A RECOMMENDED METHOD OF ANALYTICALLY DETERMINING THE COMPETENCE OF HYDRAULIC TELESCOPIC CANTILEVERED CRANE BOOMS —SAE J1078 APR94

SAE Information Report

Report of the Construction and Industrial Machinery Technical Committee approved July 1974, editorial change April 1986. Reaffirmed by the SAE Off-Road Machinery Technical Committee SC31—Cranes and Lifting Devices, April 1994.

Foreword—This reaffirmed document has been changed only to reflect the new SAE Technical Standards Board format.

1. Scope—This analysis applies to crane types as covered by Power Crane and Shovel Association Standard Number Two, Mobile Hydraulic Crane Standards and ANSI B30.15; refer to 5.1.

1.1 Purpose—This calculation method has been established to illustrate an analysis to determine the competence of hydraulic telescopic cantilevered crane booms.

2. References

2.1 Applicable Documents—The following publications form a part of this specification to the extent specified herein.

2.1.1 AISC, "Specification for the Design, Fabrication and Erection of Structural Steel for Buildings," adopted February 12, 1969. In addition, Supplement Nos. 1, 2, 3, and Commentary with additions and revisions where applicable.

2.1.2 AISI, "Specification for the Design of Cold-Formed Steel Structural Members," 1968 edition. In addition, "Commentary on the 1968 Edition," by George Winter and Supplementary Information Part II.

2.1.3 Column Research Council, "Guide to Design Criteria for Metal Compression Members," Second Printing, 1960.

2.1.4 "USS Steel Design Manual," by R. L. Brockenbrough and B. G. Johnston, November 1968 printing.

2.1.5 ANSI B30.15

2.2 Nomenclature

a = Clear distance between transverse stiffeners on side plate; also the ratio of the material yield of the web to the material yield of the compression flange

A = Actual area of section

A_e = Total effective area of section used in calculating F_a (refer to Appendix E for illustration)

A_f = Area of compression flange

A_i = Area based on inside dimensions of section (refer to Appendix E for illustration)

A_m = Area based on mean dimensions of section (refer to Appendix E for illustration)

A_o = Area based on outside dimensions of section (refer to Appendix E for illustration

A_{st} = Cross-sectional area of stiffener or pair of stiffeners

A_w = Area of both webs

b = Actual width of stiffened and unstiffened compression elements whether flange or web (refer to Appendix F for illustration)

b_e = Effective width of stiffened compression element (refer to Appendix E for illustration)

b_f = Actual flange width (refer to Appendix E for illustration)

b_m = Mean width of section or $b_w - t_w$ (refer to Appendix E for illustration)

b_w = Overall width of section (refer to Appendix E for illustration)

c_t = Distance from neutral axis to extreme tension fiber of box section (refer to Appendix E for illustration)

c_c = Distance from neutral axis to compressive fiber of box section (refer to Appendix E for illustration)

C_b = Bending coefficient dependent upon moment gradient; equal to (see Equation 1)

$$1.75+1.05\left(\frac{M_{x\,min}}{M_{x\,max}}\right)+0.3\left(\frac{M_{x\,min}}{M_{x\,max}}\right)^2 \quad \text{(Eq.1)}$$

but not more than 1.3 (refer to Appendix C for illustration)

C_c = Column slenderness ratio dividing elastic and inelastic buckling equal to (see Equation 2)

$$\sqrt{\pi^2E/\left(F_y-\sigma_{rc}\right)} \quad \text{(Eq.2)}$$

C'_c = Effective column slenderness ratio dividing elastic and inelastic buckling equal to (see Equation 3)

$$\sqrt{\frac{\pi^2E}{Q_sQ_a\left(F_y-\sigma_{rc}\right)}} \quad \text{(Eq.3)}$$

C_m = Coefficient applied to bending term in the interaction formula and dependent upon column curvature caused by applied moments; use 0.85

C_{mx} = 0.85

C_{my} = 0.85

C_v = Ratio of "critical" web stress, according to linear buckling theory, to the shear yield stress of web material

d = Overall depth of section (refer to Appendix E for illustration)

D = Factor depending upon type of transverse stiffeners

E = Modulus of elasticity 29 500 ksi

f = Computed axial and bending compression stress on appropriate flange or web

f_a = Computed axial stress based on total section area

f_b = Computed bending stress about the appropriate axis

f_c = Sum of the computed axial and side bending compressive stresses

f_{bx} = Computed bending stress about the x-x axis

f_{by} = Computed bending stress about the y-y axis

f_s = Sum of the computed torsional and vertical shear stress

f_v = Computed average web or flange shear stress

f_{vs} = Total shear transfer of stiffener(s), kips per inch of length

F_a = Allowable axial stress permitted in the absence of a bending moment

F_b = Allowable bending stress for the appropriate axis

F_{bx} = Allowable bending stress about the x-x axis if this bending moment alone existed

F'_{bx} = Allowable bending stress in compression flange of box sections as reduced for hybrid sections or because of large web depth-to-thickness ratio

F_{by} = Allowable bending stress about the y-y axis if this bending moment alone existed

F'_e = Euler stress divided by factor of safety; equal to (see Equation 4)

$$\frac{12\pi^2E}{23(Kl/r)^2} \quad \text{(Eq.4)}$$

F'_{ex} = Same as F'_e about the x-x axis

F'_{ey} = Same as F'_e about the y-y axis

F_v = Allowable web shear stress

F_y = Specified minimum yield stress of material being used, based on "yield stress" or yield strength, whichever is applicable

g = Wind load, lb/in^2, $g = 0.004\ (\text{mph})^2/144$

G = Shear modulus of elasticity 11 300 ksi

h = Clear distance between flanges (refer to Appendix E for illustration)

h_m = Mean height of section $d - (t_c + t_t)/2$ (refer to Appendix E for illustration

h_v = Vertical height of horizontal stiffener

H_o = Height to boom foot pin from ground

H_p = Height to center of pressure on boom

H_r = Reference height at which wind velocity is measured (20 ft in U.S.)

I_x = Area moment of inertia about the x-x axis

I_y = Area moment of inertia about the y-y axis

I_{st} = Moment of inertia of a pair of intermediate stiffeners, or a single intermediate stiffener, with reference to an axis in the plane of the web

I_{xe} = Effective moment of inertia about the x-x axis

I_{ye} = Effective moment of inertia about the y-y axis

J = Torsional constant; equal to (refer to Appendix D for other equations) (see Equation 5)

$$\frac{4(b_m)^2(h_m)^2}{\frac{2h_m}{t_w}+\frac{b_m}{t_c}+\frac{b_m}{t_t}} \qquad \text{(Eq.5)}$$

k = Coefficient relating linear buckling strength of a plate to its dimensions and conditions of edge support

K = Effective length factor, for cantilevered section use the value 2 unless a smaller one can be justified

K_t = Torsional length factor for cantilevered sections, use the value 4/3

l = Dimensional lengths of boom

L = Distance from tip to section in question

L_b = Actual unbraced length of section in the plane of bending

M = Bending moment about the appropriate axis

M_1 = Constant moment load about the x-x axis resulting from eccentric loading on the head

M_2 = Constant moment load about the y-y axis resulting from the side loading on the head

M_{xmin} = Smaller moment at end of unbraced length of beam-column at tip

M_{xmax} = Larger moment at end of unbraced length of beam-column at section in question

M_x = Bending moment about the x-x axis

M_y = Bending moment about the y-y axis

N = Number of parts of line

p = Wind velocity exponent

P = Externally applied load at the tip

P_a = Axial load applied to section

P_x = Lateral loading component (side load)

P_y = Vertical loading component

P_z = Axial loading component

Q_a = Ratio of effective profile area of an axially loaded member to its total profile area of A_e/A

Q_s = Axial stress reduction factor for unstiffened elements of a section; refer to Appendix F

r = Radius of gyration for appropriate axis

r_b = Radius of gyration about the axis of concurrent bending, computed on the basis of actual cross-sectional area

R = Load radius from centerline of rotation to centerline of load

R_h = Hoist cylinder reaction

R_x = Reaction loads in the lateral direction

R_y = Reaction loads in the vertical direction

R_z = Reaction loads in the axial direction

S_x = Strong axis section modulus with c taken to the compressive side

S_y = Weak axis section modulus with c taken to the compressive side

S_{xe} = Effective strong axis section modulus with c taken to the compressive side

S_{ye} = Effective weak axis section modulus with c taken to the compressive side

t = Thickness of flange or web in compression (refer to Appendix E for illustration)

t_c = Thickness of compression flange (refer to Appendix E for illustration)

t_t = Thickness of tension flange (refer to Appendix E for illustration)

t_w = Thickness of web (refer to Appendix E for illustration)

T = Torsional moment

V_p = Wind velocity (mph) at center of pressure height H_p

V_r = Wind velocity (mph) at reference height H_r

V_x = Statical shear load on section in the lateral direction

V_y = Statical shear load on section in the vertical direction

w = Component weight, lb/in

W = Total component weight

x = Subscript relating symbol to strong axis bending

Y = Ratio of yield stress of web steel to that of yield stress of stiffener steel

y = Subscript relating symbol to weak axis bending

z = Subscript relating symbol to axial loading

α = Boom centerline elevation angle relative to a horizontal plane, or the ratio of web yield stress to flange yield stress

θ = Angle between a line perpendicular to the boom axis and the hoist cylinder axis

σ_{rc} = Residual compressive stress, equal to 0.5 F_y in lieu of specific information on steel used

u = Poisson's ratio—equal to 0.3

3. Criteria—Calculations shall include the dead weight loads, rated load and a minimum side load of 2% of the rated load at the rated load radius. The side load provides for "normal" conditions of machine operation. In addition, the effect of the wind on the boom should be considered, as is provided for in the calculations.

