SAVING WEIGHT WITH SHRINK FITS

New charts and equations make it easier for designers to optimize shrink fits, saving materials and money.

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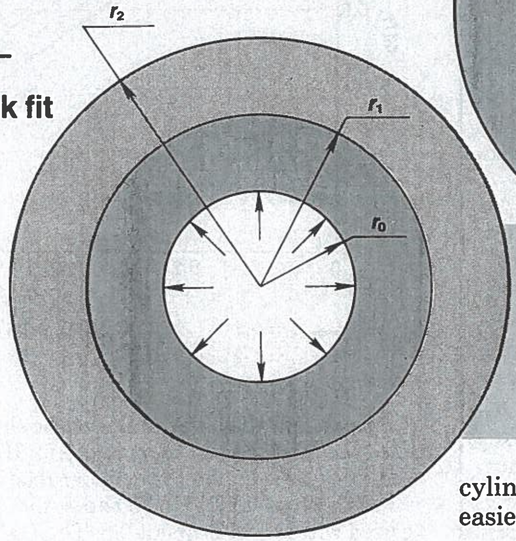
Shrink fit



Shrink-fit assemblies are often used when a prestress or residual stress is desired in an assembly. Often, this is because a weak, low-cost material can be used in a prestressed condition to duplicate the performance of a more expensive, high-strength material. Shrink fits also permit assembly weight reduction without reducing the joint's ability to resist applied loads. They are commonly used in pressure vessels, jigs, fixtures, and dies.

Many problems involved in analyzing shrink fits on thick cylinders refer to Gadolin's conditions, which were derived for shrink fits with two cylinders. These equations are based on a simple optimization procedure, without constraints on either the pressure at the interference fit or the radial thicknesses of the two cylinders. Gadolin's optimization method gives solutions as easily used expressions. However, these equations are valid only if the allowable stresses for the inner and outer cylinders are the same. And allowable stresses are generally the same only when the cylinders are made of the same materials.

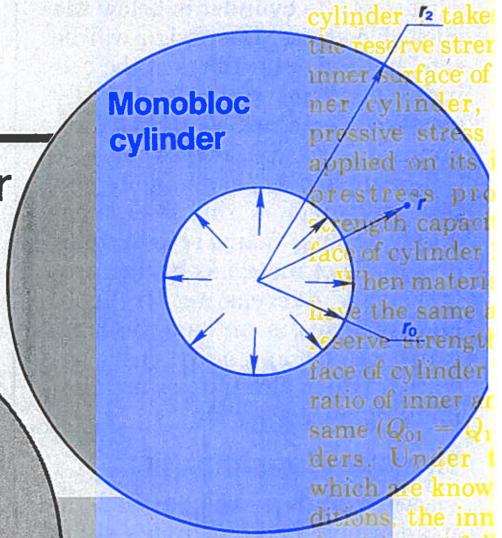
But shrink-fit assemblies of two different materials are often preferred, to take advantage of different material properties. For example, a steel shell may be used for strength over a corrosion-resistant copper or aluminum liner.

More recent studies of shrink fits account for constraints on the radial thickness of the two cylinders and the pressure at the interference fit. However, solutions obtained by modern optimization generally are of little help to a designer seeking a simple answer, unless the constraints exactly match those of a previous study.

Now, new shrink-fit studies allow Gadolin's conditions to be modified so the equations can be used even when the allowable stresses for inner and outer cylinders are not the same. Also, recently developed design data sheets allow quick and easy analysis of shrink-fit problems.

Monobloc cylinder

To analyze the shrink-fit assembly, it is best to look at its simplest case. This is not actually a shrink fit at all, but a single, or monobloc, cylinder. A monobloc cylinder is easier to produce than a shrink-fit assembly, and may be used as an alternative to a shrink fit when weight constraints are not too restricted. It is not recommended, however, when designers want to get the most out of their materials. The outer layers of the cylinder are not fully stressed, and they retain some reserve strength.



A monobloc, or single cylinder, can sometimes be substituted for a press fit. However, it may be less efficient.

Tangential and radial stresses at radius r for a monobloc cylinder with inner radius r_0 and outer radius r_2 subjected to internal pressure p_0 are given by

$$\sigma_t = \frac{p_0 r_0^2\left[1 + \frac{r_2^2}{r_2}\right]}{\left(r_2^2 - r_0^2\right)} \tag{1a}$$

$$\sigma_r = \frac{p_0 r_0^2\left[1 + \frac{r_2^2}{r_2}\right]}{\left(r_2^2 - r_0^2\right)} \tag{1b}$$

where σ_t = tangential stress at radius r and σ_r radial stress at radius r.

Tresca stress σ_T is defined as the algebraic difference between tangential and radial stress at any radius r:

$$\sigma_T = \frac{2P_0 r_0^2 r_2^2}{r^2\left[r_2^2 - r_0^2\right]} \qquad (2)$$

If the Tresca stress at the inner surface of the cylinder is below the allowable stress, the design will be safe, but the outer fibers of the cylinder material will not be fully stressed. Some reserve strength remains at any radius other than r_0.

Theoretically, a designer can take advantage of this reserve capacity by snug-fitting several cylinders made of materials having progressively decreasing strength. However, this is not practical because of the assembly costs involved. Other alternatives are simpler and more practical.

Two-cylinder shrink fit

Two cylinders can be used instead of one. It is relatively easy to compare the weight-reduction advantages of a shrink-fit assembly with an alternative design using a monobloc. In addition, designers must use a shrink fit rather than a monobloc if the inner pressure on the cylinder is greater than half the material's yield strength.

Shrink fitting cylinder 2 around cylinder 1 takes full advantage of the reserve strength capacity at the inner surface of cylinder 2. The inner cylinder, 1, sustains compressive stress before pressure is applied on its inner surface. This prestress produces a reserve strength capacity at the inner surface of cylinder 1.

When materials in both cylinders have the same allowable stress, the reserve strength at the inner surface of cylinder 2 is used fully if the ratio of inner and outer radii is the same ($Q_{01} = Q_{12}$) for the two cylinders. Under these conditions, which are known as Gadolin's conditions, the inner surface of cylinder 1 is not fully prestressed. It is instead prestressed to the same extent as the reserve strength capacity at the inner surface of cylinder 2.

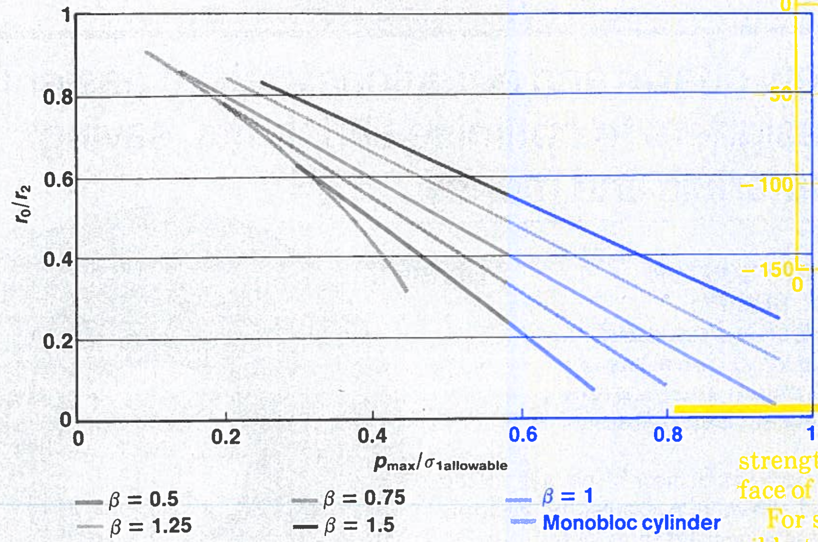
Two materials

When cylinder materials have different allowable stresses, the problem is to determine the conditions for optimum use of reserve

DATA SHEETS SIMPLIFY DESIGN

Modified Gadolin's conditions have been used to generate six design data sheets in terms of nondimensional relationships. The first plots the ratio r_0/r_2 against the ratio p_{max}/σ_{1all}. These curves have been plotted for five values of the ratio of the allowable stresses of the two cylinder materials ($\beta = 0.5$, $\beta = 0.75$, $\beta = 1$, $\beta = 1.25$, and $\beta = 1.5$). An additional curve is included for analyzing a monobloc cylinder.

