Practical Compactness and Bracing Provisions for the Design of Single Angle Beams

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ABSTRACT

This paper presents rational and practical design provisions for use with single angle beams. Compactness limits, bracing limits, and nominal moment capacity predictive equations are specified herein. Four common flexural orientations of the single angle cross section are considered. Experimentally verified nonlinear finite element modeling techniques are used extensively in this work.

INTRODUCTION

Structural steel single angles are not often thought of as being efficient and useful structural elements in flexure. Complexity in applicable design code provisions coupled with concerns about serviceability of angle beams contribute to this notion of inefficiency. Oftentimes the structural engineer is called upon to evaluate and/or retrofit an existing structure such as a tower, a roof system, or a bill board type structure. In these instances, it is very likely that angle members may be found in the critical load path. Similarly, when designing a latticed tower (where angles are often present) in a seismically prone region, it would be useful to be able to compute a collapse load for the tower based on the plastic capacity of the constituent angle members. In order to develop a collapse mechanism in the structural system adequate rotation capacity must be permitted at the locations of hinge formation. The AISC-LRFD Specification[1] recommends in its commentary that a minimum cross sectional rotation capacity of three be accommodated at a hinge location for a member proportioned with plastic analysis and design techniques. Consistent with this notion, the AISC-LRFD Specification prescribes compactness, λ_p, and bracing requirements, L_{pd} for use with plastic analysis and design methodology so as to achieve this minimum rotation capacity of three without the member experiencing excessive unloading due to local buckling or lateral-torsional buckling. Unfortunately, these compactness and bracing requirements are not applicable to the case of single angles in flexure.

The current paper will present practical compactness and bracing requirements for the evaluation of a single angle beam in light of the need to attain adequate rotation capacity for moment redistribution to occur within a given structural system. Four of the most common flexural orientations are considered. A diagrammatic representation of these four orientations is presented in Figure 1. The scope of the research is limited to the case of equal leg angles subjected to a constant moment type loading. The constant moment loading represents a worst case and these compactness and bracing requirements will be conservative when applied to the case of an angle beam subjected to a moment gradient. Issues of simple load imperfection effects on rotation capacity are also addressed. A predictive equation for the nominal moment capacity of a single angle member bent about the geometric axis such that its outstanding leg is in compression is also proposed in this paper. It is felt that this new nominal moment equation represents a more accurate alternative to the current equations contained in the AISC *Specification for Load and Resistance Factor Design of Single-Angle Members*.[9] Unless otherwise noted, all results in this paper are given with the assumption that the angle member is made from a 50 ksi yield strength mild carbon steel. The effects of material properties on certain compactness criteria are also treated in this paper.

MODELING TECHNIQUES

Nonlinear finite element modeling techniques employing the commercial multipurpose code ABAQUS are at the center of

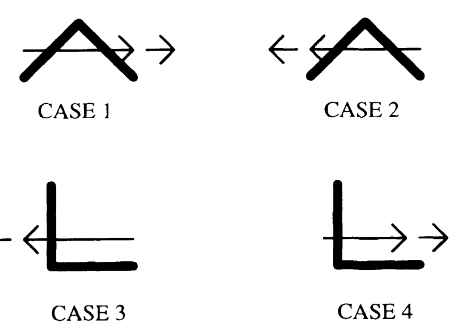

Fig. 1. Single angle flexural orientations.

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this research. Nonlinearities of both geometric and constitutive nature are considered. The single angle member is discretized into a mesh of nonlinear shell finite elements and appropriate boundary conditions are enforced. The constant moment loading of a single angle beam is achieved by considering a simply supported member subjected to two concentrated forces normal to the beam longitudinal axis. This creates a region of constant moment between the loads at the center of the member. The simply supported angle is separated into three segments. The segment at the center, which is subjected to the constant moment loading, possesses the section properties of the angle under investigation. The two end segments are modeled as being rigid. The model is braced against out-of-plane motion at the supports and the load points. A schematic of the modeling configuration is displayed in Figure 2.

In the interest of computational efficiency, the density of the finite element mesh used in the modeling is varied. The region of highest mesh density corresponds to the beam segment subjected to a constant moment loading. Figure 3 depicts a characteristic mesh which has been buckled in flexure. Initial stresses are imposed on the angle cross section to mimic the effects of the residual stresses due to hot-rolling and uneven cooling. The residual stress distribution used in the modeling is a modification of the residual stress distributions found in the literature.[2,3] Figure 4 displays the modified initial stress distribution used in the modeling.

The modeling techniques described here have been compared against results from physical experiments performed by Madugula.[4,5] The comparison between the finite element experiments and corresponding physical tests has been shown to be quite favorable.[6,7]

COMPACTNESS

The AISC-LRFD specifies limits on the plate slenderness ratios of constituent plate elements in a flexural cross section. The plate slenderness ratio associated with a cross sectional element which is suitable for use with plastic analysis and design techniques is denoted by AISC as λ_p. These λ_p values are given in Chapter B of the AISC-LRFD Specification. Suitable compactness values for use with single angles in flexure, employing plastic analysis and design methodology, are absent from these provisions. The current work addresses this omission with the proposition of compactness limits based on the results of ongoing research in the area of single angle flexural behavior.

Table 1. Governing Plate Slenderness Requirements for Use with 50 ksi Mild Steel

Test Case Number	λ_p
Case 1	14
Case 2	> 20
Case 3	13
Case 4	> 20

From research carried out with finite element models of single angle flexural members made from 50 ksi mild carbon steel, λ_p values for the four cases of single angle flexure are presented in Table 1.

In Table 1, it is noted that for Case 2 and Case 4 single angle flexure, all hot-rolled angle sections currently manufactured in the U.S. are compact for constant moment loading. A λ_p value of 13 has been assigned to Case 3 in the interest of practicality. This case is characterized as flexure about the geometric axis such that the outstanding leg is in compression. The concept of a discreet λ_p value is not strictly applicable to Case 3 since a strong interaction of local buckling and global-torsional buckling exists. This coupled behavior

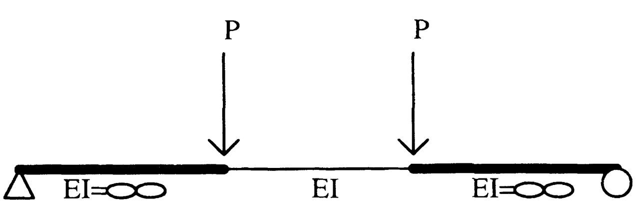

Fig. 2. Schematic of simply supported flexural model.

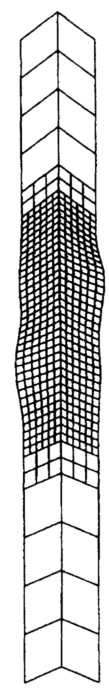

Fig. 3. Deformed single angle shell finite element mesh.

Table 2. Constitutive Parameters Used with Figure 5.					
Material	σ_y	σ_u/σ_y	$\varepsilon_{st}/\varepsilon_y$	$\varepsilon_b/\varepsilon_y$	$\varepsilon_u/\varepsilon_y$
New Japanese Steel	70 ksi	1.35	2	10	50
A514	119 ksi	1.08	2	7	17
HSLA-80	85 ksi	1.14	1	11	17
Mild Carbon Steel	50 ksi	1.5	5.5	28	45

is clearly observed in all of the finite element modeling of Case 3. This complicated coupled response is one of the motivating factors for the development of bracing requirements presented later in this paper.

MATERIAL EFFECTS ON SINGLE ANGLE COMPACTNESS

While it is recognized that a 50 ksi mild carbon steel represents the most common material encountered in current engineering practice, there has been much interest as of late in high performance steels. Specifically there are steels being developed which exhibit a high yield stress, good weldability, relatively good ductility, but exhibit almost no yield plateau and have a yield stress-to-ultimate stress ratio which is considerably lower than that for conventional steel. As a result of these differences in critical material response parameters, the suitability of these new steels for use with plastic analysis and design methodology has been questioned. A portion of the current research examines the impact of varying the steel yield stress alone on Case 1 and Case 2 compactness. Similarly, this same research examines the flexural behavior of these same single angle beams when they are manufactured from other than mild carbon steel. Specifically HSLA80 (*YR* = σ_y/σ_u = 0.88, E/E_{st} = 143), A514 (*YR* = 0.93, E/E_{st} = 4.16), and a new Japanese steel (*YR* = 0.74, E/E_{st} = 45.7), as described by Kuwamura,[8] are considered (see Figure 5 and Table 2).

