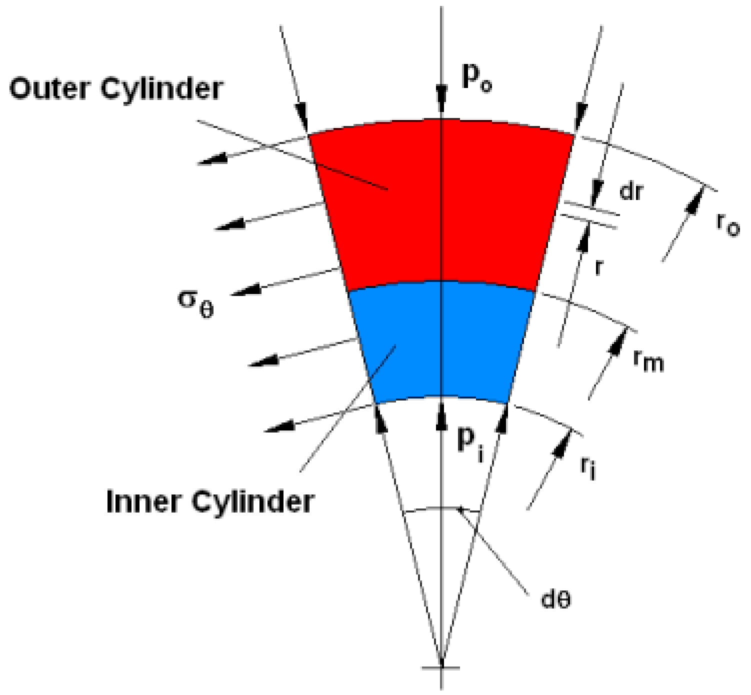
Thick-wall Cylinder Radial Interference Fit Analysis



External pressure ... $p_o := 0 \cdot MPa$

Internal pressure ... $p_i := 0 \cdot MPa$

Working temperature ... $T_w := 23 \cdot C$

Temperature difference ... $\Delta T := T_w - 23 \cdot C$ $\quad \Delta T = 0\,C$

Cylinder properties ...	Inner Cylinder ...	Outer Cylinder ...
	Drg: 123456	Drg: ABCDEF
OD at room temperature ...	$OD_1 := 114 \cdot mm$	$OD_2 := 120 \cdot mm$
ID at room temperature ...	$ID_1 := 0.0032 \cdot mm$	$ID_2 := 113.85 \cdot mm$
Elastic modulus ...	$E_1 := 210 \cdot GPa$	$E_2 := 210 \cdot GPa$
Poisson's ratio ...	$\upsilon_1 := 0.30$	$\upsilon_2 := 0.30$
Thermal expansion coefficient ...	$\alpha_1 := 12.3 \cdot 10^{-6} \cdot \frac{mm}{mm \cdot C}$	$\alpha_2 := 12.3 \cdot 10^{-6} \cdot \frac{mm}{mm \cdot C}$

Author:	Date:	Title: Thick Wall Radial Interference Fit Analysis	AC:
Check:	Date:		WV:
Company:		Dwg No.:	Page:

Ref Mechanics of Engineering Materials, by P.P.Benham & R.J.Crawford
Chapter 15 - Applications of Equilibrium and Strain-dispacement Relationships
Subject - Stress Distribution in a Pressurised Thick-walled Cylinder

Deriving radial force equilibrium ...

Elemental radial force ... per unit length $dP_r = 2 \cdot \sigma_\theta \cdot dr \cdot \sin\left(\frac{d\theta}{2}\right)$ where $\sin\left(\frac{d\theta}{2}\right)$ tends to $\frac{d\theta}{2}$

$$dP_r = \sigma_\theta \cdot dr \cdot dl \cdot d\theta$$

$$P_r = dl \cdot d\theta \cdot \int \sigma_\theta \, dr$$

For radial equilibrium ... per unit length $d\theta \cdot \int \sigma_\theta \, dr + p_o \cdot r_o \cdot d\theta - p_i \cdot r_i \cdot d\theta = 0$

