Comparison of Rotosolve spreadsheet calculated results and analytically-calculated results

I. Simple Beam Cases (calculation option 3)

Beam cases are easy to check since a simple analytical solution is readily available.

The cases in this section are called "simple" because they used the option 3 "simple" for the gyroscopic option, which means that all disk effects are ignored.

The program provides choices for boundary conditions: "hinged", "clamped", "free", which have the same meanings as in normal textbook beam calculations.

The beam was broken into 10 pieces and the lumped-MASS calculation option was set to false (continuos-mass calculation).

In all these cases, the following beam parameters were used:
Length = 1 meter
Diameter = 0.1 meter
Density = 7750 kg/m^3 (more specifically, 7750.37312)
E = 2.31E11 N/m^2 (more specifically, 230974359500)

The thee example simple beam cases were all generated with the inputs shown in the following file:
SimpleBeamDemo.xls

A. Free/Free beam,

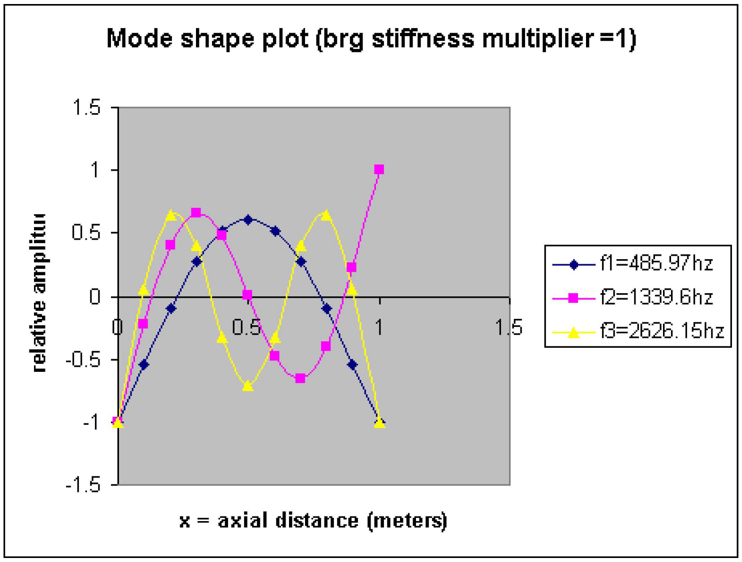
Geometry:

Results (analytical calculations shown in tab labeled "analytical calculation")

Boundary Conditions: freefree

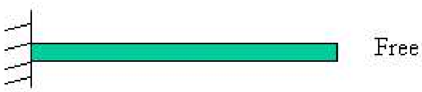
Frequency	Analytical Calculation	Program Output
1	485.971	485.971
2	1,339.598	1,339.598
3	2,626.148	2,626.148

Mode shapes



B. Clamped/Free beam (cantilevered)

Geometry as follows:

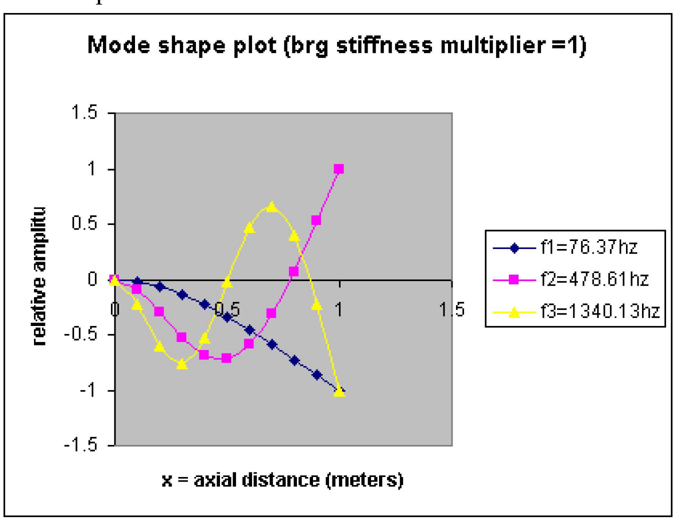


Results as follows:

