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STUDENT > # RotorHeatingSymbols.mws by electricpete
STUDENT >
STUDENT > # Compute the total I^2*R energy dissipated during the
          start
STUDENT >
STUDENT > # ================================================
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STUDENT > # ================================================
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STUDENT > #=============================================
STUDENT > # Section 1 - Symbols
STUDENT > # ============================================
STUDENT >
STUDENT > # RH = RH_unloaded = total energy dissipated in rotor only
          during an unloaded (inertia-only) start
STUDENT > # RH_loaded = total energy dissipated in rotor only during
          an loaded (inertia-plus load torque) start
STUDENT > # TotalHeating - total energy dissipated in both rotor and
          stator during the start
STUDENT > # I2 = Rotor current, referred to the stator side
STUDENT > # Im = magnetizing current
STUDENT > # Pout = motor output power
STUDENT > # T = Telec = motor torque
STUDENT > # Tmech = load torque
STUDENT > # s = slip
STUDENT > # s0 = initial slip (1)
STUDENT > # sfinal = final slip (usually close to 0)
STUDENT > # t = time
STUDENT > # t0 = initial time
STUDENT > # tfinal = final time
STUDENT > # tfinal - t0 = duration of the start
STUDENT > # w = rotational radian speed
STUDENT > # wsync = syncronous rotational radian speed
STUDENT > # KE = Kinetic Energy of the running motor and load
STUDENT > # J = rotational inertia
STUDENT >
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STUDENT > # =============================================
STUDENT > # Section 2 - Unloaded start - rotor heating only
STUDENT > #=============================================
STUDENT > restart;
STUDENT > # Rate of rotor heating is I2^2*R2
STUDENT > # Total rotor heating energy is integral over time
STUDENT > eq1:=RH=3*Int(I2^2*R2,t=t0..tfinal);
```

$$eq1 := RH = 3 \int_{t0}^{tfinal} I2^2\, R2 \, dt$$

```
STUDENT > # Change of variables
STUDENT > eq2:=RH=Int(3*I2^2*R2/Diff(s,t),s=s0..sfinal);
```

$$eq2 := RH = \int_{s0}^{sfinal} 3 \frac{I2^2\, R2}{\frac{\partial}{\partial t} s} \, ds$$

```
STUDENT > # Some calculations to determine ds/dt
STUDENT > eq3:=w=(1-s)*wsync;
```

$$eq3 := w = (1 - s)\, wsync$$

```
STUDENT > eq4:=Diff(w,t)=-wsync*Diff(s,t);
```

$$eq4 := \frac{\partial}{\partial t} w = -wsync \left(\frac{\partial}{\partial t} s \right)$$

```
STUDENT > # T = J dw/dt. This is rotational analog of F=ma =
          m*dv/dt.
STUDENT > eq5:=Diff(w,t)=T/J;
```

$$eq5 := \frac{\partial}{\partial t} w = \frac{T}{J}$$

```
STUDENT > # compare equation 4 and 5
STUDENT > eq6:=rhs(eq4)=rhs(eq5);
```

$$eq6 := -wsync \left(\frac{\partial}{\partial t} s \right) = \frac{T}{J}$$

```
STUDENT > eq7:=Diff(s,t)=solve(eq6,Diff(s,t));
```

$$eq7 := \frac{\partial}{\partial t} s = -\frac{T}{wsync\, J}$$

```
STUDENT > # Now we want expression for T
STUDENT > eq8:=T=Pout/(w);
```

$$eq8 := T = \frac{Pout}{w}$$

```
STUDENT > eq8a:=subs(w=(1-s)*wsync,eq8);
```

$$eq8a := T = \frac{Pout}{(1-s)\,wsync}$$

```
STUDENT > eq9:=Pout=3*I2^2*R2*(1-s)/s;
```

$$eq9 := Pout = 3\,\frac{I2^2\,R2\,(1-s)}{s}$$

