- Let η be the efficiency of the pump
- The pipe is very thin hence conduction is negligible
- Pipe system as a heat exchanger
- [redacted]

The power of pump:

$$P = \rho Q g h$$

Where, Q = flow rate, h = head, ρ=density, g = acceleration due to gravity

Heat transfer into water due to pump:

$$Q_{in} = \eta \times P$$

$$Q_{in} = \eta \rho Q g h$$

For time dt

$q_{in} = \eta \rho Q g h \times dt$ (Watt to Joule)

Temperature rise in water:

$$q = m C_p \Delta T$$

$$q = \rho M C_p \Delta T$$

Heat in = Heat absorbed

$q = q_{in}$

$$\rho M C_p \Delta T = \eta \rho Q g h \times dt$$

$$M C_p T - T_1 = \eta \times 3M \times g \times L \times dt$$

$$T - T_1 = \frac{3\eta g L \times dt}{c_p}$$

$$T = \frac{3\eta g L \times dt}{c_p} + T_1$$

Now assuming the pipe system as heat exchanger:

$$mC_pT - T_2 = UA \times LMTD$$

Where,

$$LMTD = \frac{T - T_0 - T_2 - T_0}{\ln \frac{T_1 - T_0}{T_2 - T_0}} = \frac{T - T_2}{\ln \frac{T_1 - T_0}{T_2 - T_0}}$$

$$U = \frac{1}{h_0 A} + \frac{1}{h_w A} = \frac{1}{A} \frac{1}{h_0} + \frac{1}{h_w}$$

m = $\rho \times 3M$

Substituting in equation

$$3\rho M C_p T - T_2 = \frac{1}{A} \frac{1}{h_0} + \frac{1}{h_w} \times A \times \frac{T - T_2}{\ln \frac{T_1 - T_0}{T_2 - T_0}}$$

Simplifying

$$\ln \frac{T - T_0}{T_2 - T_0} = \frac{1}{3\rho M C_p} \frac{1}{h_0} + \frac{1}{h_w}$$

taking log on both sides

$$\frac{T - T_0}{T_2 - T_0} = e^{\frac{1}{3\rho M C_p} \frac{1}{h_0} + \frac{1}{h_w}}$$

$$T_2 = T_0 + T - T_0 e^{-\frac{1}{3\rho M C_p} \frac{1}{h_0} + \frac{1}{h_w}}$$

$$T_2 = T_0 + T - T_0 e^{-\frac{1}{3\rho M C_p} \frac{1}{h_0} + \frac{1}{h_w}}$$

This will give us temperature T_2 at the end of the pipe at a given instant of time

We can integrate this equation over time to get temperature at required time

Now at any time dt, the temperature of the tank will be average of these two temperatures:

$$T_w = \frac{T + T_2}{2}$$

$$T_w = \frac{T + T_0 + T - T_0 e^{-\frac{1}{3\rho M C_p}\frac{1}{h_0}+\frac{1}{h_w}}}{2}$$

$$T_w = \frac{T}{2} 1 + e^{-\frac{1}{3\rho M C_p}\frac{1}{h_0}+\frac{1}{h_w}} + \frac{T_0}{2} 1 - e^{-\frac{1}{3\rho M C_p}\frac{1}{h_0}+\frac{1}{h_w}}$$

Where T will be changing with each time step and is given by:

$$T = \frac{3\eta g L \times dt}{C_p} + T_1$$