Boundary Conditions: clampedfree

Frequency	Analytical Calculation	Program Output
1	76.372	76.372
2	478.612	478.612
3	1,340.128	1,340.128

Mode shapes as follows:



C. Hinged-hinged (Simply-supported) beam

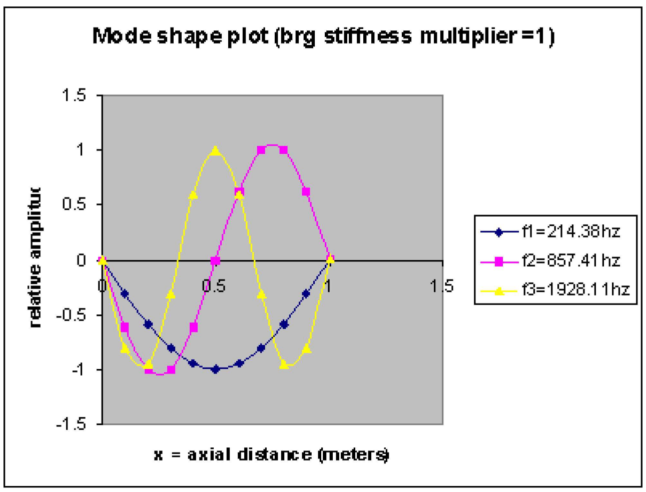
Geometry as follows:

Results as follows:

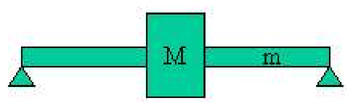
Boundary Conditions: hinged/hinged

Frequency	Analytical Calculation	Program Output
1	214.378	214.377
2	857.512	857.410
3	1,929.403	1,928.105

Mode shapes as follows:



II. Adding concentrated mass along with distributed mass.



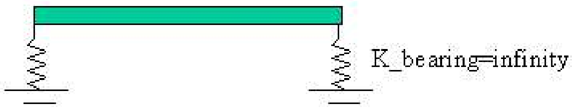
The attached file recreates the well-known scenario where a simply-supported beam with distributed beam mass m and concentrated mass M in the center causes a resonant frequency based on an effective stiffness of 48EI/L^3 and an effective mass of M + 0.5*m.

M_PLUS_Halfm.xls

Again the program calculates the result as expected.

III. Introducing bearings

Most real-world rotors will be modeled as free-free boundary conditions. It is a very simple matter to recreate the simply-supported results above by adding an additional 0-length section on the right, adding bearings to the left of the first and last rotor sections, and setting the bearing stiffness values very high, as was done in the following file: SimplySupportedFromFreeFreeWithBearings.xls



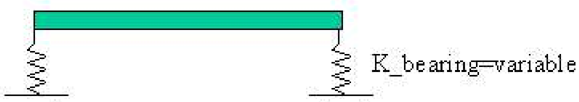
An examination of the critical speed map and the modeshapes in the above file confirms that the high bearing stiffness causes the bearings to act like rigid supports,

The frequency results match the simply-supported results above, as expected:
Results

Frequency	Analytical Calculation	Program Output
1	214.378	213.906
2	857.512	850.231
3	1,929.403	1,894.069

IV. Varying the bearing stiffness.

The model studied is the same as above, except that we have introduced a variable bearing stiffness as follows:

