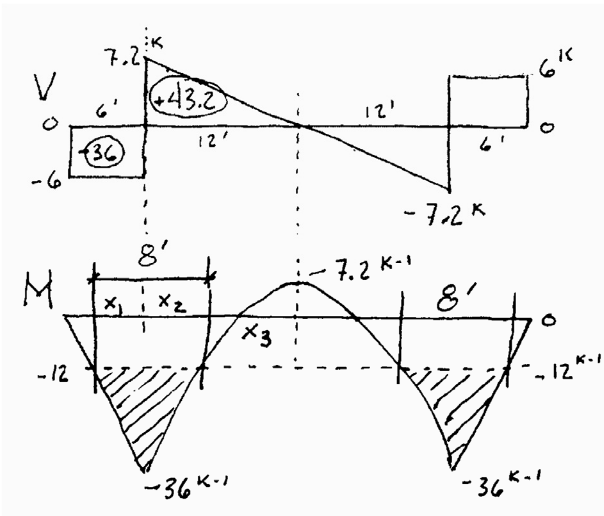
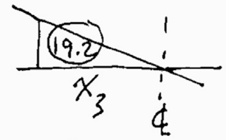


PLATE LENGTH

$\frac{36}{24} \quad \frac{6}{x_1} \quad x_1 = 4'$

SHEAR AREA = 24

$43.2 - 24 = 19.2$



$\frac{bd}{2} = 19.2$

$\frac{x_3 \left(\frac{7.2}{12} x_3\right)}{2} = 19.2$

$x_3^2 = 64 \quad x_3 = 8'$

$x_2 = 12 - x_3 = 4'$

$\therefore$ PLATE LENGTH = 8'

Applications:

Renovation in Edina, Minnesota

Four 2x8 LVLs, with two 1/2" steel plates. 18 FT span
Original house from 1949
Renovation in 2006
Engineer: Paul Voigt

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Applications:

Renovation

Chris Withers House, Reading, UK 2007
Architect: Chris Owens, Owens Galliver
Engineer: Allan Barnes

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