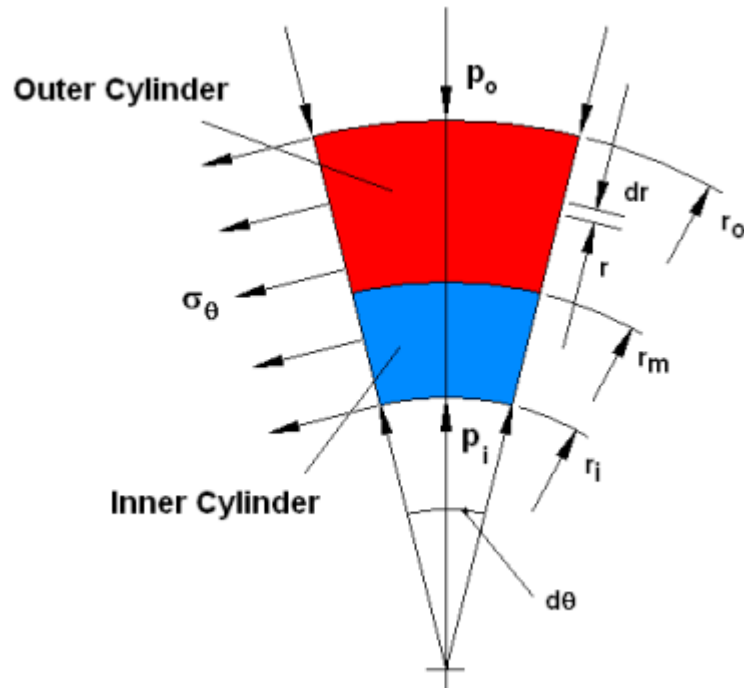


## Thick-wall Cylinder Radial Interference Fit Analysis



External pressure ...  $p_o := 0 \cdot \text{MPa}$

Internal pressure ...  $p_i := 0 \cdot \text{MPa}$

Working temperature ...  $T_w := 23 \cdot \text{C}$

Temperature difference ...  $\Delta T := T_w - 23 \cdot \text{C} \quad \Delta T = 0 \cdot \text{C}$

Cylinder properties ...

Inner Cylinder ...

Outer Cylinder ...

Drg: 123456

Drg: ABCDEF

OD at room temperature ...

$OD_1 := 114 \cdot \text{mm}$

$OD_2 := 120 \cdot \text{mm}$

ID at room temperature ...

$ID_1 := 0 \cdot \text{mm}$

$ID_2 := 113.85 \cdot \text{mm}$

Elastic modulus ...

$E_1 := 210 \cdot \text{GPa}$

$E_2 := 210 \cdot \text{GPa}$

Poisson's ratio ...

$\nu_1 := 0.30$

$\nu_2 := 0.30$

Thermal expansion coefficient ...

$\alpha_1 := 12.3 \cdot 10^{-6} \cdot \frac{\text{mm}}{\text{mm} \cdot \text{C}}$

$\alpha_2 := 12.3 \cdot 10^{-6} \cdot \frac{\text{mm}}{\text{mm} \cdot \text{C}}$

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Ref      Mechanics of Engineering Materials, by P.P.Benham & R.J.Crawford  
Chapter 15 - Applications of Equilibrium and Strain-displacement Relationships  
Subject - Stress Distribution in a Pressurised Thick-walled Cylinder

Deriving radial force equilibrium ...

Elemental radial force ...       $dP_r = 2 \cdot \sigma_\theta \cdot dr \cdot \sin\left(\frac{d\theta}{2}\right)$       where       $\sin\left(\frac{d\theta}{2}\right)$       tends to       $\frac{d\theta}{2}$   
per unit length

$$dP_r = \sigma_\theta \cdot dr \cdot dl \cdot d\theta$$

$$P_r = dl \cdot d\theta \cdot \int \sigma_\theta dr$$

For radial equilibrium ...       $d\theta \cdot \int \sigma_\theta dr + p_o \cdot r_o \cdot d\theta - p_i \cdot r_i \cdot d\theta = 0$   
per unit length

Giving ...       $\int \sigma_\theta dr = p_i \cdot r_i - p_o \cdot r_o$

Equations for inner cylinder ...      radial stress ...       $f_{r\_1}(r, A_1, B_1) := \begin{cases} \left( A_1 - \frac{B_1}{r^2} \right) & \text{if } ID_1 > 0 \cdot \text{mm} \\ A_1 & \text{otherwise} \end{cases}$

hoop stress ...       $f_{\theta\_1}(r, A_1, B_1) := \begin{cases} \left( A_1 + \frac{B_1}{r^2} \right) & \text{if } ID_1 > 0 \cdot \text{mm} \\ A_1 & \text{otherwise} \end{cases}$

