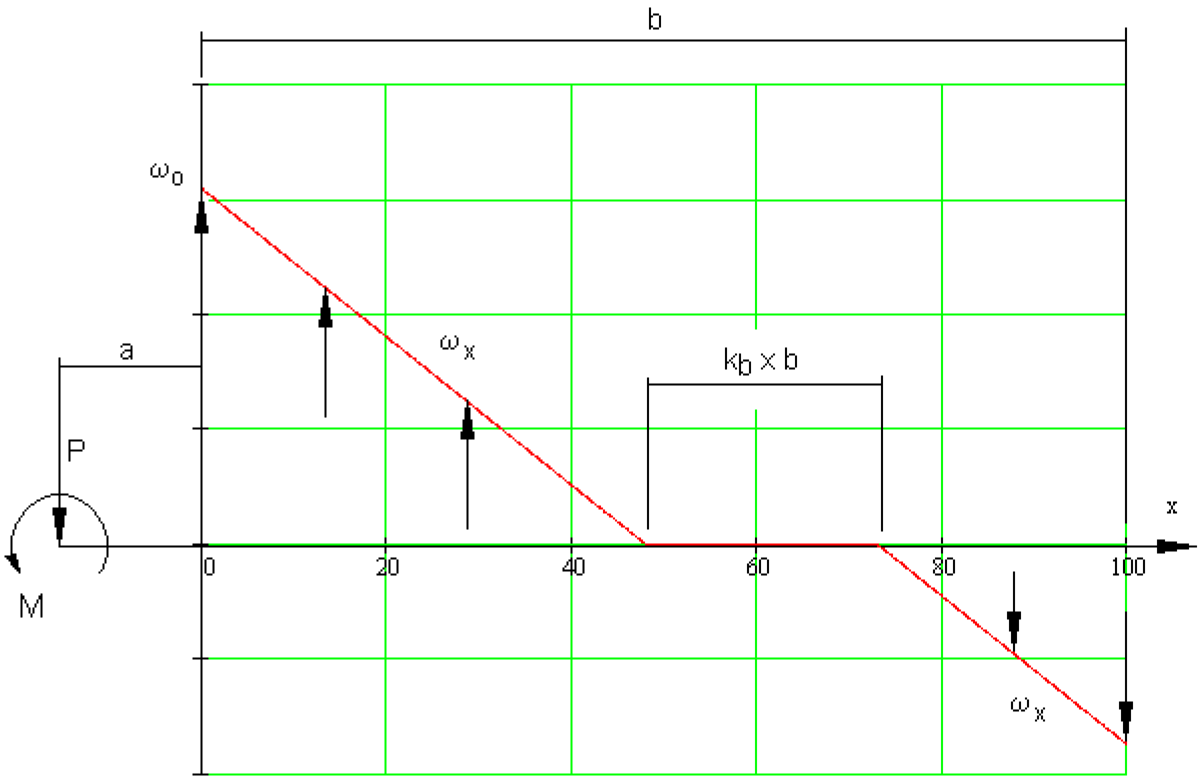
Pin Loaded Socket Analysis



$a := 20 \cdot mm$ ... point of load application on pin from mouth of socket

$b := 100 \cdot mm$ ... length of socket

$P := 20 \cdot kN$ ... applied pin loading

$M := 100 \cdot N \cdot m$ ... applied moment at free end of pin

$k_b := 0.67$ ... factor defining unloaded section length inside socket (unloaded length = b x k_b)

Distributed socket loading

$$\omega(x,\omega_o,\omega_x) := \omega_o + \omega_x \cdot x - \omega_x \cdot \left(x - \frac{-\omega_o}{\omega_x}\right) \cdot \left(x > \frac{-\omega_o}{\omega_x}\right) \ldots$$
$$+ \omega_x \cdot \left[x - \left(-\frac{\omega_o}{\omega_x} + k_b \cdot b\right)\right] \cdot \left[x \geq \left(-\frac{\omega_o}{\omega_x} + k_b \cdot b\right)\right]$$

Where ω_o is the distributed load at x = 0 and ω_x is the rate of change in distributed loading.

$$\Sigma P(\omega_o,\omega_x) := \omega_o \cdot b + \frac{1}{2} \cdot \omega_x \cdot b^2 - \frac{1}{2} \cdot \omega_x \cdot \left(b - \frac{-\omega_o}{\omega_x}\right)^2 \ldots$$
$$+ \frac{1}{2} \cdot \omega_x \cdot \left[b - \left(-\frac{\omega_o}{\omega_x} + k_b \cdot b\right)\right]^2$$

... summing distributed loading over socket length

$$\Sigma M(\omega_o,\omega_x) := \frac{1}{2} \cdot \omega_o \cdot b^2 + \frac{1}{6} \cdot \omega_x \cdot b^3 - \frac{1}{6} \cdot \omega_x \cdot \left(b - \frac{-\omega_o}{\omega_x}\right)^3 \ldots$$
$$+ \frac{1}{6} \cdot \omega_x \cdot \left[b - \left(-\frac{\omega_o}{\omega_x} + k_b \cdot b\right)\right]^3$$

... BM at end of socket length

Given $\quad \Sigma P(\omega_o,\omega_x) = P$ ... equ. 1, equating forces

$$M + P \cdot (a + b) - \Sigma M(\omega_o,\omega_x) = 0 \cdot N \cdot mm$$

... equ. 2, moment at end of pin

Solving ... $\omega_o = 2503.21 \frac{N}{mm}$ ... and ... $\omega_x = -114.98 \frac{N}{mm^2}$

$$\omega_b(x) := \left[\omega_o + \omega_x \cdot x - \omega_x \cdot \left(x - \frac{-\omega_o}{\omega_x}\right) \cdot \left(x > \frac{-\omega_o}{\omega_x}\right)\right] \ldots$$

$$+ \omega_x \cdot \left[x - \left(-\frac{\omega_o}{\omega_x} + k_b \cdot b\right)\right] \cdot \left[x \geq \left(-\frac{\omega_o}{\omega_x} + k_b \cdot b\right)\right]$$

... distributed loading

$$P_\omega(x) := P - \left[\begin{array}{l} \omega_o \cdot x + \frac{1}{2} \cdot \omega_x \cdot x^2 - \frac{1}{2} \cdot \omega_x \cdot \left(x - \frac{-\omega_o}{\omega_x}\right)^2 \cdot \left(x > \frac{-\omega_o}{\omega_x}\right) \ldots \\ + \frac{1}{2} \cdot \omega_x \cdot \left[x - \left(-\frac{\omega_o}{\omega_x} + k_b \cdot b\right)\right]^2 \cdot \left[x \geq \left(-\frac{\omega_o}{\omega_x} + k_b \cdot b\right)\right] \end{array}\right]$$

... Pin shear

$$M_\omega(x) := M + P \cdot (a + x) - \left[\begin{array}{l} \frac{1}{2} \cdot \omega_o \cdot x^2 + \frac{1}{6} \cdot \omega_x \cdot x^3 - \frac{1}{6} \cdot \omega_x \cdot \left(x - \frac{-\omega_o}{\omega_x}\right)^3 \cdot \left(x > \frac{-\omega_o}{\omega_x}\right) \ldots \\ + \frac{1}{6} \cdot \omega_x \cdot \left[x - \left(-\frac{\omega_o}{\omega_x} + k_b \cdot b\right)\right]^3 \cdot \left[x \geq \left(-\frac{\omega_o}{\omega_x} + k_b \cdot b\right)\right] \end{array}\right]$$