```
STUDENT > eq10:=subs(eq9,eq8a);
```

$$eq10 := T = 3\,\frac{I2^2\,R2}{s\,wsync}$$

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STUDENT > eq11:=subs(eq10,eq7);
```

$$eq11 := \frac{\partial}{\partial t}\,s = -3\,\frac{I2^2\,R2}{s\,wsync^2\,J}$$

```
STUDENT > eq12:=subs(eq11,eq2);
```

$$eq12 := RH = \int_{s0}^{sfinal} -s\,wsync^2\,J\,ds$$

```
STUDENT > # evaluate the integral
STUDENT > eq13 := RH = int(-s*wsync^2*J,s = s0 .. sfinal);
STUDENT >
```

$$eq13 := RH = -\frac{1}{2}\,sfinal^2\,wsync^2\,J + \frac{1}{2}\,s0^2\,wsync^2\,J$$

```
STUDENT > # Assume that sfinal ~ 0, s0=1
STUDENT > eq14:=subs({sfinal=0,s0=1},eq13);
```

$$eq14 := RH = \frac{1}{2}\,wsync^2\,J$$

```
STUDENT > # This result shows the total rotor heating during
          starting of an UNLOADED (no load torque) motor is equal to
          the kinetic energy of the system at full speed.
STUDENT >
STUDENT > # ===============================================
STUDENT > # Section 3 - Unloaded start - total rotor and stator
          heating
STUDENT > #===============================================
STUDENT >
STUDENT > # If we neglect magnetizing current Im on the basis that
          load current I2>>Im during most of the start, the total
          (rotor plus stator) I^2*R heating during the start can be
          estimated as
STUDENT > eq15:=TotalHeating=(R1+R2)/R2*rhs(eq14);
```

$$eq15 := TotalHeating = \frac{1}{2}\,\frac{(R1+R2)\,wsync^2\,J}{R2}$$

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STUDENT >
STUDENT >
STUDENT > # ==============================================
STUDENT > # Section 4 - Loaded  - rotor heating only
STUDENT > #==============================================
STUDENT >
STUDENT >
STUDENT >
STUDENT > #==== Now look at the effect of adding load======
STUDENT > # start with equation 7 rewritten as Taccel = Telec-Tmech
          instead of previous T representing Telec only.
STUDENT >
STUDENT >
STUDENT > eq16:=subs(T=Telec-Tmech,eq7);
```

$$eq16 := \frac{\partial}{\partial t} s = -\frac{Telec - Tmech}{wsync\ J}$$

```
STUDENT >
STUDENT > eq25:=subs({eq16,RH=RH_loaded},eq2);
```

$$eq25 := RH\_loaded = \int_{s0}^{sfinal} -3\,\frac{I2^2\ R2\ wsync\ J}{Telec - Tmech}\,ds$$

```
STUDENT > # Compare above RH_loaded (loaded rotor heating RH_loaded)
          to  below RH_unloaded (unloaded rotor heating RH from
          equation 2 and 7)
STUDENT > eq2a:=subs({subs(T=Telec,eq7),RH=RH_unloaded},eq2);
```

$$eq2a := RH\_unloaded = \int_{s0}^{sfinal} -3\,\frac{I2^2\ R2\ wsync\ J}{Telec}\,ds$$

```
STUDENT >
STUDENT > # Comparison of eq25 and eq2a shows that the integrand for
          the loaded start is equal to Telec/(Telec-Tmech) times the
          integrand for the unloaded start
STUDENT >
STUDENT > # Therefore we can adjust equation 12 to incorporate the
          effects of adding load as follows:
STUDENT > eq26:=RH_loaded =
          Int(-s*wsync^2*J*Telec(s)/(Telec(s)-Tmech(s)),s = s0 ..
          sfinal);
```

$$eq26 := RH\_loaded = \int_{s0}^{sfinal} -\frac{s\, wsync^2\, J\, \mathrm{Telec}(s)}{\mathrm{Telec}(s) - \mathrm{Tmech}(s)}\, ds$$

```
STUDENT >
STUDENT > # Express this in terms of the kinetic energy (at sync
          speed) KE=wsync^2*J/2
STUDENT > eq27:=RH_loaded =
          2*KE*Int(-s*Telec(s)/(Telec(s)-Tmech(s)),s = s0 ..
          sfinal);
```

$$eq27 := RH\_loaded = 2\, KE \int_{s0}^{sfinal} -\frac{s\, \mathrm{Telec}(s)}{\mathrm{Telec}(s) - \mathrm{Tmech}(s)}\, ds$$

```
STUDENT >
STUDENT > # Reverse the limits of integration (removes negative
          sign) and assume s0=1
STUDENT > eq27:=RH_loaded = 2*Int(s*Telec(s)/(Telec(s)-Tmech(s)),s =
          sfinal .. 1)*KE;
```

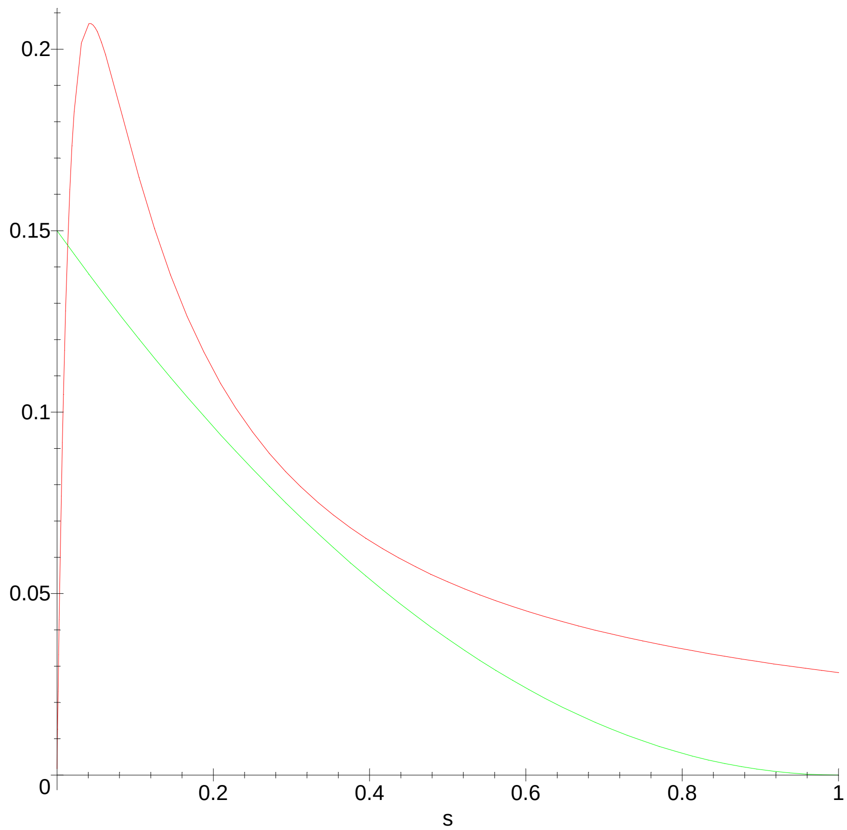
$$eq27 := RH\_loaded = 2 \int_{sfinal}^{1} \frac{s\, \mathrm{Telec}(s)}{\mathrm{Telec}(s) - \mathrm{Tmech}(s)}\, ds\, KE$$

```
STUDENT > # *** From equation 27 we find the total starting heating
          for loaded start will now be approximately the final
          kinetic energy times Integral{2s*(Telec/(Telec-Tmech))}ds.
          That integral equates to a weighted average of
          (Telec/(Telec-Tmech)) with 2s being the weighting factor.

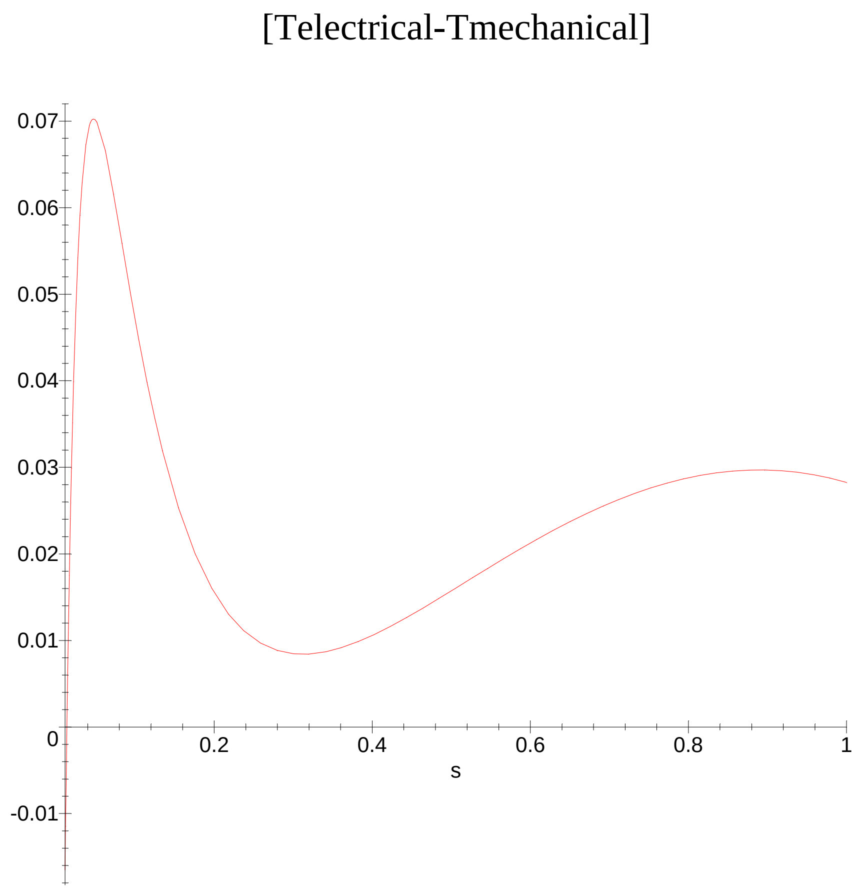
STUDENT >
STUDENT > # graphical example as follows:
STUDENT > Telec:=V^2/((R1+R2/s)^2+(X1+X2)^2)*R2/s/wsync:
STUDENT > Telec:=subs({R2=0.06,R1=1,X1=0.5,X2=0.5,V=1,wsync=1},Telec
          ):
STUDENT > Tmech:=subs(k=0.15,k*(1-s)^2):
STUDENT > mytitle:=convert([Telectrical, Tmechanical],string);
          plot({Tmech,Telec},s=0.0001..1,title=mytitle);
```

$$mytitle := [Telectrical, Tmechanical]$$

[Telectrical, Tmechanical]




```
STUDENT >
STUDENT > mytitle:=convert([Telectrical - Tmechanical],string):
          plot(Telec-Tmech,s=0.0113..1,title=mytitle);
```

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STUDENT >
STUDENT > mytitle:=convert([1 / ( Telectrical - Tmechanical
          )],string):
          plot(1/(Telec-Tmech),s=0.02..1,title=mytitle);
```
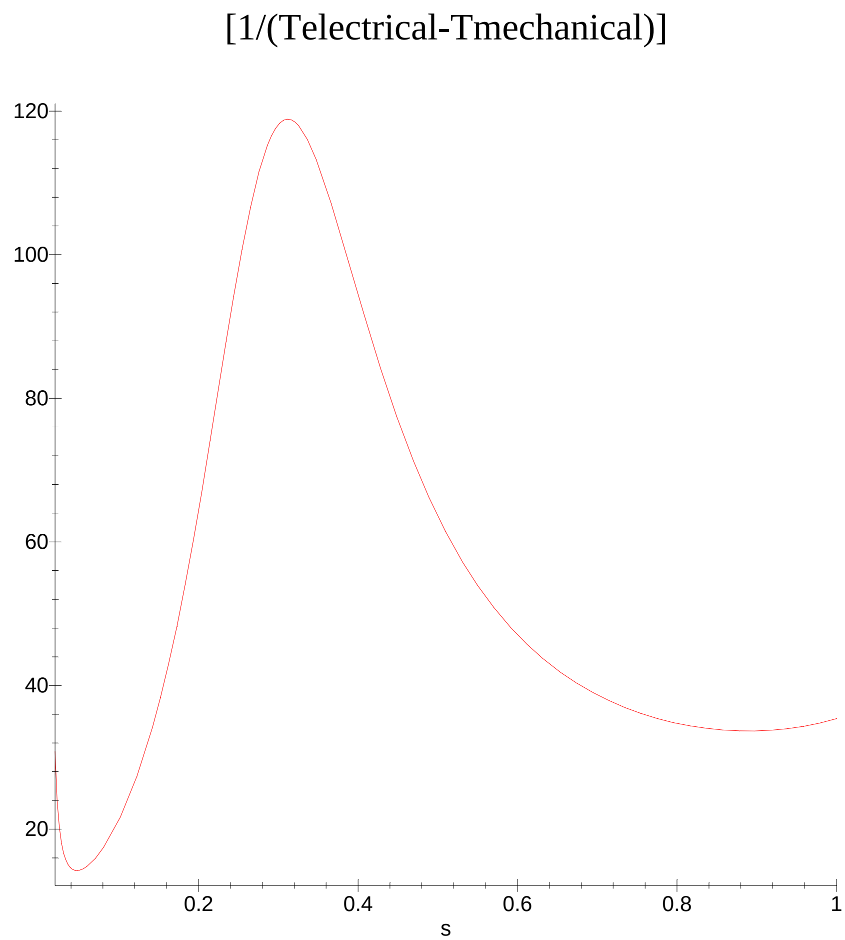
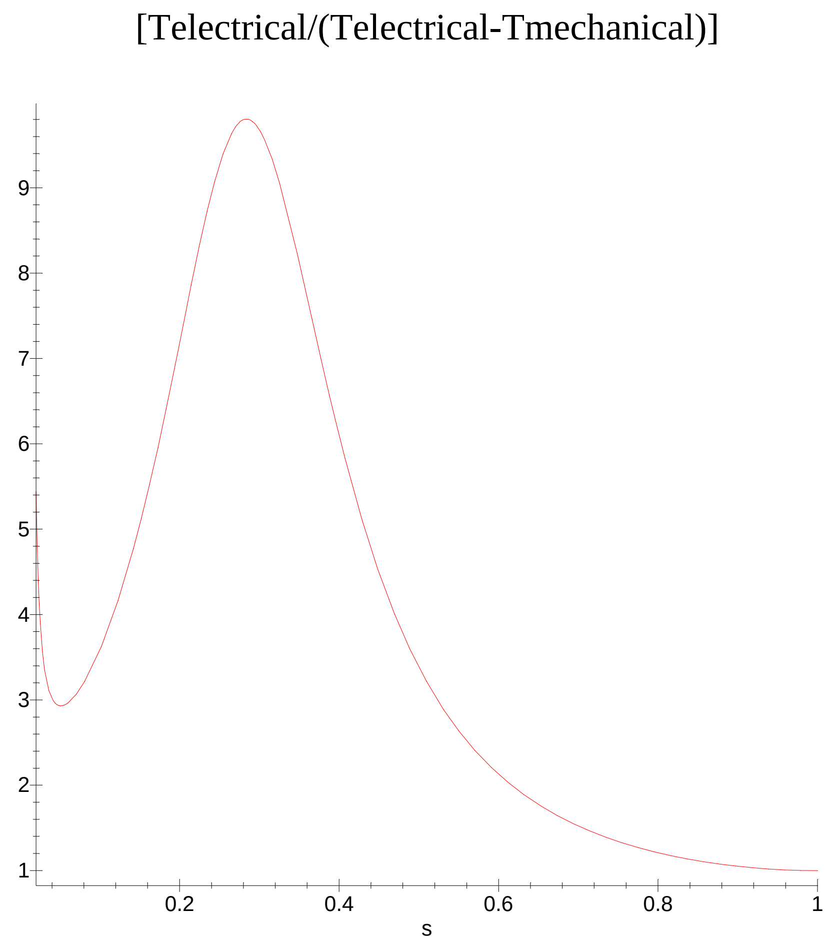
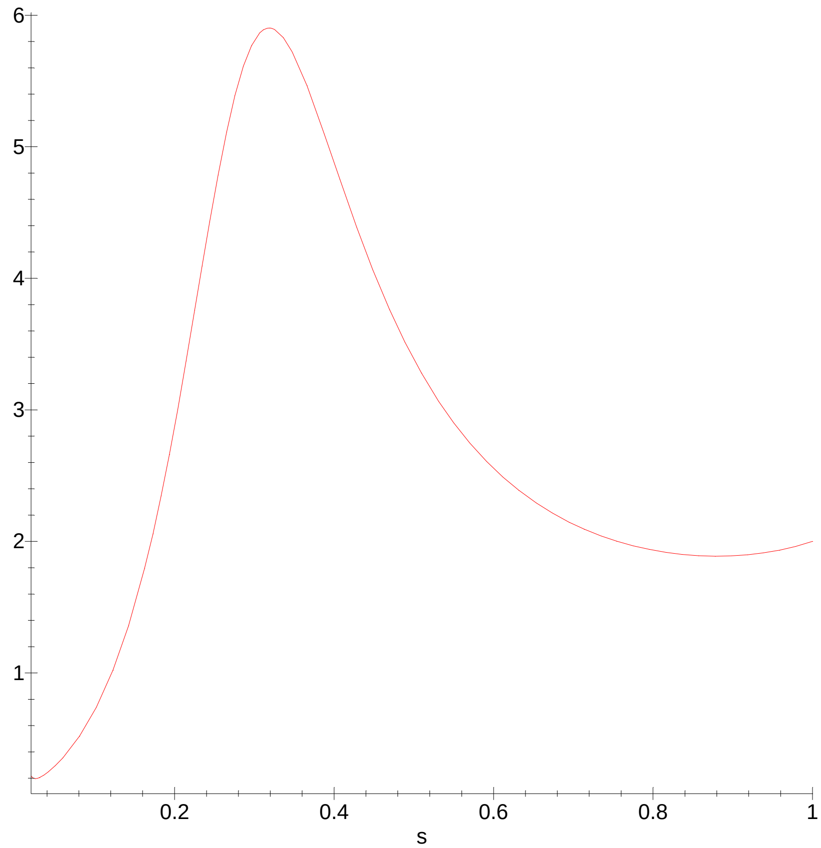




```
STUDENT >
STUDENT > mytitle:=convert([Telectrical / ( Telectrical -
          Tmechanical )],string):
          plot(Telec/(Telec-Tmech),s=0.02..1,title=mytitle);
```




```
STUDENT > mytitle:=convert([2 * s * Telectrical / ( Telectrical -
          Tmechanical )],string):
          plot(2 * s*Telec/(Telec-Tmech),s=0.02..1,title=mytitle);
```

[2*s*Telectrical/(Telectrical-Tmechanical)]



STUDENT > # Note the area under the above curve is always more than one since Te/(Te-Tm) always >=1 and Integral (2*s*1) from zero to 1 gives 1.

STUDENT >

STUDENT >

STUDENT > # ** The rotor total heating during the loaded start is the area under the above curve times the final kinetic energy.

STUDENT >

STUDENT > # ==

STUDENT > # Section 5 - Loaded start - total rotor plus stator heating

STUDENT > #==

STUDENT >

STUDENT > # Using similar logic as described for equation 15, the total (rotor plus stator) heating during a loaded start would be approx the above rotor heating times (R1+R2)/R2

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