CONTRIBUTION TO THE ANALYSIS OF SEEPAGE EFFECTS IN BACKFILLS

by

PROFESSOR HAMILTON GRAY, Sc.D., S.M.

SYNOPSIS

An analytic solution for a type of seepage commonly occurring in the backfill of retaining walls leads to a mathematical expression for water pressure on the lower boundary of the potential "sliding wedge" of earth. This pressure is the function of the angle α between the wedge boundary and the vertical, and may be written: $\frac{P_w}{\frac{1}{2}\gamma_w H^2} = F(\alpha)$ where γ_w is the unit weight of water and H the depth of the backfill. An accompanying graph of $F(\alpha)$ eliminates the necessity for plotting a flow net and for sealing the water pressure on the lower boundary of the wedge.

Une solution analytique pour une sorte de suintement se produisant fréquemment dans le remblai des murs de soutènement amène à une expression mathématique pour la pression d'eau à la limite inférieure de la masse triangulaire potentielle de glissement de terre. Cette pression est fonction de l'angle α entre la limite de la masse et la verticale et peut s'écrire $\frac{P_w}{\frac{1}{2}\gamma_w H^2} = F(\alpha)$ dans laquelle γ_w est l'unité de poids de l'eau et H la profondeur du remblai. Un graphique ci-joint de $F(\alpha)$ supprime le besoin de relever le réseau de lignes de courant et tracer à l'échelle la pression d'eau sur la limite inférieure de la masse.

Attention was first drawn to the influence of drainage on the pressure against retaining walls by Terzaghi (1936).[1] In particular, he showed that even when impoundment of water, and the consequent development of hydrostatic pressure, behind the wall was eliminated, the seepage pattern could, under certain conditions, reduce the sliding resistance of the soil mass and hence require a greater wall reaction in order to maintain equilibrium.

Since that time it seems probable that this seepage phenomenon has been taken into account in the analysis of relatively few walls. The reasons for avoiding the analysis are obviously:

(1) the extensive amount of time required to construct a reliable freehand flow net for this case;
(2) the need to determine hydraulic pressures from the flow net and to plot hydraulic pressure diagrams on potential surfaces of sliding in the backfill;
(3) the need to determine the areas of the water pressure diagrams by planimeter; and
(4) the tedious procedure required to analyse the effect of this water pressure on the amount of reaction required from the wall. In other words, the analysis appears to require more time and effort than the results would be likely to justify. Any means whereby this analysis may be simplified would therefore serve to encourage its more general use. For the case of a vertical wall sustaining a horizontal backfill, an exact solution of this seepage condition is available which wholly eliminates the first three steps enumerated above.

In Fig. 1 the x-axis represents the horizontal surface of a cohesionless permeable homogeneous and isotropic backfill on which water is falling at such a rate as to engender a steady flow toward the vertical back drain of a retaining wall. The boundary between this drain and the backfill coincides with the y-axis. The flow in the backfill is assumed to be in accordance with Darcy's law. Considerations of continuity in the field of flow then lead to Laplace's equation:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0$$

[1] See references on p. 170.

a solution of which will indicate the variation in hydraulic head, h, with position, x, y, provided this solution also satisfies the boundary conditions for this particular region of flow. The pertinent boundary conditions number three, and are as follows :—

(1) There will be no flow across the impervious horizontal base, $y = H$. This condition may be expressed by : $\frac{\partial h}{\partial y} = 0$ when $y = H$.

(2) Since seepage water will reach a properly designed drain in quantities inadequate to completely saturate the drain, it moves downward pressure-free through the highly permeable drain structure. Therefore the water pressure at the surface of the drain is merely that of the atmosphere. The hydraulic head in the drain is consequently proportional to the elevation $h = h_e = H - y$, measured for convenience above the impervious base ($y = H$), upon which the fill rests. Therefore the condition is $h = H - y$ when $x = 0$.

(3) The head along the entire positive x-axis is given by $h = H$. That is, $h = H$ when $y = 0$.

The head at any point in the field of flow ($x > 0$ and $0 < y < H$) can then be obtained by the familiar method of development in a Fourier series and can be expressed as follows :

$$h = H\left\{1 - \frac{8}{\pi^2}\sum_0^\infty \frac{(-1)^m}{(2m+1)^2}\,\epsilon^{-(2m+1)\frac{\pi x}{2H}}.\sin\,(2m+1)\frac{\pi y}{2H}\right\} \quad . \quad . \quad (1)$$

This expression satisfies Laplace's equation and all three boundary conditions and therefore is the solution for this particular seepage problem.

Typical curves for h = constant (equipotentials) are shown accurately plotted in Fig. 2. The corresponding stream lines are also shown in this Figure. These are given by setting ψ = constant in the expression :

$$\psi = H\left\{1 - \frac{8}{\pi^2}\sum_0^\infty \frac{(-1)^m}{(2m+1)^2}\,\epsilon^{-(2m+1)\frac{\pi x}{2H}}.\cos\,(2m+1)\,\frac{\pi y}{2H}\right\} \quad . \quad . \quad (2)$$

which is conjugate to 1. By means of these two formulae, a flow net consisting of mutually orthogonal equipotentials and stream lines, may be plotted to any degree of desired accuracy.

If one now considers the variation in head along planes $x = (H - y)\tan\alpha$, which radiate from point " a ", one has :

$$h = H\left\{1 - \frac{8}{\pi^2}\sum_0^\infty \frac{(-1)^m}{(2m+1)^2}\,\epsilon^{-(2m+1)\frac{\pi}{2}(1-\frac{y}{H})\tan\alpha}.\sin\,(2m+1)\,\frac{\pi y}{2H}\right\}$$

The fluid pressure head at any point is $h_p = h - h_e$ where h_e is the elevation head and is given by $h_e = H - y$. Therefore, the hydraulic pressure at any point on a plane radiating from " a " is given by :

$$\sigma_w = \gamma_w(h - H + y) = \gamma_w\left[y - \frac{8}{\pi^2}H\sum_0^\infty \frac{(-1)^m}{(2m+1)^2}\,\epsilon^{-(2m+1)\frac{\pi}{2}\left(1-\frac{y}{H}\right)\tan\alpha}.\sin\,2m+1)\,\frac{\pi y}{2H}\right] \quad . \quad . \quad (3)$$

The variation of this pressure along a particular plane, specifically that inclined at an angle of 53·6° ($\alpha = 36{\cdot}4°$) with the horizontal, is shown in Fig. 1. The maximum ordinate of the pressure curve, measured normal to the α-plane is for this case $\frac{\sigma_w}{\gamma_w H} = 0{\cdot}175$. If the pressure equation (3) is integrated over the length of any arbitrary radial plane the following result is obtained :

$$P_w = \int_0^{H \sec \alpha} \sigma_w \, dl = \sec \alpha \int_0^{H} \sigma_w \, dy$$

$$= \tfrac{1}{2}\gamma_w H^2 \left[\sec \alpha - \frac{32}{\pi^3} \cos \alpha \sum_0^\infty \frac{1}{(2m+1)^3} \left\{ \tan \alpha + (-1)^m \epsilon^{-(2m+1)\frac{\pi}{2}\tan \alpha} \right\} \right] \quad . \quad . \quad (4)$$

The ratio, $\frac{P_w}{\frac{1}{2}\gamma_w H^2}$, is a function only of the angle α. This function, $F(\alpha)$, which can be written:

$$F(\alpha) = \sec \alpha - \frac{32}{\pi^3} \cos \alpha \sum_0^\infty \frac{1}{(2m+1)^3} [\tan \alpha + (-1)^m \epsilon^{-(2m+1)\frac{\pi}{2}\tan \alpha}]$$

is shown plotted in Fig. 3. The resultant water force along this radial plane is of course directed normal to this plane.

The conventional method of analysis used by Coulomb can now be applied to the backfill. Referring to Fig. 4 one examines the conditions for equilibrium of the wedge of earth, *aob*,

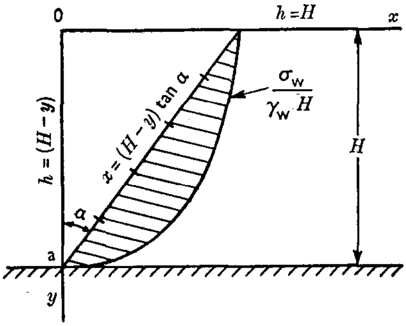

Fig. 1. Dimensions of backfill and water pressure on lower boundary of earth wedge

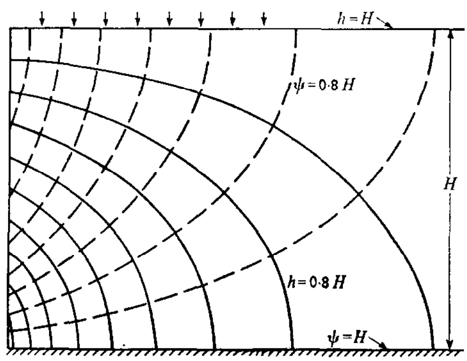

Fig. 2. Equipotentials and stream line

which is defined by the angle α. N and T represent the normal and tangential components of the " effective " or " intergranular " reaction on the inclined surface ab and Q is the external reaction required for stability. P_w is taken from Fig. 3. For equilibrium :

$$Q \cos \delta = P_w \cos \alpha + N \cos \alpha - T \sin \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and

$$W = Q \sin \delta + P_w \sin \alpha + N \sin \alpha + T \cos \alpha = \tfrac{1}{2}\gamma_w \frac{G + es}{1 + e} H^2 . \tan \alpha \quad . \quad . \quad (6)$$