Results from extensive modeling incorporating varying material properties indicate that earlier conclusions concerning the compactness of Case 2 angles is indeed valid. All hot-rolled angle sections currently manufactured in the U.S. are compact and thus suitable for plastic design considerations. This statement remains valid for single angle beams made from all steels whose material properties are listed in Table 2 for this case of loading.

To address trends in Case 1 compactness ratio as material yield stress is varied, a series of finite element models incorporating the constitutive law of Figure 5 are analyzed. The yield stress and tensile strength are varied. The other material parameters in the inelastic region remain constant. This can be thought of as simply shifting the entire inelastic region up or down as is depicted in Figure 6.

A parametric analysis is performed on the results from a large number of moment-rotation plots obtained from finite element modeling. The relationship observed between the compactness parameter, λ_p, and the material yield stress is of a linear nature as can be seen in Figure 7.

A suitably nondimensionalized design formula is presented as a means for determining the appropriate flexural compactness ratio for use with the plastic design of Case 1 single angle beams constructed from a mild carbon steel of a specified yield stress.

$$\lambda_p = 0.756\sqrt{\frac{E}{F_y}} - 1.67 \qquad (1)$$

The effects of varying the unbraced length of the constant moment region in Case 1 and Case 2 single angle beams has an impact on rotation capacity, but the effect is not as pronounced as it is for Case 3 and Case 4 flexure and thus it can be neglected.

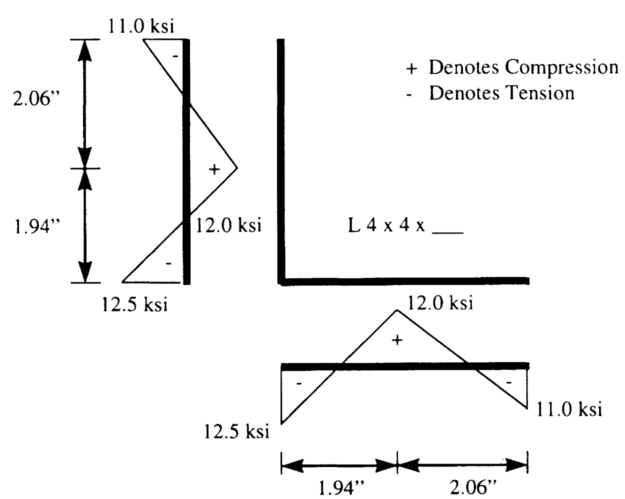

Fig. 4. Residual stress distribution used in finite element models.

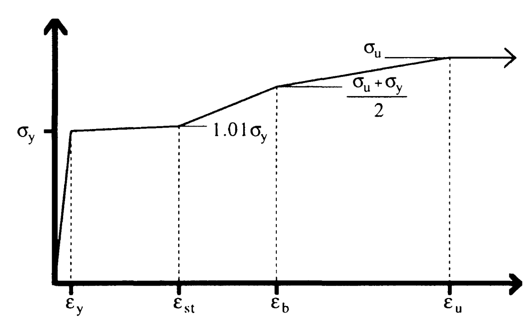

Fig. 5. Uniaxial material response parameters used in finite element models.

BRACING

The AISC-LRFD Specification prescribes a maximum unbraced length for use in plastic design, L_{pd}, for a singly or doubly symmetric flexural member. This unbraced length is prescribed to ensure that a flexural member remains free from excessive unloading, due to lateral-torsional buckling, until after a minimum rotation capacity of three is achieved. While these provisions apply for the plastic design of singly and doubly symmetric beams, no guidance for the case of single angle flexure about a geometric axis is given. The current research addresses this point by recommending provisions specifying such requirements. These bracing requirements are established from results of finite element experimentation techniques similar to those discussed in conjunction with the compactness study reported earlier.

It becomes evident upon examining the inelastic response of Case 3 finite element flexural models, that the plate slenderness ratio and unbraced length are closely interrelated in a nonlinear fashion. This nonlinear relationship is displayed in Figure 8, and approximated by the following empirically obtained (from numerical studies) design bracing equation.

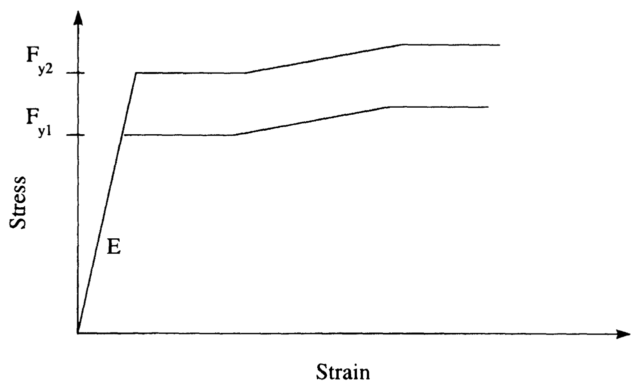

Fig. 6. Mild carbon steel schematic.

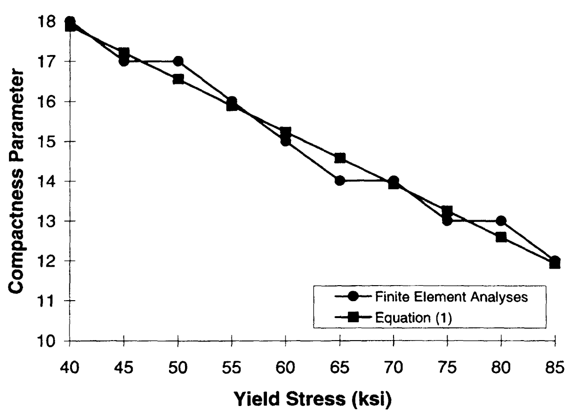

Fig. 7. Trend in Case 1 compactness as mild carbon steel yield stress varies.

$$\frac{L_{pd}}{r_y} = 195 - 20.9\left(\frac{b}{t}\right) + 0.568\left(\frac{b}{t}\right)^2 \quad (2)$$

for,

$$\frac{b}{t} \leq \lambda_p$$

Similarly, when considering bracing requirements for Case 4, a relationship between angle section properties and the maximum unbraced length is obtained. This relationship has the form:

$$\frac{L_{pd}}{r_y} = 508\left(\frac{t}{b}\right) \quad (3)$$

where

r_y = in-plane radius of gyration
b = angle leg width
t = angle leg thickness

Equation 3 is the result of interpretations made from the characteristic response of Case 4 angles. It is observed that the resisting moment of this case reaches a discrete limit where the rotation of the center test section continues to increase without a corresponding change in load. The plateau in the moment-rotation response of this case continues well past the minimum rotation requirement of three as specified for a compact angle. This plateau behavior is characteristic of Case 4 angles, and is observed over a large range of unbraced lengths. The goal of this portion of the research is to find the unbraced length of a single angle beam of known plate slenderness, such that the moment-rotation plateau corresponds to the plastic moment capacity of the section. Once the plateau is at the proper level, adequate rotation capacity is assured by the characteristic response of this case. The plateau behavior characteristic of Case 4 angle response is displayed in Figure 9. In the foregoing, both equations 2 and

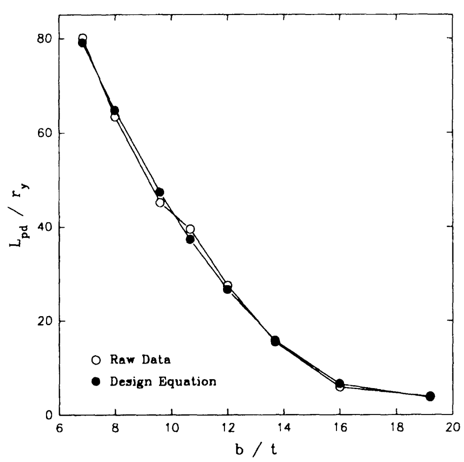

Fig. 8. Case 3 bracing-compactness interaction.

3 are developed for the constant moment case of an equal leg single angle beam made from 50 ksi mild carbon steel. It is further noted that, despite constraint on lateral rotation out-of-plane at the angle brace points which result from modeling assumptions, the entire length of the center test section is used in the development of L_{pd} from the finite element models. This appears to be valid since the failure modes observed are localized to the central portion of the unbraced section well away from the points of incidental lateral rotational restraint. Similarly, these failure modes are not at all classical manifestations of lateral-torsional buckling in that their localized nature does not appear to be significantly influenced by the out-of-plane lateral rotational boundary conditions.

The above study concerning bracing has shone a dim light on the complex interrelationship between compactness and bracing. The underlying mechanism of the interaction has yet to be identified. This complexity in the interactions of global and local buckling has created an interest in determining the validity of the AISC *Specification for Load and Resistance Factor Design of Single-Angle Members*' nominal moment capacity predictions for Case 3 flexure. The following section contains a comparison of the Case 3 nominal moment capacities, as obtained from the described finite element modeling, and those obtained from the AISC provisions. An alternative design equation, to that of the AISC, is then given.

CASE 3 NOMINAL MOMENT CAPACITY

Case 3 flexure is one of the more common single angle flexural situations encountered in engineering design practice. This flexural case is characterized by bending about the geometric axis of the angle such that the outstanding leg is in compression. The present research models this Case so as to perform a parametric analysis and thus determine the relationship between Case 3 nominal moment capacity, the plate slenderness ratio b/t, and the beam slenderness ratio L / r_z. The results of this study reveal that a linear relationship exists

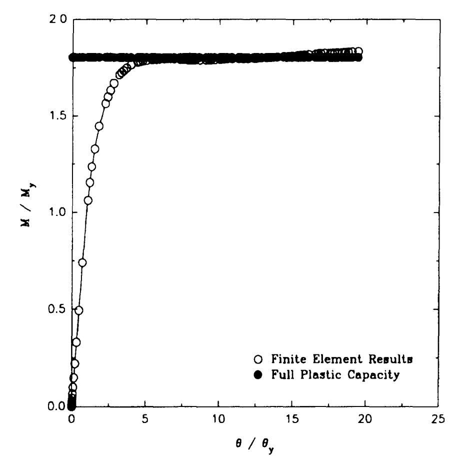

Fig. 9. Characteristic Case 4 moment-rotation limit response.

between the Case 3 nominal moment capacities and the beam slenderness ratio L / r_z at a given b/t ratio. While the nature of these lines (i.e. slope and intercept) vary as b/t changes, the quality of linearity itself remains unperturbed. A simplified design equation obtained empirically from the finite element modeling is given in Equation 4. This equation quantifies the variation in Case 3 single angle nominal moment capacity as plate slenderness and beam slenderness vary. The nominal moment predictions from this equation are plotted with the actual finite element results in Figures 10 and 11. Also present in these plots are the results from the AISC nominal moment predictions, the full plastic capacity of the Case 3 angle cross-section, and the elastic critical buckling solution. It appears that the current AISC nominal moment predictions for Case 3 flexure are quite conservative for short angle beams. Conversely, for longer angle beams there is apparently a certain degree of unconservatism in the current specification. This is not seen as critical however due to the fact that a design incorporating a Case 3 single angle beam of such length would almost certainly be controlled by deflections. The discrepancy between the capacities obtained from the nonlinear finite element modeling and those of the AISC Specification, point to a need to re-evaluate existing AISC single angle design provisions in this area.

$$\frac{M_n}{M_y} = -\frac{(b/t)^{1/3}}{804.7}\left(\frac{L}{r_z}\right) + \frac{3.690}{(b/t)^{1/4}}$$

for,

$$\frac{L_{pd}}{r_z} \le \frac{L}{r_z} \le 400, \text{ and } \frac{M_n}{M_y} \le \frac{M_p}{M_y} \tag{4}$$

IMPERFECTION SENSITIVITY

Concerns of imperfection sensitivity often arise when the notion of predicting buckling loads for structural elements is discussed. There are a number of ways to address imperfection sensitivity in finite element modeling. One way is to actually generate a slightly distorted finite element mesh and perform a nonlinear analysis on it. Another way to impose imperfections in a finite element model is to apply an imperfection loading. This latter method is used in this work as the vehicle to perform several restricted imperfection sensitivity studies on Case 1 and Case 2 single angle beams. In the standard modeling of Case 1 and Case 2 flexure, two concentrated forces are applied at the single angle shear center in such a way as to produce only in-plane bending. In the imperfection sensitivity analysis models, small out-of-plane loads are also applied to the shear center (as depicted in Figure 12) coincident with the in-plane loadings.

These load imperfections are always manifest as a constant percentage of the load level corresponding to the primary loading in a given time step as the nonlinear equilibrium path is determined. This means that just as the magnitude of the primary loading grows during the nonlinear solution process, so too does the load imperfection magnitude. In all runs incorporating these load imperfections, no out-of-plane flexural bracing is enforced in the models.

The out-of-plane load imperfection is imposed on the models with a load magnitude of either 1 percent or 10 percent that of the applied in-plane loading. Results from finite element analyses performed with a plate slenderness ratio (b/t) of 14, a beam slenderness ratio (L / r_z) of 26, and made from 45 ksi mild carbon steel are presented for Case 1 and Case 2 in Figures 13 and 14 respectively. It appears from these normalized moment-rotation plots that both Case 1 and Case 2 are relatively imperfection insensitive to the load imperfections imposed, even for the relatively severe case of an imperfection which is 10 percent of the total applied in-plane loading. It appears then, at least for Case 1 and Case 2, that perhaps concern of excessive imperfection sensitivity in these instances may be unwarranted.

CONCLUSIONS

Single angle compactness limits for use with plastic analysis and design techniques have been developed for the four most common flexural orientations encountered in practice (see Table 1). These results are valid for angles made from mild carbon steel with a 50 ksi yield stress. Similar compactness limits are also prescribed for minor principal axis flexure of angles made from mild carbon steels with other yield stresses. These are presented in Equation 1.

Bracing requirements have also been developed in this paper for single angle flexure about a geometric axis. These limits are prescribed in Equations 2 and 3. Similarly, the nominal moment capacity for geometric axis flexure of a Case 3 angle is given in Equation 4. This equation is proposed since the current research has shown that the AISC *Specification for the Load and Resistance Factor Design of Single-Angle Members* is somewhat unreliable in its Case 3 nominal moments predictions.

The current research has also demonstrated that single angles subjected to minor principal axis flexure are relatively insensitive to load imperfections. This is a welcome result when considering single angle beams for practical applications.

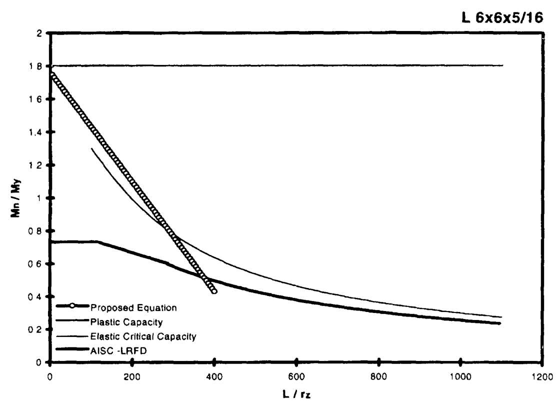

Fig. 10. Normalized nominal moment vs. beam slenderness of a Case 3 L6×6×5⁄16.

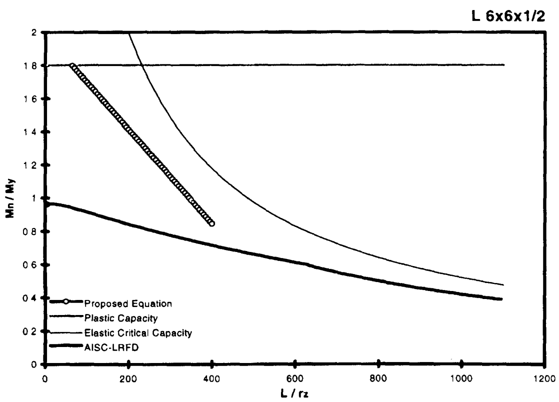

Fig. 11. Normalized nominal moment vs. beam slenderness of a Case 3 L6×6×1⁄2.