3.1 The factors of safety used herein are the recommended factors of the AISC "Specification for the Design, Fabrication and Erection of Structural Steel for Buildings," adopted February 12, 1969.

3.2 The boom shall be deemed competent when the solution of the interaction equations provided herein yield a value equal to or less than one (1.0).

4. Loads and Forces

4.1 The 2% side load provides for "normal" conditions of boom motion. No allowances have been made for dynamic loads, duty cycle operation, effects of the wind on the load lifted or operations other than lifting crane service.

4.2 All forces and loads are expressed in pounds. Dimensions are in inches. Stresses both allowable and calculated are in units of ksi. Also, the modulus of elasticity is expressed in units of ksi.

5. Analytical Determination of Stresses and Critical Loads

5.1 **Applicability**—This analysis is applicable to multisectioned "box" type booms, which are totally enclosed and cantilevered beyond the base section.

5.2 **Basis for Analysis**—The equations presented in this analysis are based on laterally unsupported beam column formulas, the solution of which are combined in interaction equations. In determining the section properties, the effective width of the plates in compression are used. The areas covered in this analysis consist of axial and torsional loading, bidirectional bending, and panel buckling. Of primary importance in the analysis are the compressive stress calculations.

The work of this committee is not intended to cover all design concepts, but rather a basic system. However, other design configurations may use alternative calculation methods when substantiated with suitable test data.

5.3 **Summary**—Where strain gage results are available they should be used to supplement the analytical data.

6. Load Moment Diagrams and Equations

6.1 Assumptions Used on Load Moment Equations

6.1.1 Wind force is negligible on head (should include effects if jib used).

6.1.2 Torque is created by the side load P on the head (would also be applicable for a jib).

6.1.3 Equations are still applicable if jib used but dimensions, weights, and center of gravity to be adjusted accordingly.

6.1.4 $P_y = P \cos \alpha$; $P_z = P \sin \alpha$

6.1.5 Winch rope fleet angle and angle relative to boom is negligible.

6.1.6 Wind force is uniformly distributed along the exposed length of the side of the section with its reaction at the center (is a valid assumption since each section considered individually).

6.1.7 That the dimensions are to the reaction points and that the tips of each section beyond these points are small in length and will not affect the validity of the equations.

6.1.8 That the axial stresses produced by the friction forces due to the section reaction ponts from one to the next are small in comparison to the other stresses, that the section support cylinders carry the axial loads.

6.1.9 That equations and formulations appearing in the foregoing analysis are for the boom in the extended position—see Figures 1 to 7. Partially retracted positions will require reformulation of some equations; as an example in Figure 4 when l_{11} is zero or negative the cylinder no longer takes the axial load at the section being considered. The moment equations would then appear as those written for reference Figure 3. Similar changes would appear in the axial load, reactions, and shear force equations.

6.2 Refer Figure 2

Load Moment Equations—Reaction of Head Forces on Tip Section (see Equations 6 to 9)

MOMENT (Equation 6)

$$M_1 = P_y l_1 + P_z l_2 - P/Nl_4 + W_1[l_5 \cos\alpha + l_6 \sin\alpha]$$
$$M_2 = P_x l_1 \qquad \text{(Eq.6)}$$
$$T = P_x l_2$$

AXIAL LOAD (Equation 7)

$$R_{z1} = P/N + P_z + W_1 \sin\alpha \qquad \text{(Eq.7)}$$

SHEAR LOADS (Equation 8)

$$V_x = -R_{x1} = P_x$$
$$V_y = -R_{y1} = W_1 \cos\alpha + P_y \qquad \text{(Eq.8)}$$

SIDE LOAD (Equation 9)

$$P_x = 0.02\,P \qquad \text{(Eq.9)}$$

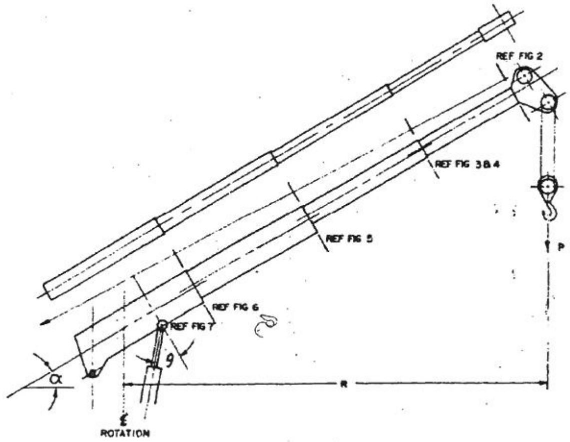


FIGURE 1—LOADING DIAGRAM—BOOM ASSEMBLY

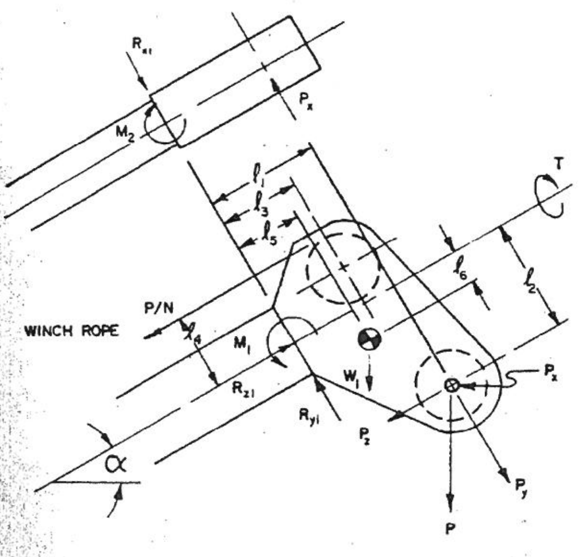


FIGURE 2—LOAD MOMENT DIAGRAM—HEAD SECTION

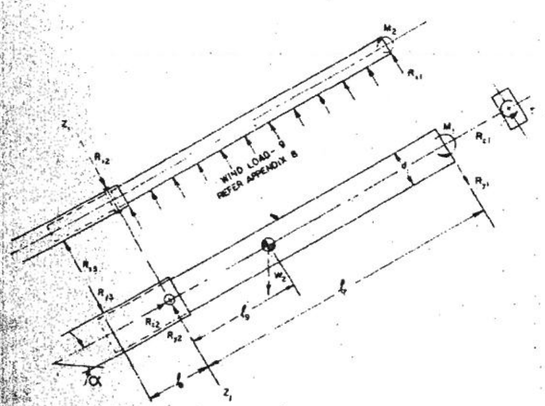


FIGURE 3—LOAD MOMENT DIAGRAM—TIP SECTION

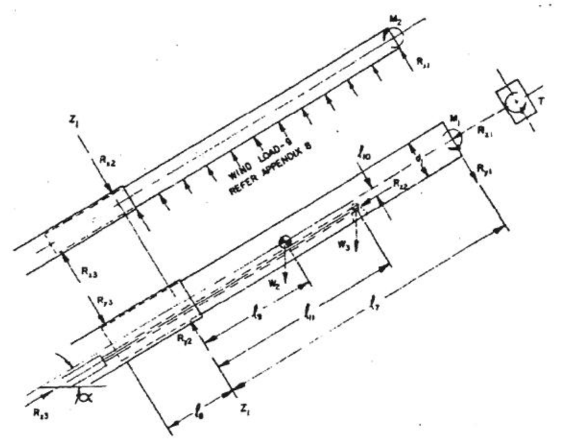


FIGURE 4—LOAD MOMENT DIAGRAM—ALTERNATE TIP SECTION

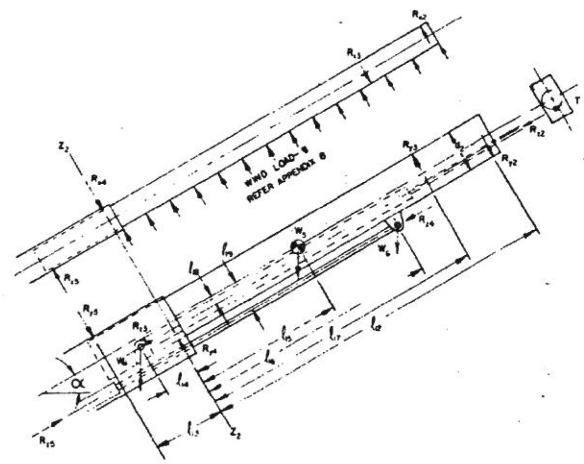


FIGURE 5—LOAD MOMENT DIAGRAM—INTERMEDIATE SECTION

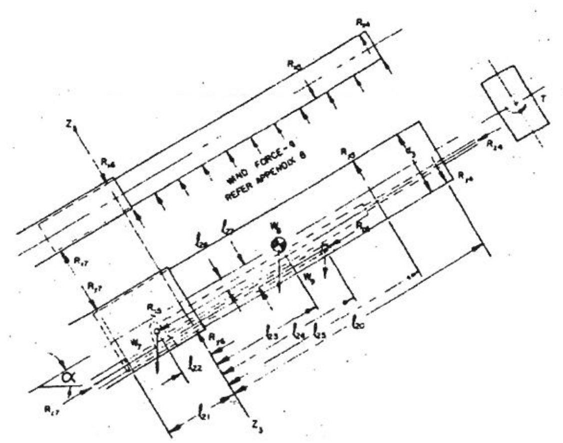


FIGURE 6—LOAD MOMENT DIAGRAM—INTERMEDIATE SECTION

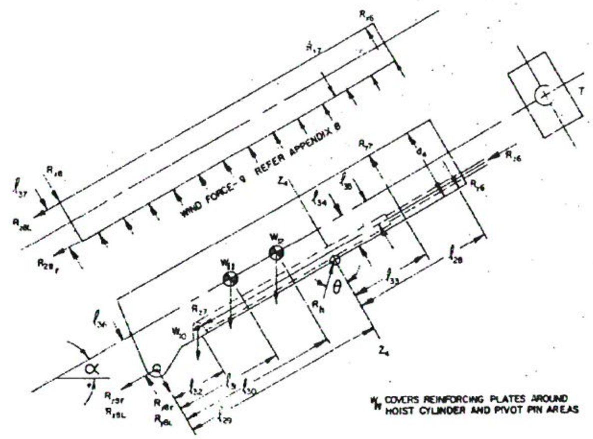


FIGURE 7—LOAD MOMENT DIAGRAM—BASE SECTION

6.3 Refer Figure 3

Load Moment Equations for Tip Section at Section Z1 - Z1 (see Equations 10 to 16)

MOMENTS (Equation 10)

$$M_x = M_1 + R_{y1} l_7 + \frac{0.5 W_2 \cos \alpha l_7^2}{l_7 + l_8}$$
$$M_y = M_2 + R_{x1} l_7 + 0.5 g d_1 l_7^2 \qquad \text{(Eq.10)}$$
$$T = P_x l_2$$

AXIAL LOAD ON PIN (Equation 11)

$$P_{Z2} = P_{Z1} + W_2 \sin \alpha \qquad \text{(Eq.11)}$$

AXIAL LOAD ON SECTION (Equation 12)

$$P_{ar} = R_{z1} + W_2 \sin \alpha \frac{l_7}{l_7 + l_8}; P_{aL} = W_2 \sin \alpha \frac{l_8}{l_7 + l_8} \qquad \text{(Eq.12)}$$

VERTICAL REACTIONS (Equation 13)

$$R_{y3} = \frac{M_x}{l_8} - 0.5 W_2 \cos \alpha \frac{l_8}{l_7 + l_8}; R_{y2} = R_{y1} + R_{y3} + W_2 \cos \alpha \qquad \text{(Eq.13)}$$

LATERAL REACTIONS (Equation 14)

$$R_{x3} = \frac{M_y}{l_8}; R_{x2} = R_{x1} + R_{x3} + g d_1 l_7 \qquad \text{(Eq.14)}$$

VERTICAL SHEAR FORCES (Equation 15)

$$V_{yr} = R_{y1} + W_2 \cos \alpha \frac{l_7}{l_7 + l_8}; V_{yL} = R_{y3} + W_2 \cos \alpha \frac{l_8}{l_7 + l_8} \qquad \text{(Eq.15)}$$

LATERAL SHEAR FORCES (Equation 16)

$$V_{xr} = R_{x1} + g d_1 l_7; V_{xL} = R_{x3} \qquad \text{(Eq.16)}$$

NOTE—Subscripts r and L refer to right and left of Section Z_1 - Z_1

6.4 Refer Figure 4

Load Moment Equations for Alternate Tip Section at Section Z_1 - Z_1 (see Equations 17 to 23)

MOMENTS (Equation 17)

$$M_x = M_1 + R_{y1} l_7 + 0.5 W_2 \cos \alpha \frac{l_7^2}{l_7 + l_8} + W_3 \cos \alpha l_{11} \mp R_{z2} l_{10}$$
$$M_y = M_2 + R_{x1} l_7 + 0.5 g d_1 l_7^2 \qquad \text{(Eq.17)}$$
$$T = P_x l_2$$

AXIAL LOAD ON CYLINDER SUPPORT (Equation 18)

$$R_{z2} = R_{z1} + W_2 \sin \alpha \qquad \text{(Eq.18)}$$

AXIAL LOAD ON SECTION (Equation 19)

$$P_{ar} = P_{aL} = \frac{W_2 l_8}{l_7 + l_8} \sin \alpha \qquad \text{(Eq.19)}$$

VERTICAL REACTIONS (Equation 20)

$$R_{y3} = \frac{M_x}{l_8} - 0.5 W_2 \cos \alpha \frac{l_8}{l_7 + l_8}; R_{y2} = R_{y1} + R_{y3} + W_2 \cos \alpha + W_3 \cos \alpha$$
$$\text{(Eq.20)}$$

LATERAL REACTIONS (Equation 21)

$$R_{x3} = \frac{M_y}{l_8}; R_{x2} = R_{x1} + R_{x3} + g d_1 l_7 \qquad \text{(Eq.21)}$$

VERTICAL SHEAR FORCES (Equation 22)

$$V_{yr} = R_{y1} + W_2 \cos \alpha \frac{l_7}{l_7 + l_8} + W_3 \cos \alpha; V_{yL} = R_{y3} + W_2 \cos \alpha \frac{l_8}{l_7 + l_8} \qquad \text{(Eq.22)}$$

LATERAL SHEAR FORCES (Equation 23)

$$V_{xr} = R_{x1} + g d_1 l_7; V_{xL} = R_{x3} \qquad \text{(Eq.23)}$$

NOTE—Subscripts r and L refer to right and left of Section Z1 - Z1

6.5 Refer Figure 5

Load Moment Equations for Intermediate Section at Section Z_2 - Z_2 (see Equations 24 to 30)

MOMENTS (Equation 24)

$$M_x = R_{y2} l_{12} - R_{y3} l_{17} + W_6 \cos \alpha l_{16} + 0.5 W_5 \cos \alpha \frac{l_{12}^2}{l_{12} + l_{13}} \mp W_5 \sin \alpha l_{19} \mp P_{x3}(l_{19} - l_{18}$$
$$M_y = R_{x2} l_{12} - R_{x3} l_{17} + 0.5 g d_2 l_{12}^2$$
$$T = P_x l_2$$
$$\text{(Eq.24)}$$

AXIAL LOAD ON CYLINDER SUPPORTS (Equation 25)

$$R_{z3} = R_{z2} + (W_3 + W_4) \sin \alpha; R_{z4} = R_{z3} + W_5 \sin \alpha \qquad \text{(Eq.25)}$$

AXIAL LOAD ON SECTION (Equation 26)

$$P_{ar} = P_{aL} = R_{z3} + W_5 \sin \alpha \frac{l_{13}}{l_{12} + l_{13}} \qquad \text{(Eq.26)}$$

VERTICAL REACTIONS (Equation 27)

$$R_{y5} = \frac{M_x}{l_{13}} - W_4 \cos \alpha \frac{l_{14}}{l_{13}} - 0.5 W_5 \cos \alpha \frac{l_{13}}{l_{12} + l_{13}};$$
$$R_{y4} = R_{y2} - R_{y3} + R_{y5} + \cos a (W_4 + W_5 + W_6) \qquad \text{(Eq.27)}$$

LATERAL REACTIONS (Equation 28)

$$R_{x5} = \frac{M_y}{l_{13}}; R_{x4} = R_{x2} - R_{x3} + R_{x5} + g d_2 l_{12} \qquad \text{(Eq.28)}$$

VERTICAL SHEAR FORCES (Equation 29)

$$V_{yr} = R_{y2} - R_{y3} + W_y \cos \alpha + W_5 \cos \alpha \frac{l_{12}}{l_{12} + l_{13}};$$
$$V_{yL} = R_{y5} + W_4 \cos \alpha + W_5 \cos \alpha \frac{l_{13}}{l_{12} + l_{13}} \qquad \text{(Eq.29)}$$

LATERAL SHEAR FORCES (Equation 30)

$$V_{xr} = R_{x2} - R_{x3} + g d_2 l_{12}; V_{xL} = R_{x5} \qquad \text{(Eq.30)}$$

NOTE—Subscripts r and L refer to the right and left of Section Z_2 - Z_2

6.6 Refer Figure 6

Load Moment Equations for Intermediate Section at Section Z_3 - Z_3 (see Equations 31 to 37)

MOMENTS (Equation 31)

$$M_x = R_{y4} l_{20} - R_{y5} l_{25} + W_9 \cos \alpha l_{24} + 0.5 W_8 \cos \alpha \frac{l_{20}^2}{l_{20} + l_{21}} \mp W_8 \sin \alpha l_{27} \mp R_{z5}(l_{27} - l$$
$$M_y = R_{x4} l_{20} - R_{x5} l_{25} + 0.5 g d_3 l_{20}^2$$
$$T = P_x l_2$$
$$\text{(Eq.31)}$$

