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Foundations for Dynamic Equipment

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This report presents to industry practitioners the various design criteria and methods and procedures of analysis, design, and construction applied to dynamic equipment foundations.

Keywords: amplitude; concrete; foundation; reinforcement; vibration.

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CHAPTER 1—INTRODUCTION

1.1—Background

Heavy machinery with reciprocating, impacting, or rotating masses requires a support system that can resist dynamic forces and the resulting vibrations. When excessive, such vibrations may be detrimental to the machinery, its support system, and any operating personnel subjected to them.

Many engineers with varying backgrounds are engaged in the analysis, design, construction, maintenance, and repair of machine foundations. Therefore, it is important that the owner/operator, geotechnical engineer, structural engineer, and equipment supplier collaborate during the design process. Each of these participants has inputs and concerns that are important and should be effectively communicated with each other, especially considering that machine foundation design procedures and criteria are not covered in building codes and national standards. Some firms and individuals have developed their own standards and specifications as a result of research and development activities, field studies, or many years of successful engineering or construction practices. Unfortunately, most of these standards are not available to many practitioners. As an engineering aid to those persons engaged in the design of foundations for machinery, the committee developed this document, which presents many current practices for dynamic equipment foundation engineering and construction.

1.2—Purpose

The committee presents various design criteria and methods and procedures of analysis, design, and construction currently applied to dynamic equipment foundations by industry practitioners.

This document provides general guidance with reference materials, rather than specifying requirements for adequate design. Where the document mentions multiple design methods and criteria in use, factors, which may influence the choice, are presented.

1.3—Scope

This document is limited in scope to the engineering, construction, repair, and upgrade of dynamic equipment foundations. For the purposes of this document, dynamic equipment includes the following:

1. Rotating machinery;
2. Reciprocating machinery; and
3. Impact or impulsive machinery.

1.4—Notation

$[C]$ = damping matrix
$[K]$ = stiffness matrix
$[K^*]$ = impedance with respect to CG
$[k]$ = reduced stiffness matrix
$[k_j']$ = battered pile stiffness matrix
$[M]$ = mass matrix
$[m]$ = reduced mass matrix
$[T]$ = transformation matrix for battered pile
$[\alpha_{ir}]$ = matrix of interaction factors between any two piles with diagonal terms $\alpha_{ii} = 1$
A = displacement amplitude
A_{head}, A_{crank} = head and crank areas, in.^{2} (mm^2)
A_p = cross-sectional area of the pile
a, b = plan dimensions of a rectangular foundation
a_o = dimensionless frequency
B_c = cylinder bore diameter, in. (mm)
B_i = mass ratio for the i-th direction
B_r = ram weight, tons (kN)
b_1, b_2 = 0.425 and 0.687, Eq. (4.15d)
c_{gi} = damping of pile group in the i-th direction
c_i = damping constant for the i-th direction
$c_i^*(adj)$ = damping in the i-th direction adjusted for material damping
c_{ij} = equivalent viscous damping of pile j in the i-th direction
D_i = damping ratio for the i-th direction
D_{rod} = rod diameter, in. (mm)
d = pile diameter
d_n = nominal bolt diameter, in. (m)
d_s = displacement of the slide, in. (mm)
E_p = Young's modulus of the pile
e_m = mass eccentricity, in. (mm)
e_v = void ratio
F = time varying force vector
F_1 = correction factor
F_{block} = the force acting outwards on the block from which concrete stresses should be calculated, lbf (N)
$(F_{bolt})_{CHG}$ = the force to be restrained by friction at the cross head guide tie-down bolts, lbf (N)
$(F_{bolt})_{frame}$ = the force to be restrained by friction at the frame tie-down bolts, lbf (N)
F_D = damper force
F_{GMAX} = maximum horizontal gas force on a throw or cylinder, lbf (N)
F_{IMAX} = maximum horizontal inertia force on a throw or cylinder, lbf (N)
F_o = dynamic force amplitude (zero-to-peak), lbf (N)
F_r = maximum horizontal dynamic force
F_{red} = a force reduction factor with suggested value of 2, to account for the fraction of individual cylinder load carried by the compressor frame ("frame rigidity factor")
F_{rod} = force acting on piston rod, lbf (N)
F_s = dynamic inertia force of slide, lbf (N)
F_{THROW} = horizontal force to be resisted by each throw's anchor bolts, lbf (N)
$F_{unbalance}$ = the maximum value from Eq. (3.18) applied using parameters for a horizontal compressor cylinder, lbf (N)

f_{i1}, f_{i2} = dimensionless stiffness and damping functions for the i-th direction, piles
f_m = frequency of motion, Hz
f_n = system natural frequency (cycles per second)
f_o = operating speed, rpm
G = dynamic shear modulus of the soil
G_{ave} = the average value of shear modulus of the soil over the pile length
G_c = the average value of shear modulus of the soil over the critical length
GE = pile group efficiency
G_l = soil shear modulus at tip of pile
G_pJ = torsional stiffness of the pile
G_s = dynamic shear modulus of the embedment (side) material
G_z = the shear modulus at depth $z = l_c/4$
H = depth of soil layer
I_i = mass moment of inertia of the machine-foundation system for the i-th direction
I_p = moment of inertia of the pile cross section
i = $\sqrt{-1}$
i = a directional indicator or modal indicator, Eq. (4.48), as a subscript
K_2 = a parameter that depends on void ratio and strain amplitude
K_{eff} = the effective bearing stiffness, lbf/in. (N/mm)
K_{ij}^* = impedance in the i-th direction with respect to motion of the CG in j-th direction
K_n = nut factor for bolt torque
K_{uu} = horizontal spring constant
$K_{u\psi}$ = coupling spring constant
$K_{\psi\psi}$ = rocking spring constant
k = the dynamic stiffness provided by the supporting media
k_{ei}^* = impedance in the i-th direction due to embedment
k_{gi} = pile group stiffness in the i-th direction
k_i = stiffness for the i-th direction
$k_i(adj)$ = stiffness in the i-th direction adjusted for material damping
k_i^* = complex impedance for the i-th direction
$k_i^*(adj)$ = impedance adjusted for material damping
k_{ij} = stiffness of pile j in the i-th direction
k_j = battered pile stiffness matrix
k_r = stiffness of individual pile considered in isolation
k_{st} = static stiffness constant
k_{vj} = vertical stiffness of a single pile
L = length of connecting rod, in. (mm)
L_B = the greater plan dimension of the foundation block, ft (m)
L_i = length of the connecting rod of the crank mechanism at the i-th cylinder
l = depth of embedment (effective)
l_c = critical length of a pile
l_p = pile length
M_h = hammer mass including any auxiliary foundation, lbm (kg)
M_r = ram mass including dies and ancillary parts, lbm (kg)
m = mass of the machine-foundation system
m_d = slide mass including the effects of any balance mechanism, lbm (kg)
m_r = rotating mass, lbm (kg)
$m_{rec,i}$ = reciprocating mass for the i-th cylinder
$m_{rot,i}$ = rotating mass of the i-th cylinder
m_s = effective mass of a spring
$(N_{bolt})_{CHG}$ = the number of bolts holding down one crosshead guide
$(N_{bolt})_{frame}$ = the number of bolts holding down the frame, per cylinder
NT = normal torque, ft-lbf (m-N)
P_{head}, P_{crank} = instantaneous head and crank pressures, psi (μPa)
P_s = power being transmitted by the shaft at the connection, horsepower (kilowatts)
R, R_i = equivalent foundation radius
r = length of crank, in. (mm)
r_i = radius of the crank mechanism of the i-th cylinder
r_o = pile radius or equivalent radius
S = press stroke, in. (mm)
S_f = service factor, used to account for increasing unbalance during the service life of the machine, generally greater than or equal to 2
S_{i1}, S_{i2} = dimensionless parameters (Table 4.2)
s = distance between piles
T = foundation thickness, ft (m)
T_b = bolt torque, lbf-in. (N-m)
T_{min} = minimum required anchor bolt tension
t = time, s
V_{max} = the maximum allowable vibration, in. (mm)
V_s = shear wave velocity of the soil, ft/s (m/s)
v = displacement amplitude
v' = velocity, in./s (cm/s)
v_h = post-impact hammer velocity, in./s (mm/s)
v_o = reference velocity = 18.4 ft/s (5.6 m/s) from a free fall of 5.25 ft (1.6 m)
v_r = ram impact velocity, ft/s (m/s)
W = strain energy
W_a = equipment weight at anchorage location
W_f = weight of the foundation, tons (kN)
W_p = bolt preload, lbf (N)
W_r = rotating weight, lbf (N)
w = soil weight density
X = vector representation of time-dependent displacements for MDOF systems
X_i = distance along the crankshaft from the reference origin to the i-th cylinder
x, z = the pile coordinates indicated in Fig. 4.9
x_r, z_r = pile location reference distances
y_c = distance from the CG to the base support
y_e = distance from the CG to the level of embedment resistance
y_p = crank pin displacement in local Y-axis, in. (mm)

Z_p = piston displacement, in. (mm)
z_p = crank pin displacement in local Z-axis, in. (mm)
α = the angle between a battered pile and vertical
α' = modified pile group interaction factor
α_1 = coefficient dependent on Poisson's ratio as given in Table 4.1
α_h = ram rebound velocity relative to impact velocity
α_i = the phase angle for the crank radius of the i-th cylinder, rad
α^*_{ij} = complex pile group interaction factor for the i-th pile to the j-th pile
α_{uf} = the horizontal interaction factor for fixed-headed piles (no head rotation)
α_{uH} = the horizontal interaction factor due to horizontal force (rotation allowed)
α_v = vertical interaction coefficient between two piles
$\alpha_{\psi H}$ = the rotation due to horizontal force
$\alpha_{\psi M}$ = the rotation due to moment
β = system damping ratio
β_i = rectangular footing coefficients (Richart, Hall, and Woods 1970), $i = v$, u, or ψ
β_j = coefficient dependent on Poisson's ratio as given in Table 4.1, j = 1 to 4
β_m = material damping ratio of the soil
β_p = angle between the direction of the loading and the line connecting the pile centers
δ = loss angle
ΔW = area enclosed by the hysteretic loop
ε_{ir} = the elements of the inverted matrix $[\alpha_{ir}]^{-1}$
ψ_i = reduced mode shape vector for the i-th mode
γ_j = coefficient dependent on Poisson's ratio as given in Table 4.1, j = 1 to 4
λ = pile-soil stiffness ratio (E_p/G_l)
μ = coefficient of friction
ν = Poisson's ratio of the soil
ν_s = Poisson's ratio of the embedment (side) material
ρ = soil mass density (soil weight density/gravitational acceleration)
ρ_a = G_{ave}/G_l
ρ_c = G_z/G_c
σ_o = probable confining pressure, lbf/ft^2 (Pa)
ω_i = circular natural frequency for the i-th mode
ω_m = circular frequency of motion
ω_n = circular natural frequencies of the system
ω_o = circular operating frequency of the machine (rad/s)
ω_{su}, ω_{sv} = circular natural frequencies of a soil layer in u and v directions

CHAPTER 2—FOUNDATION AND MACHINE TYPES

2.1—General considerations

The type, configuration, and installation of a foundation or support structure for dynamic machinery may depend on the following factors:

1. Site conditions such as soil characteristics, topography, seismicity, climate, and other effects;
2. Machine base configuration such as frame size, cylinder supports, pulsation bottles, drive mechanisms, and exhaust ducts;
3. Process requirements such as elevation requirements with respect to connected process equipment and hold-down requirements for piping;
4. Anticipated loads such as the equipment static weight, and loads developed during erection, startup, operation, shutdown, and maintenance;
5. Erection requirements such as limitations or constraints imposed by construction equipment, procedures, techniques, or the sequence of erection;
6. Operational requirements such as accessibility, settlement limitations, temperature effects, and drainage;
7. Maintenance requirements such as temporary access, laydown space, in-plant crane capabilities, and machine removal considerations;
8. Regulatory factors or building code provisions such as tied pile caps in seismic zones;
9. Economic factors such as capital cost, useful or anticipated life, and replacement or repair cost;
10. Environmental requirements such as secondary containment or special concrete coating requirements; and
11. Recognition that certain machines, particularly large reciprocating compressors, rely on the foundation to add strength and stiffness that is not inherent in the structure of the machine.

2.2—Machine types

2.2.1 *Rotating machinery*—This category includes gas turbines, steam turbines, and other expanders; turbo-pumps and compressors; fans; motors; and centrifuges. These machines are characterized by the rotating motion of impellers or rotors.

Unbalanced forces in rotating machines are created when the mass centroid of the rotating part does not coincide with the center of rotation (Fig. 2.1). This dynamic force is a function of the shaft mass, speed of rotation, and the magnitude of the offset. The offset should be minor under manufactured conditions when the machine is well balanced, clean, and without wear or erosion. Changes in alignment, operation near resonance, blade loss, and other malfunctions or undesirable conditions can greatly increase the force applied to its bearings by the rotor. Because rotating machines normally trip and shut down at some vibration limit, a realistic continuous dynamic load on the foundation is that resulting from vibration just below the trip level.

2.2.2 *Reciprocating machinery*—For reciprocating machinery, such as compressors and diesel engines, a piston moving in a cylinder interacts with a fluid through the kinematics of a slider crank mechanism driven by, or driving, a rotating crankshaft.

Individual inertia forces from each cylinder and each throw are inherently unbalanced with dominant frequencies at one and two times the rotational frequency (Fig. 2.2).

Reciprocating machines with more than one piston require a particular crank arrangement to minimize unbalanced forces and moments. A mechanical design that satisfies operating requirements should govern. This leads to piston/cylinder assemblies and crank arrangements that do not completely counter-oppose; therefore, unbalanced loads occur, which should be resisted by the foundation.

Individual cylinder fluid forces act outward on the cylinder head and inward on the crankshaft (Fig. 2.2). For a rigid cylinder and frame these forces internally balance, but deformations of large machines can cause a significant portion of the fluid load to be transmitted to the mounts and into the foundation. Particularly on large reciprocating compressors with horizontal cylinders, it is inappropriate and unconservative to assume the compressor frame and cylinder are sufficiently stiff to internally balance all forces. Such an assumption has led to many inadequate mounts for reciprocating machines.

2.2.3 *Impulsive machinery*—Equipment, such as forging hammers and some metal-forming presses, operate with regulated impacts or shocks between different parts of the equipment. This shock loading is often transmitted to the foundation system of the equipment and is a factor in the design of the foundation.

Closed die forging hammers typically operate by dropping a weight (ram) onto hot metal, forcing it into a predefined shape. While the intent is to use this impact energy to form and shape the material, there is significant energy transmission, particularly late in the forming process. During these final blows, the material being forged is cooling and less shaping takes place. Thus, pre-impact kinetic energy of the ram converts to post-impact kinetic energy of the entire forging hammer. As the entire hammer moves downward, it becomes a simple dynamic mass oscillating on its supporting medium. This system should be well damped so that the oscillations decay sufficiently before the next blow. Timing of the blows commonly range from 40 to 100 blows per min. The ram weights vary from a few hundred pounds to 35,000 lb (156 kN). Impact velocities in the range of 25 ft/s (7.6 m/s) are common. Open die hammers operate in a similar fashion but are often of two-piece construction with a separate hammer frame and anvil.