... Pin BM

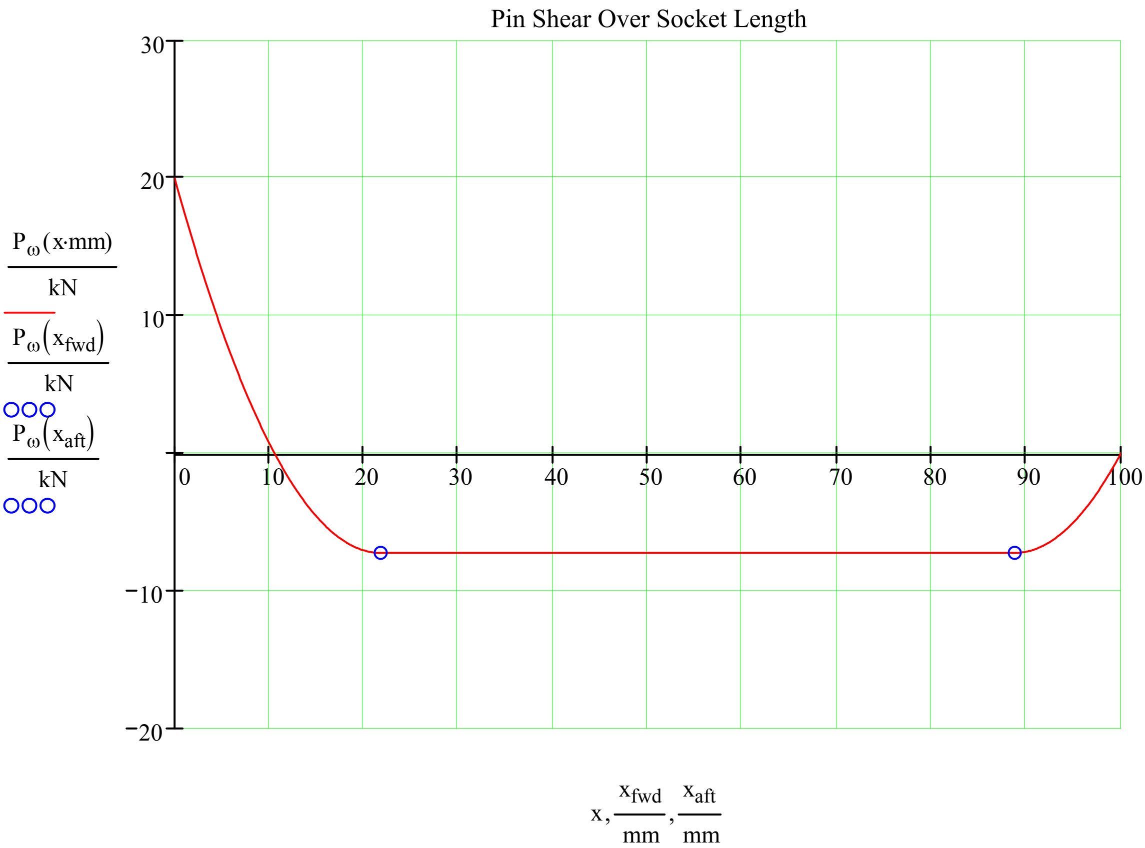
$x_p := -\left[\frac{\omega_o}{\omega_x} + \sqrt{2 \cdot \frac{P}{\omega_x} + \left(\frac{\omega_o}{\omega_x}\right)^2}\right]$ $x_p = 10.542\,\text{mm}$ ... location of max. BM