Inner cylinder radial displacement ...       $u_1(r_m, A_1, B_1) := \frac{r_m}{E_1} \cdot (f_{\theta\_1}(r_m, A_1, B_1) - \nu_1 \cdot f_{r\_1}(r_m, A_1, B_1))$

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Equations for outer cylinder ... radial stress ...  $f_{r\_2}(r, A_2, B_2) := A_2 - \frac{B_2}{r^2}$

hoop stress ...  $f_{\theta\_2}(r, A_2, B_2) := A_2 + \frac{B_2}{r^2}$

Outer cylinder radial displacement ...  $u_2(r_m, A_2, B_2) := \frac{r_m}{E_2} \cdot (f_{\theta\_2}(r_m, A_2, B_2) - \nu_2 \cdot f_{r\_2}(r_m, A_2, B_2))$

Radial interference ...  $\delta_r := 0.5 \cdot [(OD_1 - ID_2) + \Delta T \cdot (\alpha_1 \cdot OD_1 - \alpha_2 \cdot ID_2)]$   $\delta_r = 0.0750 \text{ mm}$

Thermal expansion ...  $ID_1 := ID_1 \cdot (1 + \Delta T \cdot \alpha_1)$   $ID_1 = 0 \text{ mm}$

$OD_1 := OD_1 \cdot (1 + \Delta T \cdot \alpha_1)$   $OD_1 = 114 \text{ mm}$

$ID_2 := ID_2 \cdot (1 + \Delta T \cdot \alpha_2)$   $ID_2 = 113.85 \text{ mm}$

$OD_2 := OD_2 \cdot (1 + \Delta T \cdot \alpha_2)$   $OD_2 = 120 \text{ mm}$

Hoop stress radial force ...

$$P_r(A_1, B_1, A_2, B_2, r_m) := \begin{cases} \left( \int_{0.5 \cdot ID_1}^{r_m} A_1 + \frac{B_1}{r^2} dr + \int_{r_m}^{0.5 \cdot OD_2} A_2 + \frac{B_2}{r^2} dr \right) & \text{if } ID_1 > 0 \cdot \text{mm} \\ \left( \int_{0.5 \cdot ID_1}^{r_m} A_1 dr + \int_{r_m}^{0.5 \cdot OD_2} A_2 + \frac{B_2}{r^2} dr \right) & \text{otherwise} \end{cases}$$

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Given the following boundary conditions ...

$$B_1 = \begin{cases} B_1 & \text{if } ID_1 > 0 \cdot \text{mm} \\ 0 \cdot \text{N} & \text{otherwise} \end{cases} \quad \dots \text{ setting variable } B_1 \text{ based on geometry conditions}$$

$$f_{r\_1}(0.5 \cdot ID_1, A_1, B_1) = \begin{cases} -p_i & \text{if } ID_1 > 0 \cdot \text{mm} \\ f_{r\_1}(0.5 \cdot ID_1, A_1, B_1) & \text{otherwise} \end{cases} \quad \dots \text{ equating stress to internal pressure at inner cylinder ID (including geometry conditions)}$$

$$f_{r\_2}(0.5 \cdot OD_2, A_2, B_2) = -p_o \quad \dots \text{ equating stress to external pressure at outer cylinder OD}$$

$$f_{r\_1}(r_m, A_1, B_1) = f_{r\_2}(r_m, A_2, B_2) \quad \dots \text{ equating radial stress at mating surfaces}$$

$$u_2(r_m, A_2, B_2) - u_1(r_m, A_1, B_1) = \delta_r \quad \dots \text{ equating sum of radial displacement to interference}$$

$$P_r(A_1, B_1, A_2, B_2, r_m) = 0.5 \cdot (p_i \cdot ID_1 - p_o \cdot OD_2) \quad \dots \text{ defining radial force equilibrium}$$

Stress results ... Inner Cylinder ...

Outer Cylinder ....

|                      |                                     |                                     |
|----------------------|-------------------------------------|-------------------------------------|
| ID radial stress ... | $f_{r\_ID\_1} = -13.54 \text{ MPa}$ | $f_{r\_ID\_2} = -13.54 \text{ MPa}$ |
|----------------------|-------------------------------------|-------------------------------------|

|                      |                                   |                                   |
|----------------------|-----------------------------------|-----------------------------------|
| OD radial stress ... | $f_{rOD\_1} = -13.54 \text{ MPa}$ | $f_{r\_OD\_2} = 0.00 \text{ MPa}$ |
|----------------------|-----------------------------------|-----------------------------------|

|                    |  |  |
|--------------------|--|--|
| ID hoop stress ... | $f_{\theta\_ID\_1} = -13.54 \text{ MPa}$ | $f_{\theta\_ID\_2} = 262.85 \text{ MPa}$ |
|--------------------|--|--|

