

SHEAR FACTORS FOR BEAM ANALYSIS

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Keywords

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Shear Flexibility

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Introduction

Bending, or flexure, is usually the dominant source of beam deformation.

Nevertheless, shear deformation can be significant for beams that are deep relative to their length. This deformation can be important for both static and dynamic analysis.

A shear factor is needed to calculate shear deformation. The purpose of this tutorial is to summarize shear factors as presented by several authors.

Thomson, Reference 1

A free-body diagram of a beam element is shown in Figure 1.

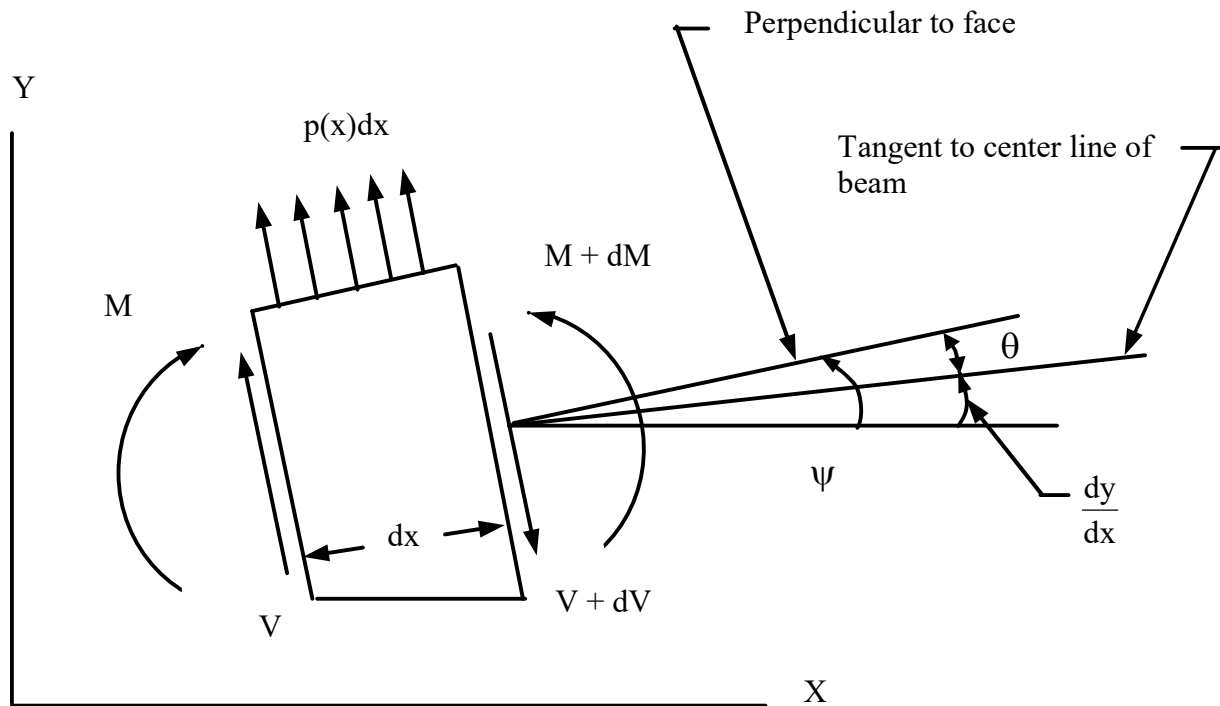


Figure 1. Effect of Shear Deformation

Let

- y = deflection of the center line of the beam
- ψ = slope due to bending
- θ = shear angle
- $p(x)$ = applied load per length
- V = shear force
- M = bending moment
- A = cross-sectional area
- E = elastic modulus
- G = shear modulus
- k = shear factor which depends on cross-section shape

There are two elastic equations of the beam.

$$\psi - \frac{dy}{dx} = \frac{V}{kAG} \quad (1)$$

$$\frac{d\psi}{dx} = \frac{M}{EI} \quad (2)$$

The shear factors for two cross-sections are given in Table 1.

Table 1. Thomson's Shear Factors	
Cross-Section	Shear Factor k
Rectangular	2/3
Circular	3/4

Lee, Reference 2

The transverse shear stiffness is

$$kAG \quad (3)$$

where

- k = shape or area factor
- A = cross-sectional area
- G = shear modulus

The transverse shear flexibility is

$$1 / kAG \quad (4)$$

Shape factors for several cross-sections are given in Table 2.

Table 2. Lee's Area or Shape Factors	
Cross-Section	Shape Factor k
Rectangular	5/6
Circular	9/10
Thin-wall Hollow Circular	1/2
Wide Flange Beam Minor Axis (approximate)	$A_f / 1.2 A$
Wide Flange Beam Major Axis (approximate)	A_w / A

- A_f = area of the flange
- A_w = area of the web
- A = area of cross-section

Gere and Timoshenko, Reference 3

Let

v_s = deflection due to shear alone

α_s = shear coefficient

V = shear force

A = cross-sectional area

G = shear modulus

The average shear stress is

$$V/A \quad (5)$$

The shear stress varies over the height of the beam. The shear coefficient α_s is the numerical factor by which the average shear stress must be multiplied to obtain the shear stress at the neutral axis.

The shearing rigidity is

$$\frac{GA}{\alpha_s} \quad (6)$$

Again, the shear stress varies throughout the height of the beam. Thus, the principle of virtual work must be used to obtain a more exact method. Specifically, the unit-load method is used. The derivation is given in Reference 3.

This approach leads to a new factor in place of the shear coefficient α_s . The new factor is the form factor f_s . The revised shearing rigidity is thus

$$\frac{GA}{f_s} \quad (7)$$

The form factor is calculated from

$$f_s = \frac{A}{I^2} \int_A \frac{Q^2}{b^2} dA \quad (8)$$

where

Q = first area moment about neutral axis

I = area moment of inertia

b = width

A = cross-sectional area

Shear coefficients and shear factors are shown in Table 3.

Table 3. Timoshenko's Shear Coefficients and Form Factors		
Cross-Section	Shear Coefficient α_s	Form Factor f_s
Rectangular	3/2	6/5
Circular	3/4	10/9
Thin-wall Hollow Circular	2	2
I-Beam or Box Section	A / A_{web}	A / A_{web}

Conclusion

Thomson's shear factors are the same as Timoshenko's shear coefficients.

Lee's shape factors are the reciprocal of Timoshenko's form factors.

Timoshenko's form factor is more accurate than his shear coefficient because the form factor accounts for shear stress variation per beam height.

References

1. W. Thomson, Theory of Vibration with Applications, Second Edition, Prentice-Hall, New Jersey, 1981.
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3. Gere and Timoshenko, Mechanics of Materials, Brooks/Cole Engineering Division, Monterey, California, 1984.