AXIAL LOAD ON CYLINDER SUPPORTS (Equation 32)

$$R_{z5} = R_{z4} + (W_6 + W_7) \sin \alpha; R_{z6} = R_{z5} + W_8 \sin \alpha \qquad \text{(Eq.32)}$$

AXIAL LOAD ON SECTION (Equation 33)

$$P_{ar} = P_{aL} = P_{z5} + W_8 \sin \alpha \frac{l_{21}}{l_{20} + l_{21}} \qquad \text{(Eq.33)}$$

VERTICAL REACTIONS (Equation 34)

$$R_{y7} = \frac{M_x}{l_{21} - W_7} \cos \alpha \frac{l_{22}}{l_{21}} - 0.5 W_8 \cos \alpha \frac{l_{21}}{l_{20} + l_{21}};$$
$$R_{y6} = R_{y4} - R_{y5} + R_{y7} + \cos a (W_7 + W_8 + W_9) \qquad \text{(Eq.34)}$$

LATERAL REACTIONS (Equation 35)

$$R_{x7} = \frac{M_y}{l_{21}}; R_{x6} = R_{x4} - R_{x5} + R_{x7} + g d_3 l_{20} \qquad \text{(Eq.35)}$$

VERTICAL SHEAR FORCES (Equation 36)

$$V_{yr} = R_{y4} - R_{y5} + W_9 \cos \alpha + W_8 \cos \alpha \frac{l_{20}}{l_{20} + l_{21}};$$
$$V_{yL} = R_{y7} + W_7 \cos \alpha + W_8 \cos \alpha \frac{l_{21}}{l_{20} + l_{21}} \qquad \text{(Eq.36)}$$

LATERAL SHEAR FORCES (Equation 37)

$$V_{xr} = R_{x4} - R_{x5} + g d_3 l_{20}; V_{xL} = R_{x7} \qquad \text{(Eq.37)}$$

NOTE—Subscripts r and L refer to right and left of Section Z_3 - Z_3

6.7 Refer Figure 7

Load Moment Equations—Base Section at Section Z_4 - Z_4 (see Equations 38 to 46)

MOMENTS (Equation 38)

$$M_x = R_{y6}l_{28} - R_{y7}l_{33} + 0.5\,w_{12}\cos\alpha\frac{l_{28}^2}{l_{28}+l_{29}}$$

$$M_y = R_{x6}l_{28} - R_{x7}l_{33} + 0.5gd_4l_{28}^2 \qquad \text{(Eq.38)}$$

$$T = P_x l_2$$

AXIAL LOAD ON CYLINDER SUPPORT (Equation 39)

$$R_{z7} = R_{z6} + (W_9 + W_{10})\sin\alpha \qquad \text{(Eq.39)}$$

AXIAL LOAD ON SECTION (Equation 40)

$$P_{ar} = P_{aL} = W_{12}\sin\alpha\frac{l_{28}}{l_{28}+l_{29}} \qquad \text{(Eq.40)}$$

VERTICAL SHEAR FORCE (Equation 41)

$$V_{yr} = R_{y6} - R_{y7} + W_{12}\cos\alpha\frac{l_{28}}{l_{28}+l_{29}};$$

$$V_{yL} = R_{y8r} + R_{y8L} + (W_{10} + W_{11})\cos\alpha + W_{12}\cos\alpha\frac{l_{29}}{l_{28}+l_{29}} \qquad \text{(Eq.41)}$$

LATERAL SHEAR FORCE (Equation 42)

$$V_{xr} = R_{x6} - R_{x7} + gd_4l_{28};\; V_{xL} = R_{x8} - gd_4l_{29} \qquad \text{(Eq.42)}$$

HOIST CYLINDER REACTIONS (Equation 43)

$$R_h = [R_{y6}(l_{28}+l_{29}) - R_{y7}(l_{29}+l_{33}) + W_{10}l_{32}\cos\alpha + W_{11}(l_{31}\cos\alpha - l_{36}\sin\alpha) + W_{12}(l_{30}\cos\alpha - l_{36}\sin\alpha) - R_{z7}(l_{36}-l_{35})] \Big/ \left[l_{29} - \left(\frac{l_{36}-l_{34}}{\text{cotan}\,\theta}\right)\right]\cos\theta \qquad \text{(Eq.43)}$$

PIVOT PIN LOADING

LATERAL REACTION (Equation 44)

$$R_{x8} = R_{x6} - R_{x7} + gd_4(l_{28}+l_{29}) \qquad \text{(Eq.44)}$$

AXIAL REACTIONS (Equation 45)

$$R_{z8r} = \frac{R_h\sin\theta}{2} + R_{x6}\frac{l_{28}+l_{29}}{l_{37}} - R_{x7}\left(\frac{l_{29}+l_{33}}{l_{37}}\right) - \frac{R_{z7}-(W_{11}+W_{12})\sin\alpha}{2} + \frac{0.5gd_4(l_{28}+l_{29})^2}{l_{37}}$$

$$R_{z8L} = \frac{R_h\sin\theta}{2} - R_{x6}\frac{l_{28}+l_{29}}{l_{37}} + R_{x7}\left(\frac{l_{29}+l_{33}}{l_{37}}\right) - \frac{R_{z7}-(W_{11}+W_{12})\sin\alpha}{2} - \frac{0.5gd_4(l_{28}+l_{29})^2}{l_{37}} \qquad \text{(Eq.45)}$$

VERTICAL REACTIONS (Equation 46)

$$R_{y8r} = 0.5[R_h\cos\theta + R_{y7} - R_{y6} - (W_{10} + W_{11} + W_{12})\cos\alpha] - P_x\frac{l_2 - l_{36}}{l_{37}} + gd_4(l_{28}+l_{29})\frac{l_{36}}{l_{37}}$$

$$R_{y8L} = 0.5[R_h\cos\theta + R_{y7} - R_{y6} - (W_{10} + W_{11} + W_{12})\cos\alpha] + P_x\frac{l_2 - l_{36}}{l_{37}} - gd_4(l_{28}+l_{29})\frac{l_{36}}{l_{37}} \qquad \text{(Eq.46)}$$

NOTE—Subscripts r and L refer to right and left of Section Z_4 - Z_4

7. *Calculation Procedure*

7.1 Step 1—Preliminary Data

A. Provide description of geometry and loading, such as boom length, working radius, boom angle, rated load, etc.

B. Identify boom arrangement.
   1. Generate shear and moment diagrams.
   2. Solve for forces and moments from Section 3.

C. Identify boom section for analysis.
   1. Determine material properties.
   2. Determine section properties.

7.2 Step 2—Calculation of Section Properties, Based on Compressive Stresses, the Actual Stress, and the Allowable Stress

7.2.1 TO DETERMINE SECTION PROPERTIES

A. Determine if plates in compression are fully effective at yield.
   1. For vertical bending loads compute the b/t ratio for the compressive flange.
   2. For side bending loads compute the b/t ratio for the compressive web.
   3. For axial loads compute the b/t ratio for both webs and both flanges. If $b/t \le 184/\sqrt{f}$, where $f = 0.6F_y$, then the entire section will be fully effective at yield. The properties can then be computed based on the actual section. Proceed to 7.2.2 for the allowable stress computations. If $b/t > 184/\sqrt{f}$, for any or all plates, the section may still be fully effective for the actual stress. AISC (1.9.2.2)

B. Determine if plates in compression are fully effective at the actual stress.
   1. Compute actual stresses based on full section properties.
      1.1 For compression flange, $f = f_a + f_{bx}$
      1.2 For compression web, $f = f_a + f_{by}$
      1.3 For axial (all plates), $f = f_a$

   Use this calculated stress for f and recompute the b/t ratios.

   Note—For the axial case the effective widths will be different. Refer to Appendix E.

   If $b/t \le 184/\sqrt{f}$, for all plates, the entire section is fully effective at a stress level 1.67 times the actual stress. Proceed to 7.2.2 for the allowable stress computations. If $b/t > 184/\sqrt{f}$, for any one or all plates, the section is not fully effective for stress level f and an effective width calculation must be made for each plate that exceeds this ratio.

C. Determine effective width of plates that have b/t ratio greater than $184/\sqrt{f}$.
   1. Calculate the effective width of plates which are not fully effective accordingly:

$$b_e = \frac{253t}{\sqrt{f}}\left(1 - \frac{50.3}{(b/t)\sqrt{f}}\right) \qquad \text{AISC (C3-1)}$$

   where:

   f is the actual stress computed from 7.2.1.B.1.1, 7.2.1.B.1.2, and 7.2.1.B.1.3 from the previous paragraph

   2. Calculate new section properties A_e, S_{xe}, and S_{ye} based on the effective widths b_e.

   Note—The effective widths be used in computing A_e do not require an iterative solution because the stress f_a is based on the actual area A.

   3. Recompute new stress levels based on new properties.
   4. Recompute new effective widths based on new stress levels.
   5. Continue until stress level stabilizes—approximately three iterations. Proceed to 7.2.2 for the allowable stress computations.

7.2.2 TO DETERMINE ALLOWABLE STRESSES

A. Allowable axial stress F_a

   Note—If the stress is a tensile value, then $F_a = 0.6F_y$, proceed to 7.2.2B.

   1. Factor Q_a

$$Q_a = \frac{\text{effective area } (A_e)}{\text{actual area } (A)} \qquad \text{AISC (C4)}$$

   where:

   A_e equals the effective area of all stiffened elements, both flanges and webs, corresponding to the actual stress

   If all plates are fully effective $Q_a = 1$.

   2. Factor Q_s; see Appendix F to determine if this computation must be made. Applies to outstanding plates free on one edge.

   3.
$$C_c = \sqrt{\frac{\pi^2 E}{Q_sQ_a(F_y - \sigma_{rc})}} \qquad \text{AISC (C5)}$$

   where:

   $Q_sQ_a \le 1.0$

   4. Compute (KL/r) of both axis and use the largest KL/r value for the F_a calculation.

$$r = \sqrt{\frac{I}{A}}$$

   5. If $(KL/r) < C'_c$—inelastic range

$$F_a = \frac{Q_sQ_a\left[1 - \frac{\sigma_{rc}(KL/r)^2}{F_y(C_c)^2}\right]F_y}{5/3 + 3/8\left(\frac{KL/r}{C_c}\right) - 1/8\left(\frac{KL/r}{C_c}\right)^3} \qquad \text{AISC (C5-1)}$$

   6. If $(KL/r) \ge C'_c$—elastic range

$$F_a = \frac{12\pi^2 E}{23(KL/r)^2} \qquad \text{AISC (1.5-2)}$$

   Note—L as used previously is the distance from the outer end of the section in question to the point where the stresses are to be calculated in that section.

B. Allowable compressive bending stresses F_b for x and y directions considering lateral torsional buckling.
   1. Inelastic lateral buckling check

$$(KL/r)\text{ equiv.} = \sqrt{\frac{5.1\,K_tLS_x}{\sqrt{JI_y}}}\Big/\sqrt{C_b} \qquad \text{CRC (4.7)}$$

   where:

   K_t = 4/3

   L = distance from tip to section in question

$$C_b = 1.75 + 1.05\left(\frac{M_{xmin}}{M_{xmax}}\right) + 0.3\left(\frac{M_{xmin}}{M_{xmax}}\right)^2;\ 1.0 \le C_b \le 1.3$$

CRC p. 101

M_{xmin} = the moment at the tip

M_{xmax} = the moment at the section in question at L

Note—Clockwise moments are positive. Counterclockwise moments are negative. Refer to Appendix C for further discussion.

2. Compute first check on allowable compressive stresses.

2.1 If (KL/r) equiv. $\le \sqrt{\frac{102\,000}{F_y}}$

$F_{bx} = 0.6F_y$
$F_{by} = 0.6F_y$

2.2 If $\sqrt{\frac{102\,000}{F_y}} < \left(\frac{KL}{r}\right)$ equiv. $\le \sqrt{\frac{51\,000}{F_y}}$

$$F_{bx} = F_y\left(\frac{2}{3} - \frac{5.1\,K_t LS_x F_y}{1\,530\,000\,C_b\,\sqrt{JI_y}}\right) \le 0.6F_y \quad \text{AISC (1.5-6a)}$$

$F_{by} = \ \ 0.6F_y$

2.3 If $\left(\frac{KL}{r}\right)_{equiv.} > \sqrt{\frac{510\,000}{F_y}}$

$F_{bx} = 170\,000/(KL/r)^2$ equiv. AISC (1.5-6b)

$F_{by} = 0.6Fy$

Note—If there are unstiffened elements on the section that result in a value for Q_s less than 1, then F_b shall be the smaller value $0.6F_yQ_s$ or that provided by 7.2.2.B.2.1, 7.2.2.B.2.2, and 7.2.2.B.2.3 multiplied by Q_s, whichever is applicable.

C. Determine if a further reduction in F_{bx} is required.

1. If web (h/t) > $760/\sqrt{F_{bx}}$, then:

$$F'_{bx} = F_{bx}\left[1.0 - 0.0005\frac{A_w}{A_f}\left(\frac{h}{t} - \frac{760}{\sqrt{F_{bx}}}\right)\right] \quad \text{AISC (1.10-5)}$$

If web (h/t) > 760 $\sqrt{5.4}/\sqrt{F_{bx}}$ and horizontal stiffeners are used and placed at 0.4 the distance between the compression flange and the neutral axis as measured from the compression flange (refer to Appendix G) then:

$$F'_{bx} = F_{bx}\left[1.0 - 0.0005\frac{A_w}{A_f}\left(\frac{h}{t} - \frac{760\sqrt{5.4}}{\sqrt{F_{bx}}}\right)\right]$$

2. And if the section is a hybrid, F_{bx} in either flange shall not exceed the previous or

$$F'_{bx} = F_{bx}\left(\frac{12 + (A_w/A_f)(3a - a^3)}{12 + 2A_w/A_f}\right) \quad \text{AISC (1.10-6)}$$

where:

a = F_y of web/F_y of flange

7.3 Step 3—Solution to the Interaction Equation(s) for the Compressive Stresses

NOTE—The actual stresses f_{bx} and f_{by} are based on the effective section properties, if applicable, S_{xe} and S_{ye}. The axial stress f_a is based on the total area (A) of the section. Also, the f_a term may be positive for some sections. Refer to 7.2.2A.

1. If $f_a/F_a \le 0.15$, then compute

$$\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \le 1.0 \quad \text{AISC (1.6-2)}$$

2. If $f_a/f_a > 0.15$, then compute both 2.1 and 2.2

2.1 $$\frac{f_a}{0.6F_y} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \le 1.0 \quad \text{AISC (1.6-1b)}$$

2.2 $$\frac{f_a}{F_a} + \frac{C_{mx} f_{bx}}{\left(1 - \frac{f_a}{F'_{ex}}\right)F_{bx}} + \frac{C_{my} f_{by}}{\left(1 - \frac{f_a}{F'_{ey}}\right)F_{by}} \le 1.0 \quad \text{AISC (1.6-1a)}$$

where:

$C_{mx} = C_{my} = 0.85$

$$F'_e = \frac{12\pi^2 E}{23\left(\frac{KL_b}{r_b}\right)^2}$$

Do for both x and y axes using their corresponding r_b.

7.4 Step 4—Calculation of the Actual and Allowable Shear Stress in the Webs

7.4.1 I. TO DETERMINE THE ACTUAL STRESSES

$$f_a = V_y/2bt + T/2A_m t$$

where:

bt = the area of one web

Am = the area based on the mean dimensions of the section. Refer to Appendix E.

7.4.2 II. TO DETERMINE THE ALLOWABLE SHEAR STRESS F_V

A. No stiffeners on the web plates if

1. h/t < 260 and/or

$$h/t \le \frac{14\,000}{\sqrt{F_y(F_y + 16.5)}} \quad \text{AISC (1.10.2)}$$

where:

F_y = material yield of the compression flange

2. k = 5.34

3. $C_v = \frac{45\,000k}{F_y(h/t)^2}$

If $C_v > 0.8$, then $C_v = \frac{190}{(h/t)}\sqrt{\frac{k}{F_y}}$ AISC (1.10-1)

4. $F_v = \frac{F_y C_v}{2.89} \le 0.4\,F_y$ AISC (1.10-1)

5. $f_s \le F_v$

B. If part A is not met, then proceed accordingly—stiffeners are required.

1. If a/h ≤ 1.5, then h/t may be as much as $\frac{2000}{\sqrt{F_y}}$. If a/h < 1.0, then h/t may be as much as $\frac{2500}{\sqrt{F_y}}$. Where F_y is the same as before, otherwise h/t is limited to $\frac{14\,000}{\sqrt{F_y(F_y + 16.5)}}$ in any case a/h ≤ $\left(\frac{260}{h/t}\right)^2$ to a maximum of 3

2. If a/h < 1.0; $k = 4 + 5.34/(a/h)^2$ AISC (1.10.5.2)
If a/h > 1.0 (up to 3); $k = 5.34 + 4/(a/h)^2$

3. $C_v = \frac{45\,000k}{F_y(h/t)^2}$; however, if $C_v > 0.8$, then

$C_v = \frac{190}{h/t}\sqrt{\frac{k}{F_y}}$

4. If $C_v \le 1.0$, then

$$F_v = \frac{F_y}{2.89}\left[C_v + \frac{1 - C_v}{1.15\sqrt{1 + (a/h)^2}}\right] \le 0.4F_y \quad \text{AISC (1.10-2)}$$

5. If $C_v > 1.0$, then
$F_v = \frac{F_y C_v}{2.89} \le 0.4F_y$

6. $f_s \le F_v$

C. Stiffener properties

1. Required cross-sectional area

$$A_{st} = \frac{1 - C_v}{2}\left(\frac{a}{h} - \frac{(a/h)^2}{\sqrt{1 + (a/h)^2}}\right) y \cdot D \cdot h \cdot t \quad \text{AISC (1.10-3)}$$

where:

Y = F_y of web/F_y of stiffener (see Appendix G)

D = 1.0 for stiffeners in pairs
= 1.8 for single angle stiffeners
= 2.4 for single plate stiffeners

Note—If F_v was computed from IIB-4 previously, then

$$A_{st} = A_{st}\frac{f_s}{F_v}$$

2. Required moment of inertia with reference to an axis in the plane of the web.

$$I_{st} \ge \left(\frac{h}{50}\right)^4 \quad \text{AISC (1.10.5.4)}$$

3. Required weld to connect stiffener to the web.

$$f_{vs} \ge h_v \sqrt{\left(\frac{F_y}{340}\right)^3} \qquad \text{AISC (1.10-4)}$$

where:

F_y = material yield of web

f_{vs} = kips per lineal inch

Note—If F_v was computed from IIB-4 previously, then

$$f_{vs} = f_{vs}\frac{f_s}{F_v}$$

7.5 Step 5—Calculation for Tensile Stresses

7.5.1 I. ACTUAL STRESS WITHOUT STIFFENERS

$$-f_a + f_{bx} + f_{by} \le 0.6F_y$$

NOTE—If the section is a hybrid, then the limit is F_{bx} from 7.2.2.C.1 and/or 2. On some sections f_a may be positive.

7.5.2 II. If stiffeners are used and if Fv came from 7.4.2.B.4, then the bending tensile stress is limited accordingly.

$$f_{bx} + f_{by} \le \left(0.825 - 0.375\frac{f_s}{F_v}\right) \le 0.6F_y \qquad \text{AISC (1.10-7)}$$

7.5.3 III. If the flanges and webs are of ASTM A 514 steel and stiffeners are used, and if

$$f_{bx} + f_{by} > 0.75F_{bx} \text{ or } F'_{bx}\text{, if applicable,}$$

then f_s shall not exceed F_v as computed from 8.4.2.B.5.

APPENDIX A

Critical flexural stress due to lateral-torsional buckling for the compression flange

$$\left(\frac{KL}{r}\right)^2_{equiv} = \frac{5.1\,K_t L S_x}{\sqrt{J I_y}} \Big/ C_b \qquad \text{(Eq.A1)}$$

(Section 1.5.1.4.4 Commentary on AISC)

Terms:

L = distance from tip to section in question

S_x = major axis section modulus to the compressive flange

I_y = minor axis moment of inertia

J = torsional constant of the beam cross section

K_t = 4/3 cantilevered section with no end restraint

$$F_{bx} = \left(\frac{2}{3} - \frac{F_y (L/r_t)^2}{1530 \times 10^3}\right) F_y \le 0.6F_y \qquad \text{AISC (1.5-6a)}$$

substitute $(KL/r)_{equiv}$ for (L/r_t) to obtain expression for F_{bx}:

$$F_{bx} = F_y\left(\frac{2}{3} - \frac{5.1\,K_t L S_x F_y}{1\,530\,000\, C_b \sqrt{J I_y}}\right) \le 0.6F_y$$

(Refer to Appendix C for value of C_b.

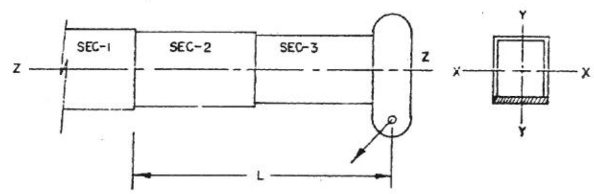


FIGURE A1

APPENDIX B

Relationship between *relative effectiveness* and width-thickness ratio for a stiffened compression element based on 0.60 F_y compressive stress level, uniformly distributed. *(Safety factor = 1.67 for k_c = 4.0)*

$$b/t = 1.9\sqrt{\frac{0.6E}{f_{max}}}\left(1 - (0.91)\frac{(0.415)}{w/t}\sqrt{\frac{0.6E}{f_{max}}}\right) \qquad \text{(Eq.B1)}$$

where:

$f_{max} \le 0.6F_y$

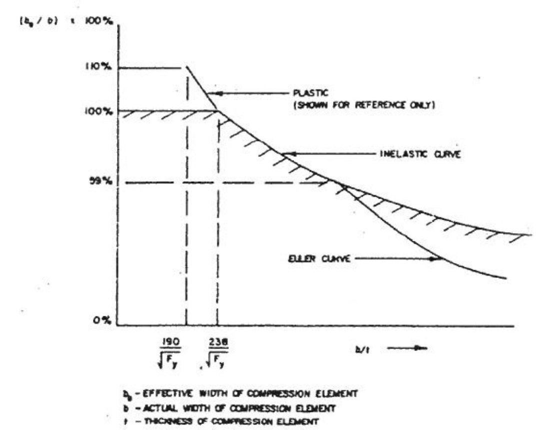


FIGURE B1

APPENDIX C
BENDING COEFFICIENT C_b

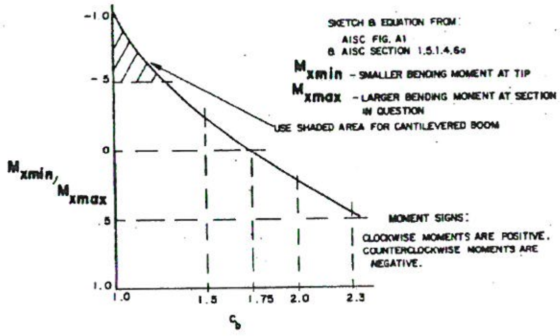


FIGURE C1

where:

$1 \leq C_b \leq 1.3$ CRC p. 101

OTHER REFERENCE—AISI Commentary—Flexural members chapter, subject—lateral buckling

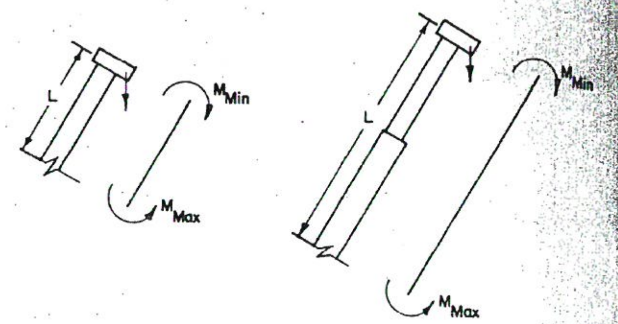


FIGURE C2

Equation:

$$C_b = 1.75 + 1.05\left(\frac{M_{x\,min}}{M_{x\,max}}\right) + 0.3\left(\frac{M_{x\,min}}{M_{x\,max}}\right)^2 \qquad \text{(Eq.C1)}$$

APPENDIX D
TORSIONAL CONSTANT J FOR CLOSED RECTANGULAR SECTIONS

General equation:

$$J = \frac{2(A_o + A_i)A_m}{\frac{2h_m}{t_w} + \frac{b_m}{t_1} + \frac{b_m}{t_2}} \qquad \text{(Eq.D1)}$$

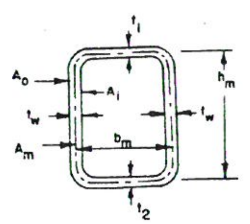


FIGURE D1

$$J = \frac{4{b_m}^2{h_m}^2}{\left(\frac{2h_m}{t_w} + \frac{b_m}{t_1} + \frac{b_m}{t_2}\right)} \qquad \text{(Eq.D3)}$$

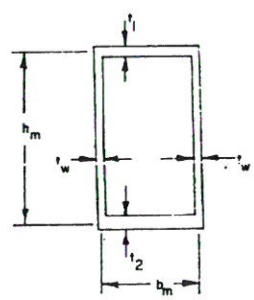


FIGURE D3

CASE I—for thin wall sections

Top and bottom flanges are the same thickness

$$J = \frac{2t_1 t_w b_m^2 h_m^2}{b_m t_w + h_m t_1} \qquad \text{(Eq.D2)}$$

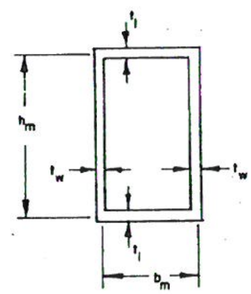


FIGURE D2

CASE II—for thin wall sections

Top and bottom flanges are not the same thickness

$$A_m = b_m \times n_m$$
$$A_i = b_f \times h \qquad \text{(Eq.D4)}$$
$$A_o = b_w \times d$$

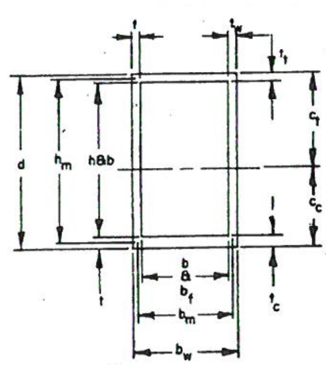
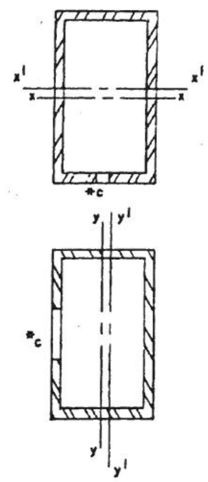


FIGURE D4

APPENDIX E
EFFECTIVE WIDTH OF CROSS SECTION

S_{xe} ; BASED ON EFFECTIVE AREA

S_{ye} ; BASED ON EFFECTIVE AREA. (AN EQUAL AREA MAY BE CONSERVATIVELY REMOVED TO FROM THE TENSILE SIDE TO MAKE THE SECTION SYMMETRICAL AND THUS FACILITATE THE SECTION MODULUS CALCULATION)



NOTE: *c INDICATES COMPRESSION SIDE

FIGURE E1

APPENDIX F
UNSTIFFENED ELEMENTS

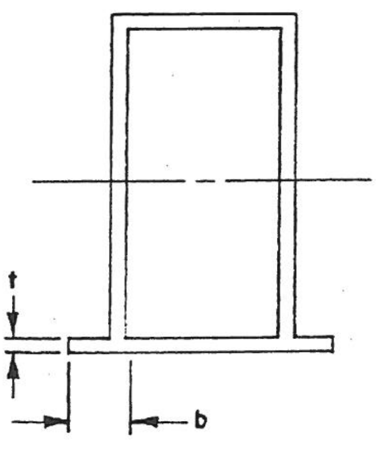


FIGURE F1

if:

$$(b/t) \le 95.0/\sqrt{F_y}\ ;\ Q_s = 1 \qquad \text{(Eq.F1)}$$

when:

$$95.0/\sqrt{F_y} < (b/t) < 176/\sqrt{F_y} \qquad \text{(Eq.F2)}$$

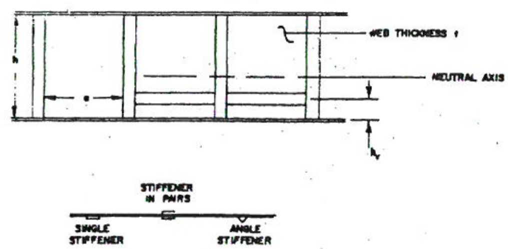
$$Q_s = 1.415 - 0.00437\,(b/t)\sqrt{F_y}$$

when:

$$(b/t) \ge 176/\sqrt{F_y} \qquad \text{(Eq.F3)}$$

$$Q_s = 20\,000/\left[F_y (b/t)^2\right]$$

APPENDIX G
PANEL STIFFENERS



h_V = 0.4 the distance between the compression flange and the neutral axis as measured from the compression flange

if a/h < 1.0:	k = 4.0 + 5.34/(a/h)2
if 1.0 < a/h ≤ 3.0:	k = 5.34 + 4.0/(a/h)2
if a/h > 3.0:	k = 5.34 (use of tension field action is not counted upon

FIGURE G1

APPENDIX H
WIND LOAD

H.1 Relationship between Wind Velocity and height.

$$V = V_r\left(H/H_r\right)^p \qquad \text{(Eq.H1)}$$

where:

V_r = wind velocity (mph) at height H_r
H_r = reference height at which wind velocity is measured (usually $H_r = 20$ ft)
H = height to center of wind pressure on boom or boom section from ground $H = H_O + H_p$
V = wind velocity (mph) at wind pressure height H
p = wind velocity exponent
for $V_r = 20$ mph at $H_r = 20$ ft, use p = 0.17
for $V_r = 50$ mph at $H_r = 20$ ft, use p = 0.25

NOTE—20 mph is for in-service condition
50 mph is four out-of-service condition

H.2 Wind load g, pounds per square inch

$$g = 0.004(V)^2/144 \qquad \text{(Eq.H2)}$$

H.3 Center of wind pressure is found for a given section at 60% of its extended length, the wind velocity for that section is found at that point.

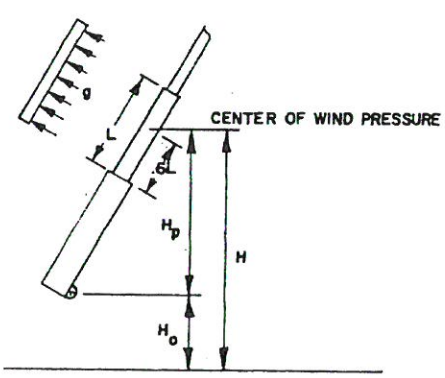
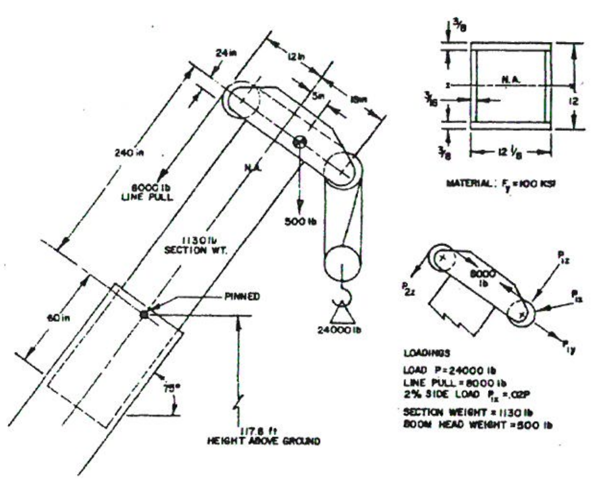


FIGURE H1

APPENDIX I
SAMPLE CALCULATIONS



WIND LOAD CALCULATION

Center of wind pressure on section (neglecting boom head wind loads)
0.6 (240-24) sin 75 deg/12 = 10.4 ft above pin
height above ground to center of wind pressure H = 10.4 + 117.6 = 128 ft
Wind Velocity V for in-service conditions

$$V = V_r(H/H_r)^p$$
$$= 20 \text{ mph } (128 \text{ ft} / 20 \text{ ft})^{0.17} = 27.4 \text{ mph} \qquad \text{(Eq.I1)}$$

Wind Load g

$$g = (0.004\ V^2 / 144) \text{ psi} = 0.004\ V^2 \text{ lb/ft}^2$$
$$= 0.004\ (27.4 \text{ mph})^2 = 3.0 \text{ lb/ft}^2 \qquad \text{(Eq.I2)}$$

FIGURE I1

SAMPLE CALCULATION—FIGURE 2

l_1 = 24 in	W_1 = 500 #
l_2 = 18 in	P = 24 000 #
l_3 = 24 in	P/N = 24 000/3 = 8000 #
l_4 = 12 in	P_x = 24 000(0.02) = 480 = 500 #
l_5 = 24 in	P_y = 24 000 cos 75 deg = 6200 #
l_6 = 5 in	P_z = 24 000 sin 75 deg = 23 200 #
α = 75 deg	

M_1 = 6200(24) + 23 200(18) - 8000(12) + 500(24 cos 75 deg + 5 sin 75 deg)
= 475 900 # in

M_2 = 480(24) = 11 520 = 11 500 # in

T = 480(18) = 8640 = 8600 # in

R_{z1} = 8000 + 23 200 + 500 sin 75 = 31 680 = 31 700 #

V_x = $-R_{x1}$ = 480 = 500 #

V_y = $-R_{y1}$ = 500 cos 75 deg + 6200 = 6329 = 6300 #

*Values rounded off to nearest 100 #

SAMPLE CALCULATION—FIGURE 3

l_7 = 240 in	W_2 = 1130 #
l_8 = 60 in	M_1 = 523 900 # in
l_9 = —	M_2 = 11 500 # in
d_1 = 12 in	T = 8600 # in
α = 75 deg	R_{x1} = 500 #
g = 3 #/ft2	R_{y1} = 6300 #
	R_{z1} = 31 700 #

$$M_x = 475\,900 + 6300(240) + 0.5(1130)\frac{(240)^2}{240+60}\cos 75\text{ deg} = 2\,016\,000\text{ \# in}$$

$$M_y = 11\,500 + 500(240) + 0.5(3/144)12(240)^2 = 138\,700\text{ \# in}$$

$$R_{z2} = 31\,700 + 1130\sin 75\text{ deg} = 32\,800\text{ \#}$$

$$P_{ar} = 31\,700 + 1130\sin 75\text{ deg}\left(\frac{240}{240+60}\right) = 32\,600\text{ \#}$$

$$P_{aL} = 1130\sin 75\text{ deg}\left(\frac{60}{240+60}\right) = 200\text{ \#}$$

$$R_{y3} = \frac{201\,600}{60} - 0.5(1130)\cos 75\text{ deg}\frac{(60)^2}{240+60} = 31\,800\text{ \#}$$

$$R_{y2} = 6300 + 31\,800 + 1130\cos 75\text{ deg} = 38\,400\text{ \#}$$

$$R_{x3} = 138\,700/60 = 2300\text{ \#}$$

$$R_{x2} = 500 + 2300 + (3/144)\,12(240) = 2900\text{ \#}$$

$$V_{yr} = 6300 + 1130\cos 75\text{ deg}\left(\frac{240}{240+60}\right) = 6500\text{ \#}$$

$$V_{yL} = 31\,800 + 1130\cos 75\text{ deg}\left(\frac{60}{240+60}\right) = 31\,900\text{ \#}$$

$$V_{xr} = 500 + (3/144)\,12(240) = 600\text{ \#}$$

$$V_{xl} = 2300\text{ \#}$$

$$M_{xmin} = \sin 75\text{ deg}\,[24\,000(18) + 500(5)] - 8000(12) = 323\,000\text{ \# in}$$

$$M_{xmax} = Mx = -2\,064\,000\text{ \# in}$$

$$\frac{M_{xmin}}{M_{xmax}} = \frac{323\,000}{-2\,016\,000} = -0.16$$

*Values rounded off to nearest 100 #

Step 1

F_y = 100 ksi for flanges and webs

Actual cross-sectional area A

$A = 2(0.375)12.12 + 2(0.187)(11.25) = 13.30\text{ in}^2$

$\overline{Y} = c_b = c_t = 12/2 = 6$; $\overline{X} = c_b = c_t = 12.12/2 = 6.06$ in

$I_x = 2(12.12)(375)^3/12 + 2(0.187)(11.25)^3/12 + 2(0.375)12.12(5.813)^2 = 351.7\text{ in}^4$

$I_y = 2(0.375)(12.12)^3/12 + 2(11.25)(0.187)^3/12 + 2(0.187)(11.25)(5.97)^2 = 261.6\text{ in}^4$

$S_x = 351.7/6 = 58.6\text{ in}^3$

$S_y = 261.6/6.06 = 43.1\text{ in}^3$

Step 2 I

$f = f_a + f_b$

$f_a = P_{ar}/A = 32\,600/13.3 = 2450$ psi

$f_{bx} = M_x/S_x = 2\,016\,000/58.6 = 34\,400$ psi

$f_{by} = M_y/S_y = 138\,700/43.1 = 3220$ psi

flange $f = 2450 + 34\,400 = 36\,850$ psi

web $f = 2450 + 3220 = 5670$ psi

A. $b/t \le 184/\sqrt{f}$

$f = 0.6F_y$ or $f = f_a + f_b$

b/t flange = (12.12 - 0.37)/0.375 = 31.3

b/t web = (12 - 0.75)/0.187 = 60.2

$f = 0.6F_y = 0.6(100) = 60$ ksi

$b/t = 184/\sqrt{60} = 23.7$

1. Axial compression

   $b/t \le 184/\sqrt{f_a} = 184/\sqrt{2.45} = 117.6$

   31.3 and 60.2 < 117.6

   section fully effective for P/A stress

2. Bending X-X

   $b/t \le 184/\sqrt{f_a + f_b}$

   flange: $184/\sqrt{36.85} = 30.3$

   b/t = 31.3 > 30.3, not fully effective

   $$b_e = \frac{253(0.375)}{\sqrt{36.85}}\left(1 - 50.3/(31.3)\sqrt{36.85}\right) = 11.42$$

   $$= \frac{253(0.375)}{\sqrt{36.85}}\left(1 - 50.3/(31.3)\sqrt{36.85}\right) = 11.42$$

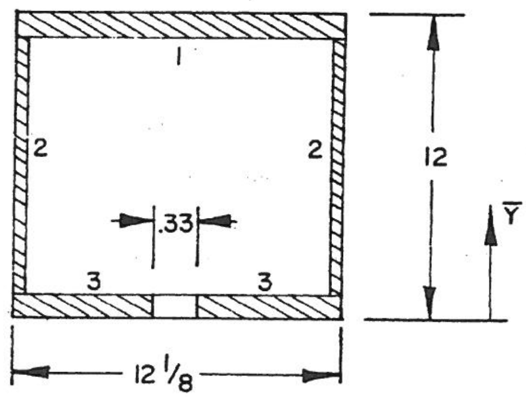


Plate	A	Y	AY	$A(\overline{Y} - Y)^2$	$I_o = bh^3/12$
(1) 3/8 x 12.12	4.55	11.81	53.7	150.9	.0
(2) 3/16 x 11.25	2.10	6.00	12.6	0	22.1
(2) 3/16 x 11.25	2.10	6.00	12.6	0	22.1
(3) 3/8 x 11.79	4.42	.18	.8	152.3	.0
	13.17	23.99	79.7	303.2	44.2

FIGURE I2

$\overline{Y} = AY/A = 79.7/13.17 = 6.05$ in

$c_c = 6.05$, $c_t = 5.95$ in

$I_{xe} = I_o + A(\overline{Y} - Y)^2 = 44.2 + 303.2 = 347.4\text{ in}^4$

compression flange $S_{xe} = I_{xe}/c_c = 347.4/6.05 = 57.4\text{ in}^3$

$f_{bx} = 2\,016\,000/57.4 = -35.1$ ksi

tension flange $S_{xe} = I_{xe}/c_t = 347.4/5.95 = 58.4\text{ in}^3$

$f_{bx} = 2\,016\,000/58.4 = +34.4$ ksi

3. Bending Y-Y

   $b/t \le 184/\sqrt{f_a + f_b}$

   web: $184/\sqrt{5.67} = 77.4$

   b/t = 60.2 < 77.4 web is fully effective

   $S_{ye} = S_y = 43.1\text{ in}^3$

   $f_{by} = M_Y/S_y = 138\,700/43.1 = \pm 3.22$ ksi

Step 2 II Allowable Stress

A. F_a allowable axial stress

$Q_a = A_e/A = 1$ section fully effective and $Q_s = 1$ for P/A = 2.47 ksi

$$C_c = \sqrt{\frac{2\pi^2 E}{Q_s Q_a F_y}} = \sqrt{\frac{2\pi^2 29\,500}{(1)(1)100}} = 76$$

KL/r value

$$X-X\ r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{351.7}{13.30}} = 5.14 \text{ in}$$

$$Y-Y\ r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{261.6}{13.30}} = 4.43 \text{ in}$$

use r_y - smaller value
$KL/r_y = 2(216)/4.43 = 97.5$
$KL/r_x = 2(216)/5.14 - 84.0$
$KL/r_y > C'_c$ 97.5 > 76 elastic range
$F_a = 12\pi^2 E/23(KL/r)^2$
$F_a = 12\pi^2\ 29.5 \times 10^3/23(97.5)^2 = 16$ ksi

B. F_b Allowable Bending Stress

$$(KL/r)_{equiv} = \sqrt{\frac{5.1\,K_t L S_x}{\sqrt{J I_y}}} \Big/ \sqrt{C_b}$$

$$J = 2t_1 t_w h_m{}^2 b_m{}^2/(t_w b_m + h_m t_1)$$

$$= \frac{2(0.375)(0.187)(11.625)^2(11.932)^2}{(0.187)(11.938)+(11.625)(0.375)} = 410 \text{ in}^4$$

$$C_b = 1.75 + 1.05\left(\frac{M_{x\,min}}{M_{x\,max}}\right) + 0.3\left(\frac{M_{x\,min}}{M_{x\,max}}\right)^2$$

$C_b = 1.75 + 1.05(-0.157) + 0.3(-0.157)^2 = 1.6$; $C_b = 1.3$

$$(KL/r)_{equiv} = \sqrt{\frac{5.1(4/3)240(58.6)}{\sqrt{410(261.6)}}} \Big/ \sqrt{1.3} = 15$$

$$\sqrt{\frac{102\,000}{F_y}} = 31.9 > \left(\frac{KL}{r}\right)_{equiv};\ F_{bx} = 0.6F_y = 60 \text{ ksi}$$

C. F_{bx}—determine if further reduction in F_{bx} is required.

$$\frac{760}{\sqrt{60}} = 98.1\ \frac{h}{t} = \frac{11.25}{0.1875} = 60$$

since $\frac{h}{t} < \frac{760}{\sqrt{F_{bx}}}$ then a further reduction is not required.

Step 3—Solution to the Interaction Equation for Compression

$$\frac{f_a}{F_a} = \frac{2.45}{16.000} = 0.149;\ \frac{f_a}{F_a} < 0.15;\ \frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \le 1.0$$

$$\frac{2.45}{16} + \frac{35.1}{60} + \frac{3.22}{60} = 0.79 < 1.0$$

Step 4—Calculation for Webs

I. To determine the actual stresses inside overlap—if this passes, the outside will pass.

$$f_s = \frac{V_{yL}}{2ht_w} + \frac{T}{2A_m t_w}$$

$$= \frac{31\,900}{2(11.25)(0.188)} + \frac{8600}{2(12.125-0.188)(12-0.375)0.188}$$

$f_s = 7.71$ ksi

II. To determine allowable shear stress F_v

1. $\frac{h}{t} = \frac{11.25}{0.188} = 60 < 260$

$$\frac{14\,000}{\sqrt{100(100+16.5)}} = 129.7 > h/t$$

2. $k = 5.34$
3. $C_v = \frac{45\,000k}{F_y(h/t)^2} = \frac{45\,000(5.34)}{100(60)^2} = 0.67 < 0.8$
4. $F_v = \frac{F_y C_v}{2.89} = \frac{100(0.67)}{2.89} = 23.18$ ksi
5. $F_v < 0.4F_y$ and $f_s < F_v$

Therefore, it is not necessary to investigate the need for stiffeners.

Step 5—Calculation for Tensile Stresses

1. Since section is not required to have stiffeners, then:
$-f_a + f_{bx} + f_{by} \le 0.6F_y$
$-2.45 + 34.4 + 3.22 = 35.17 < 0.6F_y$