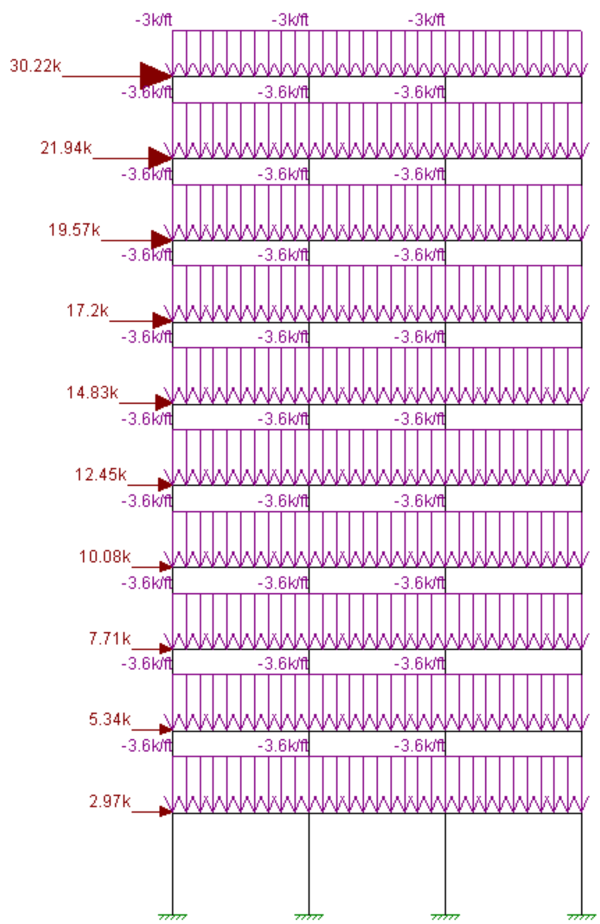
Example 2: Rayleigh Method for Natural Period

Rayleigh Method as Defined in ASCE Section 15.4.4

There are cases where an engineer wants to use a quick hand calc method to determine or verify the natural frequency of a structure. The Rayleigh method presented in ASCE -7 provides a relatively simple method of doing this which can work nicely even for relatively complex structures.

Below is a 10 story steel frame structure which has gravity loading (or mass) at each defined floor level. In additional it has been subjected to lateral wind forces at each level. The model was then solved in RISA-3D and the deflection at each floor level was determined.



Project Definition: Rayleigh Method.r3d

ASCE Equation 15.4-6 gives the Rayleigh equation for natural period calculations as the following:

$$T = 2\pi \sqrt{\frac{\sum_1^n w_i \delta_i^2}{g \sum_1^n f_i \, \delta_i}}$$

w_i = seismic weight at level i

f_i = Applied Lateral Force at level i

δ_i = Lateral Deflection at level i when subjected to applied forces.

This formula is very similar to our simplified single degree of freedom example where:

$$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{W_g * \Delta_{lat}}{g * F}}$$

The Rayleigh method just gives us a convenient way to extend this concept into multi degree of freedom systems.

One thing interesting to note is that the applied lateral forces do not need to be earthquake forces. In this case, we are using the wind loads. Below is an Excel spreadsheet summary which calculates these values for us:

Floor Level	*Floor Weight (kips)*	*Wind force (kips)*	*Deflection due to wind (inches)*	*m_i*delta_i^2*	*f_i*delta_i*
10	180	30.22	8.15	30.93	246.24
9	216	21.94	7.63	32.50	167.30
8	216	19.57	6.88	26.46	134.64
7	216	17.20	6.08	20.69	104.64
6	216	14.83	5.16	14.90	76.57
5	216	12.45	4.25	10.08	52.87
4	216	10.08	3.31	6.14	33.40
3	216	7.71	2.46	3.37	18.94
2	216	5.34	1.57	1.38	8.39
1	216	2.97	0.78	0.34	2.33
Totals	2124			146.80	845.31

Finishing off the calculation, we get:

$$T = 2\pi\sqrt{\frac{146.8}{845.31}} = 2.62\ seconds$$

We find that the Rayleigh method exactly matches exactly the value for the first mode that we get when we solve for the mode shapes and frequencies in RISA-3D as shown below.

Frequencies and Participation

	Mode	Frequency (Hz)	Period ...	SX Par...	SY Par...	SZ Par...
1	1	.3815	2.6214	78.7713		
2	2	1.0484	.9539	12.322		
3	3	1.7735	.5639	4.081		
4	Totals:			95.1742		

We can take that procedure and use it for more advanced purposes like investigating the effect of 2nd order or non-linear effects on the natural period of our structure.

Example: Adding P-Delta to the Rayleigh Solution

We have already discussed how the RISA-3D Eigen solution does not automatically account for the P-Delta effect. The Rayleigh method can be used to determine the approximate change in natural frequency due to the softening effect of these second order effects. We do this merely by including the P-Delta effect in our static analysis. For this model, the results are shown below:

Floor Level	Floor Weight (kips)	Wind force (kips)	Deflection due to wind (inches)	m_i*delta_i^2	f_i*delta_i
10	180	30.22	8.68	35.13	262.42
9	216	21.94	8.14	37.07	178.67
8	216	19.57	7.37	30.33	144.15
7	216	17.20	6.53	23.82	112.29
6	216	14.83	5.55	17.22	82.30
5	216	12.45	4.57	11.68	56.91
4	216	10.08	3.57	7.12	35.97
3	216	7.71	2.65	3.92	20.41
2	216	5.34	1.69	1.59	9.02
1	216	2.97	0.84	0.40	2.50
Totals	2124			168.27	904.63

Finishing off the calculation, we get:

$$T = 2\pi\sqrt{\frac{168.27}{904.63}} = 2.71\ seconds$$

The exercise is useful in that we can expect to see only about a 3% increase in our natural period despite the fact that our static stiffness gets reduced by about 6%.

Notes: