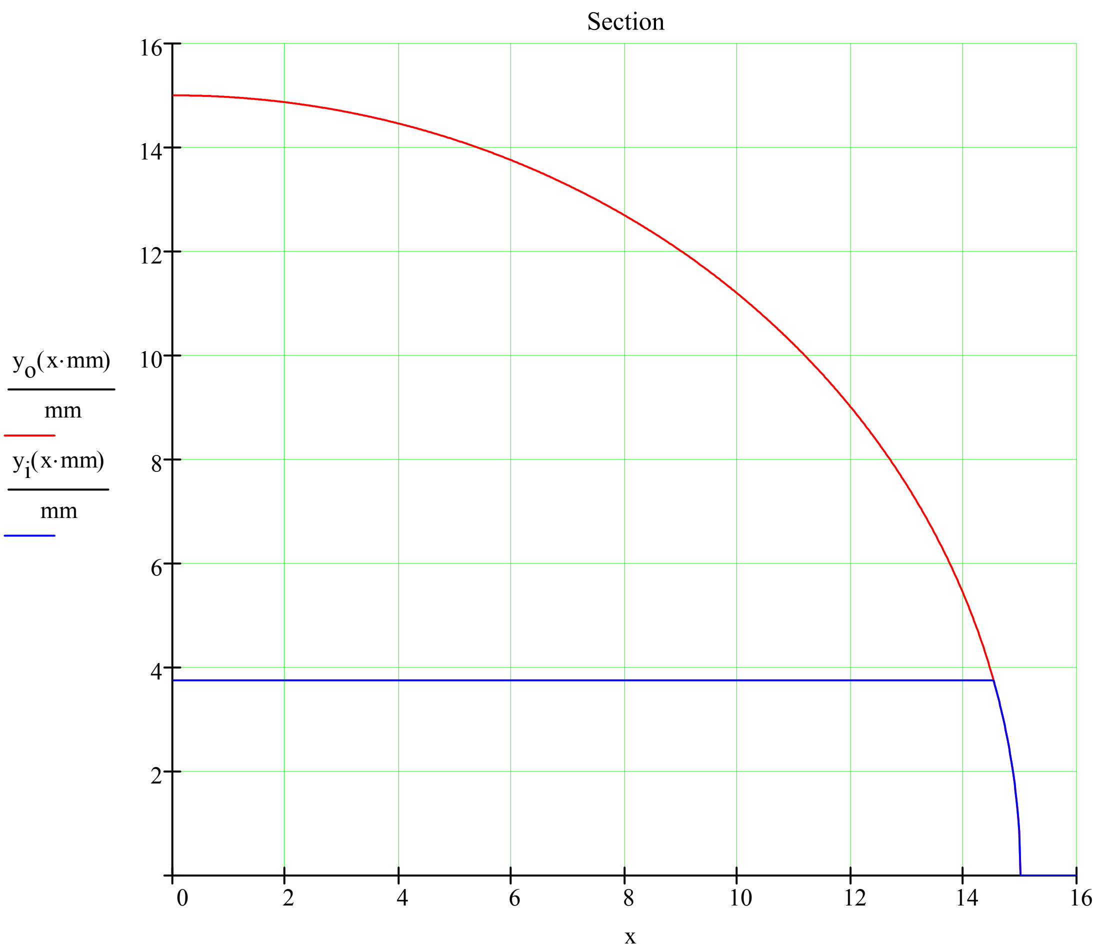
$R := 15 \cdot mm$ ... outer shaft radius

$w := 7.5 \cdot mm$ ... slot width

$y_o(x) := \mathrm{Re}\left(\sqrt{R^2 - x^2}\right)$ ... outer radius function

$y_i(x) := \min\left(\frac{w}{2}, y_o(x)\right)$ ... slot function

$L^2 + \left(\frac{t}{2}\right)^2 = R^2$ $L := \sqrt{R^2 - \left(\frac{w}{2}\right)^2}$ $L = 14.524\,mm$ ... intersection distance



$$\frac{(x-a)}{(L-a)}$$ ... linearly increasing component, for contact loading

$$\frac{\left(2\cdot\sqrt{R^2-x^2}-w\right)}{(2\cdot R-w)}$$ ... thickness factor, 1 at x=0 and 0 at x=L

$$\cos\left[\frac{\pi}{2}\cdot\frac{(2\cdot y-w)}{(2\cdot R-w)}\right]$$ ... change in shear stress direction through thickness

$$\tau(x,y,a,ks) := ks\cdot\frac{(x-a)}{(L-a)}\cdot\frac{\left(2\cdot\sqrt{R^2-x^2}-w\right)}{(2\cdot R-w)}\cdot\cos\left[\frac{\pi}{2}\cdot\frac{(2\cdot y-w)}{(2\cdot R-w)}\right]\cdot(x\geq a)$$ ... shear stress function

$dP = \tau\cdot dA$ ... elemental shear force

$dT = x\cdot dP$ ... elemental torque

$$dT = ks\cdot x\cdot\frac{(x-a)}{(L-a)}\cdot\frac{\left(2\cdot\sqrt{R^2-x^2}-w\right)}{(2\cdot R-w)}\cdot\cos\left[\frac{\pi}{2}\cdot\frac{(2\cdot y-w)}{(2\cdot R-w)}\right]\cdot dA$$ ... substituting

Integrating over section area and equating to half of the applied torque Ta

$$\frac{Ta}{2} = \frac{ks}{(2\cdot R-w)\cdot(L-a)}\cdot\int_a^L\int_{0.5\cdot w}^{\sqrt{R^2-x^2}} x\cdot(x-a)\cdot\left(2\cdot\sqrt{R^2-x^2}-w\right)\cdot\cos\left[\frac{\pi}{2}\cdot\frac{(2\cdot y-w)}{(2\cdot R-w)}\right]dy\,dx$$

$Ta := 200\cdot N\cdot m$ ... applied torque

$MPa := 10^6\cdot Pa$ ... setting stress units

$$ks(a) := \frac{Ta\cdot(2\cdot R-w)\cdot(L-a)}{2\cdot\int_a^L\int_{0.5\cdot w}^{\sqrt{R^2-x^2}} x\cdot(x-a)\cdot\left(2\cdot\sqrt{R^2-x^2}-w\right)\cdot\cos\left[\frac{\pi}{2}\cdot\frac{(2\cdot y-w)}{(2\cdot R-w)}\right]dy\,dx}$$ ... rearranged for ks

When start of contact is at middle of contact line, i.e. x = 0

$ks_0 := ks(0 \cdot mm) \qquad ks_0 = 487.1\,MPa$

$Ps_0 := \int_0^L \int_{0.5 \cdot w}^{\sqrt{R^2 - x^2}} \tau(x, y, 0mm, ks_0)\,dy\,dx \qquad Ps_0 = 13061\,N \qquad \frac{Ta}{2 \cdot Ps_0} = 7.656\,mm$

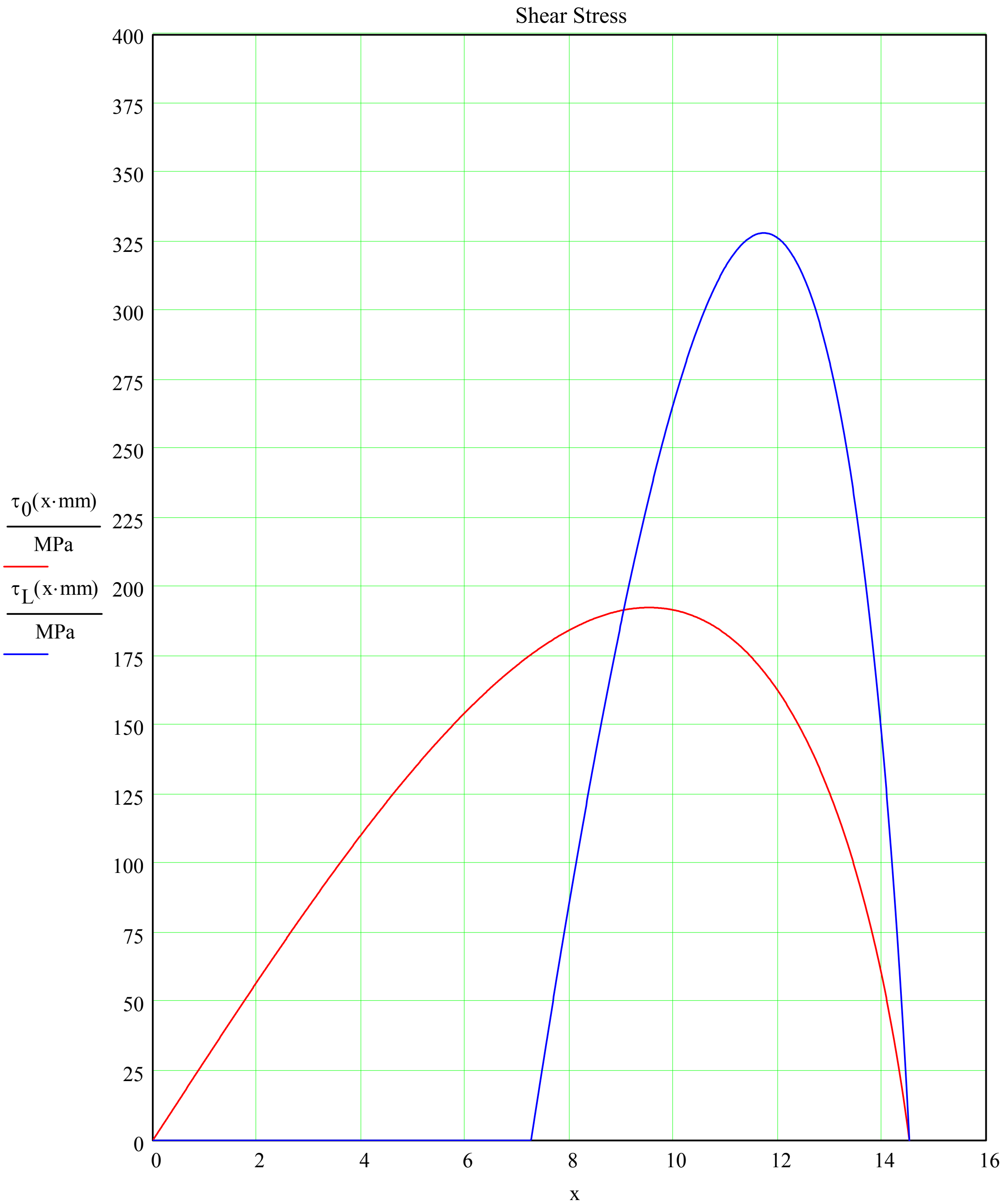
$\tau_0(x) := \tau(x, 0 \cdot mm, 0 \cdot mm, ks_0)$ ... shear stress function

When start of contact is half way along contact line, i.e. x = L/2

$ks_L := ks(0.5 \cdot L) \qquad ks_L = 1237.7\,MPa$

$Ps_L := \int_0^L \int_{0.5 \cdot w}^{\sqrt{R^2 - x^2}} \tau(x, y, 0.5 \cdot L, ks_L)\,dy\,dx \qquad Ps_L = 9261\,N \qquad \frac{Ta}{2 \cdot Ps_L} = 10.798\,mm$

$\tau_L(x) := \tau(x, 0 \cdot mm, 0.5 \cdot L, ks_L)$ ... shear stress function


Shear Stress
400
375
350
325
300
275
250
225
200
175
150
125
100
75
50
25
0
0
2
4
6
8
10
12
14
16
x
$\frac{\tau_0(x \cdot mm)}{MPa}$
$\frac{\tau_L(x \cdot mm)}{MPa}$