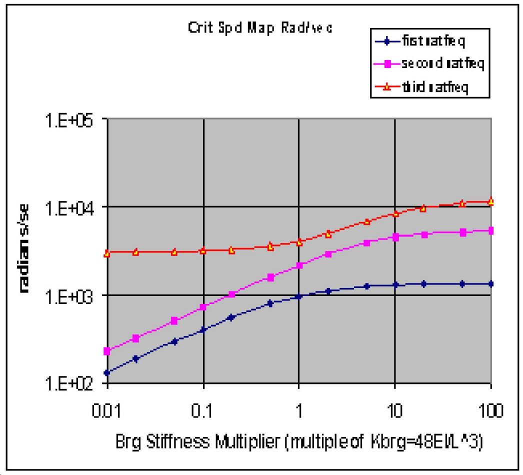


The above system is solved in the attached file: DemoShaftOnBearings2.xls

The same geometry was also solved in "Turbomachinery Rotordynamics: Phenomena, Modeling, and Analysis" By Dara Childs, page 123, which can be accessed here:
http://books.google.com/books?id=vKPfBxgQQPoC&pg=PA123&dq=%22These+modes+are+commonly+referred+to+as+stick+modes%22&sig=KKcf-5urzLoMB50PNymoxQlBzv8

A visual comparison of the critical speed maps generated by my spreadsheet with those provided by Childs shows good agreement:

My spreadsheet	Childs



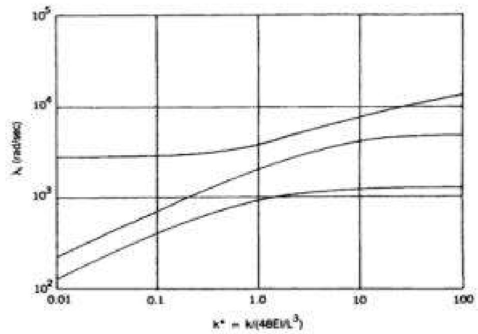
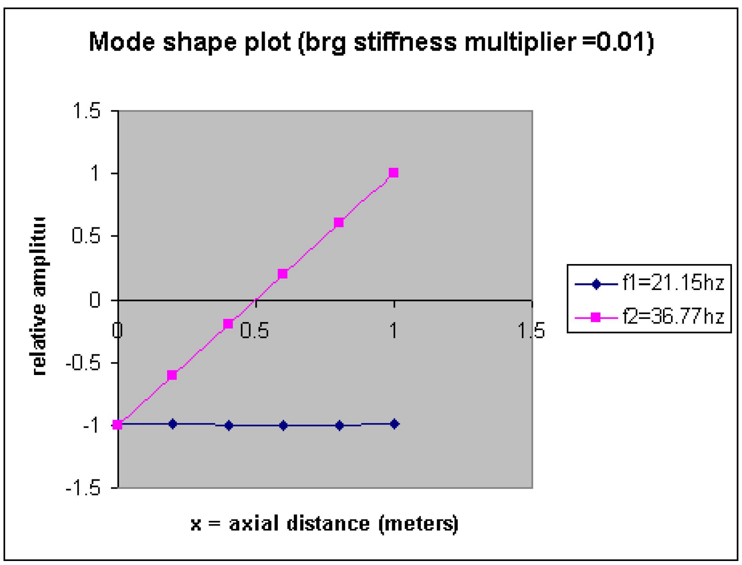


Figure 2.17 First three (lowest) rotor natural frequencies versus normalized support stiffness.

Looking toward the left of the critical speed map, we suspect that the first two modes increasing linearly on log-log plot with a slope of 0.5 are rigid-rotor modes. We confirm this with a modeshape plot of these first two frequencies (bearing multiplier 0.01):



With the rigid rotor simplification, we can analytically verify the first two resonant frequencies at a bearing stiffness multiplier of 0.01.

<u>For the first rigid rotor mode at stiffness multiplier 0.01</u>,
Kbrg = 0.01 * 48*E*I/L^3 = 543338. N/m
M = pi * rho * L * (od^2-id^2)/4 = 60.87 kg
F = sqrt(2*Kbrg/M) / (2*pi) = 21.27hz (matches program-calculated 21.15 very well).

<u>For the second rigid rotor mode at stiffness multiplier of 0.01</u>, we calculate the transverse or diametrical mass moment of inertia of the shaft (assuming all mass concentrated on the centerline... consistent with calculation mode 3) as follows:

$$Id = \frac{1}{4}\rho \cdot L \cdot \pi(R_{outer}{}^4 - R_{inner}{}^4)$$

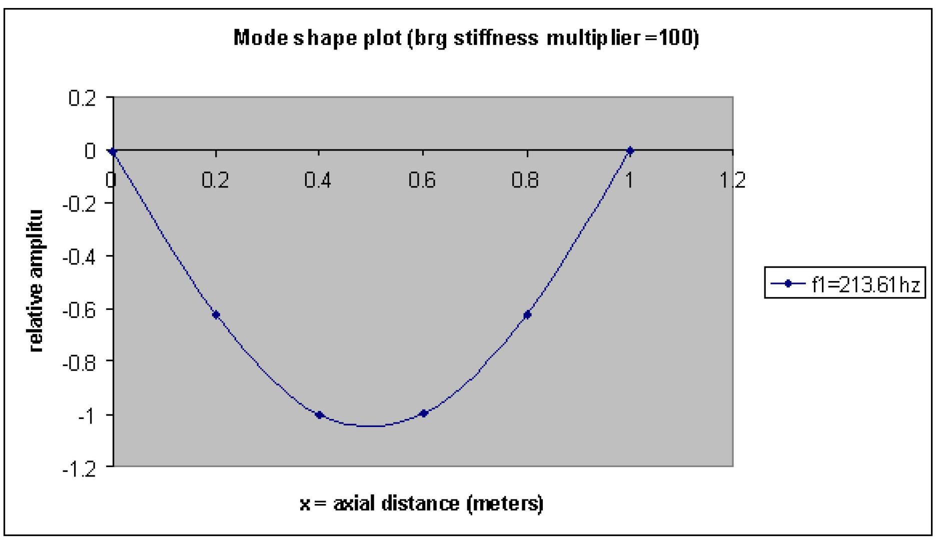
Id = 5.072363139 kg*m^2

The max kinetic energy is KE = 0.5 *Id^2 * $(\theta')^2$
For small angles, $\theta \sim \sin(\theta) = y / (L/2)$ (where y is transverse displacement at location of the bearing.
$\theta' = y' / (L/2)$
Substitute into KE equation:
KE = 0.5 *Id * $(y')^2 /(L/2)^2$
We can preserve kinetic energy by rewriting the above as
KE = 0.5 * Meffective * $(y')^2$ = 0.5 * Meffective * v^2
where Meffective = Id / $(L/2)^2$ = 20.28945255 kg

Thus from an energy standpoint, the rotary inertia Id acts like an effective mass Meffective=Id /$(L/2)^2$ at the location of one of the bearings. Since the fundamental frequency can be calculated from energy considerations (KEmax=PEmax), we can calculate the resonant frequency using this effective mass. (This approach of calculating an effective mass based on energy considerations is described in Thompson's "Mechanical Vibrations" section 2.3 or Rao's "Mechanical Vibrations" example 1.6). The relevant spring stiffness includes both bearings in parallel. The frequency is f = sqrt(2*Kb/Meffective) /(2*pi) = 36.83283813 hz. This matches the program output 36.77409375 very well (2^{nd} mode for bearing multiplier of 0.01).

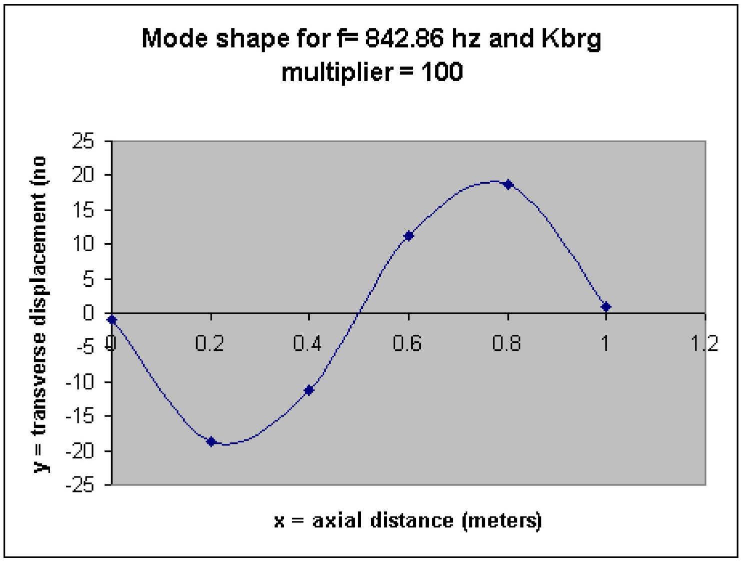
Looking toward the right side of the critical speed map toward stiffness multiplier of 100, we see a leveling of the first and second modes. We suspect these reprsent the first and second flexible rotor modes.

For the first mode at bearing stiffness multiplier 100, The modeshape plot appears to confirms a flexible rotor/rigid bearing mode.



We can check the frequency for this mode (first mode at multiplier =100) using a simply-supported beam calculation, which gives a frequency of 214.21 hz. This is reasonably close to the program-calculated resonant frequency of 213.6hz.

For the second mode at bearing stiffness multiplier 100, The modeshape plot appears to confirm a second flexible rotor/rigid bearing mode.



The theoretical modeshape of this second flexible rotor mode is sin(2π*x/L). Examining the area between x=0 and x=L/2, we find it has the same modeshape as the first flexible-rotor/rigid-bearing modeshape of a beam of length L/2. We can use this information to analytically confirm our frequency based on a simply supported beam of length L/2. The analytical solution of the half-length simply-supported beam gives 856.8hz, while the program predicts the second mode at bearing stiffness multiplier of 100 to be 842.4 hz. The small difference can be reconciled by noting that the mode shape does not come completely to 0, so there is some flexibility still present in the bearings (even at bearing stiffness multiplier of 100) which reduces the resonant frequency.

V. Adding tilting disk effects (like bump test scenario) – Calculation option 2

If we bump test a rotor with a large disk (especially overhung), the disk inertia causes the natural frequency to lower by virtue of the fact that a moment must be exerted to tilt the disk back and forth. This effect is called "rotary inertia" in beam theory, even though it is not the way we would normally use the word "rotary".

The "simply-supported" beam scenario above (1 meter beam, 0.1 m diameter, etc) was run again using option 2 to add the effects of rotary inertia. The results are shown in this file SimplySupportedWithRotaryInertaVsRaoGood.xls

As shown in the "Analytical check" tab, Rao's "Mechanical Vibrations" provides a formula for calculating the natural frequencies for this geometry (simply-supported uniform beam) including the effects of rotary inertia. The first three resonant frequencies are shown below using simple analytical calc, adding rotary inertia, and comparing to program output:

	Analytical, no rotary inertia (simple calculation)	Analytical, with rotary inertia	Program output (with rotary inertia=option 2)
f1	214.3781239	213.7199712	213.7202468
f2	857.5124957	847.1251912	847.1172571
f3	1929.403115	1877.977813	1877.775498

The rotary inertia does not play a very important role in the first mode for this geometry, but increases in importance as the higher order modeshapes introduce more nodes and more tilting. The program matches the analytical calculation very well, even at the higher mode numbers. When I increased the number of elements from 10 to 200 while keeping the total length the same, the program results matched even better (program computed f3=1877.977).

In general, we suspect rotary inertia will play in important role when there are large disks and lots of bending/tilting at the location of the disks.

VI. Gryoscopic Effects – calculation option 1

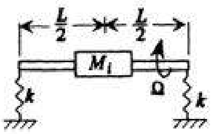
While the disk-tilting effects of option 2 tend to lower critical speed, the gryoscopic effects tend to increase critical speed. Option 1 includes both effects. This results in higher critical speeds than the simple mode (no disk effects) since the increase caused by gyroscopic effects is larger than the decrease caused by the disk tilt effects.

Rigid-rotor gyroscopically-stiffened whirl

"Formulas For Stress, Strain, And Structural Matrices", 2nd ed. by Walter D. Pilkey Table 17-1 gives the following solution for the resonant frequencies of a rigid rotor with a center disk having significant polar and transverse inertia.

7.

Long rigid rotor, elastic supports



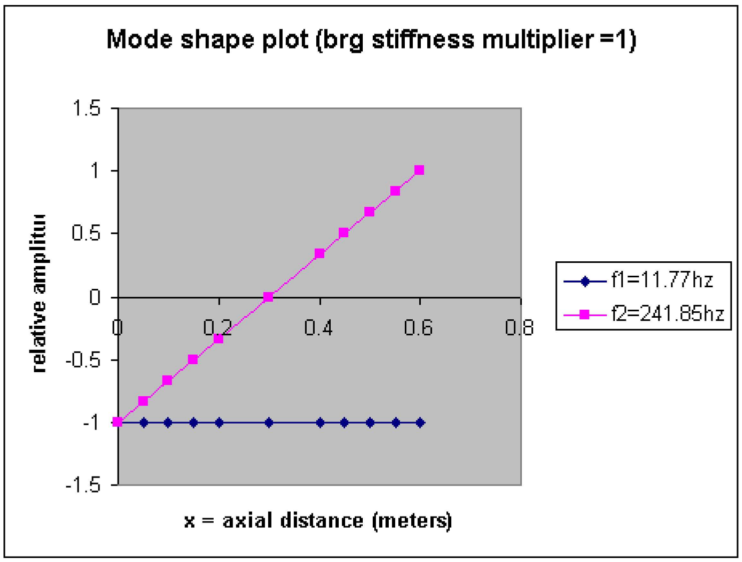
$$\omega_{c1} = \omega_{c2} = \sqrt{\frac{2k}{M_i}}$$

$$\omega_{c3} = \frac{I_{pi}}{2I_{Ti}}\Omega + \sqrt{\frac{kL^2}{2I_{Ti}} + \left(\frac{I_{pi}}{2I_{Ti}}\Omega\right)^2}$$

$$\omega_{c4} = \frac{I_{pi}}{2I_{Ti}}\Omega - \sqrt{\frac{kL^2}{2I_{Ti}} + \left(\frac{I_{pi}}{2I_{Ti}}\Omega\right)^2}$$

This was simulated in the following file RigidRotorGyroDemoWorks1.xls

The mode shapes are as follows:



Pilkey's wc1 corresponds to the first mode f1 at 11.77hz where the rigid rotor moves parallel to it's axis with a very simple solution w=sqrt(Ktotal./M).

Pilkey's wc3 corresponds to f2 and represents the forward-rotating gryoscopically-stiffened mode whose 3-d modeshape would resemble two cones with their points meeting at the center of the rotor. Note that for problems involving gryoscopic stiffening, the whirl speed changes as a function of machine speed. Therefore the analytical solution for the critical speed wc3 requires solving an implicit relationship to find the speed where the whirling frequency is equal to the machine speed (as is typical of most problems that include gyroscopic stiffening).

The parameters used for the simulation are defined in the file. The analytical calculations are shown in the analytical tab. The program results match the analytical predications reasonably well:

	Analytical	Program
f1	11.7775	11.77269
f2	244.7908	241.8507

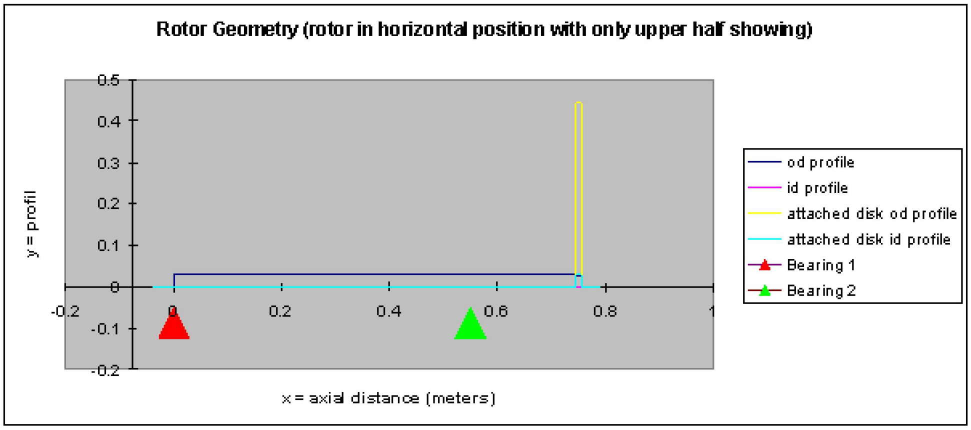
VII. Overhung rotor solved/checked for all three calculation modes

This is intended to model the an overhung rotor described in the thread "Gyroscopic effect", at

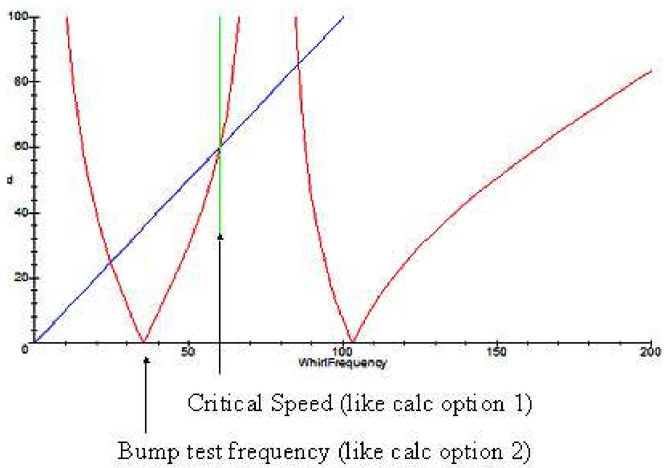
http://maintenanceforums.com/eve/forums/a/tpc/f/3751089011/m/9291040423/p/1
(except that the smaller disk and shaft stub on the left is omitted for simplicity of the analytical solution).

The rotosolve spreadsheet solution is here: [OverhungRotor.xls]

The geometry looks as follows:



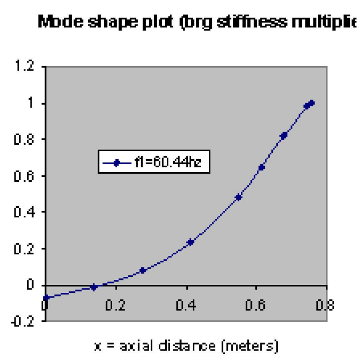
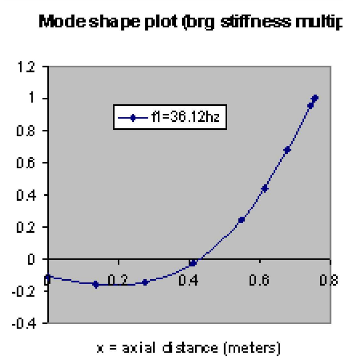
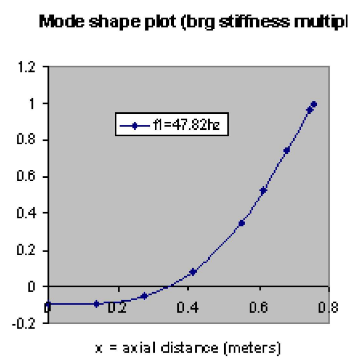
A complete analytical solution of this geometry (mode 3= simple/point mass, mode 2 = bump test, and mode 1 = critspd) is provided in the following file: [FindAlphaR1a.pdf]



My rotosolve spreadsheet was run on the same model, and the results are compared below:

	Analytical Solution (hz)	Program output (hz)
Mode 1 = simple / point mass – neglects all disk effects	47.2	47.8
Mode 2 = bump test – includes disk effects but neglects gyro	~ 35 hz	36.12
Mode 3 = Critical speed – includes gryo and disk effects	~59.5	60.44

The mode shapes for the three calculation modes are as follows:

Calc Mode 1 (crit speed)	Calc Mode 2 (bump test)	Calc Mode 3 (simple)
		

This geometry was checked with the Critspd program, and similar results were obtained.