Forging presses perform a similar manufacturing function as forging hammers but are commonly mechanically or hydraulically driven. These presses form the material at low velocities but with greater forces. The mechanical drive system generates horizontal dynamic forces that the engineer should consider in the design of the support system. Rocking stability of this construction is important. Figure 2.3 shows a typical horizontal forcing function through one full stroke of a forging press.

Mechanical metal forming presses operate by squeezing and shearing metal between two dies. Because this equipment can vary greatly in size, weight, speed, and operation, the engineer should consider the appropriate type. Speeds can vary from 30 to 1800 strokes per min. Dynamic forces from the press develop from two sources: the mechanical balance of the moving parts in the equipment and the response of the press frame as the material is sheared (snap-through forces). Imbalances in the mechanics of the equipment can occur both horizontally and vertically. Generally high-speed equipment is well balanced. Low-speed equipment is often not balanced because the inertia forces at low speeds are small. The dynamic forces generated by all of these presses can be significant as they are transmitted into the foundation and propagated from there.

2.2.4 *Other machine types*—Other machinery generating dynamic loads include rock crushers and metal shredders. While part of the dynamic load from these types of equipment tend to be based on rotating imbalances, there is also a

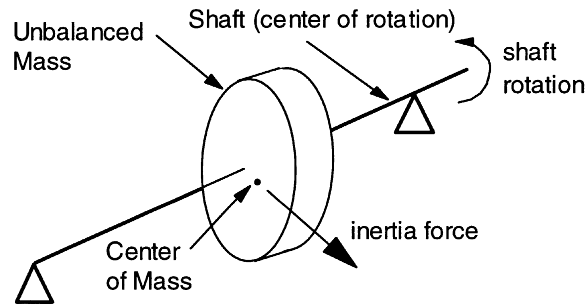

Fig. 2.1—Rotating machine diagram.

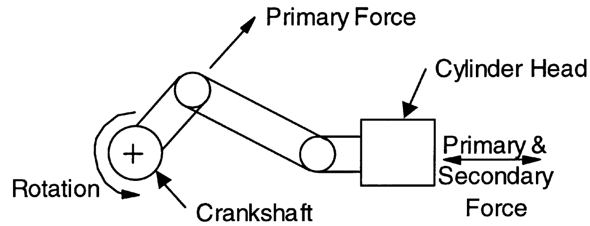

Fig. 2.2—Reciprocating machine diagram.

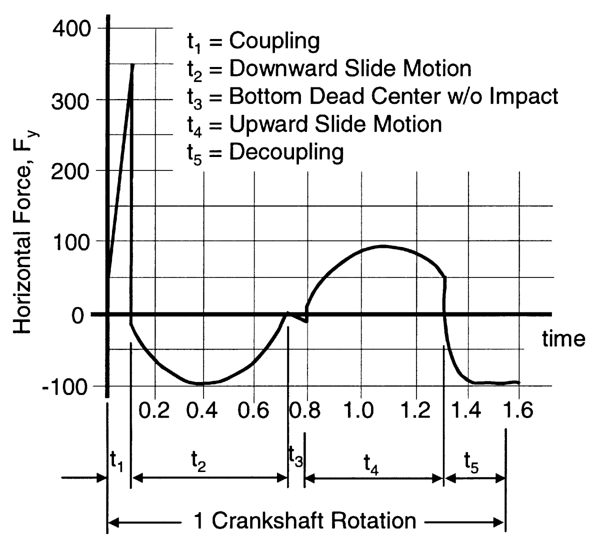

Fig. 2.3—Forcing function for a forging press.

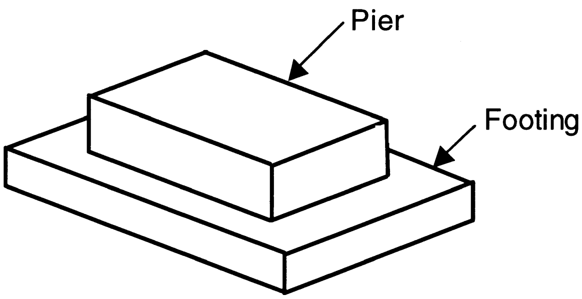

Fig. 2.4—Block-type foundation.

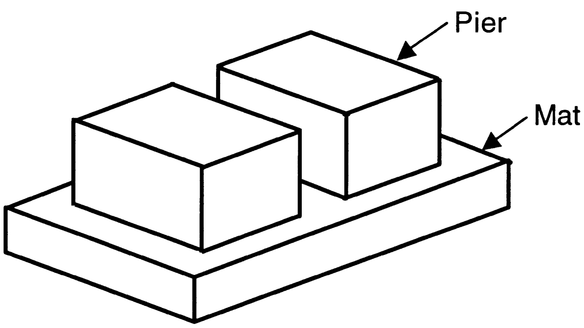

Fig. 2.5—Combined block foundation.

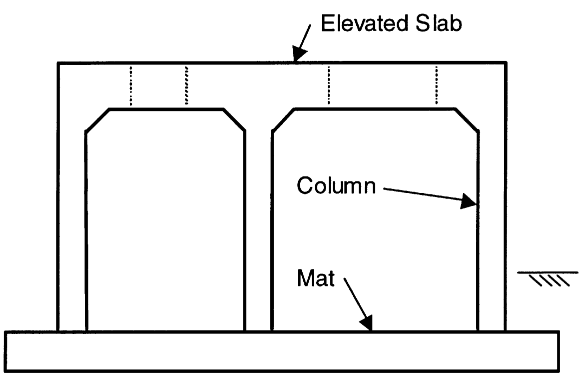

Fig. 2.6—Tabletop foundation.

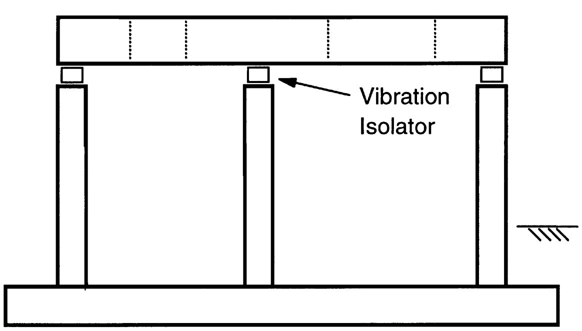

Fig. 2.7—Tabletop with isolators.

random character to the dynamic signal that varies with the particular operation.

2.3—Foundation types

2.3.1 *Block-type foundation (Fig. 2.4)*—Dynamic machines are preferably located close to grade to minimize the elevation

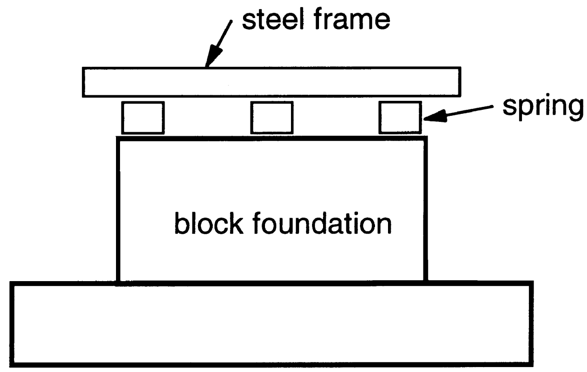

Fig. 2.8—Spring-mounted block formation.

difference between the machine dynamic forces and the center of gravity of the machine-foundation system. The ability to use such a foundation primarily depends on the quality of near surface soils. Block foundations are nearly always designed as rigid structures. The dynamic response of a rigid block foundation depends only on the dynamic load, foundation's mass, dimensions, and soil characteristics.

2.3.2 *Combined block-type foundation (Fig. 2.5)*—Combined blocks are used to support closely spaced machines. Combined blocks are more difficult to design because of the combination of forces from two or more machines and because of a possible lack of stiffness of a larger foundation mat.

2.3.3 *Tabletop-type foundation (Fig. 2.6)*—Elevated support is common for large turbine-driven equipment such as electric generators. Elevation allows for ducts, piping, and ancillary items to be located below the equipment. Tabletop structures are considered to be flexible, hence their response to dynamic loads can be quite complex and depend both on the motion of its discreet elements (columns, beams, and footing) and the soil upon which it is supported.

2.3.4 *Tabletop with isolators (Fig. 2.7)*—Isolators (springs and dampers) located at the top of supporting columns are sometimes used to minimize the response to dynamic loading. The effectiveness of isolators depends on the machine speed and the natural frequency of the foundation. Details of this type of support are provided in Section 4.5.

2.3.5 *Spring-mounted equipment (Fig. 2.8)*—Occasionally pumps are mounted on springs to minimize thermal forces from connecting piping. The springs are then supported on a block-type foundation. This arrangement has a dynamic effect similar to that for tabletops with vibration isolators. Other types of equipment are spring mounted to limit the transmission of dynamic forces.

2.3.6 *Inertia block in structure (Fig. 2.9)*—Dynamic equipment on a structure may be relatively small in comparison to the overall size of the structure. In this situation, dynamic machines are usually designed with a supporting inertia block to alter natural frequencies away from machine operating speeds and resist amplitudes by increasing the resisting inertia force.

2.3.7 *Pile foundations (Fig. 2.10)*—Any of the previously mentioned foundation types may be supported directly on soil or on piles. Piles are generally used where soft ground condi-

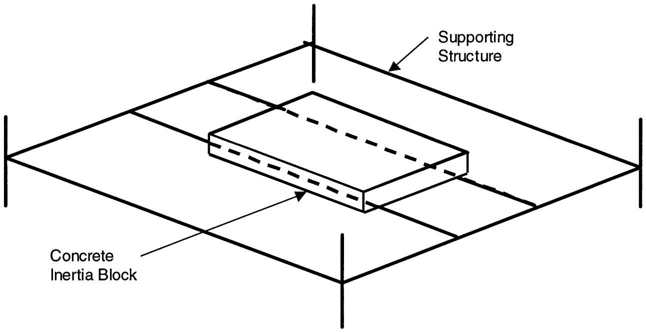

Fig. 2.9—Inertia block in structure.

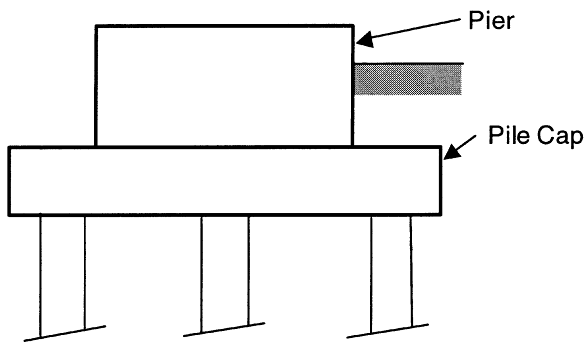

Fig. 2.10—Pile-supported foundation.

tions result in low allowable contact pressures and excessive settlement for a mat-type foundation. Piles use end bearing, frictional side adhesion, or a combination of both to transfer axial loads into the underlying soil. Transverse loads are resisted by soil pressure bearing against the side of the pile cap or against the side of the piles. Various types of piles are used including drilled piers, auger cast piles, and driven piles.

CHAPTER 3—DESIGN CRITERIA

3.1—Overview of design criteria

The main issues in the design of concrete foundations that support machinery are defining the anticipated loads, establishing the performance criteria, and providing for these through proper proportioning and detailing of structural members. Yet, behind this straightforward definition lies the need for careful attention to the interfaces between machine, mounting system, and concrete foundation.

The loads on machine foundations may be both static and dynamic. Static loads are principally a function of the weights of the machine and all its auxiliary equipment. Dynamic loads, which occur during the operation of the machine, result from forces generated by unbalance, inertia of moving parts, or both, and by the flow of fluid and gases for some machines. The magnitude of these dynamic loads primarily depends upon the machine's operating speed and the type, size, weight, and arrangement (position) of moving parts within the casing.

The basic goal in the design of a machine foundation is to limit its motion to amplitudes that neither endanger the satisfactory operation of the machine nor disturb people working in the immediate vicinity (Gazetas 1983). Allowable amplitudes depend on the speed, location, and criticality or function of the machine. Other limiting dynamic criteria affecting the design may include avoiding resonance and excessive transmissibility to the supporting soil or structure. Thus, a key ingredient to a successful design is the careful engineering analysis of the soil-foundation response to dynamic loads from the machine operation.

The foundation's response to dynamic loads can be significantly influenced by the soil on which it is constructed. Consequently, critical soil parameters, such as the dynamic soil shear modulus, are preferably determined from a field investigation and laboratory tests rather than relying on generalized correlations based on broad soil classifications. Due to the inherent variability of soil, the dynamic response of machine foundations is often evaluated using a range of values for the critical soil properties.

Furthermore, a machinery support structure or foundation is designed with adequate structural strength to resist the worst possible combination of loads occurring over its service life. This often includes limiting soil-bearing pressures to well within allowable limits to ensure a more predictable dynamic response and prevent excessive settlements and soil failures. Additionally, concrete members are designed and detailed to prevent cracking due to fatigue and stress reversals caused by dynamic loads, and the machine's mounting system is designed and detailed to transmit loads from the machine into the foundation, according to the criteria in Section 3.6.

3.2—Foundation and equipment loads

Foundations supporting reciprocating or rotating compressors, turbines, generators and motors, presses, and other machinery should withstand all the forces that may be imposed on them during their service life. Machine foundations are unique because they may be subjected to significant dynamic loads during operation in addition to normal design loads of gravity, wind, and earthquake. The magnitude and characteristics of the operating loads depend on the type, size, speed, and layout of the machine.

Generally, the weight of the machine, center of gravity, surface areas, and operating speeds are readily available from the manufacturer of the machine. Establishing appropriate values for dynamic loads is best accomplished through careful communication and clear understanding between the machine manufacturer and foundation design engineer as to the purpose, and planned use for the requested information, and the definition of the information provided. It is in the best interests of all parties (machine manufacturer, foundation design engineer, installer, and operator) to ensure effective definition and communication of data and its appropriate use. Machines always experience some level of unbalance, vibration, and force transmitted through the bearings. Under some off-design conditions, such as wear, the forces may increase significantly. The machine manufacturer and foundation design engineer should work together so that their combined knowledge achieves an integrated system structure which robustly serves the needs of its owner and operator and withstands all expected loads.

Sections 3.2.1 to 3.2.6 provide commonly used methods for determining machine-induced forces and other design loads for foundations supporting machinery. They include definitions and other information on dynamic loads to be requested from the machine manufacturer and alternative assumptions to apply when such data are unavailable or are under-predicted.

3.2.1 *Static loads*

3.2.1.1 *Dead loads*—A major function of the foundation is to support gravity (dead) loads due to the weight of the machine, auxiliary equipment, pipe, valves, and deadweight of the foundation structure. The weights of the machine components are normally supplied by the machine manufacturer. The distribution of the weight of the machine on the foundation depends on the location of support points (chocks, soleplates) and on the flexibility of the machine frame. Typically, there are multiple support points, and, thus, the distribution is statically indeterminate. In many cases, the machine manufacturer provides a loading diagram showing the vertical loads at each support point. When this information is not available, it is common to assume the machine frame is rigid and that its weight is appropriately distributed between support points.

3.2.1.2 *Live loads*—Live loads are produced by personnel, tools, and maintenance equipment and materials. The live loads used in design should be the maximum loads expected during the service life of the machine. For most designs, live loads are uniformly distributed over the floor areas of platforms of elevated support structures or to the access areas around at-grade foundations. Typical live loads vary from 60 lbf/ft^2 (2.9 kPa) for personnel to as much as 150 lbf/ft^2 (7.2 kPa) for maintenance equipment and materials.

3.2.1.3 *Wind loads*—Loads due to wind on the surface areas of the machine, auxiliary equipment, and the support foundation are based on the design wind speed for the particular site and are normally calculated in accordance with the governing local code or standard. Wind loads rarely govern the design of machine foundations except, perhaps, when the machine is located in an enclosure that is also supported by the foundation.

When designing machine foundations and support structures, most practitioners use the wind load provisions of ASCE 7. The analytical procedure of ASCE 7 provides wind pressures and forces for use in the design of the main wind-force resisting systems and anchorage of machine components.

Most structural systems involving machines and machine foundations are relatively stiff (natural frequency in the lateral direction greater than 1 Hz). Consequently, the systems can be treated as rigid with respect to the wind gust effect factor, and simplified procedures can be used. If the machine is supported on flexible isolators and is exposed to the wind, the rigid assumption may not be reasonable, and more elaborate treatment of the gust effects is necessary as described in ASCE 7 for flexible structural systems.

Appropriate consideration of the exposure conditions and importance factors is also required to be consistent with the facilities requirements.

3.2.1.4 *Seismic loads*—Machinery foundations located in seismically active regions are analyzed for seismic loads. Before 2000, these loads were determined in accordance with methods prescribed in one of various regional building codes (such as the UBC, the SBC, or the NBC) and standards such as ASCE 7 and SEAOC Blue Book.

The publication of the IBC 2000 provides building officials with the opportunity to replace the former regional codes with a code that has nationwide applicability. The seismic requirements in IBC 2000 and ASCE 7-98 are essentially identical, as both are based on the 1997 NEHRP (FEMA 302) provisions.

The IBC and its reference documents contain provisions for design of nonstructural components, including dynamic machinery, for seismic loads. For machinery supported above grade or on more flexible elevated pedestals, seismic amplification factors are also specified.

3.2.1.5 *Static operating loads*—Static operating loads include the weight of gas or liquid in the machinery equipment during normal operation and forces, such as the drive torque developed by some machines at the connection between the drive mechanism and driven machinery. Static operating loads can also include forces caused by thermal growth of the machinery equipment and connecting piping. Time-varying (dynamic) loads generated by machines during operation are covered elsewhere in this report.

Machines such as compressors and generators require some form of drive mechanism, either integral with the machine or separate from it. When the drive mechanism is nonintegral, such as a separate electric motor, reciprocating engine, and gas or steam turbine, it produces a net external drive torque on the driven machine. The torque is equal in magnitude and opposite in direction on the driver and driven machine. The normal torque (sometimes called drive torque) is generally applied to the foundation as a static force couple in the vertical direction acting about the centerline of the shaft of the machine. The magnitude of the normal torque is often computed from the following formula

$$NT = \frac{(5250)(P_s)}{f_o} \text{ lbf-ft} \qquad (3\text{-}1)$$

$$NT = \frac{(9550)(P_s)}{f_o} \text{ N-m}$$

where

NT = normal torque, ft-lbf (m-N);

P_s = power being transmitted by the shaft at the connection, horsepower (kilowatts); and

f_o = operating speed, rpm.

The torque load is generally resolved into a vertical force couple by dividing it by the center-to-center distance between longitudinal soleplates or anchor points (Fig. 3.1(a)). When the machine is supported by transverse soleplates only, the torque is applied along the width of the soleplate assuming a straight line variation of force (Fig. 3.1(b)). Normal torque can also be caused by jet forces on turbine blades. In this case it is applied to the foundation in the opposite direction from the rotation of the rotor.

The torque on a generator stator is applied in the same direction as the rotation of the rotor and can be high due to startup or an electrical short circuit. Startup torque, a property of electric motors, should be obtained from the motor manufacturer. The torque created by an electrical short circuit is considered a malfunction, emergency, or accidental load and is generally reported separately by the machinery manufacturer. Often in the design for this phenomenon, the magnitude of the emergency drive torque is determined by applying a magnification factor to the normal torque. Consultation with the generator manufacturer is necessary to establish the appropriate magnification factor.

3.2.1.6 *Special loads for elevated-type foundations*—To ensure adequate strength and deflection control, the following special static loading conditions are recommended in some proprietary standards for large equipment on elevated-type foundations:

1. Vertical force equal to 50% of the total weight of each machine;
2. Horizontal force (in the transverse direction) equal to 25% of the total weight of each machine; and
3. Horizontal force (in the longitudinal direction) equal to 25% of the total weight of each machine.

These forces are additive to normal gravity loads and are considered to act at the centerline of the machine shaft. Loads 1, 2, and 3 are not considered to act concurrently with one another.

3.2.1.7 *Erection and maintenance loads*—Erection and maintenance loads are temporary loads from equipment, such as cranes and forklifts, required for installing or dismantling machine components during erection or maintenance. Erection loads are usually furnished in the manufacturer's foundation load drawing and should be used in conjunction with other specified dead, live, and environmental loads. Maintenance loads occur any time the equipment is being drained, cleaned, repaired, and realigned or when the components are being removed or replaced. Loads may result from maintenance equipment, davits, and hoists. Environmental loads, such as full wind and earthquake, are not usually assumed to act with maintenance loads, which generally occur for only a relatively short duration.

3.2.1.8 *Thermal loads*—Changing temperatures of machines and their foundations cause expansions and contractions, and distortions, causing the various parts to try to slide on the support surfaces. The magnitude of the resulting frictional forces depends on the magnitude of the temperature change, the location of the supports, and on the condition of the support surfaces. The thermal forces do not impose a net force on the foundation to be resisted by soil or piles because the forces on any surface are balanced by equal and opposite forces on other support surfaces. Thermal forces, however, may govern the design of the grout system, pedestals, and hold downs.

Calculation of the exact thermal loading is very difficult because it depends on a number of factors, including distance between anchor points, magnitude of temperature change, the material and condition of the sliding surface, and the magnitude of the vertical load on each soleplate. Lacking a rigorous analysis, the magnitude of the frictional load may be calculated as follows

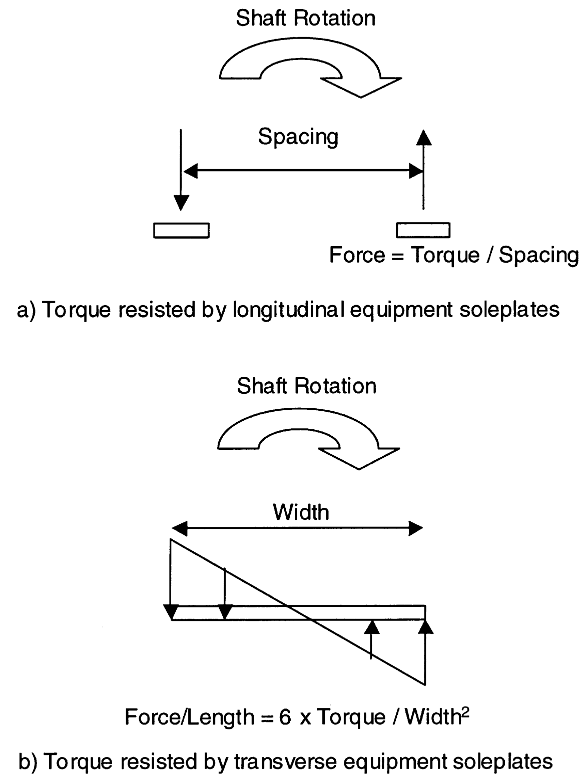

Fig. 3.1—Equivalent forces for torque loads.

$$\text{Force} = (\text{friction coefficient})(\text{load acting through soleplate}) \quad (3\text{-}2)$$

The friction coefficient generally varies from 0.2 to 0.5. Loads acting through the soleplate include: machine dead load, normal torque load, anchor bolt load, and piping loads.

Heat transfer to the foundation can be by convection across an air gap (for example, gap between sump and block) and by conduction through points of physical contact. The resultant temperature gradients induce deformations, strains, and stresses.

When evaluating thermal stress, the calculations are strongly influenced by the stiffness and restraint against deformation for the structural member in question. Therefore, it is important to consider the self-relieving nature of thermal stress due to deformation to prevent being overly conservative in the analysis. As the thermal forces are applied to the foundation member by the machine, the foundation member changes length and thereby provides reduced resistance to the machine forces. This phenomenon can have the effect of reducing the thermal forces from the machine.

Accurate determinations of concrete surface temperatures and thermal gradients are also important. Under steady-state normal operating conditions, temperature distributions across structural sections are usually linear. The air gap between the machine casing and foundation provides a significant means for dissipating heat, and its effect should be included when establishing surface temperatures.

Normally, the expected thermal deflection at various bearings is estimated by the manufacturer, based on past field measurements on existing units. The machine erector then compensates for the thermal deflection during installation.

Reports are available (Mandke and Smalley 1992; Mandke and Smalley 1989; and Smalley 1985) that illustrate the effects of thermal loads and deflections in the concrete foundation of a large reciprocating compressor and their influence on the machine.

3.2.2 *Rotating machine loads*—Typical heavy rotating machinery include centrifugal air and gas compressors, horizontal and vertical fluid pumps, generators, rotating steam and gas turbine drivers, centrifuges, electric motor drivers, fans, and blowers. These types of machinery are characterized by the rotating motion of one or more impellers or rotors.

3.2.2.1 *Dynamic loads due to unbalanced masses*—Unbalanced forces in rotating machines are created when the mass centroid of the rotating part does not coincide with the axis of rotation. In theory, it is possible to precisely balance the rotating elements of rotating machinery. In practice, this is never achieved; slight mass eccentricities always remain. During operation, the eccentric rotating mass produces centrifugal forces that are proportional to the square of machine speed. Centrifugal forces generally increase during the service life of the machine due to conditions such as machine wear, rotor play, and dirt accumulation.

A rotating machine transmits dynamic force to the foundation predominantly through its bearings (with small, generally unimportant exceptions such as seals and the air gap in a motor). The forces acting at the bearings are a function of the level and axial distribution of unbalance, the geometry of the rotor and its bearings, the speed of rotation, and the detailed dynamic characteristics of the rotor-bearing system. At or near a critical speed, the force from rotating unbalance can be substantially amplified, sometimes by a factor of five or more.

Ideally, the determination of the transmitted force under different conditions of unbalance and at different speeds results from a dynamic analysis of the rotor-bearing system, using an appropriate combination of computer programs for calculating bearing dynamic characteristics and the response to unbalance of a flexible rotor in its bearings. Such an analysis would usually be performed by the machine manufacturer. Results of such analyses, especially values for transmitted bearing forces, represent the best source of information for use by the foundation design engineer. This and other approaches used in practice to quantify the magnitude of dynamic force transmitted to the foundation are discussed in Sections 3.2.2.1a to 3.2.2.1.3e.

3.2.2.1a *Dynamic load provided by the manufacturer*—The engineer should request and the machine manufacturer should provide the following information:

Design levels of unbalance and basis—This information documents the unbalance level the subsequent transmitted forces are based on.

Dynamic forces transmitted to the bearing pedestals under the following conditions—

a) Under design unbalance levels over operating speed range;

b) At highest vibration when negotiating critical speeds;

c) At a vibration level where the machine is just short of tripping on high vibration; and

d) Under the maximum level of upset condition the machine is designed to survive (for example, loss of one or more blades).

Items a and b document the predicted dynamic forces resulting from levels of unbalance assumed in design for normal operation. Using these forces, it is possible to predict the normal dynamic vibration of the machine on its foundation.

Item c identifies a maximum level of transmitted force with which the machine could operate continuously without tripping; the foundation should have the strength to tolerate such a dynamic force on a continuous basis.

Item d identifies the higher level of dynamic force, which could occur under occasional upset conditions over a short period of time. If the machine is designed to tolerate this level of dynamic force for a short period of time, then the foundation should also be able to tolerate it for a similar period of time.

If an independent dynamic analysis of the rotor-bearing system is performed by the end user or by a third party, such an analysis can provide some or all of the above dynamic forces transmitted to the foundation.

By assuming that the dynamic force transmitted to the bearings equals the rotating unbalanced force generated by the rotor, information on unbalance can provide an estimate of the transmitted force.

3.2.2.1b *Machine unbalance provided by the manufacturer*—When the mass unbalance (eccentricity) is known or stated by the manufacturer, the resulting dynamic force amplitude is

$$F_o = m_r e_m \omega_o^2 S_f / 12 \quad \text{lbf} \tag{3-3}$$

$$F_o = m_r e_m \omega_o^2 S_f / 1000 \quad \text{N}$$

where

F_o = dynamic force amplitude (zero-to-peak), lbf (N);

m_r = rotating mass, lbm (kg);

e_m = mass eccentricity, in. (mm);

ω_o = circular operating frequency of the machine (rad/s); and

S_f = service factor, used to account for increased unbalance during the service life of the machine, generally greater than or equal to 2.

3.2.2.1c *Machine unbalance meeting industry criteria*—Many rotating machines are balanced to an initial balance quality either in accordance with the manufacturer's procedures or as specified by the purchaser. ISO 1940 and ASA/ANSI S2.19 define balance quality in terms of a constant $e_m\omega_o$. For example, the normal balance quality Q for parts of process-plant machinery is 0.25 in./s (6.3 mm/s). Other typical balance quality grade examples are shown in Table 3.1. To meet these criteria a rotor intended for faster speeds should be better balanced than one operating at a slower speed. Using this approach, Eq. (3-3) can be rewritten as

$$F_o = m_r Q \omega_o S_f / 12 \quad \text{lbf} \tag{3-4}$$

$$F_o = m_r Q \omega_o S_f / 1000 \quad \text{N}$$

Table 3.1—Balance quality grades for selected groups of representative rigid rotors (excerpted from ANSI/ASA S2.19)

Balance quality guide	Product of $e\omega$, in./s (mm/s)	Rotor types—general examples
G1600	63 (1600)	Crankshaft/drives of rigidly mounted, large, two-cycle engines
G630	2.5 (630)	Crankshaft/drives of rigidly mounted, large, four-cycle engines
G250	10 (250)	Crankshaft/drives of rigidly mounted, fast, four-cylinder diesel engines
G100	4 (100)	Crankshaft/drives of fast diesel engines with six or more cylinders
G40	1.6 (40)	Crankshaft/drives of elastically mounted, fast four-cycle engines (gasoline or diesel) with six or more cylinders
G16	0.6 (16)	Parts of crushing machines; drive shafts (propeller shafts, cardan shafts) with special requirements; crankshaft/drives of engines with six or more cylinders under special requirements
G6.3	0.25 (6.3)	Parts of process plant machines; centrifuge drums, paper machinery rolls, print rolls; fans; flywheels; pump impellers; machine tool and general machinery parts; medium and large electric armatures (of electric motors having at least 80 mm shaft height) without special requirement
G2.5	0.1 (2.5)	Gas and steam turbines, including marine main turbines; rigid turbo-generator rotors; turbo-compressors; machine tool drives; medium and large electric armatures with special requirements; turbine driven pumps
G1	0.04 (1)	Grinding machine drives
G0.4	0.015 (0.4)	Spindles, discs, and armatures of precision grinders

API 617 and API 684 work with maximum residual unbalance U_{max} criteria for petroleum processing applications. The mass eccentricity is determined by dividing U_{max} by the rotor weight. For axial and centrifugal compressors with maximum continuous operating speeds greater than 25,000 rpm, API 617 establishes a maximum allowable mass eccentricity of 10×10^{-6} in. (250 nm). For compressors operating at slower speeds, the maximum allowable mass eccentricity is

$$e_m = 0.25/f_o \quad \text{in.} \tag{3-5}$$

$$e_m = 6.35/f_o \quad \text{mm}$$

where

f_o = operating speed, rpm ≤ 25,000 rpm.

This permitted initial mass eccentricity is tighter than ISO balance quality grade G2.5, which would be applied to this type of equipment (Table 3.1, turbo-compressors) under ISO 1940. As such, the dynamic force computed from this API consideration will be quite small and a larger service factor might be used to have a realistic design force.

API 617 also identifies a limitation on the peak-to-peak vibration amplitude during mechanical testing of the compressor with the equipment operating at its maximum continuous speed ($(12{,}000/f_o)^{0.5}$ in. [$25.4(12{,}000/f_o)^{0.5}$ mm]). Some design firms use this criterion and a service factor S_f of 2.0 to compute the dynamic force amplitude as

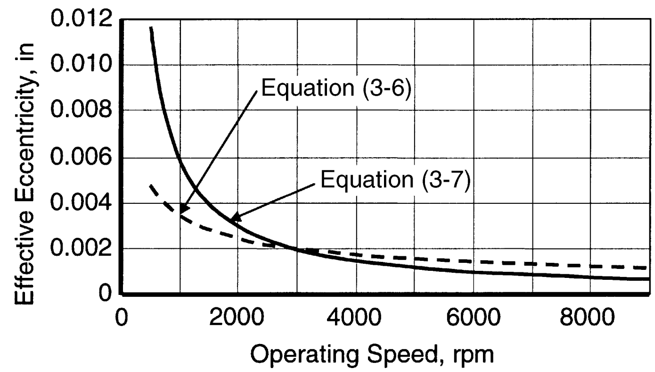

Fig. 3.2—Comparison of effective eccentricity.

$$F_o = \frac{W_r f_o^{1.5}}{322{,}000} \tag{3-6}$$

where W_r = rotating weight, lbf (N).

3.2.2.1d *Dynamic load determined from an empirical formula*—Rotating machine manufacturers often do not report the unbalance that remains after balancing. Consequently, empirical formulas are frequently used to ensure that foundations are designed for some minimum unbalance, which generally includes some allowance for increasing unbalance over time. One general purpose empirical method assumes that balancing improves with machine speed and that there is a linear relationship between the unbalanced forces and the machine speed. The zero-to-peak centrifugal force amplitude from one such commonly used expression is

$$F_o = \frac{W_r f_o}{6000} \tag{3-7}$$

Equations (3-3), (3-4), (3-6), and (3-7) appear to be very different: the exponents on the speed of rotation vary from 1 to 1.5 to 2, constants vary widely, and different variables appear. Some equations use mass, others use weight. In reality, the equations are more similar than they appear. Given the right understanding of Q as a replacement for $e\omega$, Eq. (3-3), (3-4), and (3-7) take on the same character. These equations then indicate that the design force at operating speed varies linearly with both the mass of the rotating body and the operating rotational speed. Once that state is identified, Eq. (3-3) can be adjusted to reflect the actual speed of rotation, and the dynamic centrifugal force is seen to vary with the square of the speed. Restating Eq. (3-6) and (3-7) in the form of Eq. (3-3) allows for the development of an effective eccentricity implied within these equations with the comparison shown in Fig. 3.2. Equation (3-7) produces the same result as Eq. (3-4) using Q = 0.25 in./s (6.3 mm/s), and S_f = 2.5.

The centrifugal forces due to mass unbalance are considered to act at the center of gravity of the rotating part and vary harmonically at the speed of the machine in the two orthogonal directions perpendicular to the shaft. The forces in the two orthogonal directions are equal in magnitude and 90 degrees out of phase and are transmitted to the foundation through the

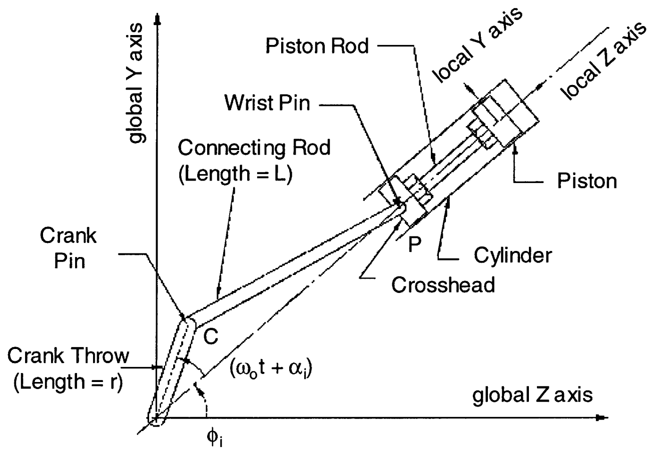

Fig. 3.3—Crack mechanism.

bearings. Schenck (1990) provides useful information about balance quality for various classes of machinery.

3.2.2.1e *Machine unbalance determined from trip vibration level and effective bearing stiffness*—Because a rotor is often set to trip on high vibration, it can be expected to operate continuously at any vibration level up to the trip limit. Given the effective bearing stiffness, it is possible to calculate the maximum dynamic force amplitude as

$$F_o = V_{max} K_{eff} \tag{3-8}$$

where

V_{max} = the maximum allowable vibration, in. (mm); and
K_{eff} = the effective bearing stiffness, lbf/in. (N/mm).

To use this approach, the manufacturer should provide effective bearing stiffness or the engineer should calculate it from the bearing geometry and operating conditions (such as viscosity and speed).

3.2.2.2 *Loads from multiple rotating machines*—If a foundation supports multiple rotating machines, the engineer should compute unbalanced force based on the mass, unbalance, and operating speed of each rotating component. The response to each rotating mass is then combined to determine the total response. Some practitioners, depending on the specific situation of machine size and criticality, find it advantageous to combine the unbalanced forces from each rotating component into a single resultant unbalanced force. The method of combining two dynamic forces is up to individual judgment and often involves some approximations. In some cases, loads or responses can be added absolutely. In other cases, the loads are treated as out-of-phase so that twisting effects are increased. Often, the operating speed of the equipment should be considered. Even if operating speeds are nominally the same, the design engineer should recognize that during normal operation, the speed of the machines will vary and beating effects can develop. Beating effects develop as two machines operate at close to the same speed. At one point in time, responses to the two machines are additive and motions are maximized. A short time later, the responses cancel each other and the motions are minimized. The net effect is a continual cyclic rising and falling of motion.

3.2.3 *Reciprocating machine loads*—Internal-combustion engines, piston-type compressors and pumps, some metal forming presses, steam engines, and other machinery are characterized by the rotating motion of a master crankshaft and the linear reciprocating motion of connected pistons or sliders. The motion of these components cause cyclically varying forces, often called reciprocating forces.

3.2.3.1 *Primary and secondary reciprocating loads*—The simplest type of reciprocating machine uses a single crank mechanism as shown in Fig. 3.3. The idealization of this mechanism consists of a piston that moves within a guiding cylinder, a crank of length *r* that rotates about a crank shaft, and a connecting rod of length *L*. The connecting rod is attached to the piston at point P and to the crank at point C. The wrist pin P oscillates while the crank pin C follows a circular path. This idealized single cylinder illustrates the concept of a machine producing both primary and secondary reciprocating forces.

If the crank is assumed to rotate at a constant angular velocity ω_o, the translational acceleration of the piston along its axis may be evaluated. If Z_p is defined as the piston displacement toward the crankshaft (local Z-axis), an expression can be written for Z_p at any time *t*. Further, the velocity and acceleration can also be obtained by taking the first and second derivatives of the displacement expression with respect to time. The displacement, velocity, and acceleration expressions for the motion of the piston are as follows

$$Z_p = \left(r + \frac{r^2}{4L}\right) - r\left(\cos\omega_o t + \frac{r}{4L}\cos 2\omega_o t\right) \tag{3-9}$$

$$\dot{Z}_p = r\omega_o\left(\sin\omega_o t + \frac{r}{2L}\sin 2\omega_o t\right) \tag{3-10}$$

$$\ddot{Z}_p = r\omega_o^2\left(\cos\omega_o t\frac{r}{L}\cos 2\omega_o t\right) \tag{3-11}$$

where

Z_p = piston displacement, in. (mm);
r = length of crank, in. (mm);
L = length of connecting rod, in. (mm);
ω_o = circular operating frequency of the machine (rad/s); and
t = time, s.

Note that the expressions contain two terms each with a sine or cosine; the term that varies with the frequency of the rotation, ω_o, is referred to as the primary term while the term that varies at twice the frequency of rotation, $2\omega_o$, is called the secondary term.

Similar expressions can be developed for the local Z-axis (parallel to piston movement) and local Y-axis (perpendicular to piston movement) motion of the rotating parts of the crank. If any unbalance in the crankshaft is replaced by a mass concentrated at the crank pin C, such that the inertia forces are the same as in the original system, the following terms for motion at point C can be written

$$y_p = -r \sin\omega_o t \quad (3\text{-}12)$$

$$\dot{y}_p = -r\,\omega \cos\omega_o t \quad (3\text{-}13)$$

$$\ddot{y}_p = r\,\omega_o^2 \sin\omega_o t \quad (3\text{-}14)$$

$$z_p = r(1 - \cos\omega_o t) \quad (3\text{-}15)$$

$$\dot{z}_p = r\omega_o \sin\omega_o t \quad (3\text{-}16)$$

$$\ddot{z}_p = r\,\omega_o^2 \cos\omega_o t \quad (3\text{-}17)$$

where

y_p = crank pin displacement in local Y-axis, in. (mm); and
z_p = crank pin displacement in local Z-axis, in. (mm).

Identifying a part of the connecting rod (usually 1/3 of its mass) plus the piston as the reciprocating mass m_{rec} concentrated at point P and designating the remainder of the connecting rod plus the crank as the rotating mass m_{rot} concentrated at point C, expressions for the unbalanced forces are as follows

Parallel to piston movement

$$F_z = (m_{rec} + m_{rot})r\omega_o^2 \cos\omega_o t + m_{rec}\frac{r^2\omega_o^2}{L}\cos 2\omega_o t \quad (3\text{-}18)$$

Perpendicular to piston movement

$$F_Y = m_{rot} r\omega_o^2 \sin\omega_o t \quad (3\text{-}19)$$

Note that Eq. (3.18) consists of two terms, a primary force

$$(m_{rec} + m_{rot})r\omega_o^2 \cos\omega_o t \quad (3\text{-}20)$$

and a secondary force

$$m_{rec} = \frac{r^2\omega_o^2}{L}\cos 2\omega_o t \quad (3\text{-}21)$$

whereas Eq. (3-19) has only a primary component.

3.2.3.2 *Compressor gas loads*—A reciprocating compressor raises the pressure of a certain flow of gas by imparting reciprocating motion on a piston within a cylinder. The piston normally compresses gas during both directions of reciprocating motion. As gas flows to and from each end, the pressure of the gas increases as it is compressed by each stroke of the piston. The increase in pressure within the cylinder creates reaction forces on the head and crank ends of the piston which alternate as gas flows to and from each end of the cylinder.

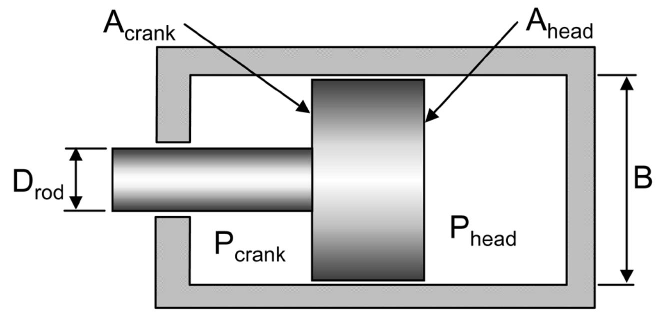

Fig. 3.4—Schematic of double-acting compressor cylinder and piston.

The gas force contributed to the piston rod equals the instantaneous difference between the pressure force acting on the head and crank end of the piston as shown in Fig. 3.4.

The following formulation can be used to estimate the maximum force acting on the piston rod of an individual double-acting cylinder

$$F_{rod} = [(P_{head})(A_{head}) - (P_{crank})(A_{crank})]\,F_1 \quad (3\text{-}22)$$

$$A_{head} = (\pi/4)B_c^2 \quad (3\text{-}23)$$

$$A_{crank} = (\pi/4)(B_c^2 - D_{rod}{}^2) \quad (3\text{-}24)$$

where

F_{rod} = force acting on piston rod, lbf (N);
A_{head}, A_{crank} = head and crank areas, in.^{2} (mm^2);
B_c = cylinder bore diameter, in. (mm);
D_{rod} = rod diameter, in. (mm);
P_{head}, P_{crank} = instantaneous head and crank pressures, psi, (MPa); and
F_1 = correction factor.

The head and crank end pressures vary continuously and the differential force takes both positive and negative net values during each cycle of piston motion. The normal approach is to establish the head and crank pressures using the maximum and minimum suction and discharge pressures. For design purposes, it is common to multiply Eq. (3-22) by a factor F_1 to help account for the natural tendency of gas forces to exceed the values based directly on suction and discharge pressures due to flow resistances and pulsations. Machines with good pulsation control and low external flow resistance may achieve F_1 as small as 1.1; for machines with low compression ratio, high pulsations, or highly resistive flow through piping and nozzles, F_1 can approach 1.5 or even higher. A reasonable working value for F_1 is 1.15 to 1.2.

Preferably, the maximum rod force resulting from gas pressures is based on knowledge of the continuous variation of pressure in the cylinder (measured or predicted). In a repair situation, measured cylinder pressure variation using a cylinder analyzer provides the most accurate value of gas forces. Even without cylinder pressure analysis, extreme operating values of suction and discharge pressure for each stage should be recorded before the repair and used in the Eq. (3-22).

On new compressors, the engineer should ask the machine manufacturer to provide values for maximum compressive and tensile gas loads on each cylinder rod and, if these are based on suction and discharge pressures, to recommend a value of F_1.

Gas forces act on the crankshaft with an equal and opposite reaction on the cylinder. Thus, crankshaft and cylinder forces globally balance each other. Between the crankshaft and the cylinder, however, the compressor frame stretches or contracts in tension or compression under the action of the gas forces. The forces due to frame deflections are transmitted to the foundation through connections with the compressor frame. When acting without slippage, the frame and foundation become an integral structure and together stretch or contract under the gas loads.

The magnitude of gas force transferred into the foundation depends on the relative flexibility of the compressor frame. A very stiff frame transmits only a small fraction of the gas force while a very flexible frame transmits most or all of the force. Similar comments apply to the transfer of individual cylinder inertia forces.

Based on limited comparisons using finite element analysis (Smalley 1988), the following guideline is suggested for gas and inertia force loads transmitted to the foundation by a typical compressor

$$F_{block} = F_{rod}/F_{red} \tag{3-25}$$

$$(F_{bolt})_{CHG} = [(F_{rod})/(N_{bolt})_{CHG}]/F_{red} \tag{3-26}$$

$$(F_{bolt})_{frame} = [(F_{unbalance}/(N_{bolt})]/F_{red} \tag{3-27}$$

where

F_{block} = the force acting outward on the block from which concrete stresses should be calculated, lbf (N);

$(F_{bolt})_{CHG}$ = the force to be restrained by friction at the cross head guide tie-down bolts, lbf (N);

$(F_{bolt})_{frame}$ = the force to be restrained by friction at the frame tie-down bolts, lbf (N);

F_{red} = a force reduction factor with suggested value of 2, to account for the fraction of individual cylinder load carried by the compressor frame ("frame rigidity factor");

$(N_{bolt})_{CHG}$ = the number of bolts holding down one crosshead guide;

$(N_{bolt})_{frame}$ = the number of bolts holding down the frame, per cylinder;

F_{rod} = force acting on piston rod, from Eq. (3-22), lbf (N); and

$F_{unbalance}$ = the maximum value from Eq. (3-18) applied using parameters for a horizontal compressor cylinder, lbf (N).

The factor F_{red} is used to simplify a complex problem, thus avoiding the application of unrealistically high loads on the anchor bolts and the foundation block. The mechanics involved in transmitting loads are complex and cannot easily be reduced to a simple relationship between a few parameters beyond the given load equations. A detailed finite-element analysis of metal compressor frame, chock mounts, concrete block, and grout will account for the relative flexibility of the frame and its foundation in determining individual anchor bolt loads and implicitly provide a value for F_{red}. If the frame is very stiff relative to the foundation, the value for F_{red} will be higher, implying more of the transmitted loads are carried by the frame and less by the anchor bolts and foundation block. Based on experience, a value of 2 for this factor is conservatively low; however, higher values have been seen with frames designed to be especially stiff.

Simplifying this approach, one report (Smalley and Harrell 1997) suggests using a finite element analysis to calculate forces transmitted to the anchor bolts. If a finite element analysis is not possible, the engineer should get from the machine manufacturer or calculate the maximum horizontal gas force and maximum horizontal inertia force for any throw or cylinder. The mounts, anchor bolts, and blocks are then designed for

$$F_{THROW} = (\text{greater of } F_{GMAX} \text{ or } F_{IMAX})/2 \tag{3-28}$$

where

F_{GMAX} = maximum horizontal gas force on a throw or cylinder, lbf (N);

F_{IMAX} = maximum horizontal inertia force on a throw or cylinder, lbf (N); and

F_{THROW} = horizontal force to be resisted by each throw's anchor bolts, lbf (N).

3.2.3.3 *Reciprocating inertia loads for multicylinder machines*—As a practical matter, most reciprocating machines have more than one cylinder, and manufacturers arrange the machine components in a manner that minimizes the net unbalanced forces. For example, rotating parts like the crankshaft can be balanced by adding or removing correcting weights. Translating parts like pistons and those that exhibit both rotation and translation, like connecting rods, can be arranged in such a way as to minimize the unbalanced forces and moments generated. Seldom, if ever, is it possible to perfectly balance reciprocating machines.

The forces generated by reciprocating mechanisms are functions of the mass, stroke, piston arrangement, connecting rod size, crank throw orientation (phase angle), and the mass and arrangement of counterweights on the crankshaft. For this reason, calculating the reciprocating forces for multicylinder machines can be quite complex and are therefore normally provided by the machine manufacturer. If the machine is an integral engine compressor, it can include, in one frame, cylinders oriented horizontally, vertically, or in between, all with reciprocating inertias.

Some machine manufacturers place displacement transducers and accelerometers on strategic points on the machinery. They can then measure displacements and accelerations at those points for several operational frequencies to determine the magnitude of the unbalanced forces and couples for multicylinder machines.

3.2.3.4 *Estimating reciprocating inertia forces from multicylinder machines*—In cases where the manufacturer's data are unavailable or components are being replaced, the engineer should use hand calculations to estimate the reciprocating forces from a multicylinder machine. One such procedure for a machine having *n* number of cylinders is discussed by Mandke and Troxler (1992). Section 3.7 summarizes this method.

3.2.4 *Impulsive machine loads*—The impulsive load generated by a forging hammer is caused by the impact of the hammer ram onto the hammer anvil. This impact process transfers the kinetic energy of the ram into kinetic energy of the entire hammer assembly. The post-impact velocity of the hammer is represented by

$$v_h = \frac{M_r}{M_h}(1 + \alpha_h)v_r \qquad (3\text{-}29)$$

where

v_h = post-impact hammer velocity, ft/s (m/s);
M_r = ram mass including dies and ancillary parts, lbm (kg);
M_h = hammer mass including any auxiliary foundation, lbm (kg);
α_h = ram rebound velocity relative to impact velocity; and
v_r = ram impact velocity, ft/s (m/s).

General experience indicates that α_h is approximately 60% for many forging hammer installations. From that point, the hammer foundation performance can be assessed as a rigid body oscillating as a single degree-of-freedom system with an initial velocity of v_h.

For metal-forming presses, the dynamic forces develop from two sources: the mechanical movement of the press components and material-forming process. Each of these forces is unique to the press design and application and needs to be evaluated with proper information from the press manufacturer and the owner.

The press mechanics often include rotating and reciprocating components. The dynamic forces from these individual pieces follow the rules established in earlier sections of this document for rotating and reciprocating components. Only the press manufacturer familiar with all the internal components can knowledgeably calculate the specific forces. Figure 2.3 presents a horizontal force time-history for a forging press. Similar presses can be expected to have similar characteristics; however, the particular values and timing data differ.

The press drive mechanisms include geared and direct-drive systems. Depending on the design, these drives may or may not be balanced. The press slide travels vertically through a set stroke of 1/2 in. (12 mm) to several inches at a given speed. Some small presses may have inclinable beds so that the slide is not moving vertically. It is often adequate to assume that the slide moves in a vertical path defined by a circularly rotating crankshaft, that is

$$d_s(t) = \frac{S}{2}\sin(\omega_o t) \qquad (3\text{-}30)$$

where

d_s = displacement of the slide, in. (mm);
S = press stroke, in. (mm); and
ω_o = circular operating frequency of the machine (rad/s).

This leads to a dynamic inertia force from the slide of

$$F_s(t) = m_d\omega_o^2\frac{S}{2}\sin(\omega_o t)/12 \quad \text{lbf} \qquad (3\text{-}31)$$

$$F_s(t) = m_d\omega_o^2\frac{S}{2}\sin(\omega_o t)/1000 \quad \text{N}$$

where

F_s = dynamic inertia force of slide, lbf (N); and
m_d = slide mass including the effects of any balance mechanism, lbm (kg).

This assumption is based on simple circular motions and simple linkages. Other systems may be in-place to increase the press force and improve the timing. These other systems may increase the acceleration of the unbalanced weights and thus alter the magnitude and frequency components of the dynamic force transmitted to the foundation.

3.2.5 *Loading conditions*—During their lives, machinery equipment support structures and foundations undergo different loading conditions including erection, testing, shutdown, maintenance, and normal and abnormal operation. For each loading condition, there can be one or more combinations of loads that apply to the structure or foundation. The following loading conditions are generally considered in design:

- Erection condition represents the design loads that act on the structure/foundation during its construction;
- Testing condition represents the design loads that act on the structure/foundation while the equipment being supported is undergoing testing, such as hydrotest;
- Empty (shutdown) represents the design loads that act on the structure when the supported equipment is at its least weight due to removal of process fluids, applicable internals, or both as a result of maintenance or other out-of-service disruption;
- Normal operating condition represents the design loading during periods of normal equipment operation; and
- Abnormal operating condition represents the design loading during periods when unusual or extreme operating loads act on the structure/foundation.

3.2.6 *Load combinations*—Table 3.2 shows the general classification of loads for use in determining the applicable load factors in strength design (ACI 318). In considering soil stresses, the normal approach is working stress design without load factors and with overall factors of safety identified as appropriate by geotechnical engineers. The load combinations frequently used for the various load conditions are as follows:

1. Erection
   a) Dead load + erection forces

Table 3.2—Load classifications for ultimate strength design

Design loads	Load classification
Weight of structure, equipment, internals, insulation, and platforms Temporary loads and forces caused by erection Fluid loads during testing and operation Thermal loads Anchor and guide loads	Dead
Platform and walkway loads Materials to be temporarily stored during maintenance Materials normally stored during operation such as tools and maintenance equipment Vibrating equipment forces Impact loads for hoist and equipment handling utilities	Live
Earthquake loads Transportation loads Snow, ice, or rain loads Wind loads	Environmental

b) Dead load + erection forces + reduced wind + snow, ice, or rain

c) Dead load + erection forces + seismic + snow, ice, or rain

2. Testing

a) Dead load + test loads

b) Dead load + test loads + live + snow, ice, or rain

c) Dead load + test loads + reduced wind + snow, ice, or rain

3. Empty (shutdown)

a) Dead load + maintenance forces + live load + snow, ice, or rain

4. Normal operation

a) Dead load

b) Dead load + thermal load + machine forces + live loads + wind + snow, ice, or rain

c) Dead load + thermal load + machine forces + seismic + snow, ice, or rain

5) Abnormal operation

a) Dead load + upset (abnormal) machine loads + live + reduced wind

It is common to only use some fraction of full wind, such as 80% in combination with erection loads and 33% for test loads, due to the short duration of these conditions (ASCE 7).

3.3—Dynamic soil properties

Soil dynamics deals with engineering properties and behavior of soil under dynamic stress. For the dynamic analysis of machine foundations, soil properties, such as Poisson's ratio, dynamic shear modulus, and damping of soil, are generally required.

Though this work is typically completed by a geotechnical engineer, this section provides a general overview of methods used to determine the various soil properties. Many references are available that provide a greater level of detail on both theory and standard practice, including Das (1993), Bowles (1996), Fang (1991), and Arya, O'Neill, and Pincus (1979). Seed and Idriss (1970) provide greater detail on items that influence different soil properties.

This section does not cover considerations that affect the suitability of a given soil to support a dynamic machine foundation. Problems could include excessive settlement caused by dynamic or static loads, liquefaction, dimensional stability of a cohesive soil, frost heave, or any other relevant soils concern.

In general, problems involving the dynamic properties of soils are divided into small and large strain amplitude responses. For machine foundations, the amplitudes of dynamic motion, and consequently the strains in the soil, are usually low (strains less than10^{-3}%). A foundation that is subjected to an earthquake or blast loading is likely to undergo large deformations and, therefore, induce large strains in the soil. The information in this report is only applicable for typical machine foundation strains. Refer to Seed and Idriss (1970) for information on strain-related effects on shear modulus and material damping.

The key soil properties, Poisson's ratio and dynamic shear modulus, may be significantly affected by water table variations. Prudence suggests that in determining these properties, such variations be considered and assessed, usually in conjunction with the geotechnical engineers. This approach often results in expanding the range of properties to be considered in the design phase.

3.3.1 *Poisson's ratio*—Poisson's ratio ν, which is the ratio of the strain in the direction perpendicular to loading to the strain in the direction of loading, is used to calculate both the soil stiffness and damping. Poisson's ratio can be computed from the measured values of wave velocities traveling through the soil. These computations, however, are difficult. The stiffness and damping of a foundation system are generally insensitive to variations of Poisson's ratio common in soils.

Generally, Poisson's ratio varies from 0.25 to 0.35 for cohesionless soils and from 0.35 to 0.45 for cohesive soils. If no specific values of Poisson's ratio are available, then, for design purposes, the engineer may take Poisson's ratio as 0.33 for cohesionless soils and 0.40 for cohesive soils.

3.3.2 *Dynamic shear modulus*—Dynamic shear modulus G is the most important soil parameter influencing the dynamic behavior of the soil-foundation system. Together with Poisson's ratio, it is used to calculate soil impedance. Refer to Section 4.2 for the discussion on soil impedance.

The dynamic shear modulus represents the slope of the shear stress versus shear strain curve. Most soils do not respond elastically to shear strains; they respond with a combination of elastic and plastic strain. For that reason, plotting shear stress versus shear strain results in a curve not a straight line. The value of G varies based on the strain considered. The lower the strain, the higher the dynamic shear modulus.

Several methods are available for obtaining useful values of dynamic shear modulus:

- Field measurements of stress wave velocities of in-place soils;
- Laboratory tests on soil samples; and
- Correlation to other soil properties.

Due to variations inherent in the determination of dynamic shear modulus values, it may be appropriate to complete more than one foundation analysis. One analysis could be completed with the minimum possible value, one could be completed using the maximum possible value, and then additional analyses could be completed with intermediate values.

3.3.2.1 *Field determination*—Field measurements are the most common method for determining the dynamic shear modulus of a given soil. These methods involve measuring the soil characteristics, in-place, as close as possible to the actual foundation location(s).

Because field determinations are an indirect determination of shear modulus, the specific property measured is the shear wave velocity. There are three different types of stress waves that can be transmitted through soil or any other elastic body.

- Compression (primary P) waves;
- Shear (secondary S) waves; and
- Rayleigh (surface) waves.

Compression waves are transmitted through soil by a volume change associated with compressive and tensile stresses. Compression waves are the fastest of the three stress waves.

Shear waves are transmitted through soil by distortion associated with shear stresses in the soil and are slower than compression waves. No volume change occurs in the soil. Rayleigh waves occur at the free surface of an elastic body; typically, this is the ground surface. Rayleigh waves have components that are both perpendicular to the free surface and parallel to the free surface and are slightly slower than shear waves.

Several methods are available for measuring wave velocities of the in-place soil:

- The cross-hole method;
- The down-hole method;
- The up-hole method; and
- Seismic reflection (or refraction).

In the cross-hole method, two vertical boreholes are drilled. A signal generator is placed in one hole and a sensor is placed in the other hole. An impulse signal is generated in one hole, and then the time the shear wave takes to travel from the signal generator to the sensor is measured. The travel time divided by the distance yields the shear wave velocity. The cross-hole method can be used to determine G at different depths (Fig. 3.5).

In the down-hole method, only one vertical borehole is drilled. A signal generator is placed at the ground surface some distance away from the borehole, and a sensor is placed in the bottom of the borehole. An impulse signal is generated, and then the time the shear wave takes to travel from the signal generator to the sensor is measured. The travel time divided by the distance yields the shear wave velocity. This method can be run several different times, with the signal generator located at different distances from the borehole each time. This permits the measuring of soil properties at several different locations, which can then be averaged to determine an average shear wave velocity (Fig. 3.6).

The up-hole method is similar to the down-hole method. The difference is that the signal generator is placed in the borehole and the sensor is placed at the ground surface.

Dynamic shear modulus and measured-in-field shear wave velocity are related as follows

$$G = \rho(V_s)^2 \tag{3-32}$$

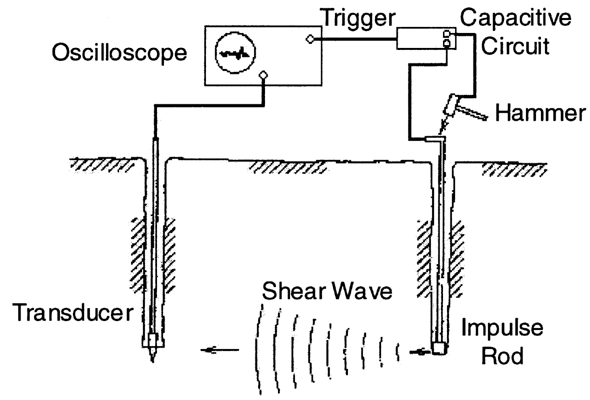

Fig. 3.5—Schematic of cross-hole technique.

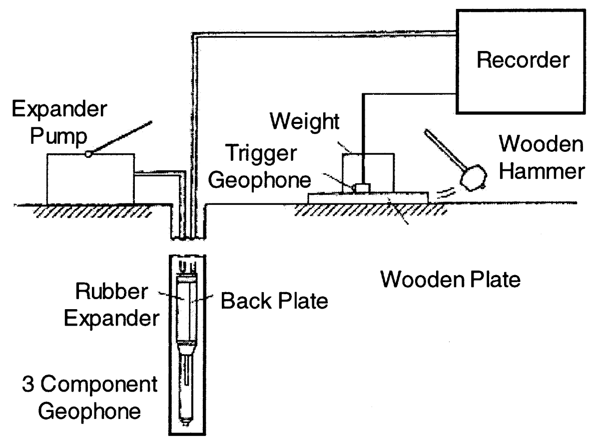

Fig. 3.6—Equipment and instrumentation for down-hole survey.

where

G = dynamic shear modulus of the soil, lbf/ft^2 (Pa);
V_s = shear wave velocity of the soil, ft/s (m/s); and
ρ = soil mass density, lbm/ft^3 (kg/m^3).

An alternative field method is to use reflection or refraction of elastic stress waves. These methods are based on the principle that when elastic waves hit a boundary between dissimilar layers, the wave is reflected or refracted. This method should only be used at locations where the soils are deposited in discrete horizontal, or nearly horizontal layers, or at locations where soil exists over top of bedrock. This method consists of generating a stress wave at one location at the ground surface and measuring the time it takes for the stress wave to reach a second location at the ground surface. The wave travels from the ground surface to the interface between differing soils layers, travels along the interface, then back to the ground surface. The time the wave takes to travel from the signal generator to the sensor is a function of the soils properties and the depth of the soil interface. One advantage of this method is that no boreholes are required. Also, this method yields an estimated depth to differing soil layers. One disadvantage is that this method cannot be used when the groundwater table is near the ground surface.

3.3.2.2 *Laboratory determination*—Laboratory tests are considered less accurate than field measurements due to the

Table 3.3—Values of K_2 versus relative density (Seed and Idriss 1970)

Relative density, %	K_2
90	70
75	61
60	52
45	43
40	40
30	34

possibility of sample disturbance. Sometimes laboratory tests are used to validate field measurements when a high level of scrutiny is required, for instance, when soil properties are required for a nuclear energy facility.

The most common laboratory test is the Resonant-Column method, where a cylindrical sample of soil is placed in a device capable of generating forced vibrations. The soil sample is exited at different frequencies until the resonant frequency is determined. The dynamic soil modulus can be calculated based on the frequency, the length of the soil sample, the end conditions of the soil sample, and the density of the soil sample. ASTM D 4015 defines the Resonant-Column method.

3.3.2.3 *Correlation to other soil properties*—Correlation is another method for determining dynamic soils properties. The engineer should be careful when using any correlation method because these are generally the least-accurate methods. The most appropriate time to consider using these methods is for preliminary design or for small noncritical applications with small dynamic loads. Correlation to other soil properties should be considered as providing a range of possible values, not providing a single exact value.

Hardin and Richart (1963) determined that soil void ratio e_v and the probable confining pressure σ_o had the most impact on the dynamic shear modulus. Hardin and Black (1968) developed the following relationships:

For round-grained sands with $e < 0.8$, dynamic shear modulus can be estimated from

$$G = \frac{31{,}530(2.17 - e_v)^2\sqrt{\sigma_o}}{1 + e_v} \text{ lbf/ft}^2 \tag{3-33}$$

$$G = \frac{218{,}200(2.17 - e_v)^2\sqrt{\sigma_o}}{1 + e_v} \text{ Pa}$$

For angular-grained materials with $e > 0.6$ and normally consolidated clays with low surface activity, dynamic shear modulus can be estimated from

$$G = \frac{14{,}760(2.97 - e_v)^2\sqrt{\sigma_o}}{1 + e_v} \text{ lbf/ft}^2 \tag{3-34}$$

$$G = \frac{102{,}140(2.97 - e_v)^2\sqrt{\sigma_o}}{1 + e_v} \text{ Pa}$$

In the previous equations,

e_v = void ratio; and

σ_o = probable confining pressure, lbf/ft^2 (Pa).

In general, relative density in sand is proportional to the void ratio. Seed and Idriss (1970) provide guidance for correlating the dynamic shear modulus to relative density in sand, along with the confining pressure

$$G = 1000K_2\sqrt{\sigma_o} \text{ lb/ft}^2 \tag{3-35}$$

$$G = 6920K_2\sqrt{\sigma_o} \text{ Pa}$$

where K_2 = a parameter that depends on void ratio and strain amplitude. Table 3.3 provides values of K_2 with respect to relative density.

3.3.3 *Damping of soil*—Damping is a phenomenon of energy dissipation that opposes free vibrations of a system. Like the restoring forces, the damping forces oppose the motion, but the energy dissipated through damping cannot be recovered. A characteristic feature of damping forces is that they lag the displacement and are out of phase with the motion. Damping of soil includes two effects—geometric and material damping.

Geometric, or radiation, damping reflects energy dissipation through propagation of elastic waves away from the immediate vicinity of a foundation and inelastic deformation of soil. It results from the practical infinity of the soil medium, and it is close to viscous in character. Refer to Chapter 4 for methods of computing geometric damping.

Material, or hysteretic, damping reflects energy dissipation within the soil itself due to the imperfect elasticity of real materials, which exhibit a hysteric loop effect under cyclic loading (Fig. 3.7). The amount of dissipated energy is given by the area of the hysteretic loop. The hysteretic loop implies a phase shift between the stress and strain because there is a stress at zero strain and vice versa, as can be seen from Fig. 3.7. The amount of dissipated energy depends on strain (displacement) but is essentially independent of frequency, as shown on Fig. 3.8.

The magnitude of material damping can be established experimentally using the hysteretic loop and the relation

$$\beta_m = \frac{1}{4\pi}\frac{\Delta W}{W} \tag{3-36}$$

where

β_m = material damping ratio;

ΔW = area enclosed by the hysteretic loop; and

W = strain energy.

Instead of an experimental determination, many practitioners use a material damping ratio of 0.05, or 5%. The material damping ratio is fairly constant for small strains but increases with strain due to the nonlinear behavior of soils.

The term material or hysteretic damping implies frequency independent damping. Experiments indicate that frequency independent hysteretic damping is much more typical of soils than viscous damping because the area of the hysteretic loop does not grow in proportion to the frequency.

3.4—Vibration performance criteria

The main purposes of the foundation system with respect to dynamic loads include limiting vibrations, internal loads, and stresses within the equipment. The foundation system also limits vibrations in the areas around the equipment where other vibration-sensitive equipment may be installed, personnel may have to work on a regular basis, or damage to the surrounding structures may occur. These performance criteria are usually established based on vibration amplitudes at key points on or around the equipment and foundation system. These amplitudes may be based on displacement, velocity, or acceleration units. Displacement limitations are commonly based on peak-to-peak amplitudes measured in mils (0.001 in.) or microns (10^{-6} m). Velocity limitations are typically based on either peak velocities or root-mean-square (rms) velocities in units of inch per second or millimeter per second. Displacement criteria are almost always frequency dependent with greater motions tolerated at slower speeds. Velocity criteria may depend on frequency but are often independent. Acceleration criteria may be constant with frequency or dependent.

Some types of equipment operate at a constant speed while other types operate across a range of speeds. The foundation engineer should consider the effect of these speed variations during the foundation design.

3.4.1 *Machine limits*—The vibration limits applicable to the machine are normally set by the equipment manufacturer or are specified by the equipment operator or owner. The limits are usually predicated on either limiting damage to the equipment or ensuring proper performance of the equipment. Limits specified by operators of the machinery and design engineers are usually based on such factors as experience or the installation of additional vibration monitoring equipment.

For rotating equipment (fans, pumps, and turbines), the normal criterion limits vibration displacements or velocities at the bearings of the rotating shaft. Excessive vibrations of the bearings increase maintenance requirements and lead to premature failure of the bearings. Often, rotating equipment has vibration switches to stop the equipment if vibrations become excessive.

Reciprocating equipment (diesel generators, compressors, and similar machinery) tends to be more dynamically rugged than rotating equipment. At the same time, it often generates greater dynamic forces. While the limits may be higher, motions are measured at bearing locations. In addition, operators of reciprocating compressors often monitor vibrations of the compressor base relative to the foundation (sometimes called "frame movement") as a measure of the foundation and machine-mounting condition and integrity.

Impulsive machines (presses, forging hammers) tend not to have specific vibration limitations as controllable by the foundation design. With these machines, it is important to recognize the difference between the inertial forces and equipment dynamics as contrasted with the foundation system dynamics. The forces with the equipment can generate significant

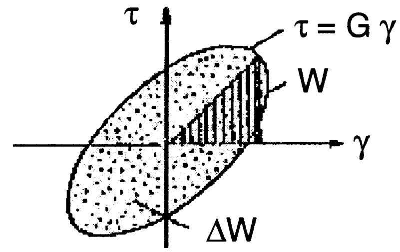

Fig. 3.7—Hysteretic loop.

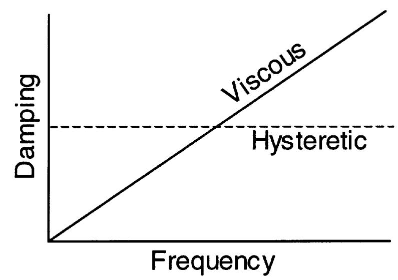

Fig. 3.8—Comparison of viscous and hysteretic damping.

accelerations and stresses that are unrelated to the stiffness, mass, or other design aspect of the foundation system. Thus, monitoring accelerations in particular on an equipment frame may not be indicative of foundation suitability or adequacy.

Researchers have presented various studies and papers addressing the issues of machinery vibration limits. This variety is reflected in the standards of engineering companies, plant owners, and industry standards. When the equipment manufacturer does not establish limits, recommendation from ISO 10816-1, Blake (1964), and Baxter and Bernhard (1967) are often followed. Most of these studies relate directly to rotating equipment. In many cases they are also applicable to reciprocating equipment. Rarely do these studies apply to impulsive equipment.

ISO publishes ISO 10816 in a series of six parts to address evaluation of machinery vibration by measurements on the nonrotating parts. Part 1 provides general guidelines and sets the overall rules with the subsequent parts providing specific values for specific machinery types. These standards are primarily directed toward in-place measurements for the assessment of machinery operation. They are not intended to identify design standards. Design engineers, however, have used predecessor documents to ISO 10816 as a baseline for design calculations and can be expected to do similarly with these more recent standards.

The document presents vibration criteria in terms of rms velocity. Where there is complexity in the vibration signal (beyond simple rotor unbalance), the rms velocity basis provides the broad measure of vibration severity and can be correlated to likely machine damage. For situations where the pattern of motion is fairly characterized by one simple harmonic, such as simple rotor unbalance, the rms velocities can be multiplied by $\sqrt{2}$ to determine corresponding peak velocity criteria. For these same cases, displacements can be calculated as

Table 3.4—Service factors from Blake (Richart, Hall, and Woods 1970)

Item	Bolted down	Not bolted down
Single-stage centrifugal pump, electric motor, fan	1.0	0.4
Typical chemical processing equipment, noncritical	1.0	0.4
Turbine, turbo-generator, centrifugal compressor	1.6	0.6
Centrifugal, stiff-shaft (at basket housing), multi-stage centrifugal pump	2.0	0.8
Miscellaneous equipment, characteristics unknown	2.0	0.8
Centrifuge, shaft-suspended, on shaft near basket	0.5	0.2
Centrifuge, link-suspended, slung	0.3	0.1

Notes: 1. Vibration is measured at the bearing housing except as noted; 2. Machine tools are excluded; and 3) Compared or measured displacements are multiplied by the appropriate service factor before comparing with Fig. 3.9.

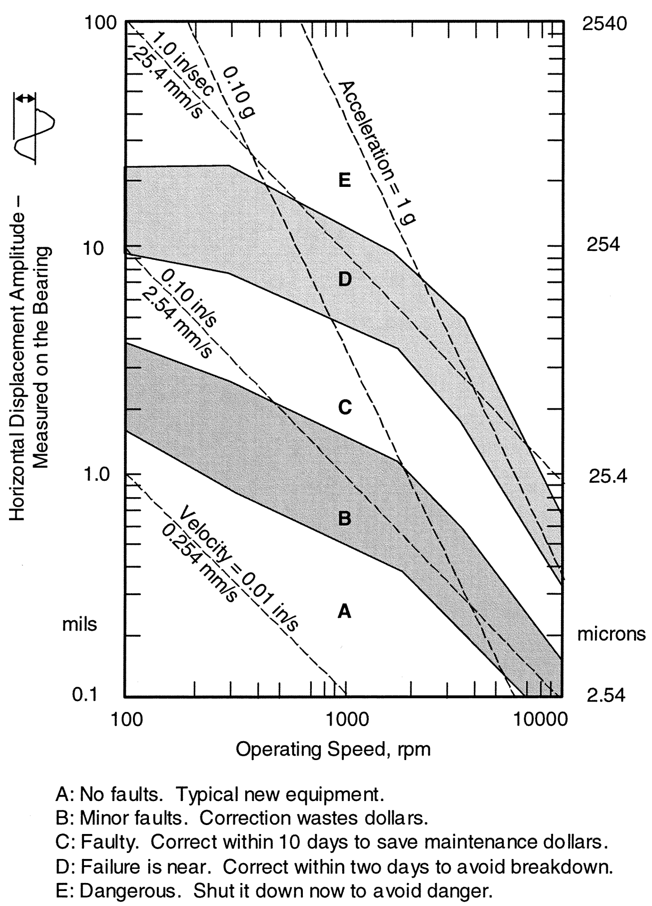

Fig. 3.9—Vibration criteria for rotating machinery (Blake 1964, as modified by Arya, O'Neill, and Pincus 1979).

$$v = \frac{v'}{\omega_m} = \frac{v'}{2\pi f_m} \quad (3\text{-}37)$$

where

v = displacement amplitude, in. (cm);

v' = velocity, in./s (cm/s);

ω_m = circular frequency of motion, rad/s; and

f_m = frequency of motion, Hz.

The rms velocity yields an rms displacement, and a peak velocity results in a zero-to-peak displacement value, which can be doubled to determine a peak-to-peak displacement value. If the motion is not a simple pure harmonic motion, a simple relationship among the rms displacement, rms velocity, peak velocity, zero-to-peak displacement, and peak-to-peak displacement does not exist.

ISO 10816-1 identifies four areas of interest with respect to the magnitude of vibration measured:

- Zone A: vibration typical of new equipment;
- Zone B: vibration normally considered acceptable for long-term operation;
- Zone C: vibration normally considered unsatisfactory for long-term operation; and
- Zone D: vibration normally considered severe enough to damage the machine.

The subsequent parts of ISO 10816 establish the boundaries between these zones as applicable to specific equipment. Part 2, ISO 10816-2, establishes criteria for large, land-based, steam-turbine generator sets rated over 67,000 horsepower (50 MW). The most general of the standards is Part 3, ISO 10816-3, which addresses in-place evaluation of general industrial machinery nominally more than 15 kW and operating between 120 and 15,000 rpm. Within ISO 10816-3, criteria are established for four different groups of machinery, and provisions include either flexible or rigid support conditions. Criteria are also established based on both rms velocity and rms displacement. Part 4, ISO 10816-4, identifies evaluation criteria for gas-turbine-driven power generation units (excluding aircraft derivatives) operating between 3000 and 20,000 rpm. Part 5 (ISO 10816-5) applies to machine sets in hydro-power facilities and pumping plants. Part 6, ISO 10816-6, provides evaluation criteria for reciprocating machines with power ratings over 134 horsepower (100 kW). The scope of Part 5 is not applicable to general equipment foundations and the criteria of Part 6 are not sufficiently substantiated and defined to be currently useful.

Another document available for establishing vibration limitation is from Lifshits (Lifshits, Simmons, and Smalley 1986). This document follows Blake's approach of identifying five different categories from No Faults to Danger of Immediate Failure. In addition, a series of correction factors are established to broaden the applicability to a wider range of equipment and measurement data.

Blake's paper (Blake 1964) has become a common basis for some industries and firms. His work presented a standard vibration chart for process equipment with performance rated from "No Faults (typical of new equipment)" to "Dangerous (shut it down now to avoid danger)." The chart was primarily intended to aid plant personnel in assessing field installations and determining maintenance plans. Service factors for different types of equipment are used to allow widespread use of the basic chart. This tool uses vibration displacement (in. or mm) rather than velocity and covers speed ranges from 100 to 10,000 rpm. Figure 3.9 and Table 3.4 present the basic chart and service factors established by Blake.

Baxter and Bernhard (1967) offered more general vibration tolerances in a paper that has also become widely referenced. Again with primary interest to the plant maintenance operations, they established the General Machinery Vibration Severity Chart, shown in Fig. 3.10, with severity ranging from extremely smooth to very rough. These are plotted as displacement versus vibration frequency so that the various categories are differentiated along lines of constant peak velocity.

The American Petroleum Institute (API) also has a series of standards for equipment common in the petrochemical industry (541, 610, 612, 613, 617, 618, and 619). ISO 10816-3 can be applied for some large electrical motors; however, most design offices do not generally perform rigorous analyses for these items.

Figure 3.11 provides a comparison of five generic standards against four corporate standards. To the extent possible, the comparisons are presented on a common basis. In particular, the comparison is based on equipment that is in service, perhaps with minor faults, but which could continue in service indefinitely. The Blake line is at the upper limit of the zone identifying operation with minor faults with a service factor of one applicable for fans, some pumps, and similar equipment. The Lifshits line separates the acceptable and marginal zones and includes a K of 0.7, reflecting equipment with rigid rotors. The ISO lines are drawn at the upper level of Zone B, normally considered acceptable for long-term operation. The ISO 10816-3 line is for large machines between 400 and 67,000 horsepower (300 kW and 50 MW) on rigid support systems. The ISO 10816-2 is for large turbines over 67,000 horsepower (50 MW).

The company standards are used for comparison to calculate motions at the design stage. For these calculations, the companies prescribe rotor unbalance conditions worse than those expected during delivery and installation. These load definitions are consistent with those presented in Section 3.2.2.1. Thus, there is a level of commonality. Company G's criteria are for large turbine applications and, thus, most comparable to the ISO 10816-2 criteria. The other company standards are for general rotating equipment. Company F permits higher motions for reciprocating equipment. In all cases the design companies standards reflect that the manufacturer may establish equipment-specific criteria that could be more limiting than their internal criteria.

Figure 3.11 shows that the corporate standards are generally below the generic standards because the generic standards are intended for in-place service checks and maintenance decisions rather than offering initial design criteria. One company is clearly more lenient for very low-speed equipment, but the corporate standards tend to be similar.

The Shock and Vibration Handbook (Harris 1996) contains further general information on such standards.

3.4.2 *Physiological limits*—Human perception and sensitivity to vibration is ambiguous and subjective. Researchers have studied and investigated this topic, but there are no clear uniform U.S. standards. In Germany, VDI 2057 provides guidance for the engineer. Important issues are the personnel expectations and needs and the surrounding environment.

ISO 2631 provides guidance for human exposure to whole-body vibration and considers different comfort levels and duration of exposure. This document does not address the extensive complexities identified in ISO 2631. Figure 3.12 presents the basic suggested acceleration limits from ISO 2631 applicable to longitudinal vibrations (vertical for a standing person). This figure reflects the time of exposure and frequency consideration for fatigue-decreased proficiency. The figure shows that people exhibit fatigue and reduced proficiency when subjected to small accelerations for long periods or greater accelerations for shorter periods. The frequency of the accelerations also impact fatigue and proficiency.

The modified Reiher-Meister figure (barely perceptible, noticeable, and troublesome) is also used to establish limits with respect to personnel sensitivity, shown in Fig. 3.13.

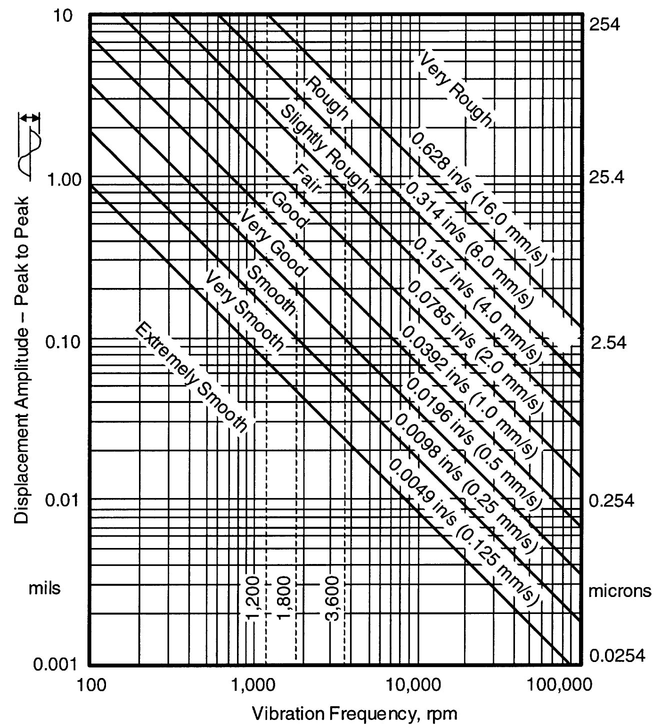

Fig. 3.10—General Machinery Vibration Severity Chart (Baxter and Bernhard 1967).

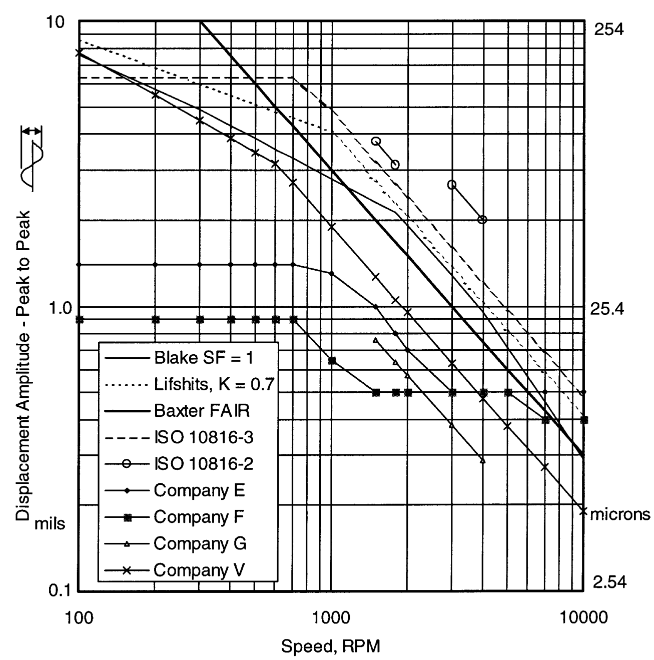

Fig. 3.11—Comparison of permissible displacements.

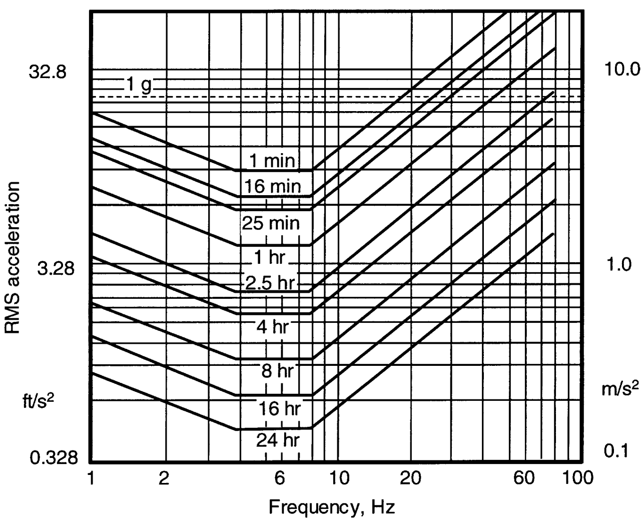

Fig. 3.12—Longitudinal acceleration limits (adapted from ISO 2631-1).

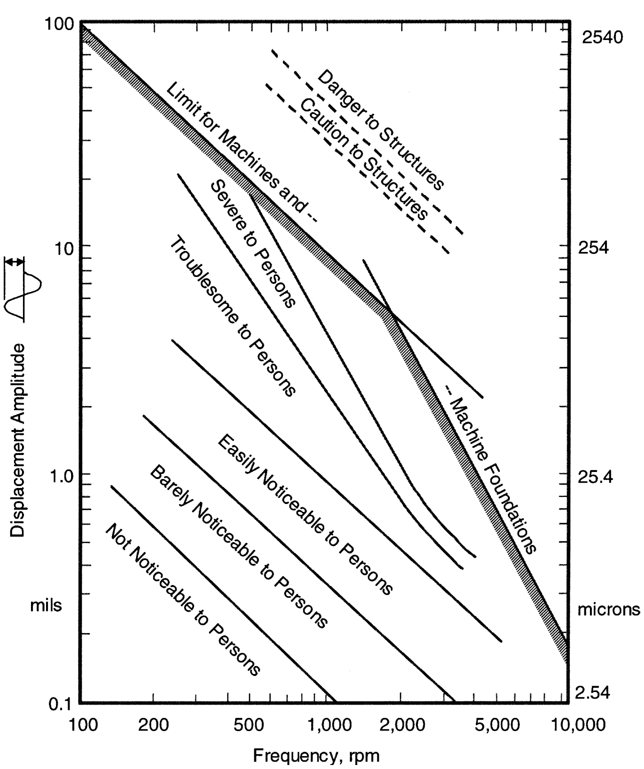

Fig. 3.13—Reiher-Meister Chart (Richart, Hall, and Woods 1970).

DIN 4150 is another standard used internationally. Part 3 defines permissible velocities suitable for assessment of short-term vibrations on structures, which are given in Table 3.5. Furthermore, Part 2 of this German standard defines limitations for allowable vibrations based on perception as a function of location (residential, light industrial) and either daytime or nighttime. Most engineering offices do not consider human perception to vibrations, unless there are extenuating circumstances (proximity to office or residential areas).

There are no conclusive limitations on the effects of vibration of surrounding buildings. The Reiher-Meister figure identifies levels of vibration from mining operations that have damaged structures.

Table 3.5—Short-term permissible values (DIN 4150-3)

Type of building	Foundation (1 to 10 Hz)	Foundation (10 to 50 Hz)	Foundation (50 to 100 Hz)	Top complete floor (all frequencies)
Industrial and commercial	0.8 in./s (20 mm/s)	0.8 to 1.6 in./s (20 to 40 mm/s)	1.6 to 2.0 in.s (40 to 50 mm/s)	1.6 in./s (40 mm/s)
Residential	0.2 in./s (5 mm/s)	0.2 to 0.6 in./s (5 to 15 mm/s)	0.6 to 0.8 in./s (15 to 20 mm/s)	0.6 in./s (15 mm/s)
Special or sensitive	0.1 in./s (3 mm/s)	0.1 to 0.3 in./s (3 to mm/s)	0.3 to 0.4 in./s (8 to 10 mm/s)	0.3 in./s (8 mm/s)

3.4.3 *Frequency ratios*—The frequency ratio is a term that relates the operating speed of the equipment to the natural frequencies of the foundation. Engineers or manufacturers require that the frequency of the foundation differ from the operating speed of the equipment by certain margins. This limitation is applied to prevent resonance conditions from developing within the dynamic soil-foundation-equipment. The formulation or presentation of frequency ratios may be based around either f_o/f_n or f_n/f_o (operating frequency to natural frequency or its inverse), and engineers or manufacturers should exercise caution to prevent misunderstandings.

A common practice among engineering firms is to compute the natural frequencies of the basic equipment-foundation and compare the values with the dynamic excitation frequency. Many companies require that the natural frequency be 20 to 33% removed from the operating speed. Some firms have used factors as low as 10% or as high as 50%. If the frequencies are well separated, no further evaluation is needed. If there is a potential for resonance, the engineer should either adjust to the foundation size or perform more refined calculations. Refined calculations may include an analysis with a deliberately reduced level of damping. The size and type of equipment play an important role in this decision process.

Frequency ratio is a reasonable design criterion, but one single limiting value does not fit all situations. Where there is greater uncertainty in other design parameters (soil stiffness, for example), more conservatism in the frequency ratio may be appropriate. Similarly, vibration problems can exist even though resonance is not a problem.

3.4.4 *Transmissibility*—A common tool for the assessment of vibrations at the design stage is a transmissibility ratio, as shown in Fig. 3.14, which is based on a single degree-of-freedom (SDOF) system with a constant speed f_o excitation force. This ratio identifies the force transmitted through the spring-damper system with the supporting system as compared with the dynamic force generated by the equipment. This ratio should be as low as possible, that is, only transmit 20% of the equipment dynamic force into the supporting system. Low transmissibility implies low vibrations in the surroundings, but this is not an absolute truth.

This transmissibility figure assumes that the damping force is directly and linearly proportional to the velocity of the SDOF. Where the system characteristics are such that the damping force is frequency dependent, the aforementioned representation is not accurate. When the damping resistance decreases at higher frequencies, the deleterious effect of damping on force transmissibility can be mitigated.

For soil or pile-supported systems, the transmissibility ratio may not be meaningful. In SDOF models of these systems, the spring and damper are provided by the soil and, while the transmissibility of the design may be low, the energy worked through these system components is motion in the surroundings that may not be acceptable.

3.5—Concrete performance criteria

The design of the foundation should withstand all applied loads, both static and dynamic. The foundation should act in unison with the equipment and supporting soil or structure to meet the deflection limits specified by the machinery manufacturer or equipment owner.

The service life of a concrete foundation should meet or exceed the anticipated service life of the equipment installed and resist the cyclic stresses from dynamic loads. Cracking should be minimized to ensure protection of reinforcing steel.

The structural design of all reinforced concrete foundations should be in accordance with ACI 318. The engineer may use allowable stress methods for nonprestressed reinforced concrete.

In foundations thicker than 4 ft (1.2 m), the engineer may use the minimum reinforcing steel suggested in ACI 207.2R.

API and the Construction Industry Institute published API Recommended Practice 686/PIP REIE 686, "Recommended Practice for Machinery Installation and Installation Design." Chapter 4 of 686/PIP REIE 686 includes design criteria for soil-supported reinforced concrete foundations that supports general and special purpose machinery. The concrete used in the foundation should tolerate its environment during placement, curing, and service. The engineer should consider various exposures such as freezing and thawing, salts of chlorides and sulfates, sulfate soils, acids, carbonation, repeated wetting and drying, oils, and high temperatures.

In addition to conventional concrete, there are many technologies available—such as admixtures, additives, specialty cements, and preblended products—to help improve placement, durability, and performance properties. These additives include water reducers, set-controlling mixtures, shrinkage-compensating admixtures, polymers, silica fumes, fly ash, and fibers.

Many foundations, whether new or repaired, require a fast turnaround to increase production by reducing downtime without compromising durability and required strength. These systems may use a combination of preblended or field-mixed concrete and polymer concrete or grout to reduce downtime to 12 to 72 h, depending on foundation volume and start-up strength requirements.

3.6—Performance criteria for machine-mounting systems

The machine-mounting system (broadly categorized as either an anchorage-type or an isolator-type) attaches the dynamic machine to its foundation. It represents a vital interface between the machine and the foundation; however, it can suffer from insufficient attention to critical detail by

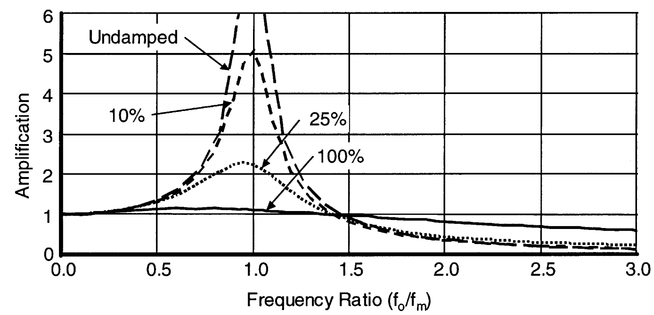

Fig. 3.14—Force transmissibility.

the foundation engineer and machinery engineer because it falls between their areas of responsibility. Anchorage-type machine-mounting systems integrate the foundation and the machine into a single structure. Isolator-type machine-mounting systems separate the machine and the foundation into two separate systems that may still dynamically interact with each other. In the processes of design, installation, and operation, the critical role of both types needs advocacy and the assurance that interface issues receive attention. The research and development of information on machine-mounting system technology by the Gas Machinery Research Council (GMRC) during the 1990s reflects the importance that this group attaches to the anchorage-type machine-mounting system. This research produced a series of reports on machine-mounting topics (Pantermuehl and Smalley 1997a,b; Smalley and Pantermuehl 1997; Smalley 1997). These reports, readily retrievable from www.gmrc.org, are essential for those responsible for dynamic machines and their foundations.

Most large machines, in spite of careful design for integrity and function by their manufacturers, can internally absorb no more than a fraction of the forces or thermal growth inherent in their function.

Those responsible for the machine-foundation interface should provide an attachment that transmits the remaining forces for dynamic integrity of the structure yet accommodates anticipated differential thermal expansion between machine and foundation. They should recognize the inherent conflicts in these requirements, the physical processes that can inhibit performance of these functions, and the lifetime constraints (such as limited maintenance and contaminating materials) from which any dynamic machine can suffer as it contributes to profitable, productive plant operation.

A dynamic machine may tend to get hotter and grow more than its foundation (in the horizontal plane). The growth can reach several tenths of an inch (0.1 in. = 2.54 mm); combustion turbine casings grow so much that they have to include deliberately installed flexibility between hotter and cooler elements of their own metallic structure. Most machines—such as compressors, steam turbines, motors, and generators—do not internally relieve their own thermal growth, so the mounting system should allow for thermal growth. Thermal growth can exert millions of pounds of force (1,000,000 lbf = 4500 kN), a level that cannot be effectively restrained.

Heat is transferred between the machine and foundation through convection, radiation, and conduction. While convection and radiation dominate in the regions where an air gap separates the machine base from the foundation, the mounting system provides the primary path for conduction.

Ten critical performance criteria can be identified as generally applicable to isolator and anchorage-type mounting systems:

1. A machine-mounting system should tolerate expected differential thermal growth across the interface. This can occur by combining strength to resist expansion forces and stresses, flexibility to accommodate the deflections, and tolerance for relative sliding across the interface (as the machine grows relative to the foundation);

2. A machine-mounting system should either absorb or transmit, across the mounting interface, those internally generated dynamic forces, resulting from the machine's operation not absorbed within the machine structure itself. These forces include both vertical and horizontal components. Flexible mounts that deflect rather than restrain the forces become an option only in cases where the machine and any rigidly attached structure have the structural rigidity needed to avoid damaging internal stresses and deflections. Large machinery may not meet this criterion;

3. A nominally rigid mount should transmit dynamic forces with only microlevel elastic deformation and negligible dynamic slippage across the interface. The dynamic forces should include local forces, such as forces from each individual cylinder, which large machines transmit to the foundation because of their flexibility. For reciprocating compressors, this criterion helps ensure that the foundation and machine form an integrated structure;

4. A machine-mounting system should perform its function for a long life—typically 25 years or more. Specifications from the operator should include required life;

5. Any maintenance and inspection required to sustain integrity of the machine-mounting system should have a frequency acceptable to the operator of the machine, for example, once per year. Engineers, installers, and operators of the machine and its foundation should agree to this maintenance requirement because the design integrity relies on the execution of these maintenance functions with this frequency;

6. The bolts that tie the machine to the mounting system, and which form an integral part of the mounting system, should have sufficient stretch and create enough normal force across all interfaces to meet the force transmission and deflection performance stated above;

7. The anchor bolt material strength should tolerate the resultant bolt tensile stresses. The mounts, soleplates, and grout layers compressed by the anchor bolt should tolerate the compressive stresses imposed on them;

8. Any polymeric material (grout or chocks) compressed by the anchor bolts should exhibit a tolerably low amount of creep to maintain bolt stretch over the time period between maintenance actions performed to inspect and tighten anchor bolts. Indeed, the machine mount should perform its function, accounting for expected creep, even if maintenance occurs less frequently;

9. The mounting system should provide a stable platform from which to align the machine. Any deflections of the mounting system that occur should remain sufficiently uniform at different points to preserve acceptable alignment of the machine. The specifications and use of adjustable chock mounts has become increasingly widespread to compensate for loss of alignment resulting from creep and other permanent deformations; and

10. The mounting system should impose tolerable loads, stresses, and deformations on the foundation itself. Appropriate foundation design to make the loads, stresses, and deformations tolerable remains an essential part of this performance criterion. Some of the loads and stresses to consider include:

- Tensile stresses in the concrete at the anchor bolt termination point, which may cause cracks;
- Shear stresses in concrete above anchor bolt termination points, which, if high enough, might result in pullout;
- Interface shear stresses between a grout layer and the concrete resulting from the typically higher expansion of polymer grout than concrete (best accommodated with expansion joints); and
- Hogging or sagging deformation of the concrete block produced by heat conduction through the mounting system. Air gaps and low conductivity epoxy chocks help minimize such deformation.

Potential conflicts requiring attention and management in these performance criteria include:

- Requirements to accommodate thermal expansion while transmitting dynamic forces; and
- Requirements to provide a large anchor bolt clamping force (so that slippage is controlled during transfer of high lateral loads) while stresses and deflections in bolt, foundation, chocks, and grout remain acceptably low.

Physical processes that can influence the ability of the mount to meet its performance criteria include:

- *Creep*—Creep of all polymeric materials under compressive load. (Creep means time-dependent deflection under load. Deflection increases with time—sometimes doubling or tripling the initial deflection);
- *Differential thermal expansion*—This can occur when two adjacent components at similar temperatures have different coefficients of thermal expansion, when two adjacent components of similar coefficients have different temperatures or a combination of both. Machine mounts with epoxy materials can experience both types of differential thermal expansion;
- *Friction*—Friction is limited by a friction coefficient. Friction defines the maximum force parallel to an interface that the interface can resist before sliding for a given normal force between the two interfacing materials;
- *Limits on friction*—The presence of oil in the interface (typically cutting the dry friction coefficient in half) causes further limits on friction;
- *Yield strength*—Yield strength of anchor bolts limits the tension available from an anchor bolt and encourages the use of high-strength anchor bolts for all critical applications;

- *Cracks*—Concrete can crack under tensile loads, and these cracks can grow with time; and
- *Oil*—Oil can pool around many machinery installations. Oil aggravates cracks in concrete, particularly under alternating stresses where it induces a hydraulic action. In many cases, oil, its additives, or the ambient materials it transports react with concrete to reduce its strength, particularly in cracks where stresses tend to be high.

Those responsible for machine mounts, as part of a foundation, should consider the aforementioned performance criteria, the conflicts that complicate the process of meeting those criteria, and the physical processes that inhibit the ability of any installation to meet the performance criteria. Other sections of this document address the calculation of loads, stresses, deflections, and the specific limits of strength implicit in different materials. The GMRC reports referred to address all these issues as they pertain to reciprocating compressors.

3.7—Method for estimating inertia forces from multicylinder machines

The local horizontal F_{zi} and vertical F_{yi} unbalanced forces for the i-th cylinder located in the horizontal plane can be written as

$$F_{zi} = (m_{rec,i} + m_{rot,i})r_i\omega_o^2\cos(\omega_o t + \alpha_i) \tag{3-38}$$

$$+ m_{rot,i}\frac{r_i^2\omega_o^2}{L_i}\cos2(\omega_o t + \alpha_i)$$

and

$$F_{yi} = m_{rec,i}r_i\omega_o^2\sin(\omega_o t + \alpha_i) \tag{3-39}$$

where

$m_{rec,i}$ = reciprocating mass for the i-th cylinder;
$m_{rot,i}$ = rotating mass of the i-th cylinder;
r_i = radius of the crank mechanism of the i-th cylinder;
L_i = length of the connecting rod of the crank mechanism at the i-th cylinder;
ω_o = circular operating frequency of the machine (rad/s);
t = time, s; and
α_i = the phase angle for the crank radius of the i-th cylinder, rad.

The primary and secondary force components are as follows

(primary)

$$F'_{zi} = (m_{rec,i} + m_{rot,i})r_i\omega_o^2\cos(\omega_o t + \alpha_i) \tag{3-40}$$

$$F'_{yi} = m_{rec,i}r_i\omega_o^2\sin(\omega_o t + \alpha_i) \tag{3-41}$$

(secondary)

$$F''_{zi} = (m_{rot,i})r_i\frac{r_i^2\omega_o^2}{L_i}\cos2(\omega_o t + \alpha_i) \tag{3-42}$$

If the i-th cylinder is oriented at angle θ_i to a global horizontal z-axis, then the primary and secondary force components, with respect to the global axis, can be rewritten as follows

(primary)

$$F_{zi}^{GP} = F'_{zi}\cos\theta_i = F'_{yi}\sin\theta_i \tag{3-43}$$

$$F_{yi}^{GP} = F'_{zi}\sin\theta_i = F'_{yi}\cos\theta_i \tag{3-44}$$

(secondary)

$$F_{zi}^{GS} = F''_{zi}\cos\theta_i \tag{3-45}$$

$$F_{yi}^{GS} = F''_{zi}\sin\theta_i \tag{3-46}$$

The resultant forces due to n cylinders in global coordinates can be calculated as follows

$$F_z^{GP} = \sum_{i=1}^{n} F_{zi}^{GP} \tag{3-47}$$

$$F_z^{GS} = \sum_{i=1}^{n} F_{zi}^{GS} \tag{3-48}$$

$$F_y^{GP} = \sum_{i=1}^{n} F_{yi}^{GP} \tag{3-49}$$

$$F_y^{GS} = \sum_{i=1}^{n} F_{yi}^{GS} \tag{3-50}$$

The resultant moments due to n cylinders in global coordinates can be determined as follows

$$M_y^{GP} = \sum_{i=1}^{n}(F_{zi}^{GP}X_i)\text{ (moment about y [vertical] axis)} \tag{3-51}$$

$$M_y^{GS} = \sum_{i=1}^{n}(F_{zi}^{GS}X_i) \tag{3-52}$$

$$M_z^{GP} = \sum_{i=1}^{n}(F_{yi}^{GP}X_i)\text{(moment about z [horizontal] axis)} \tag{3-53}$$

$$M_z^{GS} = \sum_{i=1}^{n} (F_{yi}^{GS} X_i) \tag{3-54}$$

where X_i = distance along the crankshaft from the reference origin to the i-th cylinder.

Equation (3-43) to (3-54) provide instantaneous values of time-varying inertia (shaking) forces and four time varying shaking moments for an n cylinder reciprocating machine. To visualize the time variation of these forces and moments over a revolution of the crankshaft, they can be computed at a series of crank angle values and plotted against crank angle. To obtain maximum values of the primary and secondary forces and moments (and the phase angle at which the maxima occur), they are computed at two orthogonal angles and vectorially combined as shown as follows

Maximum global horizontal primary force:

$$(F_Z^{GP})_{max} = [(F_{Z0}^{GP})^2 + (F_{Z90}^{GP})^2]^{1/2} \tag{3-55}$$

$$\text{at } \tan^{-1}(F_{Z90}^{GP}/F_{Z0}^{GP})$$

Maximum global horizontal secondary force:

$$(F_Z^{GS})_{max} = [(F_{Z0}^{GS})^2 + (F_{Z45}^{GS})^2]^{1/2} \tag{3-56}$$

$$\text{at } \tan^{-1}(F_{Z45}^{GS}/F_{Z0}^{GS})$$

Maximum global vertical primary force:

$$(F_Y^{GP})_{max} = [(F_{Y0}^{GP})^2 + (F_{Y90}^{GP})^2]^{1/2} \tag{3-57}$$

$$\text{at } \tan^{-1}(F_{Y90}^{GP}/F_{Y0}^{GP})$$

Maximum global vertical secondary force:

$$(F_Y^{GS})_{max} = [(F_{Y0}^{GS})^2 + (F_{Y45}^{GS})^2]^{1/2} \tag{3-58}$$

$$\text{at } \tan^{-1}(F_{Y45}^{GS}/F_{Y0}^{GS})$$

Maximum global horizontal primary moment:

$$(M_Z^{GP})_{max} = [(M_{Z0}^{GP})^2 + (M_{Z90}^{GP})^2]^{1/2} \tag{3-59}$$

$$\text{at } \tan^{-1}(M_{Z90}^{GP}/M_{Z0}^{GP})$$

Maximum global horizontal secondary moment:

$$(M_Z^{GS})_{max} = [(M_{Z0}^{GS})^2 + (M_{Z45}^{GS})^2]^{1/2} \tag{3-60}$$

$$\text{at } \tan^{-1}(M_{Z45}^{GS}/M_{Z0}^{GS})$$

Maximum global vertical primary moment:

$$(M_Y^{GP})_{max} = [(M_{Y0}^{GP})^2 + (M_{Y90}^{GP})^2]^{1/2} \tag{3-61}$$

$$\text{at } \tan^{-1}(M_{Y90}^{GP}/M_{Z0}^{GP})$$

Maximum global vertical secondary moment:

$$(M_Y^{GS})_{max} = [(M_{Y0}^{GS})^2 + (M_{Y45}^{GS})^2]^{1/2} \tag{3-62}$$

$$\text{at } \tan^{-1}(M_{Y45}^{GS}/F_{Y0}^{GS})$$

where subscripts 0, 45, and 90 represent the value of $\omega_o t$ used to calculate the force values listed in Eq. (3-55) to (3-62).