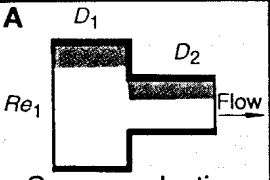
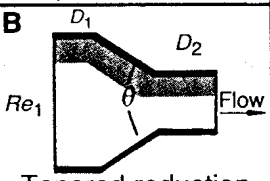
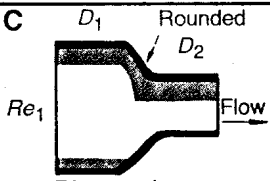
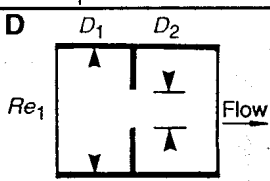
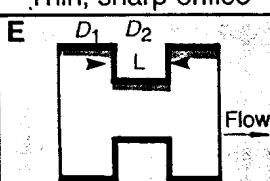
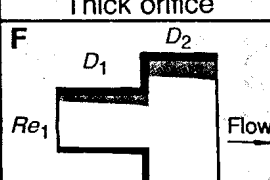
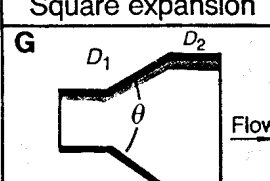
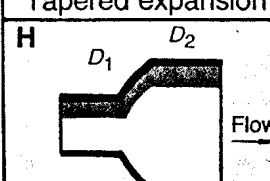


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Head loss caused by a change in pipe size at a fitting can be predicted by means of the appropriate K correlation

Fitting; Case	Inlet $Re$	$K$ — Based on inlet Velocity Head*
<b>A</b>  Square reduction	$Re_1 \leq 2,500$  $Re_1 > 2,500$	$K = \left[ 1.2 + \frac{160}{Re_1} \right] \left[ \left( \frac{D_1}{D_2} \right)^4 - 1 \right]$  $K = \left[ 0.6 + 0.48f_1 \right] \left( \frac{D_1}{D_2} \right)^2 \left[ \left( \frac{D_1}{D_2} \right)^2 - 1 \right]^{\dagger}$
<b>B</b>  Tapered reduction	All	Multiply $K$ from Case A by: $\sqrt{\sin\left(\frac{\theta}{2}\right)}$ For $45^\circ < \theta < 180^\circ$ Or $\left[ 1.6 \sin\left(\frac{\theta}{2}\right) \right]$ For $0^\circ < \theta < 45^\circ$
<b>C</b>  Pipe reducer	All	$K = \left[ 0.1 + \frac{50}{Re_1} \right] \left[ \left( \frac{D_1}{D_2} \right)^4 - 1 \right]$
<b>D</b>  Thin, sharp orifice	$Re_1 \leq 2,500$  $Re_1 > 2,500$	$K = \left[ 2.72 + \left( \frac{D_2}{D_1} \right)^2 \left( \frac{120}{Re_1} - 1 \right) \right] \left[ 1 - \left( \frac{D_2}{D_1} \right)^2 \right] \times \left[ \left( \frac{D_1}{D_2} \right)^4 - 1 \right]$  $K = \left[ 2.72 - \left( \frac{D_2}{D_1} \right)^2 \left( \frac{4,000}{Re_1} \right) \right] \left[ 1 - \left( \frac{D_2}{D_1} \right)^2 \right] \times \left[ \left( \frac{D_1}{D_2} \right)^4 - 1 \right]$
<b>E</b>  Thick orifice	All	If $L/D_2 > 5$ , use Case A and Case F; Otherwise multiply $K$ from Case D by: $\left\{ 0.584 + \frac{0.0936}{(L/D)^{1.5} + 0.225} \right\}$
<b>F</b>  Square expansion	$Re_1 \leq 4,000$  $Re_1 > 4,000$	$K = 2 \left[ 1 - \left( \frac{D_1}{D_2} \right)^4 \right]$  $K = \left[ 1 + 0.8f_1 \right] \left\{ \left[ 1 - \left( \frac{D_1}{D_2} \right)^2 \right]^2 \right\}^{\dagger}$
<b>G</b>  Tapered expansion	All	If $\theta > 45^\circ$ , use $K$ from Case F; otherwise multiply $K$ from Case F by: $\left[ 2.6 \sin\left(\frac{\theta}{2}\right) \right]$
<b>H</b>  Pipe reducer	All	Use the $K$ for Case F

\* $K$  based on outlet velocity head =  $(K \text{ from above}) \left( \frac{D_2}{D_1} \right)^4$  † $f_1$  from Moody chart or from:  $\frac{1}{\sqrt{f_1}} = -2 \log \left[ \frac{\epsilon}{3.7 D_1} + \frac{2.51}{Re_1 \sqrt{f_1}} \right]$

the system, sum up all the calculated  $\Delta H_s$ , and add to this total the changes in head due to elevation differences and shaft work done — to obtain the total head loss for the system. (The change in kinetic energy is covered in the entrance and exit losses.) The procedure is illustrated later by an example.

### Velocity head

In this article, fitting head loss is expressed in terms of velocity heads. A "velocity head" is defined as  $V^2/2g_c$ , with  $V$  being the average velocity in the pipe, and  $g_c$  the conversion between force and mass. The kinetic energy per unit volume of the flowing fluid is one velocity head in plug flow (uniform velocity profile).

For any velocity profile, the true velocity is the integral of the local velocity heads across the pipe diameter. By convention, however, it is, instead, found by first determining the average velocity — i.e., by dividing the volumetric flowrate by the cross-sectional area of the pipe, and then multiplying by a correction factor. In laminar flow, the factor is exactly two. In turbulent flow, the factor is a function of both the Reynolds number and the pipe roughness; the simple factor  $(1 + 0.8f)$  includes both of these effects. See the box for an explanation of  $f$ , the Darcy, or Moody, friction factor.

Note that when the pipe size changes, the velocity head itself changes (apart from the head loss caused by turbulence as the flow adjusts to the change). Because velocity is inversely proportional to flow area (and, thus, to diameter squared), and  $K$  is inversely proportional to velocity squared:

$$K_2 = K_1 (D_2/D_1)^4 \quad (2)$$

All  $K$ s given are based on inlet conditions.

Potential energy provides the flowing fluid with kinetic energy at the pipe entrance. However, that kinetic energy can later be recovered. So, by convention, the entrance is not charged with this acceleration loss. This means that a measured pipe pressure will be lower than calculated pressure by one velocity head. If the kinetic energy is not recovered at the pipe exit, the exit is charged with one velocity head loss — even though a pressure gage before

the exit may read the same as another after it.

**Case A: Square reduction** — Although this case for sharp, square reductions in pipe diameter was developed for a concentric reduction, the correlations should also be adequate for an eccentric reduction.

At the size change of the fitting, the flowing fluid must accelerate toward the center to enter the smaller pipe. This directs the velocity radially inward, which causes the fluid to overshoot — i.e., the fluid squeezes through an area smaller than the outlet pipe for some distance downstream of the reduction. The point of minimum flow area is called the vena contracta. Most of the "entrance" loss is actually a loss caused by the fluid expanding from this point back into the full pipe diameter.

When the velocity upstream of the fitting is negligible, the measured "permanent" head loss will be 60% of the downstream (average) velocity head in fully turbulent flow. Correcting this discrepancy in head loss caused by the upstream velocity by the factor  $1 - (D_2/D_1)^2$  provides a good fit to the data of Freeman [2] and Weisbach [8]. The head loss is further adjusted for a non-uniform velocity profile by  $1 + 0.8f$ , and converted to upstream velocity by means of  $(D_1/D_2)^4$ :

$$K = (0.6 + 0.48f)[(D_1/D_2)^4 - (D_1/D_2)^2] \quad (3)$$

In the case of laminar flow, the kinetic energy recovery after the vena contracta is negligible, and the correction for the upstream velocity becomes  $1 - (D_2/D_1)^4$ . A good fit with the data of Kaye and Rosen [5] is obtained by combining Eq. (4) with a basic factor of  $2(0.6 + 80/Re)$ :

$$K = (1.2 + 160/Re)[(D_1/D_2)^4 - 1] \quad (4)$$

**Case B: Tapered reduction** — Tapering a reduction tends to reduce the velocity in the vena contracta, reducing the head loss. The listed correction factors, from Crane Co. [9], fit the data well for concentric, conical reducers having sharp transition points.

**Case C: Rounded reduction** — Well-rounded inlets tend to minimize velocity overshooting, and thus the inlet head loss. Because the common

## FRICION FACTORS:

### THE CORRELATIONS FOR $f$ AND HOW TO SOLVE THEM

The Darcy, or Moody, friction factor has the value of  $64/Re$  when flow is laminar. (Its value is four times that of the Fanning friction factor.)

When flow is turbulent, the friction factor can be found by means of the well-known Moody diagram. It can also be calculated from the Colebrook equation, which is the basis for the Moody diagram:

$$1/f_4 = -2 \log (\epsilon/3.7D + 2.51/Re f_4)$$

Solving the Colebrook equation for  $f$  involves trial and error. There are many variant equations that can be solved explicitly. However, the trial-and-error method does converge rapidly. The simple algorithm gives at least one additional significant figure in  $f$  per iteration.

welded pipe reducer does not have abrupt transitions, its reduction loss is small. The correlation given in the table looks reasonable, but no published data are available for checking its accuracy.

**Case D: Thin orifices** — The permanent head loss from a sharp orifice is greater than the combined loss from an entrance (Case A) and an exit (Case F), because more of the kinetic energy is lost with a large abrupt expansion than at a vena contracta. However, some of the energy is recovered, so the head differential will be less when measured some distance downstream of an orifice than when measured just behind it.

The correlation for laminar flow fits the available data [6] when the orifice Reynolds number ( $Re_2$ ) is less than 2,100. The correlation for turbulent flow is very good when the pipe Reynolds number ( $Re_1$ ) exceeds 4,000. In the transition area, especially when the pipe flow is laminar and the orifice flow is turbulent, no simple correlation is reliable.

**Case E: Thick orifices** — A thick orifice is frequently installed when erosion is possible. If the orifice length-to-diameter ratio ( $L/D$ ) exceeds 0.05, the permanent head loss will be lower than that given by either Case D correlation. As the  $L/D$  ratio increases, the loss approaches the sum of Cases A and F. The correction factor given in Case E fits one set of data in the  $L/D$  range from 0.05 to 5. This factor has not been verified for laminar flow.

**Case F: Square expansion** — A laminar-flowing fluid going through a square expansion loses the difference in kinetic energy — i.e., the kinetic-energy recovery is negligible. As discussed previously, the kinetic energy in laminar flow is  $2(V^2/2g_c)$ . Simple algebra then yields the Case F laminar correlation.

In turbulent flow, a significant part of the kinetic energy is recovered for modest size changes. An assumption of conservation of momentum will result in the Case F correlation for turbulent flow. The equation fits the reported data [1] very well.

**Case G: Tapered expansion** — Expanding a fluid by means of a long cone

## Nomenclature

$D$	Inside pipe dia., ft
$d$	Inside pipe dia., in.
$f$	Moody friction factor, dimensionless
$g$	Mass/force conversion factor, (lb-mass/lb-force)(ft/s <sup>2</sup> )
$\Delta H$	Head loss, feet of fluid
$K$	Excess head loss for fitting, dimensionless
$L$	Length of pipe, ft
$Re$	Reynolds number, $VD\rho/\mu$ , dimensionless
$V$	Average velocity, ft/s
$\epsilon$	Roughness of pipe wall, ft
$\theta$	Total cone angle, deg.
$\mu$	Fluid viscosity, lb/(ft)(s)
$\rho$	Fluid density, lb/ft <sup>3</sup>

## Subscripts:

- 1 Pipe inlet
- 2 Pipe outlet

from  $D_1$  to  $D_2$  can substantially reduce the head loss. When the total cone angle ( $\theta$ ) is less than 45 deg., the Crane [9] correction factor used for Case G fits quite well. Flow separation, instead of smooth expansion, occurs if  $\theta$  exceeds 45 deg., and the loss is the same as in Case F.

If the cone is very long, the frictional loss may be important. Because it is not included in  $K$ , this loss must be calculated separately. A good approximation can be based on the mean square  $\Delta H$ . This is done by calculating the frictional loss as if the diameter of all of the cone were the same as the inlet diameter. Repeat the calculation, assuming the cone's diameter to always be the same as the outlet diameter. The mean-square loss is the square root of the product of the two calculated losses.

**Case H: Rounded expansion**—The common welded pipe reducer increases in size so quickly that flow separates. Therefore, it can be treated as a square expansion (i.e., Case F).

#### Example illustrates procedure

Calculate the required pump head for the piping system shown in the diagram on p. 90. Flow = 75,000 lb/h; fluid density = 64.30 lb/ft<sup>3</sup> (1.03 g/ml); viscosity =  $8.40 \times 10^{-4}$  lb/(ft)(s) (or 1.25 cP); piping is schedule 40, with  $\epsilon$  = 0.00015 ft and I.D./O.D. ratios for nominal 2-in., 3-in. and 4-in. pipe = 2.067/2.375, 3.068/3.500, 4.026/4.500; for all elbows, radius/diameter = 1.5.