Introducing the shearing properties of the fill: $T = N \tan \phi$, the equilibrium equations reduce to :

$$\frac{Q}{\frac{1}{2}\gamma_w H^2} = \left\{ \frac{G + es}{1 + e} \tan \alpha + F(\alpha) . (\cos \alpha . \tan [\alpha + \phi] - \sin \alpha) \right\} \Big/ \{\sin \delta + \cos \delta . \tan (\alpha + \phi)\} \quad (7)$$

For particular values of δ and ϕ this expression can be evaluated with the aid of Fig. 3 for various assumed values of α until the maximum thrust is found.

If $\delta = 0$ we have :

$$\frac{Q}{\frac{1}{2}\gamma_w H^2} = \frac{G + es}{1 + e} \tan \alpha . \cos (\alpha + \phi) + F(\alpha)\{\cos \alpha - \sin \alpha . \cot (\alpha + \phi)\} \quad . \quad . \quad (8)$$

Assuming for this special case that $G = 2{\cdot}65$, $e = 0{\cdot}65$ and $s = 1{\cdot}0$, and that $\phi = 30°$, 35° or 40°, successive trials show that the corresponding maximum values of $Q/\frac{1}{2}\gamma_w H^2$ are as given in Table 1.

The figures in column (5) are based upon the gross weight (solid plus water) of the completely saturated soil. Under such conditions, it is possible that capillary tension in the

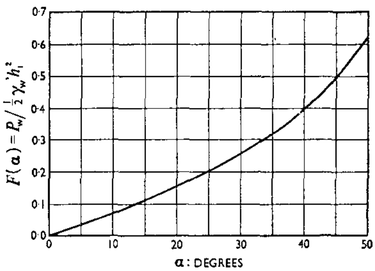

Fig. 3.

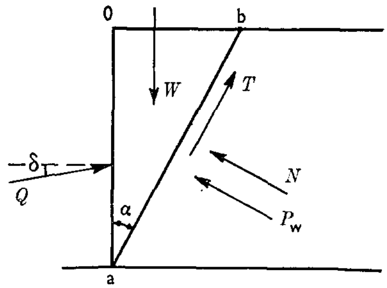

Fig. 4. Forces on sliding wedge

water will serve to reduce thrust on the wall by virtue of the " apparent " cohesion which it imparts to a granular soil. Hence the actual thrusts on the wall could be less than those indicated in column (5) and so that actual effect of seepage would be even greater than suggested by a comparison of columns (3) and (5).

It is not necessary to assume the backfill to be cohesionless, since no matter how the shearing properties of the fill are expressed, consideration of the equilibrium equations (5)

Table 1

(1)	With seepage (2)	(3)	No seepage (4)	(5)	(6)
ϕ	α_{crit}	$\frac{Q}{\frac{1}{2}\gamma_w H^2}$	α_{crit}	$\frac{Q}{\frac{1}{2}\gamma_w H^2}$	Increase : %
30°	35°	0·828	30°	0·667	24
35°	33°	0·705	27½°	0·542	31
40°	31°	0·595	25°	0·434	37

and (6) together with the shearing properties will lead to an expression analogous to equation (7) which involves $F(\alpha)$. With the aid of Fig. 3 it is relatively easy to evaluate the basic equation analogous to equation (7).

These increases are of the same order of magnitude as those reported by other investigators (Terzaghi, 1943 and Skempton, 1946) whose conclusions were apparently based on graphical flow nets and subsequent plotting and planimetering of the water-pressure distribution shown on Fig. 1. It appears that the effect of seepage becomes more pronounced, relatively, as the quality of backfill improves ; that is as ϕ increases.

REFERENCES

SKEMPTON, A. W., 1946. "Earth pressure and the stability of slopes." Second lecture in "The principles and application of soil mechanics." *Instn civ. Engrs*, pp. 31–61.

TERZAGHI, KARL, 1936. "The effect of the type of drainage of retaining walls on the earth pressure." *Proc. 1st Int. Conf. Soil Mech.*, 1 : 215–216.

TERZAGHI, KARL, 1943. "Theoretical soil mechanics." *Wiley, New York*, p. 250.