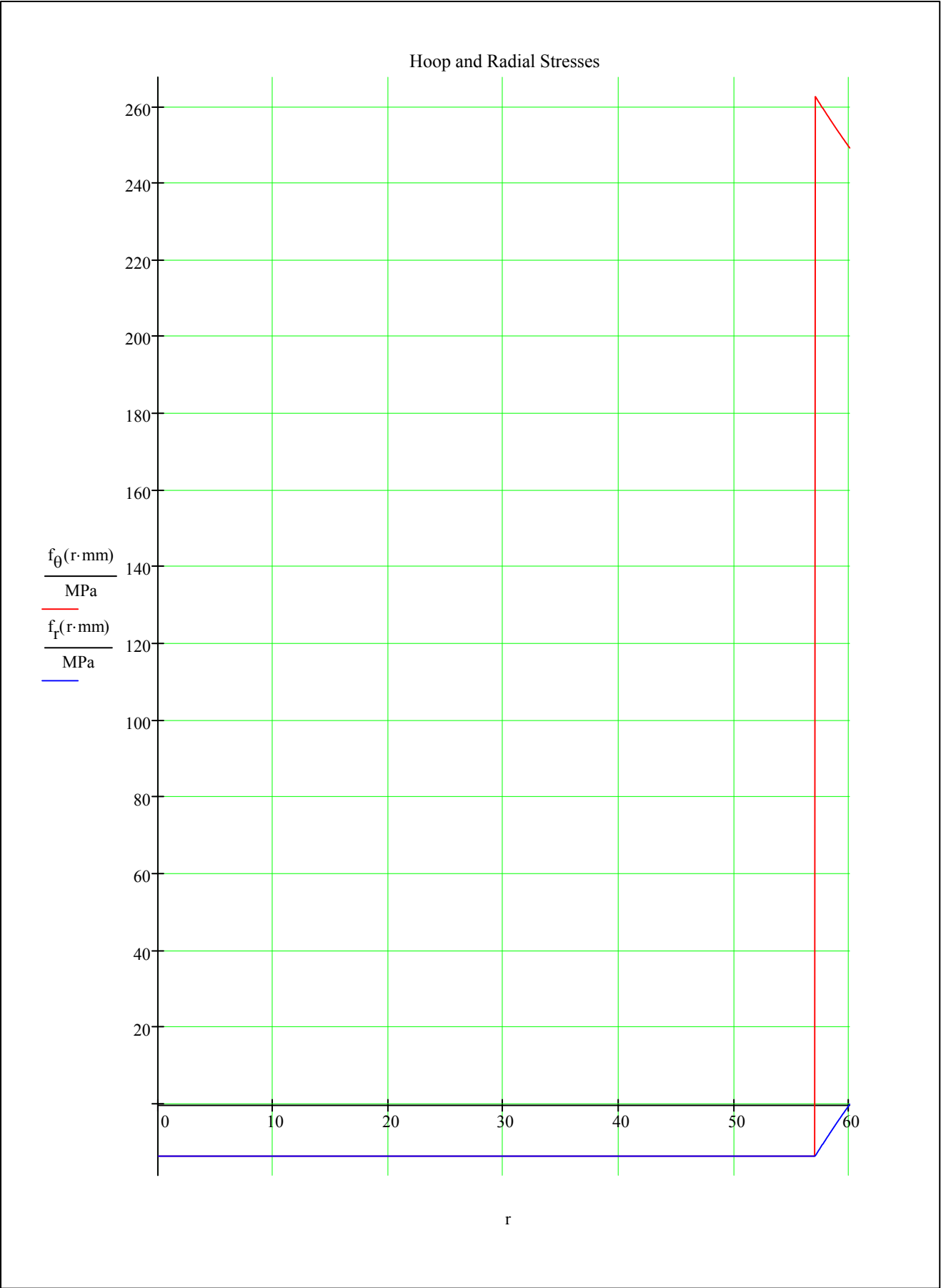
|                    |  |  |
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| OD hoop stress ... | $f_{\theta\_OD\_1} = -13.54 \text{ MPa}$ | $f_{\theta\_OD\_2} = 249.31 \text{ MPa}$ |
|--------------------|--|--|

|                               |                           |
|-------------------------------|---------------------------|
| Radius at mating surfaces ... | $r_m = 56.985 \text{ mm}$ |
|-------------------------------|---------------------------|

|                         |   |       |                                |
|-------------------------|---|-------|--------------------------------|
| Radial interference ... | $u_2(r_m, A_2, B_2) - u_1(r_m, A_1, B_1) = 0.0750 \text{ mm}$ | where | $\delta_r = 0.0750 \text{ mm}$ |
|-------------------------|---|-------|--------------------------------|

|           |  |     |   |
|-----------|--|-----|---|
| where ... | $u_1(r_m, A_1, B_1) = -0.00257 \text{ mm}$ | and | $u_2(r_m, A_2, B_2) = 0.07243 \text{ mm}$ |
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