**Head loss in the 4-in. suction line:**  $D = 0.3355$  ft (or 4.026 in.); flow area = 0.0884 ft<sup>2</sup>; velocity = 75,000/(64.30)(3,600)(0.0884) = 3.665 ft/s; velocity

head =  $V^2/2g_c = (3.665)^2/(2)(32.174) = 0.2087$  ft of fluid;  $\epsilon/D = 0.00015/0.3355 = 0.000447$ ; and  $f$  (via the Colebrook equation) = 0.0203.

$K$  values for fittings [4]:

Fitting	Number	$K_1$	$K_\infty$
Gate valve	1	500	0.15
Normal elbow	4	800	0.20
Total		3,700	0.95

Fitting  $K = 3,700/94,123 + 0.95(1 + 1/4.026) = 1.225$ .

Entrance  $K$  (converting Case A to downstream basis) =  $(0.6 + 0.48f)[1 - (D_2/D_1)^2]$ ; with  $D_2/D_1 = 0$ ,  $K = 0.6 + 0.48 \times 0.0203 = 0.610$ .

Reducer  $K$  (Case C) =  $[0.10 + (50/94,123)][(4.026/3.068)^4 - 1] = 0.198$ .

$\Sigma K = 1.225 + 0.610 + 0.198 = 2.033$ ; and  $fL/D = 0.0203 \times 80/0.3355 = 4.841$ .

$\Delta H = (\Sigma K + fL/D)(V^2/2g_c) = (2.033 + 4.841)(0.2087) = 1.435$  ft fluid.

(The 4-ft section of 3-in.-dia. pipe at the pump suction will be lumped together with the 3-in.-dia. discharge pipe.)

**Head loss in the 2-in. discharge line:**  $D = 0.17225$  ft (or 2.067 in.); flow area = 0.0233 ft<sup>2</sup>; velocity = 75,000/(64.30)(3,600)(0.0233) = 13.906 ft/s; velocity head =  $(13.906)^2/64.348 = 3.006$  ft fluid;  $Re = 13.906 \times 0.17225 \times 64.30/8.40 \times 10^{-4} = 183,355$ ;  $\epsilon/D = 0.00015/0.17225 = 0.000871$ ; and  $f = 0.0206$ .

Expansion  $K$  (Case H or F) =  $(1 + 0.8 \times 0.0206)[1 - (2.067/3.068)^2]^2 = 0.3032$ .

$\Delta H = (0.3032 + 0.0206 \times 3/0.17225)(3.006) = 1.990$  ft fluid.

**Head loss in the 3-in. discharge line:**  $D = 0.2557$  ft (or 3.068 in.); flow area = 0.0513 ft<sup>2</sup>; velocity =  $0.3240/0.0513 = 6.316$  ft/s; velocity head =  $(6.316)^2/64.348 = 0.620$  ft fluid;  $Re = 6.316 \times 0.2557 \times 64.30/8.40 \times 10^{-4} = 123,624$ ;  $\epsilon/D = 0.00015/0.2557 = 0.000587$ ; and  $f = 0.0202$ .

$K$  values for normal fittings [4]:

Fitting	Number	$K_1$	$K_\infty$
Normal elbow	4	800	0.20
Gate valve	1	500	0.15
Run-of-tee	1	150	0.05
Plug valve	1	1,000	0.25
Total		4,850	1.25

Normal fitting  $K = 4,850/123,624 + 1.25[1 + (1/3.068)] = 1.697$ .

Orifice  $K$  (Case D) =  $[2.72 - (2.000/3.068)^2(4,000/123,624)][1 - (2.000/3.068)^2][(3.068/2.000)^4 - 1] = (2.706)(0.575)(4.537) = 7.061$ .

Exit  $K$  (use Case F with  $D_1/D_2 = 0$ ) =  $(1 + 0.8 \times 0.0202)(1 - 0)^2 = 1.016$ .

$\Sigma K = 1.697 + 7.061 + 1.016 = 9.774$ .

$fL/D = 0.0202(120 + 4)/0.2557 = 9.796$ .

$\Delta H = (9.774 + 9.796)(0.619) = 12.114$ .

Frictional + fitting losses = 1.435 + 1.990 + 12.114 = 15.54 ft.

Elevation change = 60 - 40 = 20 ft.

**Required pump head** = 20 + 15.54 = 35.5 ft fluid.

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