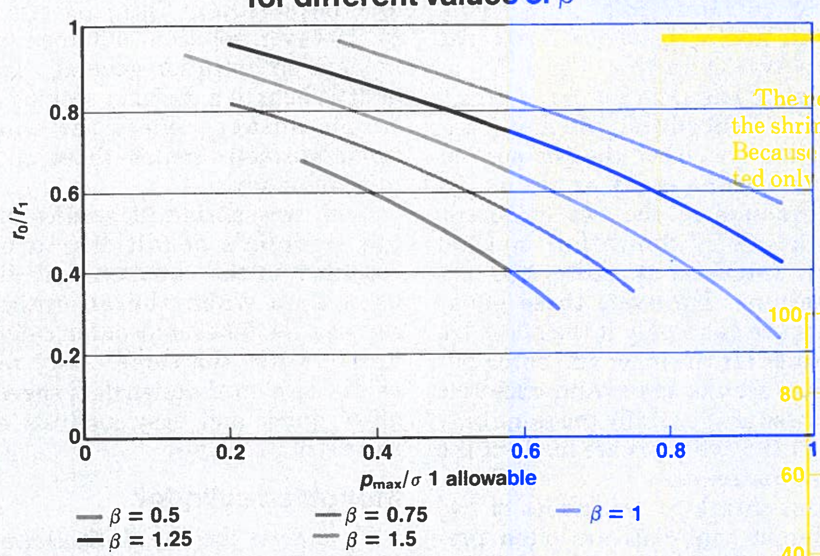
Variation of r_0/r_2 with respect to p_{max}/σ_{1all} for different values of β



Assume that the outer radius of the shrink-fit assembly is not more than five times the inner radius (r_0/r_2 is $\geqq 0.2$). If $p_{max}/\sigma_{1all} > 0.45$, the monobloc is not used, because it makes the cylinder relatively large. Comparing the results for $\beta = 1$ with those for $\beta = 0.75$ shows that the highest value of p_{max}/σ_{1all} that can be used with $\beta = 1$ is about 0.8. The highest value of p_{max}/σ_{1all} that can be used with $\beta = 0.75$ is about 0.7.

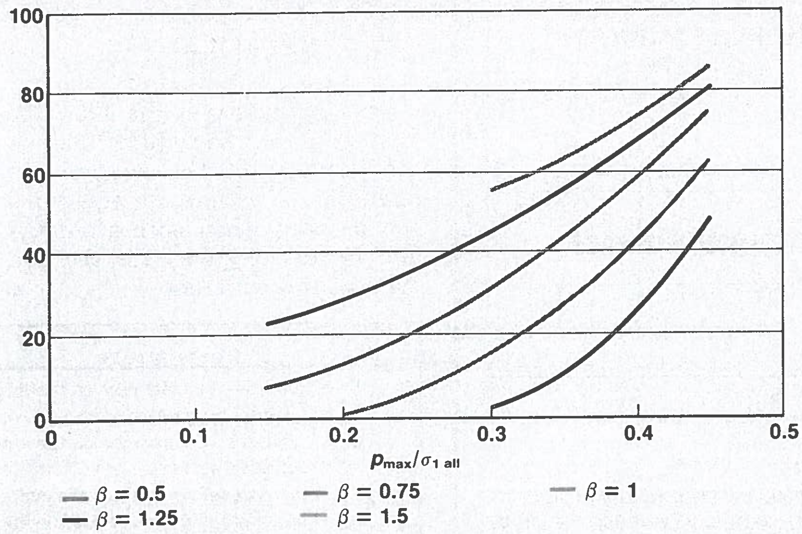
The next sheet plots the ratio r_0/r_1 of the inner and outer radii of cylinder 1 against p_{max}/σ_{1all} for different values of β.

Variation of r_0/r_1 with respect to p_{max}/σ_{1all} for different values of β



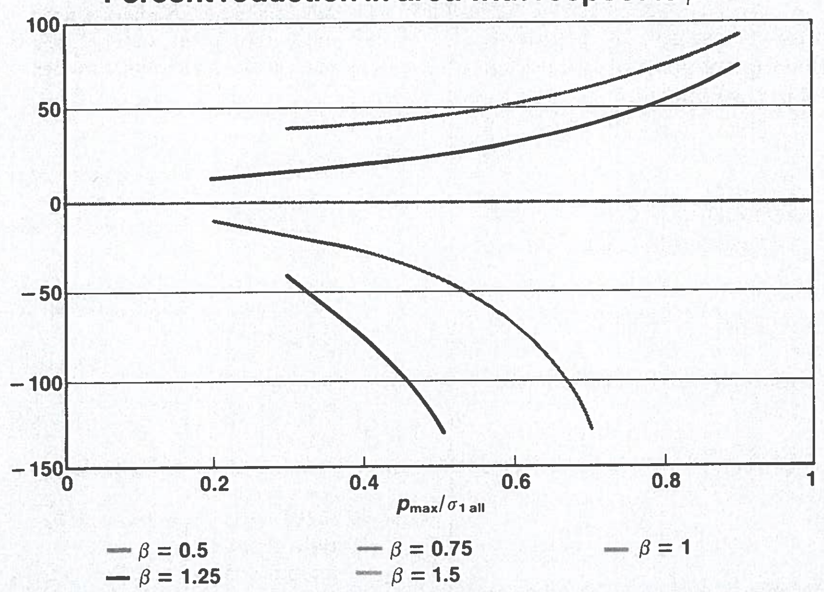
The next sheet shows the percentage reductions in the cross-sectional area of the shrink-fit as compared to a monobloc, for various values of p_{max}/σ_{1all} and β. Because a monobloc is useful only until $p_{max}/\sigma_{1all} = 0.45$, these curves are plotted only to that value. Area can be reduced by over 20% if $p_{max}/\sigma_{1all} \geqq 0.4$.

Percent reduction in area with respect to monobloc



The next sheet provides percentage reductions (or increases) in the cross-sectional area with respect to a shrink fit with $\beta = 1$. If $\beta < 1$, the area increases; if $\beta > 1$, it decreases. For example, if $p_{max}/\sigma_{1all} = 0.6$, the area is reduced about 50% if β is increased to 1.5.

Percent reduction in area with respect to $\beta = 1$



strength capacity at the inner surface of cylinder 2.

For shrink-fit cylinders, it is possible to find Tresca stresses at the inner surfaces of the cylinders. These may then be equated to the corresponding allowable stresses, σ_{1all} and σ_{2all}, to give

$$\sigma_{1all}=\sigma_{T1}=\sigma_{t1}-\sigma_{r1}=\frac{2P_0}{1-Q_{02}^2}-\frac{2P_1}{1-Q_{01}^2} \qquad (3a)$$

$$\sigma_{2all}=\sigma_{T2}=\sigma_{t2}-\sigma_{r2}=\frac{2P_0Q_{01}^2}{1-Q_{02}^2}+\frac{2P_1}{1-Q_{12}^2} \qquad (3b)$$

Equation 3b may be rewritten to solve for p_1:

$$P_1=\tfrac{1}{2}\left[\sigma_{2all}-\frac{2P_0-Q_{01}^2}{\left[1-Q_{02}^2\right]}\right]\left[1-Q_{12}^2\right] \qquad (4)$$

Substituting for p_1 in equation 3a and simplifying gives an equation for p_0 in terms of the allowable

USING THE SHEETS AND EQUATIONS

Consider a thick cylinder that has to be designed for a case where $p_{max}/\sigma_{1all} = 0.4$. The inner radius = 100 mm. The modulus of elasticity E_1 = 207 gigapascals (GPa). The allowable stress for the inner cylinder = 140 megapascals (MPa).

Because p_{max}/σ_{1all} = 0.40, a monobloc could be used instead of a shrink fit. The information in the next two tables can be easily obtained from the charts.

Finding the % reduction in area

6	r_0/r_2	r_0/r_1	% Reduction in area with respect to monobloc cylinder
Monobloc	0.45	—	0
0.50	0.50	0.59	23
0.75	0.55	0.69	42
1.00	0.60	0.77	56
1.25	0.65	0.85	66
1.50	0.69	0.92	73

Finding the radial interference parameter

β	% Reduction in area with respect to the case when $\beta = 1$	$\frac{P_1}{\sigma_{1all}}$	$\frac{\Delta E}{r_0\,\sigma_{1all}}$
0.50	−73	0.019	0.250
0.75	−31	0.038	0.400
1.00	0	0.050	0.520
1.25	22	0.052	0.620
1.50	39	0.041	0.700

