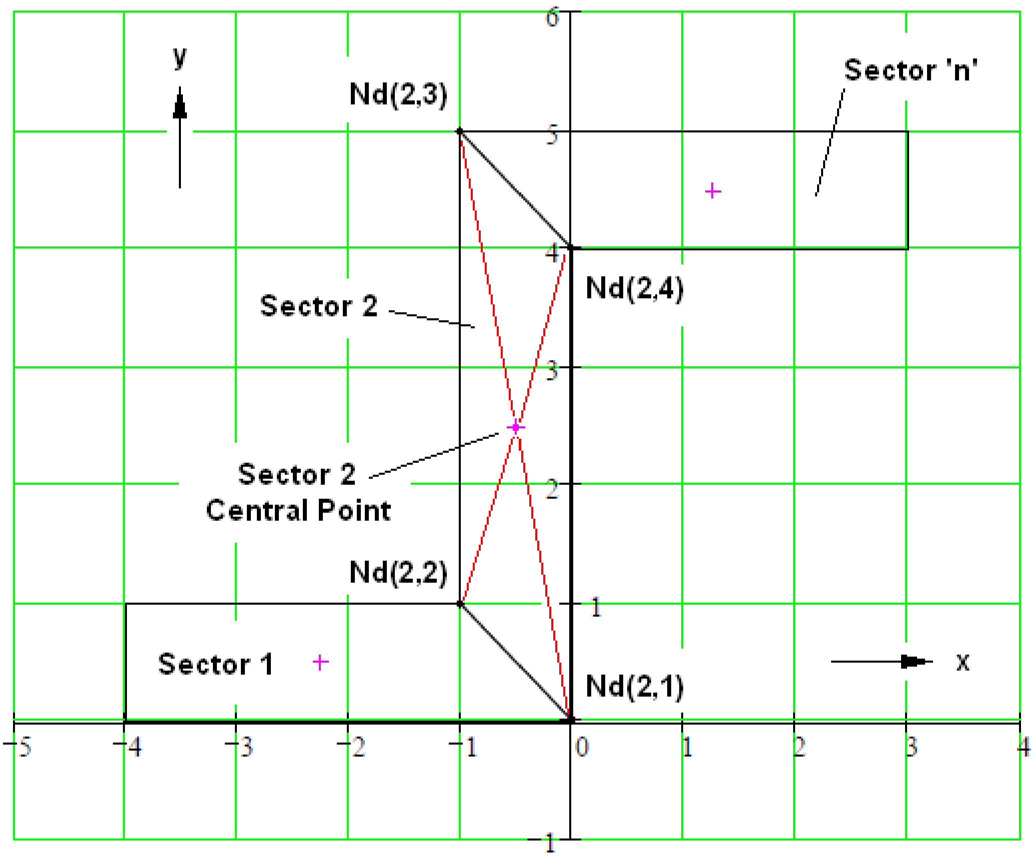
L Section Analysis



Note, a central point is identified for each sector. This point must ensure the triangles defined by the sector profile nodes and the central point are bounded by the sector profile.

For sector 'n', let $x_s(n)$ stack the x coordinates for each node, from 1 to 4, and add the first to the end, effectively closing the sector profile. The sector 'n' array will therefore consist of five (5) 'x' coordinates.

For sector 'n', let $y_s(n)$ stack the y coordinates for each node, from 1 to 4, and add the first to the end, effectively closing the sector profile. The sector 'n' array will therefore consist of five (5) 'y' coordinates.

Node coordinates... Node - Nd(n,i) where ... n = sector number
i = node number

Sector triangles defined by ... Nd(n,i), Nd(n,i+1) & Centre(n)

Note, sectors are to be ascending in order and to be adjacent to the previous.