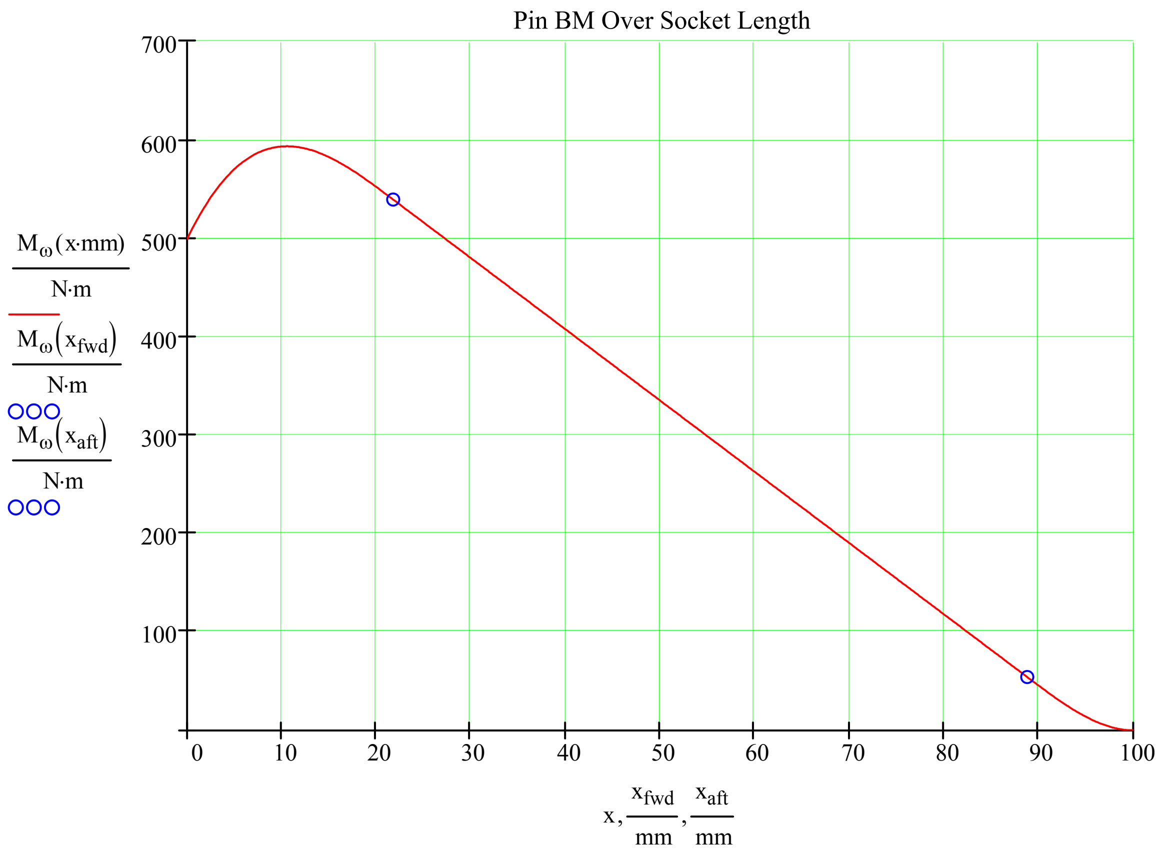
$M_{max} := M_\omega(x_p)$ $M_{max} = 594.2\,\text{N} \cdot \text{m}$ ... max. BM

$x_{fwd} := -\frac{\omega_o}{\omega_x}$ $x_{fwd} = 21.77\,\text{mm}$ ... and ... $x_{aft} := x_{fwd} + k_b \cdot b$ $x_{aft} = 88.77\,\text{mm}$

Checks ... $P_\omega(b) = 0.00\,\text{N}$ ... and ... $M_\omega(b) = -0.00\,\text{N} \cdot \text{m}$


Socket Load Distribution
3000
2500
2000
1500
1000
500
0
−500
−1000
−1500
−2000
0
20
40
60
80
100
ωb(x·mm)
N
mm
x


Pin Shear Over Socket Length
30
20
10
−10
−20
0
10
20
30
40
50
60
70
80
90
100
Pω(x·mm)
kN
Pω(xfwd)
kN
Pω(xaft)
kN
x, xfwd/mm, xaft/mm


Pin BM Over Socket Length
700
600
500
400
300
200
100
0
10
20
30
40
50
60
70
80
90
100
Mω(x·mm) / N·m
Mω(xfwd) / N·m
Mω(xaft) / N·m
x, xfwd/mm, xaft/mm