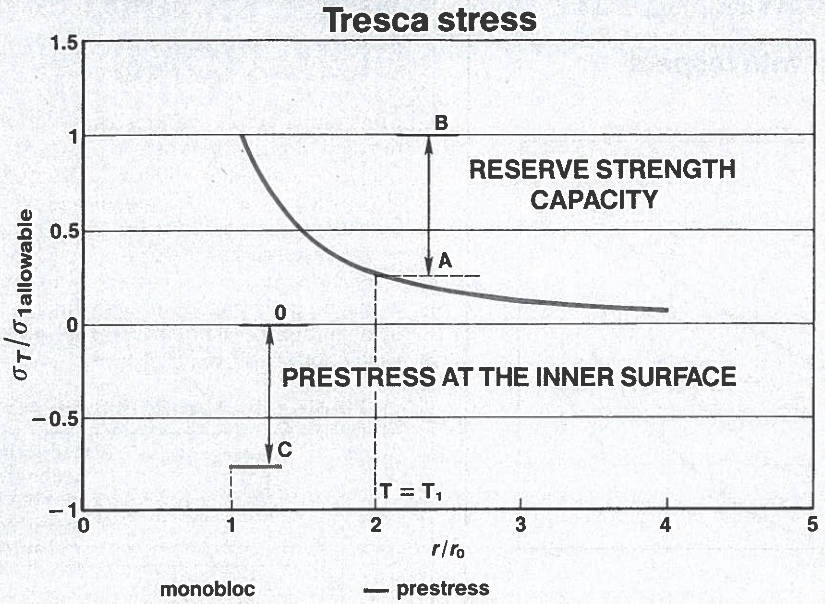
To keep the radial interference parameter <0.6, β must be $\leqq 1$. If the designer wants to reduce the area by more than 60% of the monobloc, β must be >1.

To illustrate the use of the charts, assume $\beta = 1$ and r_0 = 100 mm. From Chart 1 choose $r_0/r_2 = 0.6$, so $r_2 = 100/0.6 = 167$ mm. From Chart 2, $r_0/r_1 = 0.77$, so $r_1 = 100/0.77 = 130$ mm. From Chart 6, $\Delta E_1/(\sigma_{1all}\, r_0) = 0.52$, so $\Delta = 35\ \mu m$.

The equations may be used to design a thick cylinder given the following data: r_0 = 100 mm; p_{max} = 85 MPa; σ_{1all} = 140 MPa; σ_{2all} = 105 MPa; E_1 = 207 GPa.

From this information, p_{max}/σ_{1all} and β are calculated as: $p_{max}/\sigma_{1all} = 85/140 = 0.607$; and $\beta = 105/140 = 0.75$. Because $p_{max}/\sigma_{1all} > 0.45$, a monobloc cannot be considered.

The equation for r_2 gives its value as r_2 = 323 mm. Then r_1 = 193 mm, and the equation for p_1 gives the pressure on the outer surface of cylinder 1 as 17.5 MPa. The radial interference, Δ, is found to be 63 μm.



This graph gives the Tresca stress for a monobloc cylinder at any radius r. Monobloc cylinders will have reserve strength capacity because the outer layers of the cylinder are not fully stressed. The reserve strength is the difference between the stress at radius r and the allowable stress.

stresses of the materials of the two cylinders.

$$P_0=\frac{\sigma_{1all}}{2}\left[\left\{1-\frac{r_0^2}{r_1^2}\right\}+\beta\left\{1-\frac{r_1^2}{r_2^2}\right\}\right] \quad (5)$$

where β is $\sigma_{2all}/\sigma_{1all}$.

To determine the maximum pressure that can be applied at the inner surface of cylinder 1, equation 5 is partially differentiated with respect to r_1 and equated to zero to give

$$\frac{\partial p_0}{\partial r_1}=\frac{2r_0^2}{r_1^3}-\beta\frac{2r_1}{r_2^2}=0 \quad (6)$$

or

$$r_1=\sqrt{\frac{r_0r_2}{\sqrt{\beta}}} \quad (7)$$

Substituting this expression for r_1 in equation 5 gives the maximum pressure that can be applied on the inner surface of cylinder 1:

$$\frac{P_{max}}{\sigma_{1all}}=\frac{1}{2}\left[(1+\beta)-\frac{2r_0\sqrt{\beta}}{r_2}\right] \quad (8)$$

Often, the maximum pressure applied on the inner surface of cylinder 1 is given as one of the design criteria, and the radial thickness of cylinder 2 is found for a given inner radius of cylinder 1. The equation for finding the radius of cylinder 2 is written as

$$r_2=\frac{2r_0\sqrt{\beta}}{\left[(1+\beta)-\dfrac{2P_{max}}{\sigma_{1all}}\right]} \quad (9)$$

Because r_2 cannot be negative, the following inequality should be satisfied in the equation:

$$(1+\beta)>\frac{2P_{max}}{\sigma_{1all}} \quad (10)$$

The relationship of p_1 to σ_{1all} can be derived using equations 3a, 5, and 7:

$$\frac{P_1}{\sigma_{1all}}=\frac{1}{2}[\sqrt{\beta}-Q_{02}]^2\left[\frac{1-\sqrt{\beta}Q_{02}}{1-Q_{02}^2}\right] \quad (11)$$

The equation for the radial interference Δ is

$$\Delta=P_1\sqrt{\frac{r_0r_2}{\sqrt{\beta}}}\left[\frac{A}{E_2}+\frac{B}{E_1}\right] \quad (12)$$

where

$$A=\left[\frac{\sqrt{\beta}+Q_{02}}{\sqrt{\beta}-Q_{02}}-\nu_2\right] \quad (13)$$

and

$$B=\left[\frac{1+\sqrt{\beta}Q_{02}}{1-\sqrt{\beta}Q_{02}}-\nu_1\right] \quad (14)$$

Equations 7, 8, and 12 are the modified Gadolin's conditions when the allowable stresses for the materials of the two cylinders are different.

If the materials of the two cylinders have the same modulus of elasticity $E_1 = E_2$ and Poisson's ratio $\nu_1 = \nu_2$, equation 12 simplifies to

$$\Delta=\frac{2P_{max}}{E_1}\sqrt{r_0r_2\sqrt{\beta}}\left[\frac{\sqrt{\beta}-Q_{02}}{1+\beta-2\sqrt{\beta}Q_{02}}\right] \quad (15)$$

This analysis uses the allowable stresses of the two cylinder materials instead of the respective strengths. Therefore, the equations can be used when both strengths and factors of safety for the two cylinders are different. ■

Nomenclature

- E_1 = modulus of elasticity of the inner cylinder material
- E_2 = modulus of elasticity of the outer cylinder material
- n_1 = factor of safety for the inner cylinder
- n_2 = factor of safety for the outer cylinder
- p_0 = pressure on the inner surface of the inner cylinder
- p_{max} = maximum pressure that can be applied on the inner surface of the inner cylinder
- p_1 = pressure on the outer surface of the inner cylinder
- r_0 = inner radius of the inner cylinder
- r_1 = inner radius of the outer cylinder
- r_2 = outer radius of the outer cylinder
- Q_{01} = r_0/r_1
- Q_{02} = r_0/r_2
- Q_{12} = r_1/r_2
- S_1 = strength of the material of the inner cylinder
- S_2 = strength of the material of the outer cylinder
- β = $\sigma_{2all}/\sigma_{1all}$
- Δ = radial interference between the two cylinders
- ν_1 = Poisson's ratio of the inner cylinder material
- ν_2 = Poisson's ratio of the outer cylinder material
- σ_{r1} = radial stress at the inner surface of the inner cylinder
- σ_{r2} = radial stress at the inner surface of the outer cylinder
- σ_{t1} = tangential stress at the inner surface of the inner cylinder
- σ_{t2} = tangential stress at the inner surface of the outer cylinder
- σ_{T1} = Tresca stress at the inner surface of the inner cylinder ($\sigma_{t1} - \sigma_{r1}$)
- σ_{T2} = Tresca stress at the inner surface of the outer cylinder ($\sigma_{t1} - \sigma_{r1}$)
- σ_{1all} = allowable stress for the material of the inner cylinder ($S_{1/n1}$)
- σ_{2all} = allowable stress for the material of the outer cylinder ($S_{2/n2}$)
- σ_{1p} = prestress on the inner surface of the inner cylinder