Giving ... $\int \sigma_\theta \, dr = p_i \cdot r_i - p_o \cdot r_o$

Equations for inner cylinder ... radial stress ... $f_{r\_1}(r, A_1, B_1) := A_1 - \frac{B_1}{r^2}$

hoop stress ... $f_{\theta\_1}(r, A_1, B_1) := A_1 + \frac{B_1}{r^2}$

Inner cylinder radial displacement ... $u_1(r_m, A_1, B_1) := \frac{r_m}{E_1} \cdot \left(f_{\theta\_1}(r_m, A_1, B_1) - \upsilon_1 \cdot f_{r\_1}(r_m, A_1, B_1)\right)$

Equations for outer cylinder ... radial stress ... $f_{r\_2}(r, A_2, B_2) := A_2 - \frac{B_2}{r^2}$

hoop stress ... $f_{\theta\_2}(r, A_2, B_2) := A_2 + \frac{B_2}{r^2}$

Outer cylinder radial displacement ... $u_2(r_m, A_2, B_2) := \frac{r_m}{E_2} \cdot \left(f_{\theta\_2}(r_m, A_2, B_2) - \upsilon_2 \cdot f_{r\_2}(r_m, A_2, B_2)\right)$

Radial interference ... $\delta_r := 0.5\cdot\left[\left(OD_1 - ID_2\right) + \Delta T\cdot\left(\alpha_1\cdot OD_1 - \alpha_2\cdot ID_2\right)\right]$ $\delta_r = 0.0750\,mm$

Thermal expansion ... $ID_1 := ID_1\cdot\left(1 + \Delta T\cdot\alpha_1\right)$ $ID_1 = 0.0032\,mm$

$OD_1 := OD_1\cdot\left(1 + \Delta T\cdot\alpha_1\right)$ $OD_1 = 114\,mm$

$ID_2 := ID_2\cdot\left(1 + \Delta T\cdot\alpha_2\right)$ $ID_2 = 113.85\,mm$

$OD_2 := OD_2\cdot\left(1 + \Delta T\cdot\alpha_2\right)$ $OD_2 = 120\,mm$

Hoop stress radial force ... $P_r\left(A_1, B_1, A_2, B_2, r_m\right) := \int_{0.5\cdot ID_1}^{r_m} A_1 + \frac{B_1}{r^2}\,dr + \int_{r_m}^{0.5\cdot OD_2} A_2 + \frac{B_2}{r^2}\,dr$

Given the following boundary conditions ...

$f_{r\_1}\left(0.5\cdot ID_1, A_1, B_1\right) = -p_i$ ... equating stress to internal pressure at inner cylinder ID

$f_{r\_2}\left(0.5\cdot OD_2, A_2, B_2\right) = -p_o$ ... equating stress to external pressure at outer cylinder OD

$f_{r\_1}\left(r_m, A_1, B_1\right) = f_{r\_2}\left(r_m, A_2, B_2\right)$ ... equating radial stress at mating surfaces

$u_2\left(r_m, A_2, B_2\right) - u_1\left(r_m, A_1, B_1\right) = \delta_r$ ... equating sum of radial displacement to interference

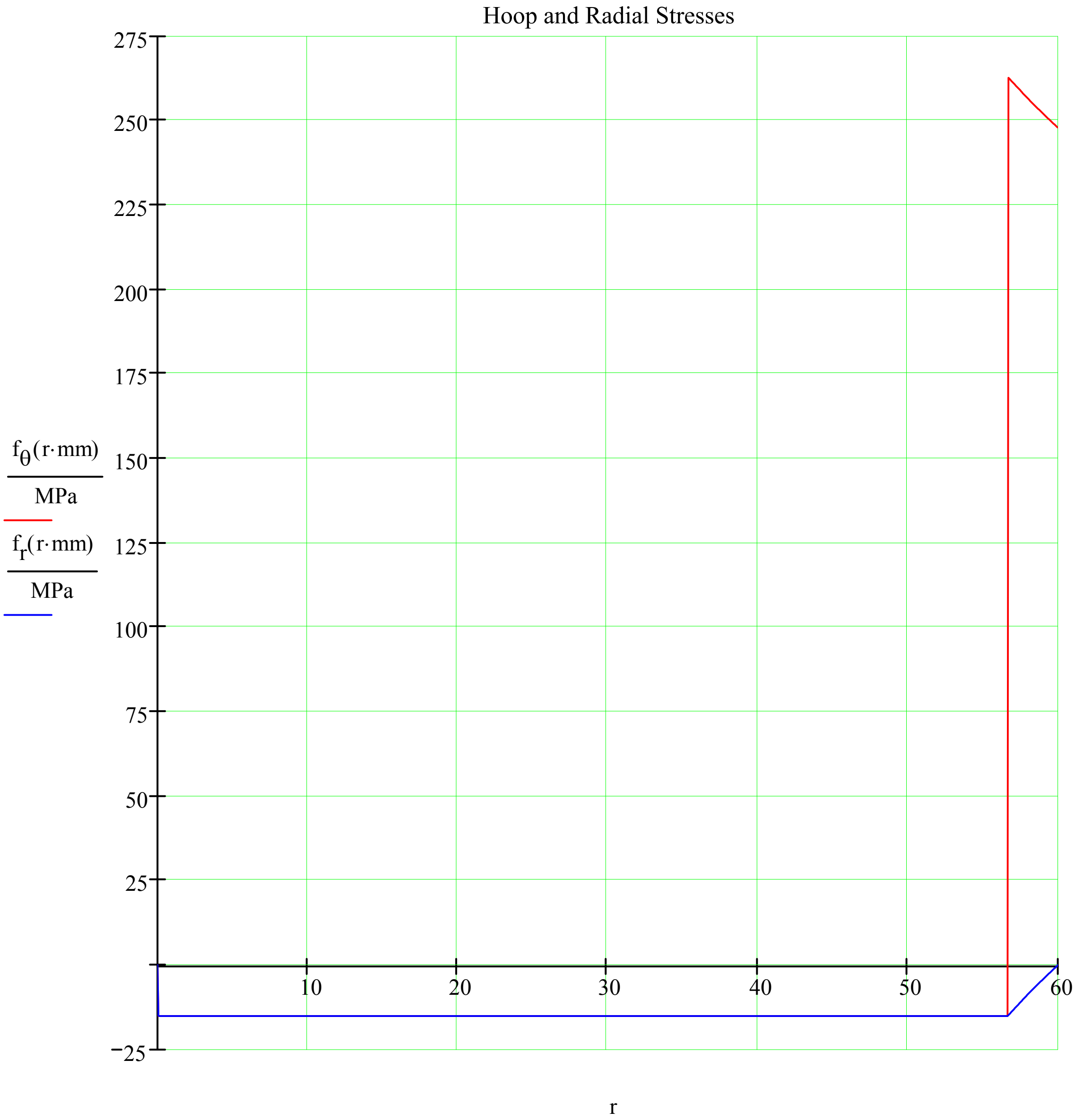
$P_r\left(A_1, B_1, A_2, B_2, r_m\right) = 0.5\cdot\left(p_i\cdot ID_1 - p_o\cdot OD_2\right)$ ... defining radial force equilibrium

Stress results ...	Inner Cylinder ...	Outer Cylinder ....
ID radial stress ...	$f_{r\_ID\_1} = 0.00\,MPa$	$f_{r\_ID\_2} = -15.07\,MPa$
OD radial stress ...	$f_{rOD\_1} = -15.07\,MPa$	$f_{r\_OD\_2} = 0.00\,MPa$
ID hoop stress ...	$f_{\theta\_ID\_1} = -30.13\,MPa$	$f_{\theta\_ID\_2} = 262.93\,MPa$
OD hoop stress ...	$f_{\theta\_OD\_1} = -15.07\,MPa$	$f_{\theta\_OD\_2} = 247.87\,MPa$

Author:	Date:	Title: Thick Wall Radial Interference Fit Analysis	AC:
Check:	Date:		WV:
Company:		Dwg No.:	Page:

ID radial stress at 0.5mm ... ID hoop stress at 0.5mm ...

Note ... $f_{r\_1}(0.5\cdot mm, A_1, B_1) = -15.07\,MPa$ ... and ... $f_{\theta\_1}(0.5\cdot mm, A_1, B_1) = -15.07\,MPa$



Radius at mating surfaces ... $r_m = 56.655\,mm$

Radial interference ... $u_2(r_m, A_2, B_2) - u_1(r_m, A_1, B_1) = 0.0750\,mm$ where $\delta_r = 0.0750\,mm$

where ... $u_1(r_m, A_1, B_1) = -0.00285\,mm$ ... and ... $u_2(r_m, A_2, B_2) = 0.07215\,mm$

Author:	Date:	Title: Thick Wall Radial Interference Fit Analysis	AC:
Check:	Date:		WV:
Company:		Dwg No.:	Page: