Thermal performance calculator according to the ISO 13786

$kJ := 10^3 \cdot J$

Assembly characteristics:

	Layer1 - Clay based plaster	Layer2 - Hempcrete
Width	$d1 := 0.01 \cdot m$	$d2 := 0.01 \cdot m$
Specific heat capacity	$c1 := 880 \cdot \frac{J}{kg \cdot K}$	$c2 := 1600 \cdot \frac{J}{kg \cdot K}$
Density	$\rho 1 := 1800 \cdot \frac{kg}{m^3}$	$\rho 2 := 300 \cdot \frac{kg}{m^3}$
Thermal conductivity	$\lambda 1 := 0.8 \cdot \frac{W}{m \cdot K}$	$\lambda 2 := 0.07 \cdot \frac{W}{m \cdot K}$

Interior layer thermal resistance $Rsi := \frac{1}{8.7} \cdot \frac{m^2 \cdot K}{W}$

Exterior layer thermal resistance $Rse := \frac{1}{23} \cdot \frac{m^2 \cdot K}{W}$

Temperature oscillation time

$Time := 24 \cdot 60 \cdot 60 \cdot sec$

$u\_value := \frac{1}{Rsi + \frac{d1}{\lambda 1} + \frac{d2}{\lambda 2} + Rse} = 3.187 \ \frac{W}{m^2 \cdot K}$

Penetration depth

$\delta 1 := \sqrt{\frac{\lambda 1 \cdot Time}{\pi \cdot \rho 1 \cdot c1}} = 0.118 \ m$

$\delta 2 := \sqrt{\frac{\lambda 2 \cdot Time}{\pi \cdot \rho 2 \cdot c2}} = 0.063 \ m$

Dimensionless penetration depth

$$\xi 1 := \frac{d1}{\delta 1} = 0.085 \qquad \xi 2 := \frac{d2}{\delta 2} = 0.158$$

Thermal transmittance matrix component

Layer1

$$z11 := \cosh(\xi 1)\cdot\cos(\xi 1) + \sqrt{-1}\cdot\sinh(\xi 1)\cdot\sin(\xi 1) = 1 + 0.007\mathrm{i}$$

$$z22 := z11 = 1 + 0.007\mathrm{i}$$

$$z12 := \frac{-\delta 1}{2\cdot\lambda 1}\cdot\left(\sinh(\xi 1)\cdot\cos(\xi 1) + \cosh(\xi 1)\cdot\sin(\xi 1) + \sqrt{-1}\cdot\left(\cosh(\xi 1)\cdot\sin(\xi 1) - \sinh(\xi 1)\cdot\cos(\xi 1)\right)\right) = \left(-0.012 - 3\mathrm{i}\cdot 10^{-5}\right)\ \frac{m^2\cdot K}{W}$$

$$z21 := \frac{-\lambda 1}{\delta 1}\cdot\left(\sinh(\xi 1)\cdot\cos(\xi 1) - \cosh(\xi 1)\cdot\sin(\xi 1) + \sqrt{-1}\cdot\left(\cosh(\xi 1)\cdot\sin(\xi 1) + \sinh(\xi 1)\cdot\cos(\xi 1)\right)\right) = (0.003 - 1.152\mathrm{i})\ \frac{W}{m^2\cdot K}$$

Layer2

$$z11\_2 := \cosh(\xi 2)\cdot\cos(\xi 2) + \sqrt{-1}\cdot\sinh(\xi 2)\cdot\sin(\xi 2) = 1 + 0.025\mathrm{i}$$

$$z22\_2 := z11\_2 = 1 + 0.025\mathrm{i}$$

$$z12\_2 := \frac{-\delta 2}{2\cdot\lambda 2}\cdot\left(\sinh(\xi 2)\cdot\cos(\xi 2) + \cosh(\xi 2)\cdot\sin(\xi 2) + \sqrt{-1}\cdot\left(\cosh(\xi 2)\cdot\sin(\xi 2) - \sinh(\xi 2)\cdot\cos(\xi 2)\right)\right) = (-0.143 - 0.001\mathrm{i})\ \frac{m^2\cdot K}{W}$$

$$z21\_2 := \frac{-\lambda 2}{\delta 2}\cdot\left(\sinh(\xi 2)\cdot\cos(\xi 2) - \cosh(\xi 2)\cdot\sin(\xi 2) + \sqrt{-1}\cdot\left(\cosh(\xi 2)\cdot\sin(\xi 2) + \sinh(\xi 2)\cdot\cos(\xi 2)\right)\right) = (0.003 - 0.349\mathrm{i})\ \frac{W}{m^2\cdot K}$$

Construction thermal transmittance matrix

Layer1

$$Z1 := \begin{bmatrix} z11 & z12 \cdot \frac{W}{m^2 \cdot K} \\ \frac{m^2 \cdot K}{W} \cdot z21 & z22 \end{bmatrix}$$

Layer2

$$Z2 := \begin{bmatrix} z11\_2 & z12\_2 \cdot \frac{W}{m^2 \cdot K} \\ \frac{m^2 \cdot K}{W} \cdot z21\_2 & z22\_2 \end{bmatrix}$$

$$Z := Z1 \cdot Z2 = \begin{bmatrix} 1 + 0.036\mathrm{i} & -0.155 - 0.003\mathrm{i} \\ 0.037 - 1.501\mathrm{i} & 0.998 + 0.197\mathrm{i} \end{bmatrix}$$

Boundary layer thermal transmittance matrix

$$Zs2 := \begin{bmatrix} 1 & -Rsi \cdot \frac{W}{m^2 \cdot K} \\ 0 & 1 \end{bmatrix}$$

$$Zs1 := \begin{bmatrix} 1 & -Rse \cdot \frac{W}{m^2 \cdot K} \\ 0 & 1 \end{bmatrix}$$

Construction thermal transmittance matrix from environment to environment

$$Zee := Zs2 \cdot Z \cdot Zs1 = \begin{bmatrix} 0.995 + 0.209\mathrm{i} & -0.313 - 0.034\mathrm{i} \\ 0.037 - 1.501\mathrm{i} & 0.996 + 0.262\mathrm{i} \end{bmatrix}$$

Decrement factor

$$f := \frac{\left| \mathrm{Im}\left(\frac{1}{Zee_{0,1}} \right) \right| \cdot \frac{W}{m^2 \cdot K}}{u\_value} = 0.108$$

Internal area heat capacity, *kJ/m2K*

$$k1 := \left| \frac{Zee_{1,1} - 1}{Zee_{0,1}} \right| \cdot \frac{24 \cdot hr \cdot W}{2 \cdot \pi \cdot m^2 \cdot K} = \left(11.43 \cdot 10^3\right) \frac{J}{m^2 \cdot K}$$

External area heat capacity, *kJ/m2K*

$$k2 := \left| \frac{Zee_{0,0} - 1}{Zee_{0,1}} \right| \cdot \frac{24 \cdot hr \cdot W}{2 \cdot \pi \cdot m^2 \cdot K} = \left(9.12 \cdot 10^3\right) \frac{J}{m^2 \cdot K}$$

Periodic thermal transmittance, *W/m2K*:

$$Y12 := \left| \mathrm{Im}\left(\frac{1}{Zee_{0,1}} \right) \right| \cdot \frac{W}{m^2 \cdot K} = 0.345 \frac{W}{m^2 \cdot K}$$

Time shift periodic thermal transmittance, *hr*:

$$\Delta t\_f := \frac{\mathrm{atan2}\left(\mathrm{Re}\left(\frac{1}{Zee_{0,1}} \right), \mathrm{Im}\left(\frac{1}{Zee_{0,1}} \right) \right) \cdot 24 \cdot hr}{2 \cdot \pi} - 12 \cdot hr = -0.416 \ hr$$

Time shift internal side, *hr*:

$$\Delta int := \frac{\mathrm{atan2}\left(\mathrm{Re}\left(-1 \cdot \frac{Zee_{1,1}}{Zee_{0,1}} \right), \mathrm{Im}\left(-1 \cdot \frac{Zee_{1,1}}{Zee_{0,1}} \right) \right) \cdot 24 \cdot hr}{2 \cdot \pi} = 0.566 \ hr$$

Internal thermal admittance, *W/m2K*:

$$Y22 := \left| -1 \cdot \frac{Zee_{1,1}}{Zee_{0,1}} \right| \cdot \frac{W}{m^2 \cdot K} = 3.268 \ \frac{W}{m^2 \cdot K}$$

External thermal admittance, *W/m2K*:

$$Y11 := \left| -1 \cdot \frac{Zee_{0,0}}{Zee_{0,1}} \right| \cdot \frac{W}{m^2 \cdot K} = 3.227 \ \frac{W}{m^2 \cdot K}$$

Timeshift external side, *hr*:

$$\Delta ext := \frac{\operatorname{atan2}\left(\operatorname{Re}\left(-1 \cdot \frac{Zee_{0,0}}{Zee_{0,1}}\right), \operatorname{Im}\left(-1 \cdot \frac{Zee_{0,0}}{Zee_{0,1}}\right)\right) \cdot 24 \cdot hr}{2 \cdot \pi} = 0.375 \ hr$$

Algorithmisation of multilayered assembly calculation

$d1 = 0.01\ \boldsymbol{m}$ $d2 = 0.01\ \boldsymbol{m}$

$n := 100$

$Internal :=$ for $i \in 1..n$

$d1_i \leftarrow 0.1 \cdot i \cdot d1$

$d2_i \leftarrow 0.5 \cdot i \cdot d2$

$u\_value_i \leftarrow \dfrac{1}{Rsi + \dfrac{d1_i}{\lambda 1} + \dfrac{d2_i}{\lambda 2} + Rse}$

$\xi 1_i \leftarrow \dfrac{d1_i}{\delta 1}$

$\xi 2_i \leftarrow \dfrac{d2_i}{\delta 2}$

$z11_i \leftarrow \cosh(\xi 1_i) \cdot \cos(\xi 1_i) + \sqrt{-1} \cdot \sinh(\xi 1_i) \cdot \sin(\xi 1_i)$

$z22_i \leftarrow z11_i$

$z12_i \leftarrow \dfrac{-\delta 1}{2 \cdot \lambda 1} \cdot \left(\sinh(\xi 1_i) \cdot \cos(\xi 1_i) + \cosh(\xi 1_i) \cdot \sin(\xi 1_i) + \sqrt{-1} \cdot \left(\cosh(\xi 1_i) \cdot \sin(\xi 1_i) - \sinh(\xi 1_i) \cdot \cos(\xi 1_i)\right)\right)$

$z21_i \leftarrow \dfrac{-\lambda 1}{\delta 1} \cdot \left(\sinh(\xi 1_i) \cdot \cos(\xi 1_i) - \cosh(\xi 1_i) \cdot \sin(\xi 1_i) + \sqrt{-1} \cdot \left(\cosh(\xi 1_i) \cdot \sin(\xi 1_i) + \sinh(\xi 1_i) \cdot \cos(\xi 1_i)\right)\right)$

$z11\_2_i \leftarrow \cosh(\xi 2_i) \cdot \cos(\xi 2_i) + \sqrt{-1} \cdot \sinh(\xi 2_i) \cdot \sin(\xi 2_i)$

$z22\_2_i \leftarrow z11\_2_i$

$z12\_2_i \leftarrow \dfrac{-\delta 2}{2 \cdot \lambda 2} \cdot \left(\sinh(\xi 2_i) \cdot \cos(\xi 2_i) + \cosh(\xi 2_i) \cdot \sin(\xi 2_i) + \sqrt{-1} \cdot \left(\cosh(\xi 2_i) \cdot \sin(\xi 2_i) - \sinh(\xi 2_i) \cdot \cos(\xi 2_i)\right)\right)$

$$z21\_2_i \leftarrow \frac{-\lambda 2}{\delta 2} \cdot \left(\sinh\left(\xi 2_i\right) \cdot \cos\left(\xi 2_i\right) - \cosh\left(\xi 2_i\right) \cdot \sin\left(\xi 2_i\right) + \sqrt{-1} \cdot \left(\cosh\left(\xi 2_i\right) \cdot \sin\left(\xi 2_i\right) + \sinh\left(\xi 2_i\right) \cdot \cos\left(\xi 2_i\right) \right) \right)$$

$$Z1_i \leftarrow \begin{bmatrix} z11_i & z12_i \cdot \frac{W}{m^2 \cdot K} \\ z21_i \cdot \frac{m^2 \cdot K}{W} & z22_i \end{bmatrix}$$

$$Z2_i \leftarrow \begin{bmatrix} z11\_2_i & z12\_2_i \cdot \frac{W}{m^2 \cdot K} \\ z21\_2_i \cdot \frac{m^2 \cdot K}{W} & z22\_2_i \end{bmatrix}$$

$$Z_i \leftarrow Z1_i \cdot Z2_i$$

$$Zee_i \leftarrow Zs2 \cdot Z_i \cdot Zs1$$

$$f \leftarrow \begin{cases} \dfrac{\left| Im\left(\dfrac{1}{\left(Zee_i\right)_{0,1}} \right) \right|}{u\_value_i} & \text{if } u\_value_i \le 0.25 \cdot \dfrac{W}{m^2 \cdot K} \wedge d1_i > 0 \cdot m \wedge d2_i > 0 \cdot m \wedge d1_i + d2_i \le 0.6 \cdot m \\ 0 & \text{else} \end{cases}$$

$$k1 \leftarrow \begin{cases} \left| \dfrac{\left(Zee_i\right)_{1,1} - 1}{\left(Zee_i\right)_{0,1}} \right| \cdot \dfrac{24 \cdot 3600}{2 \cdot \pi \cdot 1000} & \text{if } u\_value_i \le 0.25 \cdot \dfrac{W}{m^2 \cdot K} \wedge d1_i \ne 0 \cdot m \wedge d2_i \ne 0 \cdot m \wedge d1_i + d2_i \le 0.6 \cdot m \\ \text{else} \end{cases}$$

else

0

$Internal \leftarrow augment\left(f \cdot \frac{W}{m^2 \cdot K}, k1 \cdot \frac{J}{m^2 \cdot K}\right)$

$Internal$

$Internal = ?$