FORMULAS FOR STRESS, STRAIN, AND STRUCTURAL MATRICES

SECOND EDITION

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From case 2 of Table 18-4, part A, the deflection is expressed by

$$w = -\frac{R}{8\pi D} r^2(\ln r - 1) + C_1 \frac{r^2}{4} + F_w \tag{1}$$

Table 18-4, part B, gives F_w for the concentrated ring force W:

$$F_w = < r - a >^0 \frac{Wa}{4D}\left[(r^2 + a^2)\ln\frac{r}{a} - (r^2 - a^2)\right] \tag{2}$$

According to Table 18-4, part C, the reaction R and constant C_1 are given by

$$\begin{aligned} R &= -\frac{16\pi D}{a_L^2}\bar{F}_w - \frac{8\pi D}{a_L}\bar{F}_\theta \\ C_1 &= -\frac{8(\ln a_L - \frac{1}{2})}{a_L^2}\bar{F}_w - \frac{4(\ln a_L - 1)}{a_L}\bar{F}_\theta \end{aligned} \tag{3}$$

Insertion of $\bar{F}_w = F_{w|r=a_L}$ and $\bar{F}_\theta = F_{\theta|r=a_L}$ from Table 18-4, part B, into (3) gives

$$\begin{aligned} C_1 &= \frac{Wa}{D}(1 - \beta^2 + 2\beta^2 \ln\beta)[\ln a + (1 - \beta^2)\ln\beta] \\ R &= 2\pi Wa(1 - \beta^2 + 2\beta^2 \ln\beta) \end{aligned} \tag{4}$$

where $\beta = a/a_L$. Substitution of (2) and (4) into (1) provides the expression for the deflection at any radius r.

Buckling Loads

When a circular plate is subjected to a static in-plane radial force (per unit length) P, the plate equation is

$$D\,\nabla^4 w = P\,\nabla^2 w \tag{18.9}$$

in which the Laplacian operators are written in polar coordinates. For certain critical values of the in-plane load, the plate will buckle transversely even though transverse loads may not be present. A critical-load value is associated with each buckled mode shape.

Majumdar [18.3] studied the buckling of a circular plate clamped at the outer edge, free at the inner edge, and loaded with a uniform radial compressive force applied at the outside edge. It is shown that for small ratios of inner to outer radius, the plate buckles in a radially symmetric mode. When the ratio of the inner to outer radius exceeds a certain value, the minimum buckling load corresponds to buckling modes with nodes, which are the loci of points for which the displacements in the buckling modes are zero, along a circumference. The number of nodes depends on the ratio of the inner and outer radii.

Formulas for the critical load of several circular plate configurations are listed in Table 18-5. These critical loads are compressive in-plane forces per unit length applied at the outer edge. The stress corresponding to the critical load should be less than the yield strength of the material of the plate in order for the buckling load to be valid.

Techniques for obtaining solutions to circular plate stability problems are discussed in Ref. [18.1].

Example 18.3 Critical In-Plane Loads of a Circular Plate Compute the critical in-plane compressive force for a circular plate with no center hole for both (a) pinned and (b) clamped outer edges if the buckled shape has neither nodal diameters nor circles. Let $E = 3.0 \times 10^7$ lb/in^2, $h = 1.0$ in., $\nu = 0.3$, and $a_L = 36$ in., so that $D = Eh^3/[12(1-\nu^2)] = 2.75 \times 10^6$ lb-in. Also, calculate the stress level corresponding to buckling to assure that the yield stress has not been reached.

(a) For pinned edges, from case 1 of Table 18-5,

$$P_{\text{cr}} = 0.426\pi^2 \frac{D}{a_L^2} = 0.426\pi^2(2.75 \times 10^6)/(36)^2 \tag{1}$$

$$= 0.89 \times 10^4 \text{ lb/in.}$$

The stress corresponding to the stress resultant P_{cr} is given by

$$\sigma_{\text{cr}} = P_{\text{cr}}/h = 0.89 \times 10^4/1.0 = 8900 \text{ lb/in}^2 \tag{2}$$

(b) For clamped edges, from case 2 of Table 18-5,

$$P_{\text{cr}} = (1.49)\frac{\pi^2 D}{a_L^2} = 1.49\pi^2(2.75 \times 10^6)/(36)^2 = 3.12 \times 10^4 \text{ lb/in.} \tag{3}$$

$$\sigma_{\text{cr}} = P_{\text{cr}}/h = 3.12 \times 10^4/1.0 = 31{,}200 \text{ lb/in}^2 \tag{4}$$

Natural Frequencies

The formulas for natural frequencies in a number of cases of uniform thickness circular plates are listed in Table 18-6. The nodes, which are the loci of points along which the mode shape displacements are zero, occur along diameters, numbered n, of the plate or along concentric circles, numbered s, centered at the plate center. A particular mode is chosen by specifying the number of nodal diameters and nodal concentric circles. It can be observed that the fundamental frequency does not always correspond to the smallest s and n. Also, except for certain small values of

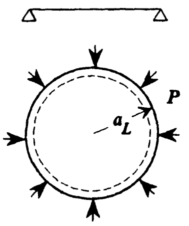
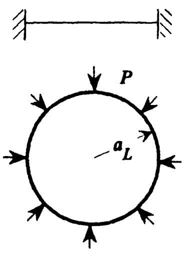
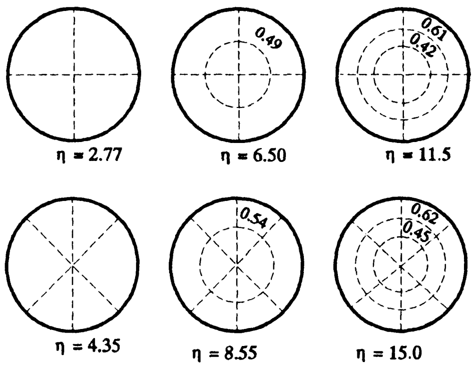
TABLE 18-5 CRITICAL IN-PLANE FORCES FOR CIRCULAR PLATES

Notation

E = modulus of elasticity
ν = Poisson's ratio
c = buckling coefficient
P_{cr} = buckling load (force per unit length)

$$\beta = a_0/a_L \qquad P' = \frac{\pi^2 D}{a_L^2} \qquad D = \frac{Eh^3}{12(1-\nu^2)}$$

Nodal circle or nodal diameter refer to the circle or diameter in the plane of the plate for which the displacement is zero in a buckling mode shape.

Conditions	Buckling Loads
1. Simply supported outer boundary 	Applies for no nodal circles or nodal diameters present in the buckling mode shapes, $P_{\mathrm{cr}} = 0.426P'$ Ref. [18.5]
2. Clamped outer boundary 	**1.** Applies for no nodal circles or nodal diameters present in the buckling mode shapes, $P_{\mathrm{cr}} = 1.49P'$ **2.** Applies when nodal circles and/or nodal diameters are present in the buckling mode shapes, $P_{\mathrm{cr}} = \eta \times 1.49P'$  Nodal circles and diameters are shown. The numbers on the dashed nodal circles refer to the percentage of a_L (e.g., $0.49a_L$). Refs. [18.5]–[18.8]