Applied Bending Moments

Axial force ... $P_z := 1000 \cdot N$

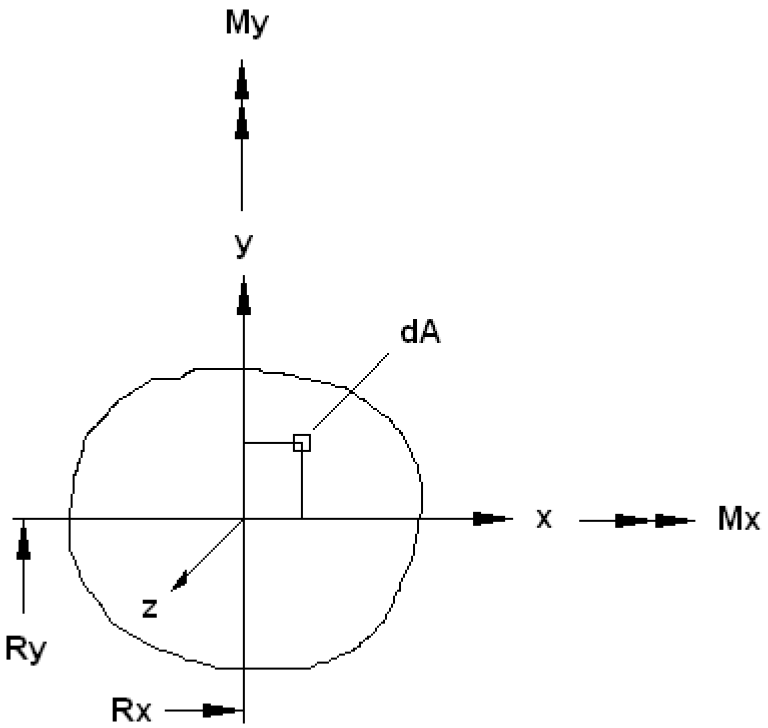
Shear force in the x axis direction ... $V_x := 1000 \cdot N$

Shear force in the y axis direction ... $V_y := 0 \cdot N$

Moment about x axis ... $M_x := 0 \cdot N \cdot m$

Moment about y axis ... $M_y := 1 \cdot N \cdot m$

Applied moment sign convention



Rx and Ry are radius of curvatures.

Section geometry input. Four nodal coordinates for each of the sector profiles making up the complete section are to be given.

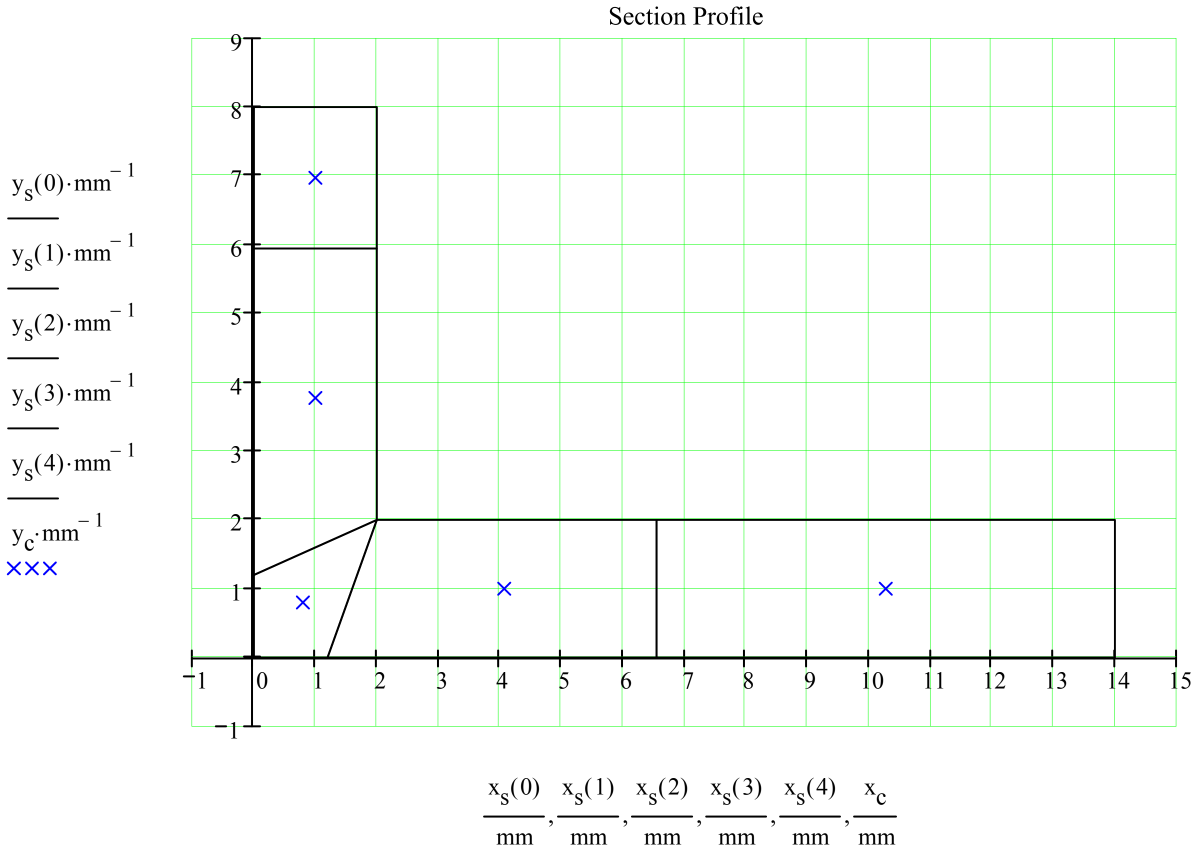
$xy_{se} :=$

"Node No."	"1"	"1"	"2"	"2"	"3"	"3"	"4"	"4"
"Node Coord. (mm)"	"x"	"y"	"x"	"y"	"x"	"y"	"x"	"y"
"Sector No 1"	0	5.944	0	8	2	8	2	5.944
"Sector No 2"	0	1.2	0	5.944	2	5.944	2	2
"Sector No 3"	0	0	0	1.2	2	2	1.2	0
"Sector No 4"	1.2	0	2	2	6.546	2	6.546	0
"Sector No 5"	6.546	0	6.546	2	14	2	14	0

Number of rectangular sectors ... $n_s = 5$

Sector centres ... $x_{c_n} = \frac{1}{4} \cdot \sum_{j=0}^{3} x_s(n)_j$ $\qquad x_c = \begin{pmatrix} 1.00 \\ 1.00 \\ 0.80 \\ 4.07 \\ 10.27 \end{pmatrix} mm$

$y_{c_n} = \frac{1}{4} \cdot \sum_{j=0}^{3} y_s(n)_j$ $\qquad y_c = \begin{pmatrix} 6.97 \\ 3.77 \\ 0.80 \\ 1.00 \\ 1.00 \end{pmatrix} mm$



Note, adjust the graph $y_s(n)$ and $x_s(n)$ entries depending on the number of sectors.

Application of The Equimomental System

... n = section sector number (0 to 'n_s -1')
i = sector triangle number (0 to 3, i.e. four triangles)
j = triangle length number (0 to 2, i.e. three lengths)

x and y co-ordinates for each sector ... triangle (4 triangles per sector)

$$x_t(n,i) := \begin{pmatrix} x_s(n)_i \\ x_s(n)_{i+1} \\ x_{c_n} \\ x_s(n)_i \end{pmatrix} \qquad y_t(n,i) := \begin{pmatrix} y_s(n)_i \\ y_s(n)_{i+1} \\ y_{c_n} \\ y_s(n)_i \end{pmatrix}$$

Individual sector ... line length

$$l_t(n,i,j) := \sqrt{\left(x_t(n,i)_{j+1} - x_t(n,i)_j\right)^2 + \left(y_t(n,i)_{j+1} - y_t(n,i)_j\right)^2}$$

Area using 'Hero's formula' ...

$$s_t(n,i) := \frac{1}{2}\cdot\sum_{j=0}^{2} l_t(n,i,j)$$ ... semi-perimeter

Sector 'n', triangle 'i' area ...

$$A_t(n,i) := \sqrt{s_t(n,i)\cdot\left(s_t(n,i) - l_t(n,i,0)\right)\cdot\left(s_t(n,i) - l_t(n,i,1)\right)\cdot\left(s_t(n,i) - l_t(n,i,2)\right)}$$

$$A_{s_n} := \sum_{i=0}^{3} A_t(n,i) \qquad A_s = \begin{pmatrix} 4.112 \\ 8.688 \\ 2.400 \\ 9.892 \\ 14.908 \end{pmatrix} mm^2$$ ... individual sector areas

$$\Sigma A := \sum A_s \qquad \Sigma A = 40.00\,mm^2$$

$$\Sigma Ax := \frac{1}{6} \cdot \sum_{n=0}^{n_s-1} \sum_{i=0}^{3} \sum_{j=0}^{2} \left[A_t(n,i) \cdot \left(x_t(n,i)_j + x_t(n,i)_{j+1}\right)\right] \qquad \Sigma Ax = 208.000\ mm^3$$

$$\Sigma Ay := \frac{1}{6} \cdot \sum_{n=0}^{n_s-1} \sum_{i=0}^{3} \sum_{j=0}^{2} \left[A_t(n,i) \cdot \left(y_t(n,i)_j + y_t(n,i)_{j+1}\right)\right] \qquad \Sigma Ay = 88.000\ mm^3$$

Centre of area for section ... x co-ordinate ... $x'_s := \frac{\Sigma Ax}{\Sigma A}$ $x'_s = 5.2000\ mm$

y co-ordinate ... $y'_s := \frac{\Sigma Ay}{\Sigma A}$ $y'_s = 2.2000\ mm$

1st moment of area ... about the x axis for each sector

$$Ax_n := \frac{1}{3} \cdot \sum_{i=0}^{3} \sum_{j=0}^{2} \left[A_t(n,i) \cdot \left[\frac{1}{2} \cdot \left(y_t(n,i)_j \ldots + y_t(n,i)_{j+1}\right) - y'_s\right]\right] \qquad Ax = \begin{pmatrix} 19.622 \\ 13.604 \\ -3.200 \\ -12.137 \\ -17.890 \end{pmatrix} mm^3$$

$$Q_{x_n} := \sum_{k=0}^{n} Ax_k \qquad Q_x = \begin{pmatrix} 19.6225 \\ 33.2267 \\ 30.0267 \\ 17.8896 \\ -0.0000 \end{pmatrix} mm^3$$

1st moment of area ... about the y axis for each sector

$$Ay_n := \frac{1}{3} \cdot \sum_{i=0}^{3} \sum_{j=0}^{2} \left[A_t(n,i) \cdot \left[\frac{1}{2} \cdot \left(x_t(n,i)_j \ldots + x_t(n,i)_{j+1}\right) - x'_s\right]\right] \qquad Ay = \begin{pmatrix} -17.270 \\ -36.756 \\ -10.400 \\ -11.202 \\ 75.628 \end{pmatrix} mm^3$$

$$Q_{y_n} := \sum_{k=0}^{n} Ay_k \qquad Q_y = \begin{pmatrix} -17.2704 \\ -54.0267 \\ -64.4267 \\ -75.6283 \\ -0.0000 \end{pmatrix} mm^3$$

2nd moment of area about x axis for rectangular sectors ...

$$Ixx := \frac{1}{3}\cdot\sum_{n=0}^{n_s-1}\sum_{i=0}^{3}\sum_{j=0}^{2}\left[A_t(n,i)\cdot\left[\frac{1}{2}\cdot\left(y_t(n,i)_j + y_t(n,i)_{j+1}\right) - y'_s\right]^2\right]$$

$Ixx = 179.73\,mm^4$

2nd moment of area about y axis for rectangular sectors ...

$$Iyy := \frac{1}{3}\cdot\sum_{n=0}^{n_s-1}\sum_{i=0}^{3}\sum_{j=0}^{2}\left[A_t(n,i)\cdot\left[\frac{1}{2}\cdot\left(x_t(n,i)_j + x_t(n,i)_{j+1}\right) - x'_s\right]^2\right]$$

$Iyy = 763.73\,mm^4$

Product moment of area for rectangular sectors ...

$$Ixy := \frac{1}{3}\cdot\sum_{n=0}^{n_s-1}\sum_{i=0}^{3}\sum_{j=0}^{2}\left[A_t(n,i)\cdot\left[\frac{1}{2}\cdot\left(y_t(n,i)_j + y_t(n,i)_{j+1}\right) - y'_s\right]\cdot\left[\frac{1}{2}\cdot\left(x_t(n,i)_j + x_t(n,i)_{j+1}\right) - x'_s\right]\right]$$

$Ixy = -201.60\,mm^4$

Principal axes ...

Ref. Mechanics of Engineering Materials, by P.P.Benham & R.J.Crawford, 1987
Appendix A - Properties of Areas

Minimum principal moment of area ...

$$I_{min} := \frac{1}{2}\cdot(Ixx + Iyy) - \sqrt{\frac{1}{4}\cdot(Ixx - Iyy)^2 + Ixy^2} \qquad I_{min} = 116.90\,mm^4$$

Maximum principal moment of area ...

$$I_{max} := \frac{1}{2}\cdot(Ixx + Iyy) + \sqrt{\frac{1}{4}\cdot(Ixx - Iyy)^2 + Ixy^2} \qquad I_{max} = 826.57\,mm^4$$

Max. / Min. principal axes ...

$$\theta_p := \frac{1}{2}\cdot atan\left(\frac{2\cdot Ixy}{Iyy - Ixx}\right) \qquad \theta_p = -17.31\,deg$$

$$y_{max}(x) := y'_s - \frac{x - x'_s}{\tan(\theta_p)}$$ ... max. principal axis

$$y_{min}(x) := y'_s + \tan(\theta_p)\cdot(x - x'_s)$$ ... min. principal axis

Note, x and y coordinates are taken relative to x'_s, y'_s.

strain in z direction ... $\varepsilon_z = \frac{y}{Ry} + \frac{x}{Rx}$

stress in z direction ... $\sigma_z = E \cdot \left(\frac{y}{Ry} + \frac{x}{Rx} \right)$

Moment about x axis ... $dMx = y \cdot dPz$ ... where ... $dPz = \sigma_z \cdot dA$

$$dMx = y \cdot \sigma_z \cdot dA$$

$$dMx = E \cdot \left(\frac{y^2}{Ry} + \frac{x \cdot y}{Rx} \right) \cdot dA$$

$$Mx = \frac{E}{Ry} \cdot \int y^2 \, dA + \frac{E}{Rx} \cdot \int x \cdot y \, dA$$

$Mx = \frac{E}{Ry} \cdot Ixx + \frac{E}{Rx} \cdot Ixy$ ... or ... $Mx = k_y \cdot Ixx + k_x \cdot Ixy$

Moment about y axis ... $-dMy = x \cdot dPz$

$$-dMy = x \cdot \sigma_z \cdot dA$$

$$-dMy = E \cdot \left(\frac{x \cdot y}{Ry} + \frac{x^2}{Rx} \right) \cdot dA$$

$$-My = \frac{E}{Ry} \cdot \int x \cdot y \, dA + \frac{E}{Rx} \cdot \int x^2 \, dA$$

$-My = \frac{E}{Ry} \cdot Ixy + \frac{E}{Rx} \cdot Iyy$ ... or ... $-My = k_y \cdot Ixy + k_x \cdot Iyy$

In matrix format ... $\begin{pmatrix} Ixy & Ixx \\ Iyy & Ixy \end{pmatrix} \cdot \begin{pmatrix} k_x \\ k_y \end{pmatrix} = \begin{pmatrix} Mx \\ -My \end{pmatrix}$

$$\begin{pmatrix} k_x \\ k_y \end{pmatrix} = \begin{pmatrix} Ixy & Ixx \\ Iyy & Ixy \end{pmatrix}^{-1} \cdot \begin{pmatrix} Mx \\ -My \end{pmatrix}$$

$$\begin{pmatrix} k_x \\ k_y \end{pmatrix} = \begin{bmatrix} \dfrac{-(Ixy \cdot Mx + Ixx \cdot My)}{Ixx \cdot Iyy - Ixy^2} \\ \dfrac{Iyy \cdot Mx + Ixy \cdot My}{Ixx \cdot Iyy - Ixy^2} \end{bmatrix}$$

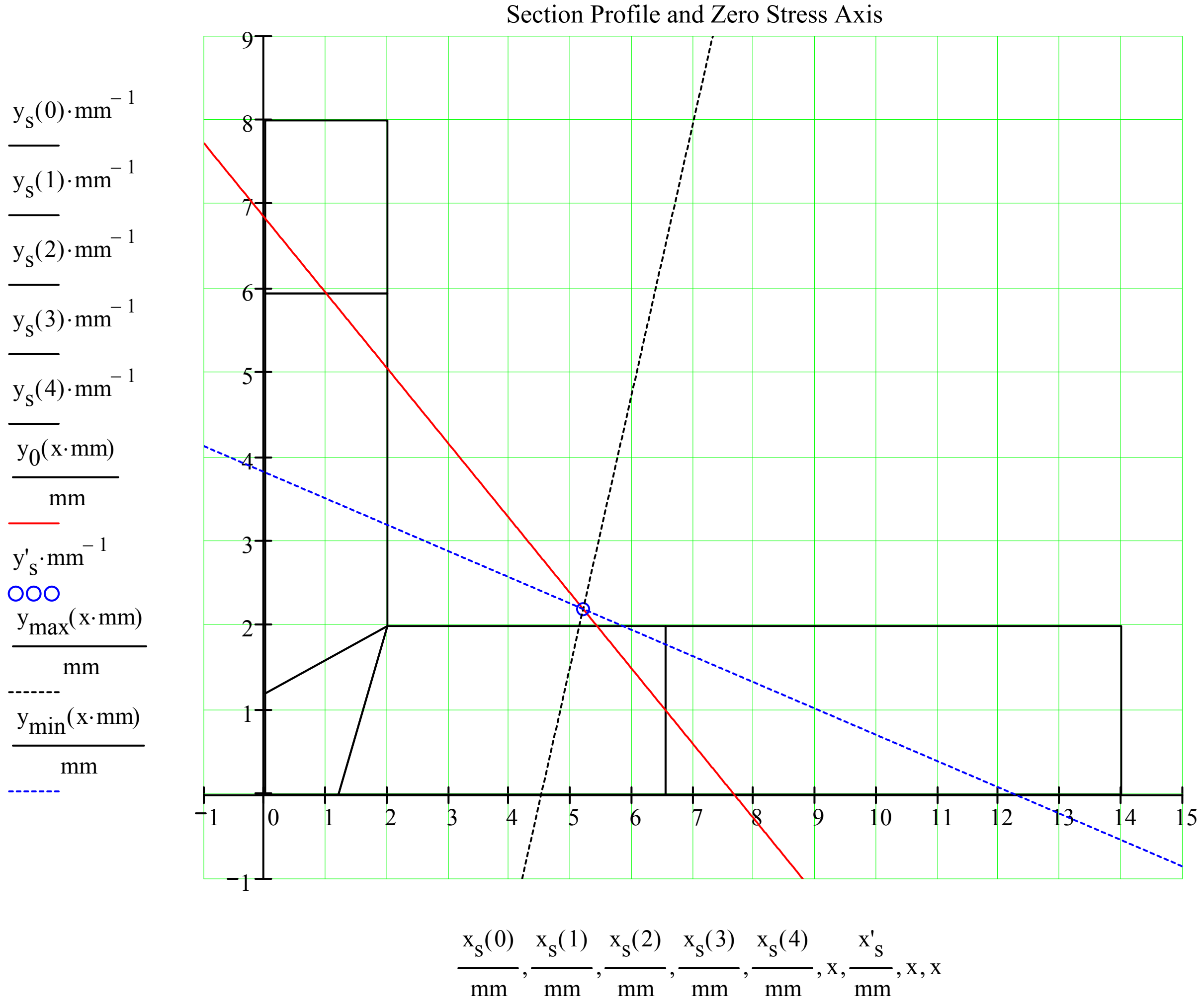
Section bending stress ... $\sigma_{zb}(x, y) = E \cdot \left[\dfrac{(y - y'_s)}{Ry} + \dfrac{(x - x'_s)}{Rx} \right]$ ... relative to x'_s, y'_s

$$\sigma_{zb}(x, y) = \frac{(Iyy \cdot M_x + Ixy \cdot M_y) \cdot (y - y'_s)}{Ixx \cdot Iyy - Ixy^2} - \frac{(Ixy \cdot M_x + Ixx \cdot M_y) \cdot (x - x'_s)}{Ixx \cdot Iyy - Ixy^2}$$

Equating stress to zero and rearranging for y.

Zero stress axis ... $y_0(x) := y'_s + \dfrac{(Ixy \cdot M_x + Ixx \cdot M_y)}{(Iyy \cdot M_x + Ixy \cdot M_y)} \cdot (x - x'_s)$

$$\operatorname{atan}\left[\frac{(Ixy \cdot M_x + Ixx \cdot M_y)}{(Iyy \cdot M_x + Ixy \cdot M_y)} \right] = -41.72\,deg$$



Note, adjust the graph $y_s(n)$ and $x_s(n)$ entries depending on the number of sectors.

Coordinates ... (mm)

$xy_{se} =$

"Node No."	"1"	"1"	"2"	"2"	"3"	"3"	"4"	"4"
"Node Coord. (mm)"	"x"	"y"	"x"	"y"	"x"	"y"	"x"	"y"
"Sector No 1"	0	5.944	0	8	2	8	2	5.944
"Sector No 2"	0	1.2	0	5.944	2	5.944	2	2
"Sector No 3"	0	0	0	1.2	2	2	1.2	0
"Sector No 4"	1.2	0	2	2	6.546	2	6.546	0
"Sector No 5"	6.546	0	6.546	2	14	2	14	0

Bending stress ... (MPa)

$\sigma_{zb} =$

"Node No"	1	2	3	4
"Sector No 1"	1.86	−2.43	−6.15	−1.86
"Sector No 2"	11.76	1.86	−1.86	6.37
"Sector No 3"	14.26	11.76	6.37	12.03
"Sector No 4"	12.03	6.37	−2.09	2.09
"Sector No 5"	2.09	−2.09	−15.95	−11.78

Shear Flow

Ref. Niu, Chaper 6, Part (B), Eq.6.5.3 (Page 156)

$q = -(K_3 \cdot V_x - K_1 \cdot V_y) \cdot Q_y - (K_2 \cdot V_y - K_1 \cdot V_x) \cdot Q_x$ ... Eq. 6.5.3 - shear flow

where ... Q_x, Q_y - first moment of areas about the x and y axes, respectively

$$K_1 := \frac{Ixy}{Ixx \cdot Iyy - Ixy^2} \qquad K_1 = -2.0864 \times 10^{-3}\, mm^{-4}$$

$$K_2 := \frac{Iyy}{Ixx \cdot Iyy - Ixy^2} \qquad K_2 = 7.9040 \times 10^{-3}\, mm^{-4}$$

and ... $$K_3 := \frac{Ixx}{Ixx \cdot Iyy - Ixy^2} \qquad K_3 = 1.8601 \times 10^{-3}\, mm^{-4}$$

$q_s =$

"-"	"Shear Flow (N/mm)"
"Sector No 1"	−8.816
"Sector No 2"	31.171
"Sector No 3"	57.192
"Sector No 4"	103.351
"Sector No 5"	0.000

... shear flow at end of each segment
(+ve shear flow is -y and +x)

Axial stress ... $$\sigma_{za} := \frac{P_z}{\Sigma A} \qquad \sigma_{za} = 25.00\,MPa$$