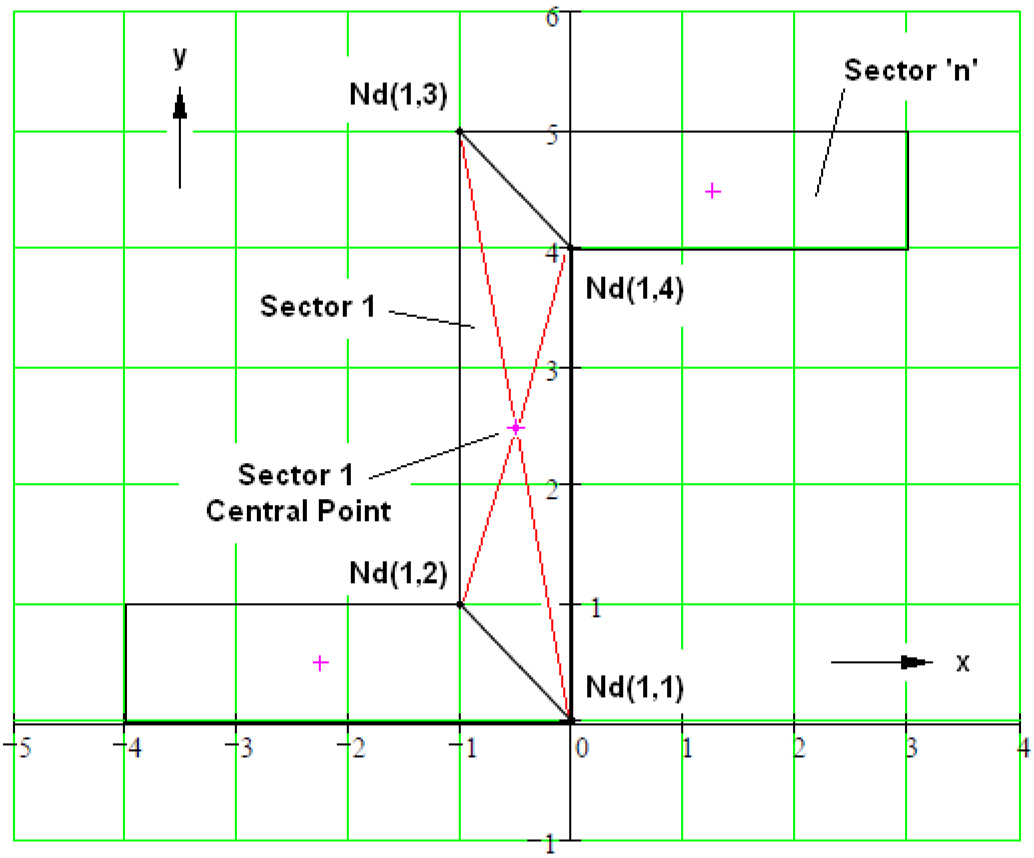
Section Analysis



Note, a central point is identified for each sector. This point must ensure the triangles defined by the sector profile nodes and the central point are bounded by the sector profile.

For sector 'n', let $x_n(n)$ stack the x coordinates for each node, from 1 to 4, and add the first to the end, effectively closing the sector profile. The sector 'n' array will therefore consist of five (5) 'x' coordinates.

For sector 'n', let $y_n(n)$ stack the y coordinates for each node, from 1 to 4, and add the first to the end, effectively closing the sector profile. The sector 'n' array will therefore consist of five (5) 'y' coordinates.

Node coordinates... Node - Nd(n,i) where ... n = sector number
i = node number

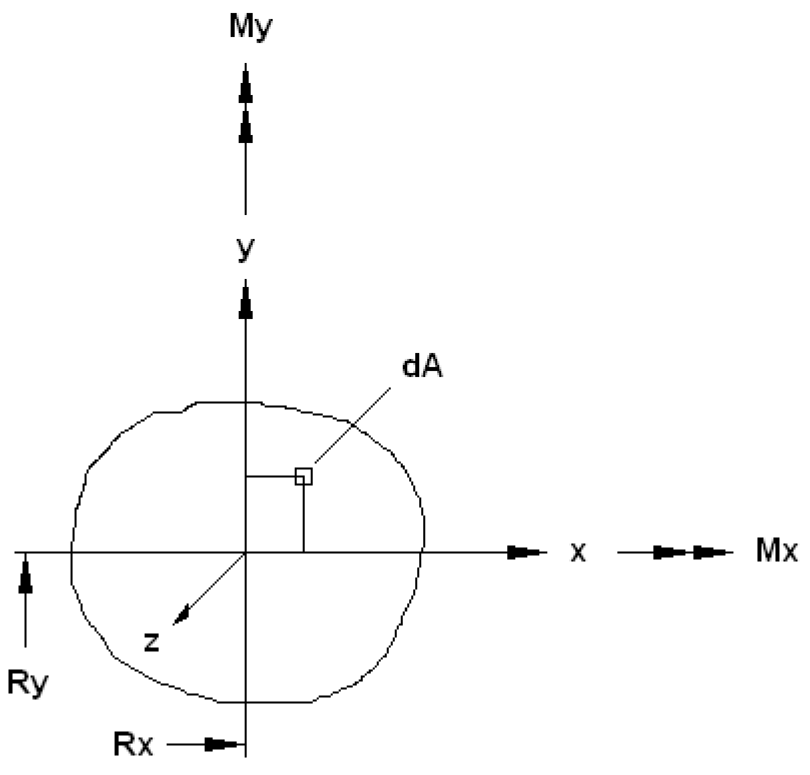
Sector triangles defined by ... Nd(n,i), Nd(n,i+1) & Centre(n)

Applied Bending Moments

Moment about x axis ... $Mx := 0 \cdot N \cdot m$

Moment about y axis ... $My := 1 \cdot N \cdot m$

Applied moment sign convention



Rx and Ry are radius of curvatures.

Section geometry input. Four nodal coordinates for each of the sector profiles making up the complete section are to be given.

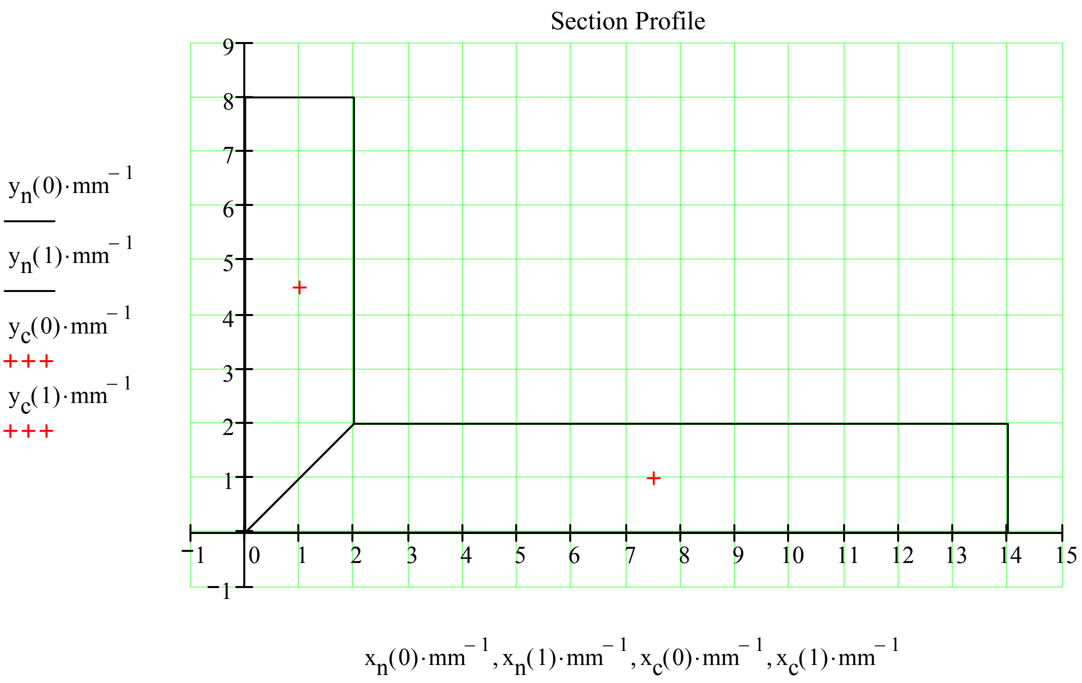
$xy_{en} :=$

"Node No"	"-"	"Sector 1"	"Sector 2"
1	"X (mm)"	0	0
1	"Y (mm)"	0	0
2	"X (mm)"	0	2
2	"Y (mm)"	8	2
3	"X (mm)"	2	14
3	"Y (mm)"	8	2
4	"X (mm)"	2	14
4	"Y (mm)"	2	0

Number of rectangular sectors ... $n_s = 2$

Sector centres ... $x_c(n) = \frac{1}{4} \cdot \sum_{j=0}^{3} x_n(n)_j$ $\quad x_c(n) = \begin{pmatrix} 1.00 \\ 7.50 \end{pmatrix} \text{mm}$

$y_c(n) = \frac{1}{4} \cdot \sum_{j=0}^{3} y_n(n)_j$ $\quad y_c(n) = \begin{pmatrix} 4.50 \\ 1.00 \end{pmatrix} \text{mm}$



Application of The Equimomental System

Note, adjust the graph $y_n(n)$ and $x_n(n)$ entries depending on the number of sectors.

x and y co-ordinates for each sector ... triangle (4 triangles per sector)

$$x_t(n,i) := \begin{pmatrix} x_n(n)_i \\ x_n(n)_{i+1} \\ x_c(n) \\ x_n(n)_i \end{pmatrix} \qquad y_t(n,i) := \begin{pmatrix} y_n(n)_i \\ y_n(n)_{i+1} \\ y_c(n) \\ y_n(n)_i \end{pmatrix}$$

... n = section sector number (0 to 'n_s -1')
i = sector triangle number (0 to 3, i.e. four triangles)

Individual sector ... line length

$$l_t(n,i,j) := \sqrt{\left(x_t(n,i)_{j+1} - x_t(n,i)_j\right)^2 + \left(y_t(n,i)_{j+1} - y_t(n,i)_j\right)^2}$$

... n = section sector number (0 to 'n_s -1')
i = sector triangle number (0 to 3, i.e. four triangles)
j = triangle length number (0 to 2, i.e. three lengths)

Area using 'Hero's formula' ...

$$s_t(n,i) := \frac{1}{2}\cdot\sum_{j=0}^{2} l_t(n,i,j) \qquad s_t(0,0) = 8.1249\,\text{mm}$$

... semi-perimeter

Sector 'n', triangle 'i' area ...

$$A_t(n,i) := \sqrt{s_t(n,i)\cdot\left(s_t(n,i) - l_t(n,i,0)\right)\cdot\left(s_t(n,i) - l_t(n,i,1)\right)\cdot\left(s_t(n,i) - l_t(n,i,2)\right)}$$

$$A_s(n) := \sum_{i=0}^{3} A_t(n,i) \qquad A_s(n) = \begin{pmatrix} 14.00 \\ 26.00 \end{pmatrix}\text{mm}^2$$

$$\Sigma A := \sum_{n=0}^{n_s-1} \left(A_s(n)\right) \qquad \Sigma A = 40.000\,\text{mm}^2$$

$$\Sigma Ax := \frac{1}{6} \cdot \sum_{n=0}^{n_s - 1} \sum_{i=0}^{3} \sum_{j=0}^{2} \left[A_t(n,i) \cdot \left(x_t(n,i)_j + x_t(n,i)_{j+1} \right) \right] \qquad \Sigma Ax = 208.000\,\text{mm}^3$$

$$\Sigma Ay := \frac{1}{6} \cdot \sum_{n=0}^{n_s - 1} \sum_{i=0}^{3} \sum_{j=0}^{2} \left[A_t(n,i) \cdot \left(y_t(n,i)_j + y_t(n,i)_{j+1} \right) \right] \qquad \Sigma Ay = 88.000\,\text{mm}^3$$

Centre of area for section ... x co-ordinate ... $x'_s := \frac{\Sigma Ax}{\Sigma A}$ $x'_s = 5.200\,\text{mm}$

y co-ordinate ... $y'_s := \frac{\Sigma Ay}{\Sigma A}$ $y'_s = 2.200\,\text{mm}$

2nd moment of area about x axis for rectangular sectors ...

$$Ixx := \frac{1}{3} \cdot \sum_{n=0}^{n_s - 1} \sum_{i=0}^{3} \sum_{j=0}^{2} \left[A_t(n,i) \cdot \left[\frac{1}{2} \cdot \left(y_t(n,i)_j + y_t(n,i)_{j+1} \right) - y'_s \right]^2 \right] \qquad Ixx = 179.73\,\text{mm}^4$$

2nd moment of area about y axis for rectangular sectors ...

$$Iyy := \frac{1}{3} \cdot \sum_{n=0}^{n_s - 1} \sum_{i=0}^{3} \sum_{j=0}^{2} \left[A_t(n,i) \cdot \left[\frac{1}{2} \cdot \left(x_t(n,i)_j + x_t(n,i)_{j+1} \right) - x'_s \right]^2 \right] \qquad Iyy = 763.73\,\text{mm}^4$$

Product moment of area for rectangular sectors ...

$$Ixy := \frac{1}{3} \cdot \sum_{n=0}^{n_s - 1} \sum_{i=0}^{3} \sum_{j=0}^{2} \left[A_t(n,i) \cdot \left[\frac{1}{2} \cdot \begin{pmatrix} y_t(n,i)_j \;... \\ + y_t(n,i)_{j+1} \end{pmatrix} - y'_s \right] \cdot \left[\frac{1}{2} \cdot \begin{pmatrix} x_t(n,i)_j \;... \\ + x_t(n,i)_{j+1} \end{pmatrix} - x'_s \right] \right]$$

$Ixy = -201.60\,\text{mm}^4$

Principal axies ...

Ref. Mechanics of Engineering Materials, by P.P.Benham & R.J.Crawford, 1987
Appendix A - Properties of Areas

Minimum principal moment of area ...

$$I_{min} := \frac{1}{2}\cdot(Ixx + Iyy) - \sqrt{\frac{1}{4}\cdot(Ixx - Iyy)^2 + Ixy^2} \qquad I_{min} = 116.90\,mm^4$$

Maximum principal moment of area ...

$$I_{max} := \frac{1}{2}\cdot(Ixx + Iyy) + \sqrt{\frac{1}{4}\cdot(Ixx - Iyy)^2 + Ixy^2} \qquad I_{max} = 826.57\,mm^4$$

Max. / Min. principal axes ...

$$\theta_p := \frac{1}{2}\cdot atan\left(\frac{2\cdot Ixy}{Iyy - Ixx}\right) \qquad \theta_p = -17.31\,deg$$

$$y_{max}(x) := y'_s - \frac{x - x'_s}{\tan(\theta_p)}$$ ... max. principal axis

$$y_{min}(x) := y'_s + \tan(\theta_p)\cdot(x - x'_s)$$ ... min. principal axis

Note, x and y coordinates are taken relative to x'_s, y'_s.

strain in z direction ... $\varepsilon_z = \frac{y}{Ry} + \frac{x}{Rx}$

stress in z direction ... $\sigma_z = E \cdot \left(\frac{y}{Ry} + \frac{x}{Rx}\right)$

Moment about x axis ... $dMx = y \cdot dPz$ ... where ... $dPz = \sigma_z \cdot dA$

$$dMx = y \cdot \sigma_z \cdot dA$$

$$dMx = E \cdot \left(\frac{y^2}{Ry} + \frac{x \cdot y}{Rx}\right) \cdot dA$$

$$Mx = \frac{E}{Ry} \cdot \int y^2 \, dA + \frac{E}{Rx} \cdot \int x \cdot y \, dA$$

$Mx = \frac{E}{Ry} \cdot Ixx + \frac{E}{Rx} \cdot Ixy$ ... or ... $Mx = ky \cdot Ixx + kx \cdot Ixy$

Moment about y axis ... $-dMy = x \cdot dPz$

$$-dMy = x \cdot \sigma_z \cdot dA$$

$$-dMy = E \cdot \left(\frac{x \cdot y}{Ry} + \frac{x^2}{Rx}\right) \cdot dA$$

$$-My = \frac{E}{Ry} \cdot \int x \cdot y \, dA + \frac{E}{Rx} \cdot \int x^2 \, dA$$

$-My = \frac{E}{Ry} \cdot Ixy + \frac{E}{Rx} \cdot Iyy$ ... or ... $-My = ky \cdot Ixy + kx \cdot Iyy$

In matrix format ... $\begin{pmatrix} Ixy & Ixx \\ Iyy & Ixy \end{pmatrix} \cdot \begin{pmatrix} kx \\ ky \end{pmatrix} = \begin{pmatrix} Mx \\ -My \end{pmatrix}$

$$\begin{pmatrix} kx \\ ky \end{pmatrix} = \begin{pmatrix} Ixy & Ixx \\ Iyy & Ixy \end{pmatrix}^{-1} \cdot \begin{pmatrix} Mx \\ -My \end{pmatrix}$$

$$\begin{pmatrix} k_x \\ k_y \end{pmatrix} = \begin{bmatrix} \dfrac{-(Ixy \cdot Mx + Ixx \cdot My)}{Ixx \cdot Iyy - Ixy^2} \\ \dfrac{Iyy \cdot Mx + Ixy \cdot My}{Ixx \cdot Iyy - Ixy^2} \end{bmatrix}$$

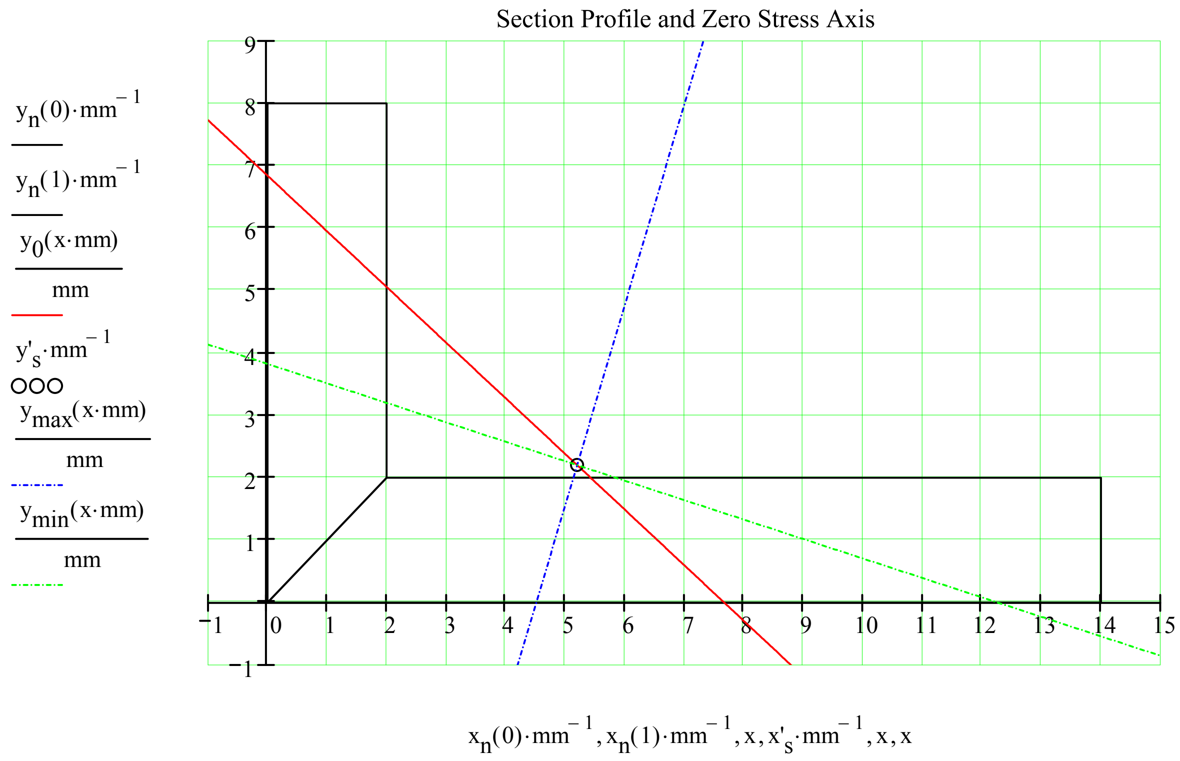
Section stress ... $\sigma_z(x,y) = E \cdot \left[\dfrac{(y - y'_s)}{Ry} + \dfrac{(x - x'_s)}{Rx} \right]$ ... relative to x'_s, y'_s

$$\sigma_z(x,y) = \frac{(Iyy \cdot Mx + Ixy \cdot My) \cdot (y - y'_s)}{Ixx \cdot Iyy - Ixy^2} - \frac{(Ixy \cdot Mx + Ixx \cdot My) \cdot (x - x'_s)}{Ixx \cdot Iyy - Ixy^2}$$

Equating stress to zero and rearranging for y.

Zero stress axis ... $y_0(x) := y'_s + \dfrac{(Ixy \cdot Mx + Ixx \cdot My)}{(Iyy \cdot Mx + Ixy \cdot My)} \cdot (x - x'_s)$

$$\operatorname{atan}\left[\frac{(Ixy \cdot Mx + Ixx \cdot My)}{(Iyy \cdot Mx + Ixy \cdot My)}\right] = -41.72 \, deg$$



Coordinates ... $xy_{en} =$

"Node No"	"-"	"Sector 1"	"Sector 2"
1	"X (mm)"	0	0
1	"Y (mm)"	0	0
2	"X (mm)"	0	2
2	"Y (mm)"	8	2
3	"X (mm)"	2	14
3	"Y (mm)"	8	2
4	"X (mm)"	2	14
4	"Y (mm)"	2	0

Note, adjust the graph $y_n(n)$ and $x_n(n)$ entries depending on the number of sectors.

Stress ... $\sigma_z =$

"-"	"Sector 1"	"Sector 2"
"Node No"	"(MPa)"	"(MPa)"
1	14.26	14.26
2	−2.43	6.37
3	−6.15	−15.95
4	6.37	−11.78