Pipe flow rate, pressure and friction factor in isothermal compressible gas flow

The flow in long pipelines closely approximates isothermal conditions. The pressure drop in such lines is often large compared to inlet pressure and solution of this problem is outside the limitations of the Darcy equation. An accurate solution is made by compressible isothermal equation where weight flow w in [kg/s] is:

$$w^2 = \left[\frac{A^2}{v_1\left(\frac{f\,L}{D} + 2\ln\frac{p_1}{p_2}\right)}\right]\left[\frac{p_1^2 - p_2^2}{p_1}\right]$$

where is:

$p_{1,2}$ - pressure on the begging and on the end of pipe line; w - mass flow rate; v_1 - specific volume; f - friction factor; L - pipe length; D - internal pipe diameter; A - pipe cross section area;

Substituting specific volume using equation of state pressure drop due to friction can be written as:

$$p_1^2 - p_2^2 = Z_m R T\left(\frac{w}{A}\right)^2\left(f\,\frac{L}{D} + 2\ln\frac{p_1}{p_2}\right)$$

where is:

$p_{1,2}$ - pressure on the begging and on the end of pipe line; w - mass flow rate; Z_m - mean compressibility factor; R - gas constant; T - temperature; f - friction factor; L - pipe length; D - internal pipe diameter; A - pipe cross section area;

Both two above equations are developed with following assumptions: