Comparison of Geometric Axis and Principal Axis Bending in Single Angles

By Whitney McNulty, P.E., SECB

The provisions of Section F10 of the American Institute of Steel Construction (AISC) Specification 360-05 permit some single angles to be designed for flexure using either geometric axis or principal axis bending. This leads to the question of whether one method produces flexural capacities greater than the other, and if so, whether the difference is significant enough to worry about. This article will answer that question, and the results may surprise you.

Uniqueness

Single angles in flexure are unlike any of the other standard rolled shapes used by engineers because the geometric axes of the cross section are not aligned with the principal axes. This has a significant effect on the flexural behavior of the angle since loads that are parallel to the geometric axes produce biaxial bending about the principal axes. Despite this, AISC Specification 360-05 permits two categories of single angles to be designed for flexure using geometric axis bending. The first is any angle with continuous lateral-torsional restraint, and the second includes only equal leg angles without lateral-torsional restraint or lateral-torsional restraint only at the point of maximum moment.

Principal axis bending can also be used to design these two categories of angles, and this creates a unique condition in the Specification – namely, that there are two alternative methods to calculate the same design strength.

Geometric vs. Principal Axes

Before we jump into the comparison of the two analysis methods, let's review some basic principles that are important to this discussion. Recall from elementary mechanics that the principal axes of a cross section are that pair of mutually perpendicular axes about which are found the largest and smallest moments of inertia. They are important because an unrestrained compression member has its greatest buckling resistance about the major principal axis and its greatest tendency to buckle about the minor principal axis. Knowing the location of these axes is crucial to safe and economical design.

An axis of symmetry will always be a principal axis, so all of the standard shapes in the AISC Steel Manual, except single angles, have their principal axes aligned with the major elements of the cross section. Equal leg angles have an axis of symmetry located 45° between the legs, so this becomes one of the principal axes. In this case it is the major axis. Referring to *Figure 1*, the major principal axis is labeled the W axis and the minor principal axis is labeled the Z axis. Unequal leg angles have no axis of symmetry, so the major principal axis is located at angle α between the legs (*Figure 2*). The AISC manual provides the tangent of angle α with the single angle section properties.

The geometric axes are simply that pair of mutually perpendicular axes parallel to the flanges and webs of a cross section. These are the familiar X and Y axes. When the principal axes and geometric axes are the same, or are at least parallel to each other, working with those cross sections becomes much easier. Loads tend to be applied about the geometric axes, so it is not necessary to transform them into components about some other set of axes to perform an analysis. This also means that it is not necessary to consider biaxial stresses, so designs can be completed in fewer steps. Important section properties are significantly easier to calculate about the geometric axes since determining them becomes simply a matter of working with rectangles. Engineers have become accustomed to working with shapes that have their geometric and principal axes aligned, so when working with single angles they may not even realize that significant differences exist.

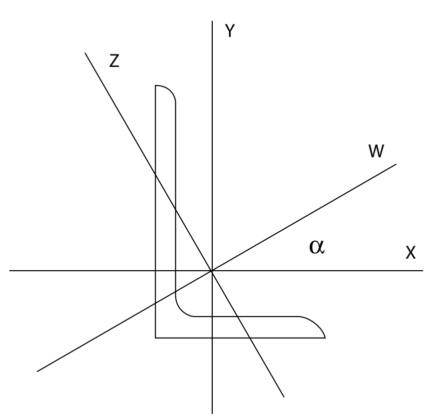

Figure 2: Location of the principal axes for an unequal leg angle.

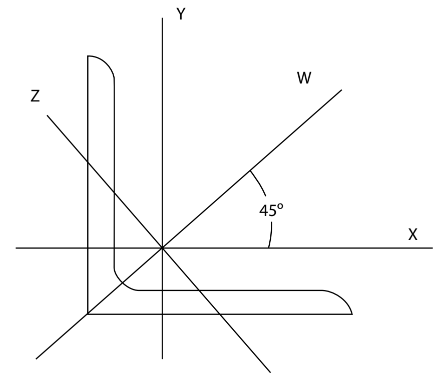

Figure 1: Equal Leg Angles – X and Y are the geometric axes, W and Z are the principal axes. The principal axes are located 45° from the geometric axes for all equal leg angles.

Assumptions

The requirements for the flexural design of single angles using either geometric axis or principal axis bending are found in Section F10 of the AISC Specification. There are three limit states that apply to both methods:

- Yielding
- Lateral-Torsional Buckling
- Leg Local Buckling

The first category of single angle that can be designed using geometric axis bending is any angle with continuous lateral-torsional restraint. This applies to both equal leg and unequal leg angles. Since this continuous lateral restraint eliminates the flexural limit state where the difference between the two methods has its greatest effect, it is not worth comparing the capacities predicted between geometric axis and principal axis bending for these angles. Since geometric axis bending is easier to analyze, use it whenever these angles are encountered.

The second category is the one that interests us. It includes only equal leg angles without lateral-torsional restraint or lateral-torsional restraint only at the point of maximum moment. The AISC Specification permits these angles to be designed for flexure using either geometric axis or principal axis bending. Unequal leg angles without continuous lateral torsional restraint must be designed using principal axis bending. Geometric axis bending is not permitted to be used when analyzing these angles.

STRUCTURAL PRACTICES

practical knowledge beyond the textbook

It might be tempting to predict that the principal axis bending provisions will produce higher strengths than the geometric axis provisions. After all, the geometric axis provisions are approximations and should be conservative, while the principal axis provisions are based on more realistic behavior and thus should be more accurate. To test this conclusion we need to look at the capacities determined by each method and compare the results. We will make the following assumptions:

- The angle is simply supported
- The loads are uniformly distributed and applied parallel to the vertical leg
- There is no lateral-torsional restraint along the length of the angle

The calculations will also recognize whether the vertical leg is pointing up or down and whether the angle is compact or noncompact, since these conditions have an effect on the capacity.

Compact Equal Leg Angles

To begin, we first need to calculate the nominal moment capacity, M_n, using both geometric axis and principal axis bending. *Table 1* presents the values for a typical compact cross section, L4x4x5/16. A range of lengths is used to illustrate how the capacity is affected by the span. Although this table does not provide the ability to compare geometric axis and principal axis bending directly, it does let us make some observations. We can see that the orientation of the vertical leg has no effect on the principal axis capacity. This can be explained by recognizing that the flexural capacity about the major axis will always be limited by the capacity of the compression leg under lateral-torsional buckling. Leg local buckling does not apply to compact cross sections, and the lateral-torsional buckling capacity will never exceed the yield capacity because of the limit found in AISC Equation F10-3. Since equal leg angles are symmetrical about the major axis, (*Figure 3, page 20*), one leg will always be in compression regardless of the orientation of the vertical leg, and it does not matter which one it is. On the other hand, it is reasonable to expect some difference in capacity about the minor axis, since the orientation of the legs with respect to the load will produce tension and compression in different parts of the cross section (*Figure 3*).

So why are the values in *Table 1* the same for both orientations of the vertical leg for minor axis bending? When the vertical leg is down, the leg tips are in tension about the minor axis and yielding is the only limit state. When the vertical leg is up, the leg tips are in compression for minor axis bending, so the limit states

Table 1: M_n for a Compact Cross Section

4x4x5/16 Compact	M_n, Geometric Axis Vertical Leg:		M_n, Principal Axis, inch-kips			
			Vertical Leg Up		Vertical Leg Down	
Length, ft.	Up	Down	Major Axis	Minor Axis	Major Axis	Minor Axis
4	51.8	54.9	107.3	50.4	107.3	50.4
6	50.7	54.9	99.1	50.4	99.1	50.4
8	49.5	54.9	92.2	50.4	92.2	50.4
10	48.2	54.9	86.2	50.4	86.2	50.4
12	47.0	54.5	80.7	50.4	80.7	50.4
14	45.8	52.8	75.7	50.4	75.7	50.4
16	44.6	51.2	71.0	50.4	71.0	50.4

are leg local buckling and yielding. But for a compact section, leg local buckling does not apply and yielding again becomes the only limit state. This produces the same capacity for flexure about the minor axis regardless of vertical leg orientation.

Using the results in *Table 1 (page 19)*, we can conclude that the principal axis bending capacity is independent of the orientation of the vertical leg for all compact cross sections. On the other hand, the results show that the same is not true for geometric axis bending. This gives us another question: Which orientation is better for geometric axis bending?

The answer is obvious by looking at *Table 1*. For geometric axis bending, the vertical leg down orientation has a higher capacity. Why? In both vertical leg orientations, lateral-torsional buckling is the controlling limit state. In the vertical leg down orientation, the other possible limit state is yielding. However, the lateral-torsional buckling capacity will never exceed the yield capacity because the provisions in Section F10.2 of the AISC specification limit the lateral-torsional buckling capacity to 80 percent of the yield moment for geometric axis bending. In the vertical leg up orientation, the other possible limit state is leg local buckling. But since the cross section is compact, leg local buckling does not apply. So if both orientations are subject to lateral-torsional buckling, why are the capacities different?

The explanation has to do with the issue of maximum tension or compression at the toe. AISC Equations (F10 4a) and (F10-4b) for lateral-torsional buckling are nearly identical to each other, they only differ at the very end by the term of "-1" or "+1". The choice of which equation applies is based on whether the maximum stress in the angle is tension or compression. Since the neutral axis in an equal leg angle subjected to geometric axis bending is closer to the horizontal leg, this is the X axis in *Figure 1 (page 18)*, the maximum stress in the cross section will always occur at the tip of the vertical leg. The lateral-torsional buckling equations recognize whether this stress is tension or compression and the capacity changes accordingly. The greater capacity obviously results when the leg tip is in tension. So now the orientation of the vertical leg is important for achieving maximum flexural capacity. For compact equal leg angles, the vertical leg down orientation puts the leg tip in tension, so it has the higher capacity.

Notice something else from *Table 1* about geometric axis bending – the capacity for the vertical leg down orientation does not change until the unbraced length hits a threshold limit, 12 feet in this case. What's happening here is that for lengths less than this limit, the result of AISC lateral-torsional buckling equation (F10-3) is exceeding $1.5M_y$, so the $1.5M_y$ limit is controlling. At the threshold length, we reach the point where the result of AISC Equation (F10-3) no longer exceeds $1.5M_y$ and the capacity changes accordingly.

The original question comparing geometric axis and principal axis bending strength requires another table. We need to convert the nominal moments from *Table 1* into equivalent uniformly distributed loads in the vertical direction. Remember, we are assuming in both cases that the angle is simply supported and that the loads are applied parallel to the vertical leg. Those results are given in *Table 2*.

So what does this show us? We already knew that the principal axis loads were going to be the same regardless of the orientation of the vertical leg because of symmetry and classification of this section as compact. We also knew that the geometric axis vertical leg down orientation was going to be better than the geometric axis vertical leg up orientation. What we did not know was how geometric axis bending was going to compare to principal axis bending.

Table 2 shows that the geometric axis provisions give capacities greater than the principal axis provisions. Even the weaker geometric

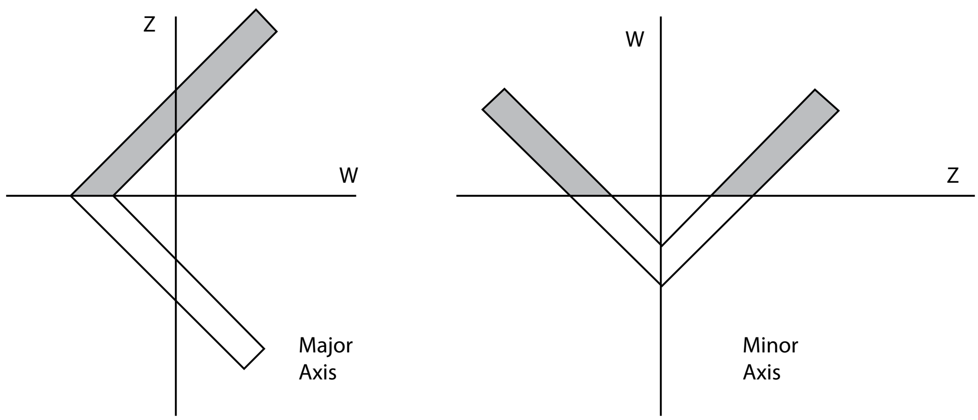

Figure 3: Compression zones are shown shaded for bending about the major W axis and minor Z axis of an equal leg angle when subject to positive moments.

Table 2: Maximum Vertical Uniform Load (kips/ft.)

4x4x5/16 Compact	Geometric Axis Vertical Leg Orientation		Principal Axis Vertical Leg Orientation	
Length, ft.	Up	Down	Up	Down
4	2.16	2.29	2.02	2.02
6	0.94	1.02	0.87	0.87
8	0.52	0.57	0.48	0.48
10	0.32	0.37	0.30	0.30
12	0.22	0.25	0.20	0.20
14	0.16	0.18	0.15	0.15
16	0.12	0.13	0.11	0.11

Table 3: M_n for a Noncompact Cross Section

L 4x4x¼	M_n, Geometric Axis		M_n, Principal Axis, inch – kips			
Noncompact	Vertical Leg:		Vertical Leg Up		Vertical Leg Down	
Length, ft.	Up	Down	Major Axis	Minor Axis	Major Axis	Minor Axis
4	38.6	43.3	84.0	41.8	84.0	41.8
6	37.7	43.3	76.4	41.8	76.4	41.8
8	36.8	43.3	70.1	41.8	70.1	41.8
10	35.8	43.3	64.4	41.8	64.4	41.8
12	34.7	43.3	59.4	41.8	59.4	41.8
14	33.7	41.7	54.7	41.8	54.7	41.8
16	32.7	40.2	50.3	41.8	50.3	41.8

axis orientation, vertical leg up, is better than the flexural capacity based on the principal axis provisions. Thus, it can be concluded that the geometric axis provisions give capacities greater than the principal axis provisions for compact equal leg angles. It is easier to perform a geometric axis analysis since no new section properties are required and the load does not have to be resolved into principal axis components. This is good news to practicing engineers looking for easy and accurate ways to do things.

Noncompact Equal Leg Angles

What about the noncompact equal leg angles? There are only seven such angles in the AISC manual. The major difference when compared to the compact angles is that the leg local buckling provisions will apply, and these may change the results. For a typical noncompact cross section, L4x4x¼, *Table 3* shows that, as with compact equal leg angles, the principal axis bending capacities are the same for either orientation of the vertical leg. The vertical leg down orientation produces larger geometric axis bending capacities than the vertical leg up orientation, and the geometric axis bending capacity of the vertical leg down orientation does not change until we reach the threshold length where the lateral-torsional buckling capacity drops below $1.5M_y$.

In *Table 4*, we see that the capacity based on the principal axis provisions is nearly identical to the geometric axis provisions for the vertical leg up orientation. However, the geometric axis bending vertical leg down configuration still provides the largest capacity. The conclusion here is that for equal leg angles the geometric axis provisions should be used whenever possible.

Table 4: Equivalent Uniform Loads (kips/ft.)

L 4x4x¼ Noncompact	Equivalent Uniform Load Geometric Axis		Equivalent Uniform Load Principal Axis	
Length, ft.	Up	Down	Up	Down
4	1.61	1.80	1.64	1.64
6	0.70	0.80	0.71	0.71
8	0.38	0.45	0.39	0.39
10	0.24	0.29	0.24	0.24
12	0.16	0.20	0.16	0.16
14	0.11	0.14	0.11	0.11
16	0.09	0.10	0.08	0.08

Conclusion

To summarize, the geometric axis provisions should be used for the flexural design of all equal leg angles. The designer must pay attention to AISC Specification Section F10.2, which covers lateral-torsional buckling, since the particular conditions of the angle being analyzed will dictate which equation applies. When given the choice, use an equal leg angle in the vertical leg down orientation. This results in flexural tension at the tip of the vertical leg and maximizes the bending capacity. The principal axis provisions do not produce greater capacities for equal leg angles when compared to the geometric axis provisions, so they should only be used to design unequal leg angles without continuous lateral torsional restraint. The geometric axis provisions cannot be used to design unequal leg angles that do not have continuous lateral-torsional restraint. ▪

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To learn how to perform geometric and principal axis flexural analyses of single angles, tune in to Whitney McNulty's webinar on November 9. Register at **www.ncsea.com**.