ACKNOWLEDGMENTS

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APPENDIX

Design Example 1

The suitability of applying plastic analysis and design methodology to the single angle beam, shown below, is evaluated.

Given:

Angle Section is L6×6×5⁄8.

Steel yield stress is 50 ksi.

Bracing is provided at supports and load points. L_b = 7 ft.

Stiffener plates are provided at supports and load points.

Flexure is about geometric axis such that the upright leg is in compression.

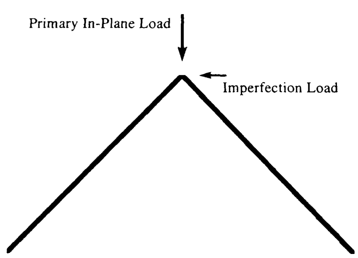
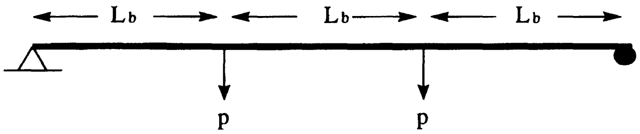


Fig. 12. Principal axis flexure—load imperfection schematic (Case 1 shown).

Loading is applied at shear center.



$b / t = 9.6 \leq 13$

Thus section is compact according to Table 1

$L_b = 84$ in.
$r_y = 1.84$ in

$$\frac{L_{pd}}{1.84} = 195 - 20.9(9.6) + 0.568(9.6)^2$$

which results in $L_{pd} = 86$ in. > 84 in.

It is concluded that this angle beam is compact, adequately braced, and thus suitable for use with plastic analysis and design techniques.

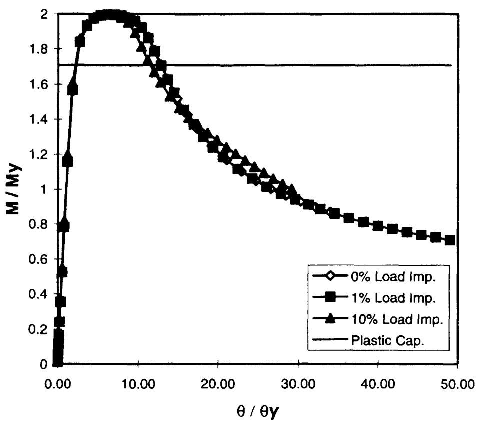


Fig. 13. Case 1 load imperfection results $b / t = 14$, $L / r_z = 26$, $F_y = 45$ ksi.

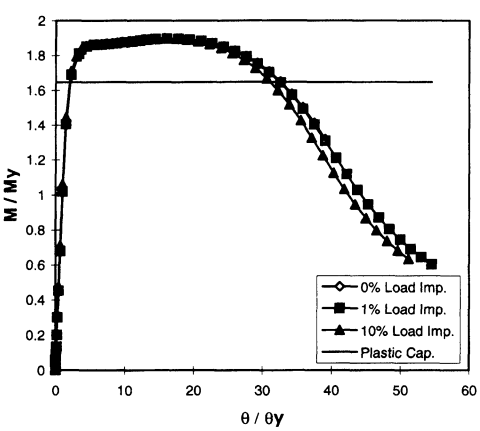


Fig. 14. Case 2 load imperfection results $b / t = 20$, $L / r_z = 26$, $F_y = 45$ ksi.

Design Example 2

Consider the angle section and loading arrangement from Design Example 1. Increase the unbraced length to be 24 ft-7-in.

$L_b = 295$ in. $\quad r_y = 1.84$ in. $\quad r_z = 1.18$ in.
$S = 5.66$ in.^{3} $\quad Z = 10.2$ in.^{3}
$M_y = 280$ k-in. $\quad M_p = 510$ k-in.

$$\frac{L_b}{r_z} = 250 \leq 400 \quad \textbf{o.k.}$$

$$\frac{M_n}{280} = -\frac{(9.6)^{1/3}}{804.7}\left(\frac{295}{1.18}\right) + \frac{3.690}{(9.6)^{1/4}}$$

which results in a $M_n = 402$ k-in.

$M_n = 402$ k-in. ≤ 510 k-in. **o.k.**

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